Two credulous notions for acceptance of arguments in abstract frameworks

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Abstract

Abstract argumentation systems are formalisms for defeasible reasoning where some components remain unspecified, the structure of arguments being the main abstraction. In this work, we use an extended argumentation framework where two kinds of defeat relation are present, in order to define two basic semantic notions: a fixpoint operator and an argument extension based in the concept of *progressive defeat paths*. These mechanisms constitute a credulous approach to characterize sets of possible accepted arguments.

Keywords: Defeasible argumentation, abstract frameworks, argument extensions.

1 INTRODUCTION

In formal systems of defeasible argumentation, arguments for and against a proposition are produced and evaluated to verify the acceptability of that proposition. The main idea in these systems is that any proposition will be accepted as true if there exists an argument that supports it, and this argument is acceptable according to an analysis between it and its counterarguments. This analysis requires a process of comparison of conflicting arguments, in order to decide which one is preferable. After this dialectical analysis over the set of arguments in the system, some of them will be *acceptable* or *justified* arguments, while others not. Argumentation is widely used in nonmonotonic reasoning [3] and it is suitable to model dialogues between intelligent agents [1, 8].

Abstract argumentation systems [12, 2, 5] are formalisms for argumentation, where some components remains unspecified, usually the structure of arguments. In this kind of systems, the emphasis is put on semantic notions, basically the task of finding the set of accepted arguments. Most of them are based on a single abstract notion called *attack relation*, and several argument extensions are defined as sets of possible accepted arguments. However, the task of comparing arguments to establish a preference is not always successful. Finding a preferred argument is essential to determine a defeat relation. In the next section, we present an abstract framework for argumentation where conflicts and preference between arguments are considered. This structure is the basis for the modelization of well formed argumentation lines.

2 ABSTRACT ARGUMENTATION FRAMEWORK

Our argumentation framework is formed by four elements: a set of arguments, and three basic relations between arguments.

Definition 1 *An abstract argumentation framework (AF) is a quartet* $\langle AR, \sqsubseteq, \mathbf{C}, \mathbf{R}\rangle$*, where* AR *is a finite set of arguments,* $\sqsubseteq$ *is the subargument relation,* $\mathbf{C}$ *is a symmetric and anti-reflexive binary conflict relation between arguments,* $\mathbf{C} \subseteq AR \times AR$*, and* $\mathbf{R}$ *is a preference relation among arguments.*

In this framework, arguments are abstract entities [2] that will be denoted using calligraphic uppercase letters. The symbol $\sqsubseteq$ denotes subargument relation: $\mathcal{A} \sqsubseteq \mathcal{B}$ means "$\mathcal{A}$ is a subargument of $\mathcal{B}$". Any argument $\mathcal{A}$ is considered a superargument and a subargument of itself. Any subargument $\mathcal{B} \sqsubseteq \mathcal{A}$ such that $\mathcal{B} \neq \mathcal{A}$ is said to be a non-trivial subargument, denoted by symbol $\sqsubset$. The following notation will be also used: given an argument $\mathcal{A}$ then $\mathcal{A}^-$ will represent a subargument of $\mathcal{A}$, and $\mathcal{A}^+$ will represent a superargument of $\mathcal{A}$. When no confusion may arise, subscript index will be used for distinguishing different subarguments or superarguments of $\mathcal{A}$.

The conflict relation between two arguments $\mathcal{A}$ and $\mathcal{B}$ denotes the fact that these arguments cannot be accepted simultaneously since they contradict each other. For example, two arguments $\mathcal{A}$ and $\mathcal{B}$ that support complementary conclusions l and $\neg l$ cannot be accepted together. The set of all pairs of arguments in conflict on Φ is denoted by $\mathbf{C}$. Given a set of arguments S, an argument $\mathcal{A} \in S$ is said to be in conflict in S if there is an argument $\mathcal{B} \in S$ such that $(\mathcal{A}, \mathcal{B}) \in \mathbf{C}$. The set *Conf*($\mathcal{A}$) is the set of all arguments $\mathcal{X} \in AR$ such that $(\mathcal{A}, \mathcal{X}) \in \mathbf{C}$. A common sense property states that conflict relations are propagated to superarguments. That is, if $(\mathcal{A}, \mathcal{B}) \in \mathbf{C}$, then $(\mathcal{A}, \mathcal{B}^+) \in \mathbf{C}$, $(\mathcal{A}^+, \mathcal{B}) \in \mathbf{C}$, and $(\mathcal{A}^+, \mathcal{B}^+) \in \mathbf{C}$, for any superargument $\mathcal{A}^+$ of $\mathcal{A}$ and $\mathcal{B}^+$ of $\mathcal{B}$. This is called *conflict inheritance*.

The constraints imposed by the conflict relation lead to several sets of possible accepted arguments. For example, if $AR = \{\mathcal{A}, \mathcal{B}\}$ and $(\mathcal{A}, \mathcal{B}) \in \mathbf{C}$, then $\{\mathcal{A}\}$ is a set of possible accepted arguments, and so is $\{\mathcal{B}\}$. Therefore, some way of deciding among all the possible outcomes must be devised. In order to accomplish this task, the relation $\mathbf{R}$ is introduced in the framework and it will be used to evaluate arguments, modelling a preference criterion based on a measure of strength.

Definition 2 *Given a set of arguments* AR*, an* argument comparison criterion $\mathbf{R}$ *is a binary relation on* AR*. If* $\mathcal{A}\mathbf{R}\mathcal{B}$ *but not* $\mathcal{B}\mathbf{R}\mathcal{A}$ *then* $\mathcal{A}$ *is preferred to* $\mathcal{B}$*, denoted* $\mathcal{A} \succ \mathcal{B}$*. If* $\mathcal{A}\mathbf{R}\mathcal{B}$ *and* $\mathcal{B}\mathbf{R}\mathcal{A}$ *then* $\mathcal{A}$ *and* $\mathcal{B}$ *are arguments with equal relative preference, denoted* $\mathcal{A} \equiv \mathcal{B}$*. If neither* $\mathcal{A}\mathbf{R}\mathcal{B}$ *or* $\mathcal{B}\mathbf{R}\mathcal{A}$ *then* $\mathcal{A}$ *and* $\mathcal{B}$ *are incomparable arguments, denoted* $\mathcal{A} \bowtie \mathcal{B}$*.*

As the comparison criterion is treated abstractly, we do not assume any property of $\mathbf{R}$, but $monotonicity$ as explained later. Any concrete framework may establish additional rationality requirements for decision making.

