Abstract

In this work, contexts for extended argumentation frameworks (EAF) are defined. A context for an EAF is another framework where original arguments, conflicts and preferences are kept, while introducing new arguments leading to new defeat relations. Thus, the context may interfere with the original classification of arguments, inducing new set of extensions. These semantic change in the outcome of an extended framework in a particular context is characterized, and Dung’s acceptability concept is analyzed on this basis.

1 Introduction

Argumentation has became an important subject of research in Artificial Intelligence and it is also of interest in several disciplines, such as Logic, Philosophy and Communication Theory. This wide range of attention is due to the constant presence of argumentation in many activities, most of them related to social interactions between humans, as in civil debates, legal reasoning or every day dialogues. Basically, an argument is a piece of reasoning that supports a claim from certain evidence. The tenability of this claim must be confirmed by analyzing other arguments for and against such a claim. In formal systems of defeasible argumentation, a claim will be accepted if there exists an argument that supports it, and this argument is acceptable according to an analysis between it and its counterarguments. After this dialectical analysis is performed over the set of arguments in the system, some of them will be acceptable, justified or warranted arguments, while others will be not. The study of the acceptability of arguments is the search for rationally based positions of acceptance in a given scenario of arguments and their relationships. It is one of the main concerns in Argumentation Theory.

Abstract argumentation systems [5, 12, 6, 1, 2] are formalisms for argumentation where some components remain unspecified, being the structure of an argument the main abstraction. In this kind of system, the emphasis is put on the semantic notion of finding
the set of accepted arguments. Most of them are based on the single abstract concept
of attack represented as an abstract relation, and extensions are defined as sets of possibly
accepted arguments. For two arguments \( A \) and \( B \), if \((A, B)\) is in the attack relation,
then the acceptance of \( B \) is conditioned by the acceptance of \( A \), but not the other way
around. It is said that argument \( A \) attacks \( B \), and it implies a priority between conflicting
arguments. It is widely understood that this priority is related to the argument strengths.
Several frameworks do include an argument order \([1, 3, 4]\), although this order is used at
another level, as the classic attack relation is kept.

In \([8, 7]\) an extended abstract argumentation framework (EAF) is introduced, where
two kinds of defeat relations are present. These relations are obtained by applying a
preference criterion between conflictive arguments. The conflict relation is kept in its most
basic, abstract form: two arguments are in conflict simply if both arguments cannot be
accepted simultaneously. The preference criterion subsumes any evaluation on arguments
and it is used to determine the direction of the attack. This argument comparison, however,
is not always succesful and therefore attacks, as known in classic frameworks, are no
longer valid.

An argumentation framework \( \Phi \) is basically the modelization of a knowledge base
conformed by arguments. These arguments interact each other and then several possible
outcomes as sets of accepted arguments are obtained. However, it is possible for this outcome
to be different when new arguments are taken into account. These new arguments are
considered the context of the framework \( \Phi \). For example, when a person is judged in
a regular trial, several arguments for and against its innocence are exposed by the district
attorney and by the defender lawyer. This set of arguments, say Case, is about the
assumptions and facts of the particular case. Another set of arguments, however, is taked
into account: those produced by the juror and the judge. Thus, the set Case is placed in a
special context: the actual trial. If the person is declared guilty, its lawyers may appeal to
an upper level of Justice Court. Basically, they want to expose its arguments in a different
context, in order to plead the defended not guilty.

We think situations like above may be modeled using extended abstract frameworks.
This paper is organized as follows. In Section 2 our extended argumentation framework
is presented. In Section 3 the notion of contexts for EAF is introduced. In Section 4
the behaviour of contexts is analyzed according to Dung’s acceptability semantics \([5]\).
Finally, the conclusions and future work are presented in Section 5.

2 Extended Argumentation Framework

In our extended argumentation framework three relations are considered: conflict, sub-
argument and preference between arguments. The definition follows:

**Definition 1.**
An extended abstract argumentation framework (EAF) is a quartet \( \Phi=\langle AR, \sqsubseteq, C, R \rangle \),
where \( AR \) is a finite set of arguments, and \( \sqsubseteq, C \) and \( R \) are binary relations over \( AR \)
denoting respettively subarguments, conflicts and preferences between arguments.

