Probabilistic Algorithm for Computed Tomography

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ABSTRACT

A probabilistic algorithm for on-line tomographic reconstruction of ellipse-like images is presented. The algorithm takes advantage of the characteristic preferential direction of the objects, constructing a guidance function to select the angles for subsequent radiographic projections. The simulation results confirm that the technique reduces the number of projections required to achieve a given quality limit.

I. INTRODUCTION

Recent advances in radiation technology have risen new challenges in image processing and recognition. Radiation pulses provide unique characteristics compared with other radiation devices, namely very short flashes (10 nanoseconds) of high intense beams. This feature opens interesting possibilities in industry and medicine. Very short radiation pulses have been proposed in recent years for ultra-fast tomographic scannings to obtain fast cross-sectional information (Casali et al, 1995). Three-dimensional images of the internal structure of key components or critical parts in production lines have an enormous potential from the point of view of quality control. One can imagine a step in a production line where the quality of certain critical components is automatically monitored by means of tomographic identification of material defects. However, this step will introduce some time cost, which should be minimized if the technique is to be applied in real production lines.

Essentially, a computed tomography is a tridimensional image of an object constructed from a certain number of photographs of the attenuated radiation passing through the object at different angles. In order to construct a perfect tomography, infinite projections are required. However, certain images can be reconstructed from a finite number of projections (although with some distortion). The present work is oriented to develop an image processing system, which takes advantage of radiation flashes, optimizing the emission-detection-reconstruction procedure. An optimization technique based in a probabilistic algorithm for the assessment of the best projection-angles is presented. The proposed algorithm is applied to design a strategy for an interactive on-line scanning of elliptical objects from limited projection data.

II. ANGLE SPACING DISTRIBUTION

Let us consider the ellipse shown in Fig.1, representing a cut of an object with certain attenuation properties. A radiation beam of intensity I_o passing through at a given angle θ produces a projection profile I on a screen located behind the object. This profile is related to the local attenuation distribution $f(x,y)$ by:

$$g(s,\theta) = ln\left(\frac{I_0}{I}\right) = \boldsymbol{R}\, f(x,y) \qquad (1)$$

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where $\boldsymbol{R}$ is the Radon transform. Considering the image digitalized in pixels *(i,j)*, and projected through by *N* parallel beams at *M* different angles, Eq. (1) is written as:

$$g_{mn} = R_{mnij} f_{ij} \tag{2}$$

Given a set of projection profiles g_{mn} , the original image f_{ij} can be reconstructed solving Eq. (2) by numerical methods. This is known as the algebraic reconstruction technique (Herman 1980), and it was the procedure followed whenever a reconstruction is required along the present paper.

Usually, the *M* projection angles are distributed uniformly in the interval $(0,\pi)$. However, this is not always the best strategy. Figure 2 compares the reconstructions of an ellipse (a) using four projections, equally spaced (b) and with an appropriate selection of the perspectives (c). It can be seen that substantial quality differences can be achieved with a careful selection of the view angles, and is reasonable to take this into account for the design of an intelligent control of the angles the object will be rayed through.

The particular characteristic of the angle distribution of Fig. 2c is that most of the angles accumulate close to the direction of the main axis of the ellipse. In this example the ellipse is known prior to the reconstruction. In a real tomographic process the position and orientation of the object is not known *a priori*, and consequently a suitable algorithm should be provided to estimate these parameters.

Let us consider a digital representation, e_{ij} , of an ellipse in a square domain:

$$e_{ij} = \begin{cases} 1 & inside\, the\, ellipse \\ 0 & otherwise \end{cases}$$

The position of the center of the ellipse can be calculated determining the position of the *center of mass*, $(x_c\ ,y_c\ )$ of the image:

$$(\boldsymbol{x_c}, \boldsymbol{y_c}) = \frac{\sum\limits_{pixels} (\boldsymbol{x_{ij}}, \boldsymbol{y_{ij}})\boldsymbol{e_{ij}}}{\sum\limits_{pixels} \boldsymbol{e_{ij}}} \tag{3}$$