Example 1 $\Phi = \langle AR, \sqsubseteq, \mathbf{C}, \mathbf{R}\rangle$ *where* $AR = \{\mathcal{A}, \mathcal{B}, \mathcal{C}, \mathcal{D}, \mathcal{E}\}$, $\mathbf{C} = \{\{\mathcal{A}, \mathcal{B}\}, \{\mathcal{B}, \mathcal{C}\}, \{\mathcal{C}, \mathcal{D}\}\}$, $\{\mathcal{C}, \mathcal{E}\}\}$[1] *and* $\mathcal{A} \succ \mathcal{B}, \mathcal{B} \succ \mathcal{C}$, $\mathcal{E} \bowtie \mathcal{C}$ *and* $\mathcal{C} \equiv \mathcal{D}$. *is an AF according to definition 1.*

For two arguments $\mathcal{A}$ and $\mathcal{B}$ in AR, such that the pair $(\mathcal{A}, \mathcal{B})$ belongs to $\mathbf{C}$ the relation $\mathbf{R}$ is considered. If a concrete preference is made ($\mathcal{A} \succ \mathcal{B}$ or $\mathcal{B} \succ \mathcal{A}$), then a defeat relation is established. It is said that the preferred argument is a *proper defeater* of the non-preferred argument. If the arguments are *indifferent* according to $\mathbf{R}$, then they have the *same* relative conclusive force. For example, if the preference criterion establishes that smaller arguments are preferred, then two arguments of the same size are indifferent. On the other hand, arguments may be *incomparable*. For example, if the preference criterion states that argument $\mathcal{A}$ is preferred to $\mathcal{B}$ whenever the premises of $\mathcal{A}$ are included in the premises of $\mathcal{B}$, then arguments with disjoint sets of premises are incomparable. This situation must be understood as a natural behaviour. When two conflictive arguments are indifferent or incomparable according to $\mathbf{R}$, the conflict between these two arguments remains unresolved.

When two conflictive arguments are indistinguishable or incomparable, the conflict between these two arguments remains unresolved. Due to this situation and to the fact that the conflict relation is a symmetric relation, each of the arguments is *blocking* the other one and it is said that both of them are *blocking defeaters* [9, 11]. An argument $\mathcal{B}$ is said to be a *defeater* of an argument $\mathcal{A}$ if $\mathcal{B}$ is a blocking or a proper defeater of $\mathcal{A}$. In example 1, argument $\mathcal{A}$ is a proper defeater of argument $\mathcal{B}$, while $\mathcal{C}$ is a blocking defeater of $\mathcal{D}$ and vice versa, $\mathcal{D}$ is a blocking defeater of $\mathcal{C}$.

Abstract frameworks can be depicted as graphs, with different types of arcs. We use the arc ($\cdots\bullet$) to denote the subargument relation. An arrow ($\rightarrow$) is used to denote proper defeaters and a double-pointed arrow ($\longleftrightarrow$) connects blocking defeaters. In figure 1, a simple framework is shown. Argument $\mathcal{C}$ is a subargument of $\mathcal{A}$. Argument $\mathcal{B}$ is a proper defeater of $\mathcal{C}$ and $\mathcal{D}$ is a blocking defeater of $\mathcal{B}$ and viceversa.

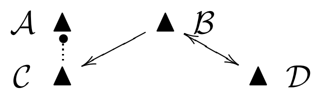

Figure 1: Defeat graph

Some authors leave the preference criteria unspecified, even when it is one of the most important components in the system. However, in many cases it is sufficient to establish a set of properties that the criteria must exhibit. A very reasonable one states that an argument is as strong as its weakest subargument [12]. We formalize this idea in the next definition.

Definition 3 (Monotonic preference relation) *A preference relation* $\mathbf{R}$ *is said to be monotonic if, given* $\mathcal{A} \succ \mathcal{B}$*, then* $\mathcal{A} \succ \mathcal{C}$*, for any argument* $\mathcal{B} \sqsubseteq \mathcal{C}$.

We will assume from now on that the criterion $\mathbf{R}$ included in Φ is monotonic. This is important because any argument $\mathcal{A}$ defeated by another argument $\mathcal{B}$ should also be defeated by another argument $\mathcal{B}^+$. In figure 1, argument $\mathcal{B}$ defeats $\mathcal{C}$, but it should also be a defeater of $\mathcal{A}$, because $\mathcal{C}$ is its subargument. A defeat arc from $\mathcal{B}$ to $\mathcal{A}$ may be drawn in the graph, although redundant.

[1] When describing elements of $\mathbf{C}$, we write $\{\mathcal{A}, \mathcal{B}\}$ as an abbreviation for $\{(\mathcal{A}, \mathcal{B}), (\mathcal{B}, \mathcal{A})\}$, for any arguments $\mathcal{A}$ and $\mathcal{B}$ in AR.

3 FIXPOINT SEMANTIC FOR EXTENDED DEFEAT

Arguments can be classified as *accepted* arguments or *non-accepted* or *rejected* arguments according to their context in the framework. Any set of accepted arguments should not contain arguments in conflict. A set of arguments S is said to be *conflict free* if for all $\mathcal{A}, \mathcal{B} \in S$ then $(\mathcal{A}, \mathcal{B}) \notin \mathbf{C}$. In example 1 the set $\{\mathcal{A}, \mathcal{C}\}$ is a conflict free set.