Arguments are abstract entities, as in \([5]\), that will be denoted using calligraphic uppercase letters, possibly with indexes. In this work, the subargument relation is not relevant for the topic addressed. Basically, it is used to model the fact that arguments may
include inner pieces of reasoning that can be considered arguments by itself, and it is of special interest in dialectical studies [9]. Hence, unless explicitly specified, in the rest of the paper $\sqsubseteq = \emptyset$. The conflict relation $C$ states the incompatibility of acceptance between arguments. Given a set of arguments $S$, an argument $A \in S$ is said to be in conflict in $S$ if there is an argument $B \in S$ such that $\{A, B\} \in C$. The relation $R$ is introduced in the framework and it will be used to evaluate arguments, modelling a preference criterion based on a measure of strength.

**Definition 2.**

*Given a set of arguments $AR$, an argument comparison criterion $R$ is a binary relation on $AR$. If $ARB$ but not $BR.A$ then $A$ is strictly preferred to $B$, denoted $A > B$. If $ARB$ and $BR.A$ then $A$ and $B$ are indifferent arguments with equal relative preference, denoted $A \equiv B$. If neither $ARB$ or $BR.A$ then $A$ and $B$ are incomparable arguments, denoted $A \nRightarrow B$.*

For two arguments $A$ and $B$ in $AR$, such that the pair $\{A, B\}$ belongs to $C$ the relation $R$ is considered. In order to elucidate conflicts, the participant arguments must be compared. Depending on the preference order, two notions of argument defeat are derived.

**Definition 3.**

*Let $\Phi = \langle AR, \sqsubseteq, C, R \rangle$ be an EAF and let $A$ and $B$ be two arguments such that $\{A, B\} \in C$. If $A > B$ then it is said that $A$ is a proper defeater of $B$. If $A \equiv B$ or $A \nRightarrow B$, it is said that $A$ is a blocking defeater of $B$, and viceversa. An argument $B$ is said to be a defeater of an argument $A$ if $B$ is a blocking or a proper defeater of $A$.*

**Example 1.** 

Let $\Phi_1 = \langle AR, \sqsubseteq, C, R \rangle$ be an EAF where $AR = \{A, B, C, D, E\}$, $\sqsubseteq = \emptyset$, $C = \{\{A, B\}, \{B, C\}, \{C, D\}, \{C, E\}\}$ and $A > B, B > C, E \nRightarrow C, C \equiv D$.

Extended abstract frameworks can also be depicted as graphs, with different types of arcs, called *EAF-graphs*. We use to represent arguments as black triangles. An arrow ($\rightarrow$) is used to denote proper defeaters. A double-pointed straight arrow ($\leftrightarrow$) connects blocking defeaters considered equivalent in strength, and a double-pointed zig-zag arrow ($\overleftrightarrow{\rightarrow}$) connects incomparable blocking defeaters. In Figure 1, the framework $\Phi_1$ is shown. Argument $A$ is a proper defeater of $B$. Argument $B$ is a proper defeater of $C$, and $E$ is an incomparable blocking defeater of $C$ and viceversa. Argument $D$ and $C$ are blocking defeaters being equivalent in strength.

![Figure 1: EAF-graph of framework $\Phi_1$](image)

In the next section we formally present *contexts* for extended argumentation frameworks and several semantic notions around this concept.
3 Contexts

An extended argumentation framework may be considered in different contexts, where its elements are still valid and well-defined, but interacting with new arguments. This is formalized as follows.