A useful indicator of the projection angles that provide information about the ellipse orientation is the standard deviation of the image respect to a line passing through $(x_c\ ,y_c\ )$, that is:

$$\sigma(\theta) = \sqrt{\frac{\sum\limits_{pixels} e_{ij} d_{ij}^2(\theta)}{N_{pixels}}} \tag{4}$$

where $d_{ij}(\theta)$ represents the distance of the pixel (i,j) from a line with an orientation θ. Figure 3 shows the standard deviation of the ellipse shown in Fig. 2a. The minimum σ corresponds to the direction of the main axis. Since better reconstructions can be achieved accumulating more views about the main axis, the function $\sigma(\theta)$ is useful to construct an estimate of the optimum angle distribution, that is:

$$p(\theta) = \frac{e^{-\sigma(\theta)}}{\int_0^\pi e^{-\sigma(\theta')} d\theta'} \qquad (5)$$

Equation (5) can be seen as the energy distribution of a collection of thermodynamic states, where $e^{-\sigma(\theta)}$ plays the role of a Boltzmann factor, and $\sigma(\theta)$ corresponds to an energy to be minimized (REF Chandler). Figure 3 shows the distribution, $p(\theta)$.

III. INTERACTIVE TOMOGRAPHY

In real tomographies the proposed guidance angle distribution, $p(\theta)$, cannot be used, for the actual object is unknown. Nevertheless, it is still possible to construct successive approaches of $p(\theta)$ using partial reconstructions, which can be applied to determine successive best choices. However, the estimates of $p(\theta)$ should be complemented by a filter to prevent the occurrence of redundant angles that add little information to the reconstruction. For simplicity, the *"I-already-got-that"* filter is incorporated into the guidance function as a modulation factor that creates dark zones around the angles previously visited. Therefore, calculating the $\sigma_n(\theta)$ from the partial reconstruction using n projections, the guidance function for the choice of the next angle is:

$$p_{n+1}(\theta) = ke^{-\beta\sigma_n(\theta)} \prod_{i=1}^{n} \frac{|\theta - \theta_i|}{5^o + |\theta - \theta_i|}$$

where θ_i is the angle projection in the step-i, and k is an appropriate normalization coefficient.

The complete algorithm is summarized in the following set of rules:

1) Start with two projections.
2) Reconstruct a partial image, $e_{ij}(n)$.
3) Calculate an estimate of $\sigma_n(\theta)$ applying Eq. (4) to the previous partial reconstruction.
4) Calculate the guidance function, $p_n(\theta)$ applying Eq. (5).
5) Generate a new projection angle randomly with probability distribution $p_n(\theta)$.
6) Repeat rules (2) to (5) while the quality enhancement between $e_{ij}(n)$ and $e_{ij}(n-1)$, exceeds certain convergence criterion.

III. RESULTS AND DISCUSSION

The probabilistic algorithm was implemented in C++ and applied to an ellipse with aspect ratio ε = 0.5. Figures 4 show the evolution of $p_n(\theta)$ and $\sigma_n(\theta)$ during the reconstruction sequence of the ellipse shown in Fig. 2a. In the first step (Fig. 4a), the shadow resulting from the partial reconstruction using two projections is used to produce a preliminary estimate of the orientation and to assess where should be rayed the next view —*i.e.* angles with larger $p_1(\theta)$. Figure 4b shows the third step in the reconstruction. Two new angles where already taken (83^o and 105^o), and consequently the filter algorithm reduces the probability in the corresponding neighbourhoods. Finally, in the seventh step, the quality of the image is highly improved, and the maxima of the guidance function, $p_7(\theta)$, indicate the views that would provide more information in future shots.

The proposed algorithm was compared against tomographies using equally spaced angles in order to assess its efficiency. The quality of the reconstruction can be measured using a distortion indicator defined by:

$$d = \sqrt{\sum_{ij}\left(e_{ij} - e'_{ij}\right)^2} \qquad (6)$$

where e'_{ij} is the reconstruction of the original pixel e_{ij}.

The performance is measured by counting the number of projections needed to reduce the distortion below a given value. Ultimately, this metric will represent savings in irradiation doses or monitoring costs, depending on the particular application of the tomography. Figure 5 compares the performance of the present algorithm with uniform spacing, for the ellipse shown in Fig. 2a. It can be seen that the same distortion limit can be achieved more efficiently using the proposed algorithm. Figure 6 compares two reconstructions from seven projections using uniform spacing and the probabilistic algorithm The difference in quality is quite evident.

IV. CONCLUSIONS

A probabilistic algorithm has been applied to the design of an adaptive procedure for on-line tomographic reconstructions of ellipse-like images. The algorithm takes advantage of the characteristic preferential direction of the objects, constructing a probability distribution to guide the selection of the angles for subsequent radiographic projections. The simulation results confirm that the technique allows the reduction of the number of projections required to achieve a given quality reconstruction limit. Furthermore, the conceptual procedure can be extended to more general objects involving appropriate guidance functions, covering families of objects sharing similar shapes.