Given a set of arguments S, two kinds of arguments are easily identified as accepted arguments: first, those arguments not involved in any conflict in S; second, those arguments actually involved in a conflict, but preferred to the arguments that are in conflict with them, according to relation $\mathbf{R}$. Both kinds of special arguments are called *defeater free* arguments. An argument $\mathcal{A}$ is defeater-free in a set S if no argument in S is a defeater of $\mathcal{A}$. Defeater-free arguments must be accepted, since no (preferred) contradictory information is provided in the framework. Note that this classification is relative to the set in which the argument is included. The semantic of $\mathbf{C}$ states that when an argument $\mathcal{A}$ is accepted, any argument in *Conf*$(\mathcal{A})$ should be rejected. The following definition captures a subset of arguments that should be rejected in the framework.

Definition 4 *Let S be a set of arguments in $\langle AR, \sqsubseteq, \mathbf{C}, \mathbf{R}\rangle$. An argument $\mathcal{A} \in S$ is said to be suppressed in S if one of the following cases hold: (a) there is a defeater-free argument $\mathcal{B}$ in S such that $\mathcal{B}$ is a proper defeater of $\mathcal{A}$, or (b) there is a blocking defeater $\mathcal{B}$ of $\mathcal{A}$ in S, and there is no other argument $\mathcal{C}$ ($\mathcal{C} \neq \mathcal{A}$) in S such that $\mathcal{C}$ is a defeater of $\mathcal{B}$.*

The first case is clear since any argument involved in a conflict must be suppressed when its counterpart in this conflict is accepted (has no defeater). The second case reflects the situation in which two arguments are taking part of an unsolved conflict and from the point of view of one of them ($\mathcal{A}$) its opponent is not attacked by a third argument. The argument $\mathcal{A}$ should be suppressed since the threat of $\mathcal{B}$ cannot be avoided, despite other attacks on $\mathcal{A}$. Note that if $\mathcal{A}$ is only defeated by $\mathcal{B}$ then both arguments should be suppressed because the blocking condition is symmetrical.

Given a set S of arguments it is as easy to identify obviously suppressed arguments as it is to identify inevitably accepted ones. The following function $\Upsilon : 2^{AR} \longrightarrow 2^{AR}$ characterizes the set of arguments not directly suppressed in a given set S.

$$\Upsilon(S) = \{\mathcal{A} : \mathcal{A} \in S \text{ and } \mathcal{A} \text{ is not suppressed in } S\}$$

It is easy to see that if S is a conflict-free set of arguments, then $S = \Upsilon(S)$. However, the converse is not true, as shown in the next example:

Example 2 *Let $\langle AR, \sqsubseteq, \mathbf{C}, \mathbf{R}\rangle$ be an AF, where $AR = \{\mathcal{A}, \mathcal{B}, \mathcal{C}, \mathcal{D}\}$ and $\mathbf{C} = \{\{\mathcal{A}, \mathcal{B}\}, \{\mathcal{B}, \mathcal{C}\}, \{\mathcal{C}, \mathcal{D}\}, \{\mathcal{D}, \mathcal{A}\}\}$ and for all arguments $\mathcal{X}$ and $\mathcal{Y}$, $\mathcal{X} \bowtie \mathcal{Y}$. No argument in AR is a defeater-free argument, therefore $\Upsilon(AR) = AR$.*

By definition, $\Upsilon(S)$ includes some (or all) of the arguments in S. In the set $\Upsilon(S)$ some arguments may now be classified as *defeater-free* arguments, since its defeaters are suppressed arguments in S. It is then possible to repeatedly apply function Υ to the set of arguments in the framework. This process may continue until a fixpoint is reached.

Definition 5 *Υ^n is defined as: Υ^0 is AR, and $\Upsilon^{(n+1)} = \Upsilon \circ \Upsilon^n$. The set of arguments $\Upsilon^k, k \geq 0$ such that $\Upsilon^k = \Upsilon^{k+1}$ is denoted Υ^ω.*

Example 3 *Let $\Phi_2 = \langle AR, \sqsubseteq, \mathbf{C}, \mathbf{R}\rangle$ be an AF where $AR = \{\mathcal{A}, \mathcal{B}, \mathcal{C}, \mathcal{D}\}$, $\mathbf{C} = \{\{\mathcal{A}, \mathcal{B}\}, \{\mathcal{B}, \mathcal{C}\}, \{\mathcal{C}, \mathcal{D}\}\}$ and $\mathcal{A} \equiv \mathcal{B}$, $\mathcal{B} \bowtie \mathcal{C}$ and $\mathcal{C} \succ \mathcal{D}$. In this framework, $\Upsilon^1 = \{\mathcal{A}, \mathcal{D}, \mathcal{C}\}$, because $\mathcal{B}$ is a suppressed argument, as $\mathcal{A}$ is a blocking defeater not defeated by a third argument. $\Upsilon^2 = \{\mathcal{A}, \mathcal{C}\}$ because $\mathcal{D}$ is defeated by $\mathcal{C}$ which is now defeater-free in Υ^1. Because $\Upsilon^2 = \Upsilon^3$ then $\Upsilon^\omega = \{\mathcal{A}, \mathcal{C}\}$.*

Trivially, no argument is suppressed in Υ^ω. An argument in Υ^ω which is not in conflict with any other argument in the same set is an accepted argument. The set of accepted arguments in Υ^ω is denoted $\Upsilon^{\omega+}$. Therefore, if Υ^ω is a conflict-free set (as in example 3, but not in example 2), then any argument in Υ^ω is an *accepted* argument.

The previously defined conflict inheritance leads to a common sense property of argumentation frameworks. For any argument $\mathcal{A}$, if $\mathcal{A} \in \Upsilon^{\omega+}$ then $\mathcal{B} \in \Upsilon^{\omega+}$ for all $\mathcal{B} \sqsubseteq \mathcal{A}$. Suppose $\mathcal{A}_1 \sqsubseteq \mathcal{A}$ is not in Υ^ω. Then $\mathcal{A}_1$ is a suppressed argument, because one of the conditions of definition 4 holds in some $\Upsilon^i, i > 0$. But if $\mathcal{A}_1$ is suppressed in Υ^i then also $\mathcal{A}$ is suppressed in Υ^i because they share defeaters (because of conflict inheritance) and therefore is also suppressed. The reader is referred to [7] for the role of subarguments in well structured argumentation, using the framework of definition 1.