Definition 4.
Let $\Phi = \langle AR_1, \sqsubseteq_1, C_1, R_1 \rangle$ be an extended argumentation framework. A context for $\Phi$ is a tuple $\langle AR_2, \sqsubseteq_2, C_2, R_2 \rangle$ such that

- $AR_1 \subseteq AR_2$,
- For any pair of conflicting arguments $(A, B) \in C_2$ such that $A, B \in AR_1$ then $(A, B) \in C_1$.
- If $AR_2 B$ for any pair of arguments $A, B \in AR_1$, then $AR_1 B$.
- For any arguments $X, Y \in AR_2$ such that $X \sqsubseteq_2 Y$, if $X \in AR_2 - AR_1$ then $Y \in AR_2 - AR_1$.

Arguments in $AR_2 - AR_1$ are called contextual arguments.

Definition 4 states that a context for an extended argumentation framework $\Phi_1$ is just another extended framework $\Phi_2$ where

1. all of the arguments in $\Phi_1$ are included in $\Phi_2$, and
2. no conflict between arguments of $\Phi_1$ is added by $\Phi_2$, and
3. preferences established in $\Phi_1$ remain intact in $\Phi_2$, and
4. $\Phi_2$ may include new superarguments of arguments in $\Phi_1$, but not the other way around.

Any framework is said to be a context for itself or a self-context.

In Figure 2 the general idea of framework context is shown. New arguments are present, which are able to defeat or to be defeated by arguments in $\Phi_1$. Note that if $\Phi_2$ is a context for $\Phi_1$ then $\Phi_1$ is a restriction of $\Phi_2$, in the sense of [10], taking into account subarguments. That is, the set $AR_1$ must be structurally complete: it includes the subarguments of all of its elements.

![Figure 2: EAF $\Phi_2$ is a context for EAF $\Phi_1$](image-url)
Example 2. Let $\Phi_1 = \langle AR_1, \sqsubseteq_1, C_1, R_1 \rangle$ be an extended abstract framework where $AR_1 = \{A, B, C\}$, $C_1 = \{\{A, B\}, \{B, C\}\}$, $A \succ B$ and $B \bowtie C$. The following framework $\Phi_2 = \langle AR_2, \sqsubseteq_2, C_2, R_2 \rangle$ is a context for $\Phi_1$, where $AR_2 = \{A, B, C, D, E\}$, $C_2 = \{\{A, B\}, \{B, C\}, \{C, D\}, \{A, E\}\}$, $A \succ B$, $B \bowtie C$, $E \succ A$ and $D \equiv C$, is a context for $\Phi_1$.

The context $\Phi_X$ for an extended framework $\Phi$ may also be placed in a new context $\Phi_Y$. Even more, this new context $\Phi_Y$ is also a context for $\Phi$.

Proposition 1. Let $\Phi_1$ and $\Phi_2$ be two extended argumentation frameworks such that $\Phi_2$ is a context for $\Phi_1$. Then every extended framework $\Phi_3$ such that $\Phi_3$ is a context for $\Phi_2$, it is also a context for $\Phi_1$.

Proof. Obvious from the definition.

In order to evaluate the outcome of an argumentation framework in a particular context, we use the notation adopted by Baroni & Giacomin in [11], where semantic extensions are studied.

Definition 5. [11]
Given a generic argumentation semantic $S$, the set of extensions prescribed by $S$ for an EAF $\Phi$ is denoted as $E_S(\Phi)$.

The set of argument extensions induced by an EAF may change when its arguments are challenged by new arguments in the context. It is possible that an argument is no longer present in every extension, or to be included in a new one. Even more, an entire extension may not be valid in a specific context. The following definition introduces new terminology in relevant cases.

Definition 6.
Let $\Phi_1 = \langle AR_1, \sqsubseteq_1, C_1, R_1 \rangle$ be an extended argumentation framework and let $\Phi_2 = \langle AR_2, \sqsubseteq_2, C_2, R_2 \rangle$ be a context for $\Phi_1$. Let $S$ be an argumentation semantic.