REFERENCES

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Casali, P. Chirco, M. Zanarini, *Advanced Imaging Techniques: a new deal for neutron physics*, Nuovo Cimento, V. 18, N^0 10, 1995.

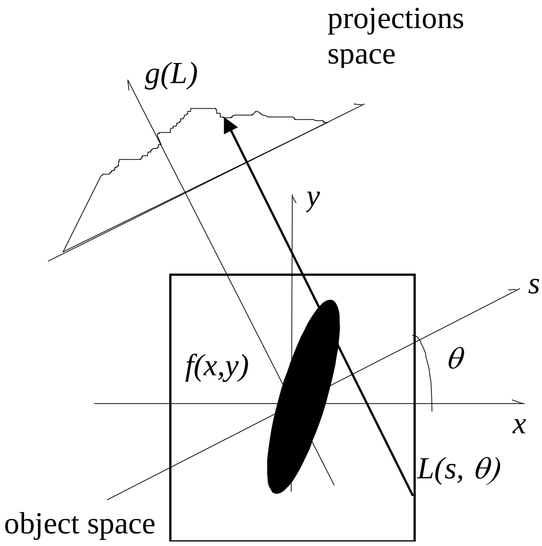
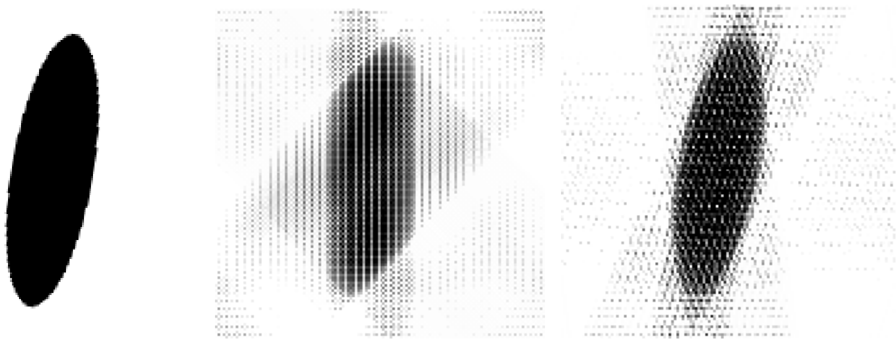


Figure 1. Image space *f(x,y)* and projection space *g(s, θ)*.



(a) *(b)* *(c)*

Figure 2. Reconstruction of an ellipse using 4 projections. (a) original image, (b) uniform spacing (0^0,45^0,90^0,135^0), (c) non-uniform spacing (0^0,63^0, 80^0,98^0).