In the framework of example 2, no arguments should be accepted as it is not possible to establish a concrete preference. Here, Υ^ω is not a conflict-free set. This is related to the presence of some special arguments involved in a cicle of defeaters, a common situation called a *fallacy*. Any argument involved in a fallacy is usually called *fallacious*. The most important premise in defeasible argumentation is that an argument must be *accepted* only when none of its defeaters are. However, no fallacious argument can exhibit this property, because at least one of its defeaters is also a fallacious argument [2]. Therefore, any argument of this kind should not be accepted. An AF is said to contain a fallacy if Υ^ω is not a conflict-free set of arguments.

4 DEFEAT PATHS

In [2], several semantic notions are defined. Other forms of clasifying arguments as $accepted$ or $rejected$ can be found in [5, 6]. From a procedural point of view, when evaluating the acceptance of an argument, a set of conflict-related arguments are considered. An important structure of this process is captured in the following definition.

Definition 6 (Defeat path) *A* defeat path *λ of an argumentation framework $\langle AR, \sqsubseteq, \mathbf{C}, \mathbf{R}\rangle$ is a finite sequence of arguments $[A_1, A_2, \ldots, A_n]$ such that argument A_{i+1} is a defeater of argument A_i for any $0 < i < n$. The number of arguments in the path is denoted $|\lambda|$. A defeat path for an argument $\mathcal{A}$ is any defeat path starting with $\mathcal{A}$.*

A defeat path is a sequence of defeating arguments. The length of the defeat path is important for acceptance purposes, because an argument $\mathcal{A}$ defeated by an argument $\mathcal{B}$ may be reinstated by another argument $\mathcal{C}$. In this case, it is said that argument $\mathcal{C}$ *defends* $\mathcal{A}$ against $\mathcal{B}$. If the length of a defeat path for argument $\mathcal{A}$ is odd, then the last argument in the sequence is playing a *supporting* or *defender* role. If the length is even, then the last argument is playing an *interfering* or *attacker* role [10, 4].

The notion of defeat path is very simple and only requires that any argument in the sequence must defeat the previous one. Under this unique constraint, which is the basis of argumentation processes, it is possible to obtain some controversial structures, as shown in the next examples.

[2] Because any non-fallacious defeater has been previously suppressed.

Example 4 *Let* $\Phi = \langle AR, \sqsubseteq, \mathbf{C}, \mathbf{R}\rangle$ *an argumentation framework where* $AR = \{\mathcal{A}, \mathcal{B}, \mathcal{C}\, \mathcal{A}_1{}^-, \mathcal{A}_2{}^-\}$, $\mathbf{C}$=$\{\{\mathcal{A}_1{}^-, \mathcal{B}\}, \{\mathcal{B}, \mathcal{C}\}, \{\mathcal{A}_2{}^-, \mathcal{C}\}, \{\mathcal{A}_1{}^-, \mathcal{C}\} \ldots\}$ *and* $\mathcal{B} \succ \mathcal{A}$, $\mathcal{C} \succ \mathcal{B}$, $\mathcal{A}_2{}^- \bowtie \mathcal{C}$, $\mathcal{A} \bowtie \mathcal{C}$.

By conflict inheritance, if $(\mathcal{A}_1{}^-, \mathcal{B}) \in \mathbf{C}$ *then also* $(\mathcal{A}, \mathcal{B}) \in \mathbf{C}$. *The same is true for* $(\mathcal{A}, \mathcal{C})$*, due the inclusion of* $(\mathcal{A}_1{}^-, \mathcal{C})$ *in* $\mathbf{C}$.

The sequence $\lambda_1 = [\mathcal{A}, \mathcal{B}, \mathcal{C}, \mathcal{A}]$ *is a defeat path in* Φ*, because* $\mathcal{B}$ *is a proper defeater of* $\mathcal{A}$*,* $\mathcal{C}$ *is a proper defeater of* $\mathcal{B}$ *and* $\mathcal{A}$ *and* $\mathcal{C}$ *are blocking defeaters of each other. The argument* $\mathcal{A}$ *appears twice in the sequence, as the first and last argument. Note that in order to analyze the acceptance of* $\mathcal{A}$*, it is necessary to analyze the acceptance of every argument in* λ*, including* $\mathcal{A}$*. This is a circular defeat path for* $\mathcal{A}$.

The sequence $\lambda_2 = [\mathcal{A}, \mathcal{B}, \mathcal{C}, \mathcal{A}_1{}^-]$ *is also a defeat path, because* $\mathcal{A}_1{}^-$ *and* $\mathcal{C}$ *are blocking defeaters of each other. Note that even when no argument is repeated in the sequence, the subargument* $\mathcal{A}_1{}^-$ *was already taken into account in the path, as argument* $\mathcal{B}$ *is its defeater. This sequence may be considered another circular defeat path for* $\mathcal{A}$.

The sequence $\lambda_3 = [\mathcal{A}, \mathcal{B}, \mathcal{C}, \mathcal{A}_2{}^-]$ *is a defeat path in* Φ*, because* $\mathcal{A}_2{}^-$ *and* $\mathcal{C}$ *are blocking defeaters of each other. In this case, a subargument* $\mathcal{A}_2{}^-$ *of* $\mathcal{A}$ *appears in the defeat path for* $\mathcal{A}$*. However, this is not a controversial situation, as* $\mathcal{A}_2{}^-$ *was not involved in any previous conflict in the sequence. Argument* $\mathcal{B}$ *is defeating* $\mathcal{A}$ *just because* $(\mathcal{A}_1{}^-, \mathcal{B}) \in \mathbf{C}$*, and is not related to* $\mathcal{A}_2{}^-$*. Defeat path* λ_3 *is correctly structured.*

The initial idea of restricting the inclusion of arguments previously considered in the sequence is not enough. Even more, example 4 shows that forbidding the inclusion of subarguments is not accurate, because valid argumentation lines (as path λ_3) are thrown apart. Two main problematic situations must be taken into account, *direct* and *indirect* reinsertion of arguments. In the first case, an argument appears again in the sequence as a defeater of a new argument. In the second case, an argument is reinserted by including a superargument in the sequence. Both situations are controversial and some well-formed structure must be devised.