- $\Phi_2$ is said to $S$-confirm $\Phi_1$ if $E_S(\Phi_1) = E_S(\Phi_2)$.
- $\Phi_2$ is said to $S$-preserve $\Phi_1$ if for every extension $X \in E_S(\Phi_1)$, there is an extension $Y \in E_S(\Phi_2)$ such that $X \subseteq Y$. Every argument in $X$ is said to be preserved by $\Phi_2$.
- $\Phi_2$ is said to $S$-expand $\Phi_1$ if $\Phi_2$ $S$-preserve $\Phi_1$ and every extension $Y \in E_S(\Phi_2)$ is a superset of an extension $X$ in $E_S(\Phi_1)$.
- $\Phi_2$ is said $S$-revise $\Phi_1$ if exists an extension $X$ in $E_S(\Phi_1)$ such that no extension in $E_S(\Phi_2)$ is a superset of $X$. The set $X$, as an extension, is said to be revised by $\Phi_2$. Also it is said that $\Phi_2$ revises $\Phi_1$ in $X$.

The following table summarizes the concepts presented in Definition 6, and captures the essential meaning of every case.
<table>
<thead>
<tr>
<th>Concept</th>
<th>Meaning</th>
</tr>
</thead>
<tbody>
<tr>
<td>S-confirm</td>
<td>No extension is changed or added</td>
</tr>
<tr>
<td>S-preserve</td>
<td>The same alternatives of acceptance are available, but some extensions of $\Phi_2$ may propose new sets of arguments for acceptance.</td>
</tr>
<tr>
<td>S-expand</td>
<td>There is always an extension of $\Phi_2$ that includes a valid alternative of acceptance for $\Phi_1$ according to $S$.</td>
</tr>
<tr>
<td>S-revise</td>
<td>The alternative $X$ of acceptance in $\Phi_1$ is no longer valid in $\Phi_2$ as a whole, i.e. the extension is “broken” or discarded by $\Phi_2$.</td>
</tr>
</tbody>
</table>

It is clear that any $EAF \Phi$ S-confirms $\Phi$. When a context $\Phi_X$ S-confirm an $EAF \Phi_Y$ then every argument in $\Phi_X$ (if any) is defeated by at least an argument in an extension of $\Phi_Y$. Simple frameworks and contexts exhibiting these properties are shown in Example 3 and Figure 3.

![Figure 3: Frameworks and contexts](image)

**Example 3.** Consider the three frameworks depicted in Figure 2. In each case, arguments in $\Phi_1$ are shown as black triangles and arguments in context $\Phi_2$ are shown as white triangles. Let $P$ be the admissibility-based preferred semantics and let $E_{P(\Phi)}$ the set of all preferred extensions of framework $\Phi$. In the following table the preferred extensions and context properties are shown:

<table>
<thead>
<tr>
<th>Example</th>
<th>$E_{P(\Phi_1)}$</th>
<th>$E_{P(\Phi_2)}$</th>
<th>Properties of $\Phi_2$ with respect to $\Phi_1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a)</td>
<td>${{A,C}, {B,C}}$</td>
<td>${{D,B,C}}$</td>
<td>$P$-revise, not $P$-preserve, not $P$-expand</td>
</tr>
<tr>
<td>(b)</td>
<td>${{B}}$</td>
<td>${{B,C}, {B,D}}$</td>
<td>$P$-preserve, $P$-expand, not $P$-revise</td>
</tr>
<tr>
<td>(c)</td>
<td>${{A,B}, {A,C}}$</td>
<td>${{A,B,C}, {A,C,D}, {A,F}}$</td>
<td>$P$-preserve, not $P$-expand, not $P$-revise</td>
</tr>
</tbody>
</table>

In the next section the acceptability semantics defined in [5] is analyzed for extended abstract frameworks and its contexts.
The argumentation framework defined by Dung in [5] is the core of argument basic semantic notions. Its framework only includes arguments and attacks as a binary relation on arguments, the basic elements for semantics elaborations. The main contribution is the formalization of several argument extensions capturing rational sets of acceptance. The key notion is acceptability of arguments, defined here for extended abstract frameworks.

**Definition 7.**

Let \(<AR, \sqsubseteq, C, R>\) be an EAF. An argument \(A \in AR\) is acceptable with respect to a set of arguments \(S \subseteq AR\) if and only if every argument \(B\) defeating \(A\) is defeated by an argument in \(S\).