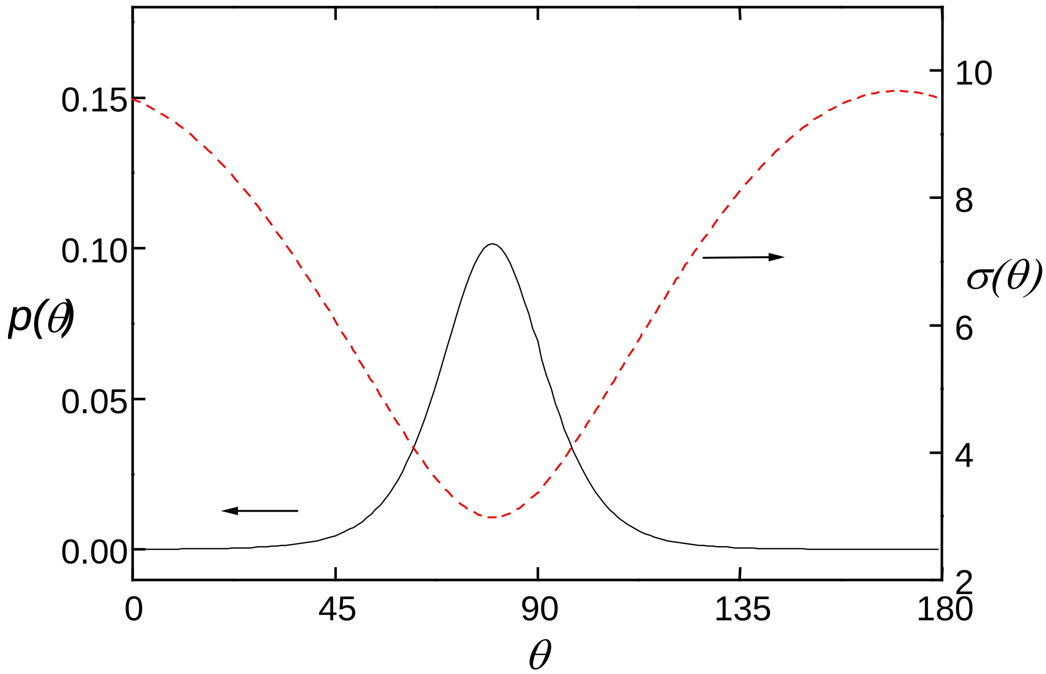


Figure 3. Deviation *σ(θ)* and optimum angle distribution *p(θ)* of image 2a.

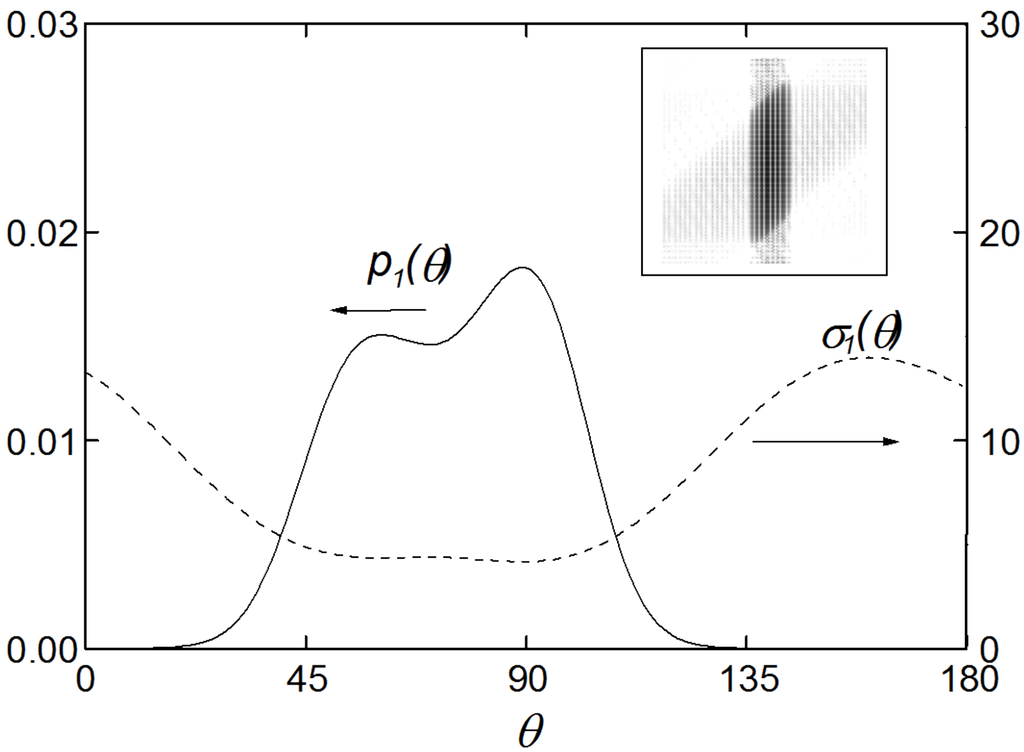


Figure 4a. Partial reconstruction (step 1)