5 PROGRESSIVE DEFEAT PATHS

In this section, we present the concept of progressive defeat paths, a notion related to *acceptable argumentation lines* defined for a particulary concrete system in [4]. First, we formalize the consequences of removing an argument from a set of arguments. This is needed because it is important to identify the set of arguments available for use in evolving defeat paths.

Suppose S is a set of available arguments used to construct a defeat path λ. If an argument $\mathcal{A}$ in S is going to be discarded in that process (i. e., its information content is not taken into account), then every argument that includes $\mathcal{A}$ as a subargument should be discarded too. Let S be a set of arguments and $\mathcal{A}$ an argument in S. The operator $\triangleq$ is defined as $S \triangleq \mathcal{A} = S - Sp(\mathcal{A})$ where $Sp(\mathcal{A})$ is the set of all superarguments of $\mathcal{A}$.

As stated before, conflict relations are propagated through superarguments: if $\mathcal{A}$ and $\mathcal{B}$ are in conflict, then $\mathcal{A}^+$ and $\mathcal{B}$ are also conflictive arguments. On the other hand, whenever two arguments are in conflict, it is always possible to identify conflictive subarguments. This notion can be extended to defeat relations. Let $\mathcal{A}$ and $\mathcal{B}$ be two arguments such that $\mathcal{B}$ is a defeater of $\mathcal{A}$. Then both arguments are in conflict and $\mathcal{A} \not\succ \mathcal{B}$. By conflict inheritance, there may exist a subargument $\mathcal{A}_i \sqsubset \mathcal{A}$ such that $(\mathcal{B}, \mathcal{A}_i) \in \mathbf{C}$. It is clear, as $\mathbf{R}$ is monotonic, that $\mathcal{A}_i \not\succ \mathcal{B}$, and therefore $\mathcal{B}$ is also a defeater of $\mathcal{A}_i$. Thus, for any pair of conflictive arguments $(\mathcal{A}, \mathcal{B})$ there is always a pair of conflictive arguments $(\mathcal{C}, \mathcal{D})$ where $\mathcal{C} \sqsubseteq \mathcal{A}$ and $\mathcal{D} \sqsubseteq \mathcal{B}$. Note that possibly $\mathcal{C}$ or $\mathcal{D}$ are trivial subarguments, that is the reason for the existence of the pair to be assured.

Definition 7 (Core conflict) *Let $\mathcal{A}$ and $\mathcal{B}$ be two arguments such that $\mathcal{B}$ is a defeater of $\mathcal{A}$. A* core conflict *of $\mathcal{A}$ and $\mathcal{B}$ is a pair of arguments $(\mathcal{A}_i, \mathcal{B})$ where (i) $\mathcal{A}_i \sqsubseteq \mathcal{A}$, (ii) $\mathcal{B}$ is a defeater of $\mathcal{A}_i$ and (iii) there is no other argument $\mathcal{A}_j \sqsubset \mathcal{A}_i$ such that $\mathcal{A}_j$ is defeated by $\mathcal{B}$.*

The core conflict is the underlying cause of a conflict relation between two arguments, due to the inheritance property. Observe that the core conflict is not necessarily unique.

It is possible to identify the real disputed subargument, which is causing other arguments to fall in conflict. In figure 1, argument $\mathcal{B}$ defeats $\mathcal{A}$ because it is defeating one of its subarguments $\mathcal{C}$. The core conflict of $\mathcal{A}$ and $\mathcal{B}$ is $\mathcal{C}$. In this case the defeat arc between the superarguments may not be drawn.

Definition 8 (Disputed subargument) *Let $\mathcal{A}$ and $\mathcal{B}$ be two arguments such that $\mathcal{B}$ is a defeater of $\mathcal{A}$. A subargument $\mathcal{A}_i \sqsubseteq \mathcal{A}$ is said to be a* disputed subargument *of $\mathcal{A}$ with respect to $\mathcal{B}$ if $\mathcal{A}_i$ is a core conflict of $\mathcal{A}$ and $\mathcal{B}$.*

The notion of *disputed subargument* is very important in the construction of defeat paths in dialectical processes. Suppose argument $\mathcal{B}$ is a defeater of argument $\mathcal{A}$. It is possible to construct a defeat path $\lambda = [\mathcal{A}, \mathcal{B}]$. If there is a defeater of $\mathcal{B}$, say $\mathcal{C}$, then $[\mathcal{A}, \mathcal{B}, \mathcal{C}]$ is also a defeat path. However, $\mathcal{C}$ should not be a disputed argument of $\mathcal{A}$ with respect to $\mathcal{B}$, as circularity is introduced in the path. Even more, $\mathcal{C}$ should not be an argument that $includes$ that disputed argument, because that path can always be extended by adding $\mathcal{B}$ again.

The set of arguments available to be used in the construction of a defeat path is formalized in the following definition.

Definition 9 (Defeat domain) *Let $\Phi = \langle AR, \sqsubseteq, \mathbf{C}, \mathbf{R}\rangle$ be an AF and let $\lambda = [\mathcal{A}_1, \mathcal{A}_2, \ldots, \mathcal{A}_n]$ be a defeat path in Φ. The function $D^i(\lambda)$ is defined as*

- $D^1(\lambda) = AR$
- $D^k(\lambda) = D^{k-1}(\lambda) \triangleq \mathcal{B}_n$*, where $\mathcal{B}_n$ is the disputed subargument of $\mathcal{A}_{k-1}$ with respect to $\mathcal{A}_k$ in the sequence, with $2 \leq k \leq n$.*

The defeat domain discards controversial arguments for a given path. The function $D^k(\lambda)$ denotes the set of arguments that can be used to extend the defeat path λ at stage k, i. e., to defeat the argument $\mathcal{A}_k$. Choosing an argument from $D^k(\lambda)$ avoids the introduction of previous disputed arguments in the sequence. It is important to remark that if an argument including a previous disputed subargument is reintroduced in the defeat path, it is always possible to reintroduce its original defeater.