Defeaters mentioned in Definition 7 may be either proper or blocking ones. It is also said that \(S\) is defending \(A\) against its attackers. The defense or reinstatement of arguments is a central concept on argumentation. Extensions are required to be free of inner conflicts, and thus the following definition is needed.

**Definition 8.**

A set of arguments \(S \subseteq AR\) is said to be conflict-free if for all \(A, B \in S\) it is not the case that \({A, B}\) \(\in C\).

As said before, in Dung’s approach several semantic notions are defined as argument extensions leading to rational positions of acceptance. These extensions can also be applied to extended frameworks and are summarized in the following definition.

**Definition 9.**

A set of arguments \(S \subseteq AR\) is said to be

- admissible if it is conflict-free and defends all its elements.
- a preferred extension if \(S\) is a maximal (for set inclusion) admissible set.
- a complete extension if \(S\) is admissible and it includes every acceptable argument w.r.t. \(S\).
- a grounded extension if and only if it is the least (for set inclusion) complete extension.
- a stable extension if \(S\) is conflict-free and it attacks each argument not in \(S\).

The grounded extension of a framework \(\Phi\), denoted \(GE_{\Phi}\), is also the least fixpoint of a simple monotonic characteristic function:

\[ F_{AF}(S) = \{A : A \text{ is acceptable wrt } S\} \]

Several modifications to the classic Dung’s framework are proposed in the literature, and new semantic notions were introduced. For example, in [6] the original framework is kept, while presenting a new argument extension. In [1], preferences between arguments are added to the framework and new semantic considerations are made.

The following proposition uses Definition 6 applied to preferred and grounded extensions.
Proposition 2. Let $\Phi_1 = \langle AR_1, \sqsubseteq_1, C_1, R_1 \rangle$ be an extended argumentation framework and let $\Phi_2 = \langle AR_2, \sqsubseteq_2, C_2, R_2 \rangle$ be a context for $\Phi_1$. Let $\mathcal{P}$ and $\mathcal{G}$ be the preferred and grounded semantics respectively.

- If $\Phi_2 \mathcal{P}$-preserves $\Phi_1$ then every argument in an extension $X \in \mathcal{E}_\mathcal{P}(\Phi_1)$ is acceptable with respect to $AR_2$.
- If $\Phi_2 \mathcal{G}$-preserves $\Phi_1$ then also $\Phi_2 \mathcal{G}$-expands $\Phi_1$.

Proof. If $\Phi_2 \mathcal{P}$-preserves $\Phi_1$ then every argument included in a preferred extension $X$ of $\Phi_1$ is also included in a preferred extension of $\Phi_2$ and therefore is acceptable with respect to a set in $AR_2$. The grounded extension is unique (being a skeptical notion), and thus if $\Phi_2 \mathcal{G}$-preserves $\Phi_1$, then the grounded extension of $\Phi_2$ includes every argument in the grounded extension of $\Phi_1$. As these are the only sets in $\mathcal{E}_\mathcal{P}(\Phi_1)$ and $\mathcal{E}_\mathcal{P}(\Phi_2)$, then $\Phi_2 \mathcal{G}$-expands $\Phi_1$. \qed

As stated in Proposition 1, a context $\Phi_3$ for a framework $\Phi_2$ being a context for $\Phi_1$, is in turn a context for $\Phi_1$. As $\Phi_2$ and $\Phi_3$ are taking into account new arguments with respect to $\Phi_1$, the extensions may vary among these frameworks. For a semantic notion $S$, an argument $A$ may be in an extension $X_1$ of $\mathcal{E}_S(\Phi_1)$, but not in any extension of $\mathcal{E}_S(\Phi_2)$. Later on, it is possible for $A$ to be included in an extension of $\mathcal{E}_S(\Phi_3)$, resembling argument reinstatement. The following proposition relates this situation in the particular case of acceptability semantics.