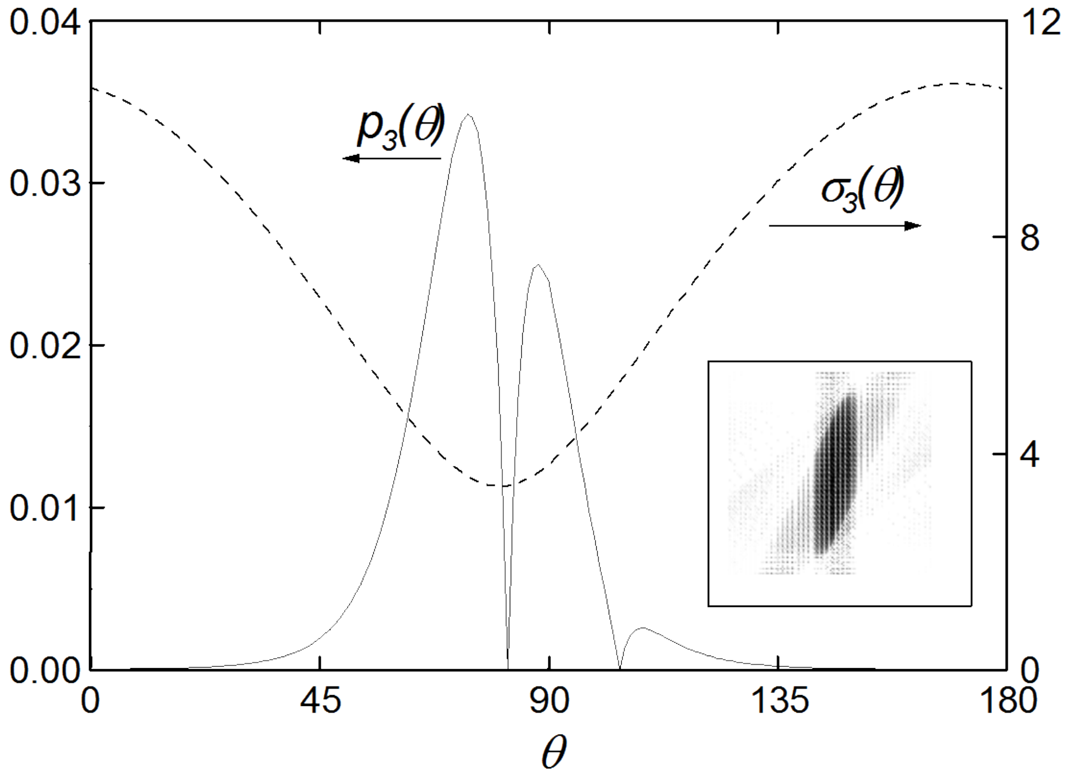


Figure 4b. Partial reconstruction (step 3)

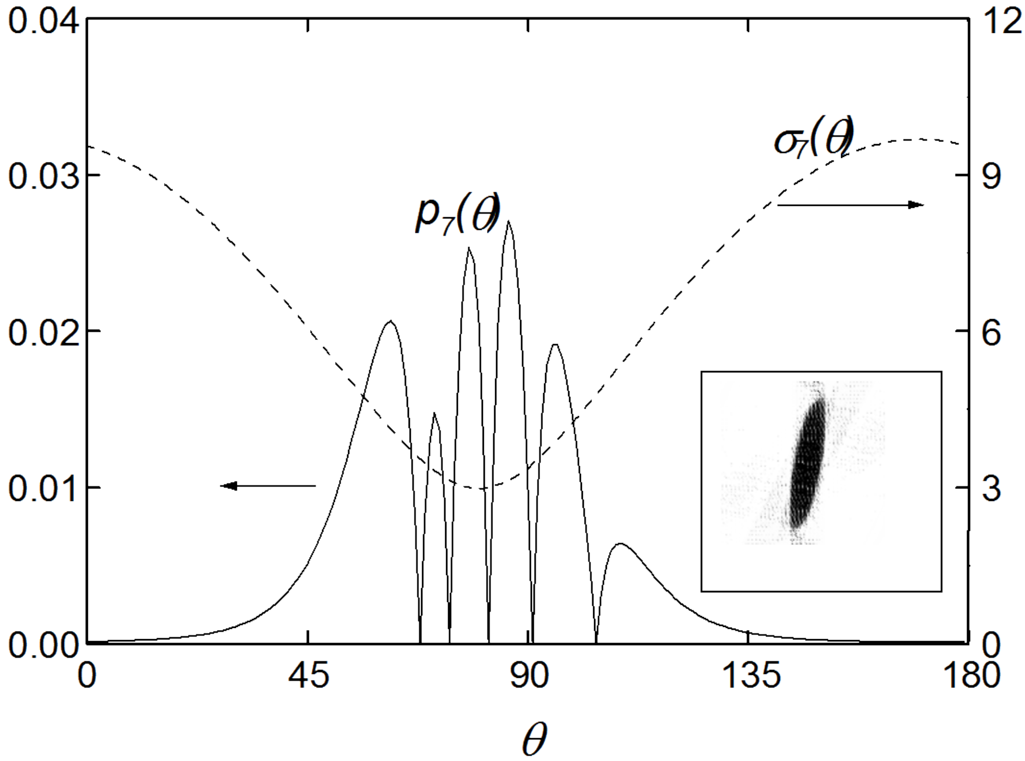


Figure 4c. Partial reconstruction (step 7)