Therefore, in order to avoid controversial situations, any argument $\mathcal{A}_i$ of a defeat path λ should be in $D^{i-1}(\lambda)$. Selecting an argument outside this set implies the repetition of previously disputed information. The following definition characterizes well structured sequences of arguments, called *progressive defeat paths*.

Definition 10 (Progressive defeat path) *Let $\langle AR, \sqsubseteq, \mathbf{C}, \mathbf{R}\rangle$ be an argumentation framework. A progressive defeat path is defined recursively in the following way:*

- $[\mathcal{A}]$ *is a progressive defeat path, for any $\mathcal{A} \in AR$.*
- *If $\lambda = [\mathcal{A}_1, \mathcal{A}_2, \ldots, \mathcal{A}_n]$, $n \geq 1$ is a progressive defeat path, then for any defeater $\mathcal{B}$ of $\mathcal{A}_n$ such that $\mathcal{B} \in D^n(\lambda)$, $\lambda' = [\mathcal{A}_1, \mathcal{A}_2, \ldots, \mathcal{A}_n, \mathcal{B}]$ is a progressive defeat path.*

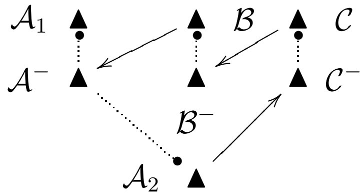

Figure 2: Controversial Situation

Progressive defeat paths are free of circular situations and guarantees progressive argumentation (in the sense of using always new arguments), as desired on every dialectical process. Note that it is possible to include a subargument of previous arguments in the sequence, as long as it is not a disputed subargument.

In figure 2 a controversial abstract framework is shown. For space reasons we do not provide the formal specification, although it can be deduced from the graph. There are seven arguments $\mathcal{A}_1, \mathcal{A}_2, \mathcal{A}^-, \mathcal{B}, \mathcal{B}^-, \mathcal{C}, \mathcal{C}^-$. There exists an infinite defeat path $[\mathcal{A}_1, \mathcal{B}, \mathcal{C}, \mathcal{A}_2, \mathcal{B}, \mathcal{C}..]$ which is not progressive. Lets construct a progressive defeat path λ for argument $\mathcal{A}_1$. We start with $\lambda = [\mathcal{A}_1]$. The pool of arguments used to select a defeater of $\mathcal{A}_1$ is $D^1(\lambda) = \{\mathcal{A}_2, \mathcal{A}^-, \mathcal{B}, \mathcal{B}^-, \mathcal{C}, \mathcal{C}^-\}$. The only defeater belonging to $D^1(\lambda)$ is $\mathcal{B}$, with disputed subargument $\mathcal{A}^-$, so we add it to λ. Now $\lambda = [\mathcal{A}_1, \mathcal{B}]$ and the pool of available arguments is $D^2(\lambda) = \{\mathcal{B}, \mathcal{B}^-, \mathcal{C}, \mathcal{C}^-\}$, where $\mathcal{A}^-$ and its superarguments were removed. $\mathcal{C} \in D^2(\lambda)$ is a defeater of $\mathcal{B}$ so we add it to the path and now $\lambda = [\mathcal{A}_1, \mathcal{B}, \mathcal{C}]$. The potential defeater arguments are now in $D^3(\lambda) = \{\mathcal{C}, \mathcal{C}^-\}$. As there are no defeaters of $\mathcal{C}$ in $D^3(\lambda)$, then the path can not be extended. Thus, the resulting sequence $[\mathcal{A}_1, \mathcal{B}, \mathcal{C}]$ is a progressive defeat path.

6 EXTENSIONS BASED ON PROGRESSIVE PATHS

In Dung's approach [2] several semantic notions are defined as argument extensions. The set of accepted arguments is characterized using the concept of *acceptability*. An argument $\mathcal{A} \in AR$ is *acceptable* with respect to a set of arguments S if and only if every argument $\mathcal{B}$ attacking $\mathcal{A}$ is attacked by an argument in S. It is also said that S is defending $\mathcal{A}$ against its attackers, and this is a central notion on argumentation. A set R of arguments is a *complete extension* if R defends every argument in R. A set of arguments G is a *grounded extension* if and only if it is the least (with respect to set inclusion) complete extension. The grounded extension is also the least fixpoint of a simple monotonic function:

$$F_{AF}(S) = \{\mathcal{A} : \mathcal{A} \text{ is acceptable wrt } S\}.$$

The framework of figure 2 may be completed with inherited defeat relations. For example, an arc from $\mathcal{B}$ to $\mathcal{A}_1$ can be drawn, as shown in figure 3 (argument positions are relocated in order to simplify the graph). A cycle is produced involving arguments $\mathcal{B}$, $\mathcal{C}$ and $\mathcal{A}_2$. According to a skeptical point of view, the grounded extension of the completed framework is the empty set, and no argument is accepted. Dung's extensions may be applied to this framework, considering proper defeat as the classic attack relation. In this case, there are no stable nor preferred extensions due to the cycle. However, as a non-conflictive relation is present, a new premise must be stated: if an argument is accepted, then all of its subarguments are accepted. Therefore, any extension including, for example, argument $\mathcal{A}_1$ should also include argument $\mathcal{A}^-$.

When considering subarguments, new semantic extensions can be introduced in order to capture sets of possible accepted arguments.

We will focus here in the impact of progressive defeat paths in the acceptance of arguments.

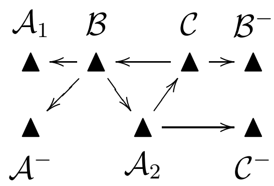

Figure 3: Completed framework

Definition 11 (Dialectical space) *Let $\langle AR, \sqsubseteq, \mathbf{C}, \mathbf{R}\rangle$ be an AF. The dialectical space of an argument $\mathcal{A} \in AR$ is the set $\mathcal{SP}_{\mathcal{A}} = \{\lambda | \lambda$ is a defeat path for $\mathcal{A}\}$.*

The dialectical space for a given argument is formed by all of the defeat paths for that argument.