Proposition 3. Let $\Phi_1$ and $\Phi_2$ be two extended argumentation frameworks such that $\Phi_2$ is a context for $\Phi_1$ and let $\mathcal{G}$ be the grounded extension semantics. If $\Phi_2 \mathcal{G}$-revise $\Phi_1$, it is possible to construct a context $\Phi_3$ for $\Phi_2$ such that $\Phi_3 \mathcal{G}$-expand $\Phi_1$.

Proof. If $\Phi_2 \mathcal{G}$-revise $\Phi_1$, then a subset $S \subseteq GE_{\Phi_1}$ is not included in $GE_{\Phi_2}$, due to new defeaters in $AR_2 - AR_1$. Let $S' = \{D_1, D_2, ..., D_n\}$ be the set of these defeaters. The extended framework $\Phi_3 = (AR_3, \sqsubseteq_3, C_3, R_3)$ is constructed as following:

- $AR_3 = AR_2 \cup \{Z_1, Z_2, ..., Z_n\}$ where $Z_i, 1 \leq i \leq n$ is a new argument not appearing in $AR_2$.
- $\sqsubseteq_3 = \emptyset$,
- $C_3 = C_2 \cup \bigcup_i \{D_i, Z_i\}$
- $Z_i \succ D_i$ for all $D_i, Z_i$

As any argument $D_i$ is defeated by a defeater-free argument ($Z_i$), any threat over $S$ introduced by $\Phi_2$ is no longer valid, and then every argument in $S$ is included in a grounded extension again. Also every $Z_i$ is in the grounded extension as they are defeater-free arguments. \qed
4.1 Non-relevant arguments

Some arguments in a context may not be relevant for changes in semantic extensions. Basically, these are arguments such that its inclusion is not a threat for other arguments. Of course, this notion is considered under a particular semantic notion $S$. In the following definition, non-relevant arguments are presented according to the classical grounded extension.

Definition 10.

Let $\Phi_1 = \langle AR_1, \sqsubseteq_1, C_1, R_1 \rangle$ be an extended argumentation framework and let $\Phi_2 = \langle AR_2, \sqsubseteq_2, C_2, R_2 \rangle$ be a context for $\Phi_1$. Let $A$ be an argument in the grounded extension $GE_{\Phi_1}$. A contextual argument $X$ is said to be non-relevant for $A$ if

- $X$ does not directly nor indirectly defeats $A$, or
- whenever $X$ directly or indirectly defeats $A$, it is directly or indirectly defeated by an argument in $GE_{\Phi}$

Non relevant arguments for $A$ are those contextual arguments not being able to avoid the inclusion of $A$ in the grounded extension of the context. This is important in several scenarios. Following the introductory analogy if Justice trials, non-relevant arguments are the main target of lawyers. These arguments may be viewed as a useless argument used by a member of the juror. It is useless because, even when defeating an argument in the case, it is already defeated by an argument in that case. These arguments are important in different ways. For example, a defender lawyer may want to introduce enough arguments to defeat any contextual argument defeating an argument exposed by himself. He is trying to maximize the number of non-relevant contextual arguments in that sense. On the other hand, he also wants to avoid the defeat of juror’s arguments defeating arguments exposed by the District attorney. In this sense, he is trying to minimize the number of non-relevant arguments. Of course, they do not know a priori any of the contextual arguments. All they can do is to produce a set of arguments good enough to face any court.

5 Conclusions

An argumentation framework $\Phi$ is basically the model of a knowledge base based on arguments. These arguments interact each other and then several possible outcomes, as sets of accepted arguments, are obtained. However, it is possible for this outcome to be different when new arguments are taken into account. These new arguments are considered the context of the framework $\Phi$. In this work, we formally defined contexts for extended argumentation frameworks ($EAF$). In general terms, a context for an $EAF$ is another framework where original arguments, conflicts and preferences are kept, while new arguments are introduced, possibly leading to new defeat relations. As new argument interactions are present, the context may apply changes in the original classification of arguments, inducing new set of extensions. These semantic change in the outcome of an extended framework in a particular context was characterized, and Dung’s acceptability concept was analyzed on this basis.
References