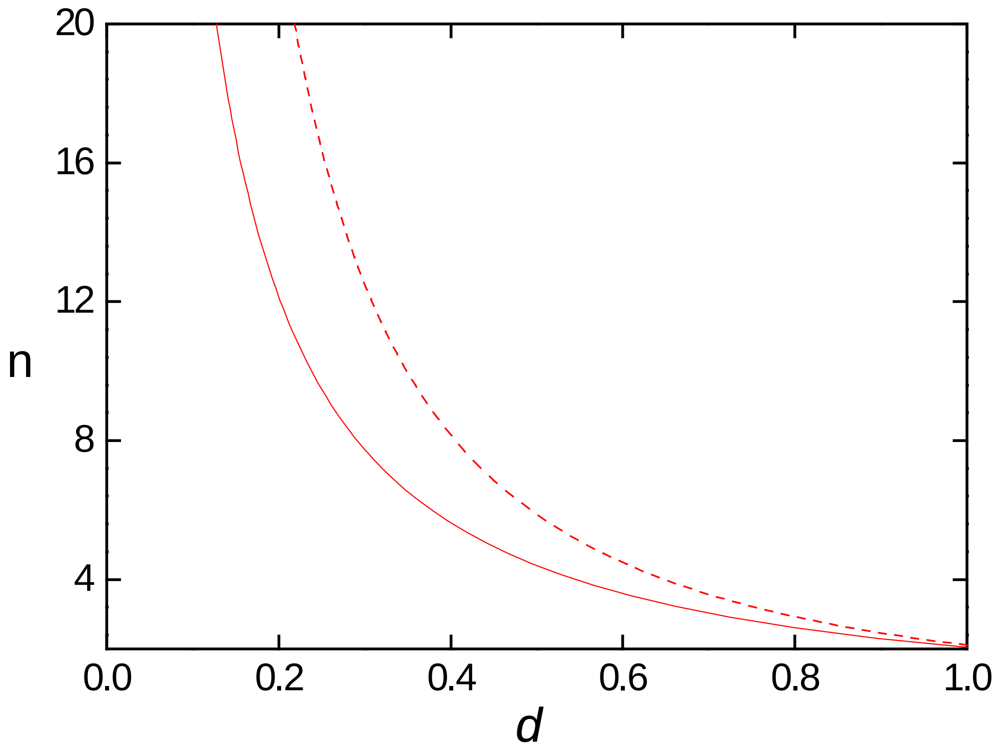


Figure 5. Number of projections required to achieve a given distortion. Uniform spacing (dashed line), probabilistic algorithm (solid line).

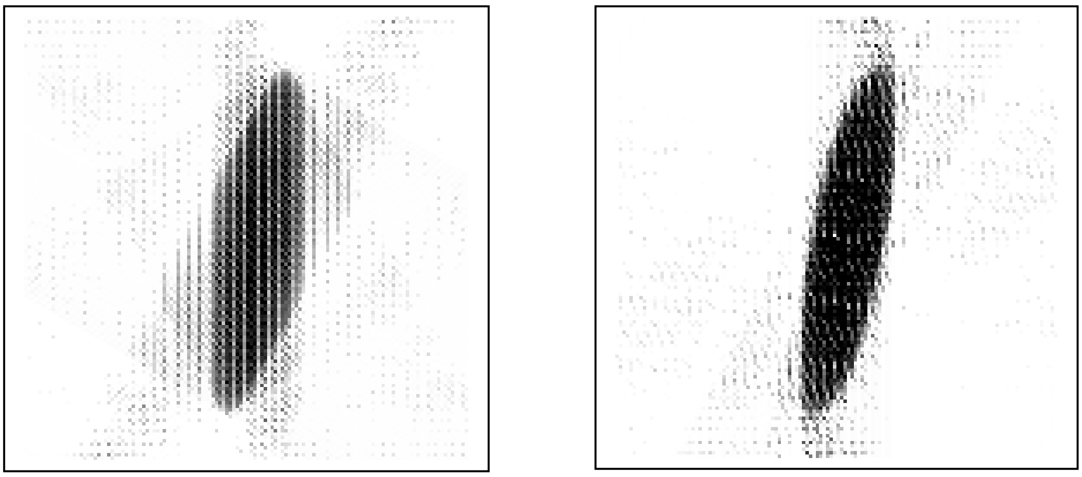


Figure 6. Reconstructions with seven projections. Uniform spacing (left), probabilistic algorithm (right)