Example 5 *In the simple argumentation framework of figure 4, $\mathcal{SP}_{\mathcal{A}} = \{[\mathcal{A}, \mathcal{B}], [\mathcal{A}, \mathcal{C}, \mathcal{D}, \mathcal{E}]\}$ and $\mathcal{SP}_{\mathcal{B}} = \{[\mathcal{B}]\}$.*

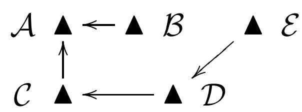

Figure 4: Simple framework

The dialectical space may be infinite, if cycles are present. In figure 2 every argument has an infinite set of defeat paths. Consider the path $[\mathcal{B}, \mathcal{C}, \mathcal{A}_2]$. Because of the cycle, $[\mathcal{B}, \mathcal{C}, \mathcal{A}_2, \mathcal{B}]$ is also a defeat path. Therefore, defeat paths of any lenght may be constructed. In fact, every dialectical space in this framework is infinite.

Cycles of defeaters are very common in argumentation, usually called *fallacies*. The status of fallacious arguments cannot be determined, although they are not considered accepted as they are controversial in the framework. In many cases, using skeptical semantic concepts [2], an argument that is not taking part of a cycle cannot be accepted due to a fallacy. This is the case of argument $\mathcal{A}_1$ in figure 2. A credulous semantic may be defined using progressive defeat paths.

Several definitions are needed. We consider only progressive argumentation in order to evaluate the acceptance of an argument. Maximality of paths is important because all possible arguments must be taken into account.

Definition 12 (Progressive dialectical space) *Let $\mathcal{A}$ be an argument. The progressive reduction of $\mathcal{SP}_{\mathcal{A}}$, denoted $\mathcal{SP}^R_{\mathcal{A}}$, is the set of all maximal progressive defeat paths for $\mathcal{A}$.*

A notion of acceptability analogous to [2] may be defined, using a progressive dialectical space. As usual, an argument $\mathcal{A}$ is said to be defended by a set of arguments S if every defeater of $\mathcal{A}$ is defeated by an element of S. The defense of $\mathcal{A}$ by S occurs in a path $\lambda = [\mathcal{A}_1, \ldots, \mathcal{A}_n]$ if $\mathcal{A} = \mathcal{A}_i$, $1 \leq i \leq n$ and the defender argument $\mathcal{A}_{i+2}$ is in S.

Definition 13 (Defense) *Given an argument $\mathcal{A}$, a set P of defeat paths and a set of arguments S, $\mathcal{A}$ is said to be acceptable with respect to S in P if for every defeater $\mathcal{B}$ of $\mathcal{A}$, S defends $\mathcal{A}$ against $\mathcal{B}$ in at least one element of P.*

If $\mathcal{A}$ is defended against $\mathcal{B}$ in at least one defeat path in P then argument $\mathcal{B}$ is no longer a threat for $\mathcal{A}$, no matter what is the situation in other defeat paths. In the framework of figure 4, argument $\mathcal{C}$ is defended by $\{\mathcal{E}\}$ in the defeat path $[\mathcal{A}, \mathcal{C}, \mathcal{D}, \mathcal{E}]$ and therefore $\mathcal{C}$ is acceptable with respect to $\{\mathcal{E}\}$ in $P = \{[\mathcal{A}, \mathcal{C}, \mathcal{D}, \mathcal{E}]\}$. This fact cannot be changed adding new defeaters for $\mathcal{D}$. Argument $\mathcal{A}$, however, is not acceptable in $\mathcal{SP}^R_{\mathcal{A}}$, because it cannot be defended in $[\mathcal{A}, \mathcal{B}]$.

Definition 14 (Grounded extension) *Let P be a set of defeat paths. The grounded extension of P is the least fixpoint of the function $F_P(S) = \{\mathcal{A} : \mathcal{A}$ is acceptable wrt S in $P\}$.*

The grounded extension for a set of defeat paths is analogous to the Dung's grounded extension for basic argumentation frameworks. In the framework of figure 2, $\mathcal{SP}^R_{\mathcal{A}_1} = \{[\mathcal{A}_1, \mathcal{B}, \mathcal{C}]\}$. In this set, $F(\emptyset) = \{\mathcal{C}\}$, $F(\{\mathcal{C}\}) = \{\mathcal{C}, \mathcal{A}_1\}$ and $F(\{\mathcal{C}, \mathcal{A}_1\}) = \{\mathcal{C}, \mathcal{A}_1\}$. Then, the grounded extension of $\mathcal{SP}^R_{\mathcal{A}_1}$ is $\{\mathcal{C}, \mathcal{A}_1\}$.

Definition 15 (Warranted extension) *Let Φ = $\langle AR, \sqsubseteq, \mathbf{C}, \mathbf{R}\rangle$ be an argumentation framework. A set of arguments $S \subseteq AR$ is said to be a warranted extension, if every argument $\mathcal{X}$ in S belongs to the grounded extension of $\mathcal{SP}^R_{\mathcal{X}}$. Every argument of S is said to be warranted in Φ.*

In the framework of figure 2, $\{\mathcal{A}_1, \mathcal{A}^-, \mathcal{B}^-, \mathcal{C}^-\}$ is the warranted extension, as all of those arguments are in the grounded extension of its own progressive dialectical space.

The two main semantic concepts presented in this framework are related. Both notions classify non-fallacious arguments as accepted ones. The warranted extension is, however, more credulous than the fixpoint semantic as it is considering only pruned argumentation lines (progressive defeat paths) to evaluate the acceptance of an argument. Arguments affected by a fallacy are not accepted in Υ^ω. Consider the framework of example 3. Here $\Upsilon^\omega = \emptyset$, while the warranted extension is $\{\mathcal{A}_1, \mathcal{A}^-, \mathcal{B}^-, \mathcal{C}^-\}$.

On the other hand, the characteristic function F_{AF} defined in [2] adopts a skeptical position with respect to our semantic notions. Although under this operator the framework of 3 has no accepted arguments, it is based in classic attack relations. This leads to mutual defeaters when two arguments are attacking each other.

The fixpoint semantic defined here results more credulous than the Dung's grounded extension, as it can be noted in example 3, where according to Dung the grounded extension (the least fixpoint of F_{AF}) is the empty set. In our framework, if the relation $\mathbf{R}$ does not lead to incomparable or indistinguishable arguments, then the final result is the basic argumentation framework free of cycles, as there are only proper defeaters corresponding to the attack relation of Dung. In this case, both fixpoint operators are equivalent.

The following proposition confirm the credulous profile of the warranted extension based on progressive argumentation.

Proposition 1 *Let Φ=$\langle AR, \sqsubseteq, \mathbf{C}, \mathbf{R}\rangle$ be an argumentation framework. Every conflict-free argument in $\Upsilon^\omega(AR)$ is included in the warranted extension of Φ.*

Proof: Let $\mathcal{A}$ be an argument in $\Upsilon^n = \Upsilon^{n+1} = \Upsilon^\omega(AR)$ that is not in conflict with any other argument in that set. Then every defeater of $\mathcal{A}$ in Φ (if any) is a suppresed argument in Υ^{n-1}, that is, every defeater is defeated by a defeater-free argument in Υ^{n-1}. Therefore, $\mathcal{A}$ in Υ^n is defended by arguments in Υ^{n-1}. Due to the fact that Υ is monotonic, any argument that became free of defeaters in Υ^k is free of defeaters in Υ^j, for all $j > k$, and therefore every defender of $\mathcal{A}$ is an accepted argument. These defenders are all in $\mathcal{SP}^R_{\mathcal{A}}$ and then $\mathcal{A}$ is in the grounded extension of that set.

Moreover, if the abstract argumentation framework is free of cycles (as in example 4), then every non-accepted argument is suppressed. The set Υ^ω is a coherent set of arguments, and the warranted extension by itself. The concept of well-founded frameworks presented in [2] is suitable to be applied to our framework.

Definition 16 *An argumentation framework $\langle AR, \sqsubseteq, \mathbf{C}, \mathbf{R}\rangle$ is well-founded if there exists no infinite defeat path.*

In well-founded argumentation frameworks the two semantic concepts presented in this paper coincide, as stated in the next proposition.

Proposition 2 *Let Φ=$\langle AR, \sqsubseteq, \mathbf{C}, \mathbf{R}\rangle$ be a well-founded argumentation framework. Then $\Upsilon^\omega(AR)$ is the warranted extension of Φ.*

Proof: *If Φ is well-founded, then no cycles are present, and every dialectial space of an argument is finite. Even more, there are only proper defeaters, as blocking defeaters always produce infinite paths due to symmetry. Let $\mathcal{A}$ be an argument in Φ. Suppose there is a defeat path λ for $\mathcal{A}$ such that λ is not progressive. Then an argument $\mathcal{A}_j$ in λ is the superargument of another argument $\mathcal{A}_i$ in the sequence, $i < j$. As $\mathcal{A}_{i+1}$ defeats $\mathcal{A}_i$ and $\mathcal{A}_i \sqsubseteq \mathcal{A}_j$ then $\mathcal{A}_{i+1}$ is also a defeater of $\mathcal{A}_j$. Then there is a subsequence of arguments $\mathcal{A}_{i+1}, \ldots, \mathcal{A}_j, \mathcal{A}_{i+1}$ which is a cycle of defeaters, and then Φ is not well founded, contradicting the premise. Therefore, every defeat path for $\mathcal{A}$ is progressive, that is, $\mathcal{SP}_{\mathcal{A}} = \mathcal{SP}^R_{\mathcal{A}}$.*
We will show that if an argument $\mathcal{A}$ is in Υ^ω then it is in the grounded extension of $\mathcal{SP}_{\mathcal{A}}$. The converse is omitted for space reasons. Suppose $\mathcal{A}$ is in Υ^ω. Then $\mathcal{A}$ became a defeater-free argument in Υ^k, for some $k > 0$. Its defeaters were suppressed arguments in Υ^{k-1}, that is, they are defeated by defeater-free (d.f.) arguments in Υ^{k-1}. Let D be this set of d.f. arguments of Υ^{k-1}, it is clear that every element of D is part of a defeat path in $\mathcal{SP}_{\mathcal{A}}$. As every defeater of $\mathcal{A}$ is defeated by an argument in D, then $\mathcal{A}$ is acceptable with respect to D in $\mathcal{SP}_{\mathcal{A}}$. All the elements in D are defeater-free arguments in Υ^{k-1} and (following the previous analysis for $\mathcal{A}$) therefore acceptable wrt Υ^{k-2}. Any d.f. argument in Υ^{k-2} is acceptable wrt Υ^{k-3}, and so on. Defeater-free arguments in Υ^0 are acceptable with respect to the empty set, so they are in warranted extension. As all of its defenders are in the warranted extension, then d.f. arguments of Υ^1 are also in that extension. The same is true for d.f. arguments in Υ^2, Υ^3,...,Υ^k, so $\mathcal{A}$ is in the warranted extension of Φ.

The warranted extension is a credulous alternative for accepting arguments when controversial situations (cycles) are present. The equivalence condition stated in proposition 2 shows that if the framework is free of controversy, then the warranted extension is as credulous as the semantic based on suppressed arguments.

7 CONCLUSIONS

Abstract argumentation systems are formalisms for argumentation, where some components remains unspecified, usually the structure of arguments. In the dialectical process carried out to identify accepted arguments in the system, some controversial situations may be found, related to the reintroduction of arguments in this process, causing a circularity that must be treated in order to avoid an infinite analysis process. Some systems apply a single restriction to argumentation lines: no previously considered argument is reintroduced in the process. In this work, an abstract framework with conflicts and preference criteria is used to define two semantic notions. First, the classic fixpoint semantic for extended defeat relations. Second, an argument extension based on the concept of progressive defeat paths, where superarguments of previously disputed arguments are discarded. These mechanisms constitute a credulous approach to characterize sets of possible accepted arguments.

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