

# Techniques to determine the magnitude and direction error of GPS system

Juan Toloza<sup>1,3</sup>, Nelson Acosta<sup>1</sup> and Armando De Giusti<sup>2,4</sup>

<sup>1</sup>Facultad de Ciencias Exactas  
Universidad Nacional del Centro de la Provincia de Buenos Aires

<sup>2</sup>Facultad de Informática  
Universidad Nacional de La Plata

<sup>3</sup>Scholarship ANPCyT

<sup>4</sup>CONICET

**Abstract.** The Global Positioning System (GPS) allows locating an object in any part of the World with a certain degree of accuracy. Some precision activities need to operate with a sub-metric accuracy [1]. In this article, techniques are introduced to determine the magnitude and direction error of GPS system. With this error vector it is possible to correct any low cost standard GPS receiver to improve the positional accuracy.

**Keywords:** GPS accuracy, relative positioning, DGPS, error correction GPS.

## 1 Introduction

The functioning principle of the Global Positioning System (GPS) is based on measuring ranges of distances between the receiver and the satellites [2] [3]. The GPS has an architecture that is divided into three segments: spatial, control and user's [4]. The first is composed of 24 satellites over 20 thousand kms away from the Earth, divided into 6 orbital levels and each satellite period is of about 12 hours. The second segment is composed of Earth stations distributed around the planet, controlling that satellites do not deviate from their trajectory. Finally, GPS receivers are in the user's segment, which in general use two frequencies: L1 at 1575,42 Mhz and L2 at 1227,60 Mhz [5].

The accuracy in longitude and latitude coordinates is of 10-15 meters 95% of the readings [6]. Sometimes, it is more precise, but it depends on a variety of factors that include from the deviation or the delay of the signal when cross the atmosphere, the bouncing of the signal in buildings or its concealment due to the presence of trees, low accuracy of clocks and noise in the receiver. In altitude the accuracy is reduced to 50% regarding the obtained in terms of longitude and latitude [7]. Systems that enhance positional accuracy are: the DGPS (Differential GPS), AGPS (Assisted GPS), RTK (Real-Time Kinematic), e-Dif (extended Differential), amongst others.

- a. In the DGPS system it must be paid an amount for the service and for corrections be valid, receiver must be relatively near a DGPS station;

generally less than 1,000 km. The achieved accuracy can be of a few meters [7-10]. The correction signal can not be received if it is a mountainous zone.

- b.* In the case of AGPS it is necessary to have mobile devices with active data connection or cell phone like GPRS, Ethernet or WiFi [11]. It is used in the cases where there is a weak signal due to a surrounding of buildings or trees; this implies having a not much precise position. Standard GPS receivers, in order to triangulate and position, need a certain time of cold start [12][13].
- c.* For use the RTK system, it is paid for the service and, besides, it is very expensive to acquire the infrastructure. This is a technique used in topography, marine navigation and in agricultural automatic guidance in the use of measurements of signals carrying navigators with GPS, GLONASS (Globalnaya Navigatsionnaya Sputnikovaya Sistema) and/or Galileo's signals, where only one reference station provides correction in real time, obtaining a sub-metric accuracy [14].
- d.* The last case, e-Dif system, is autonomous and it process files with RINEX (Receiver Independent Exchange) format, which was created to unify data of different receivers manufacturers [15]. It generates autonomous corrections regarding a coordinate of arbitrary reference and it extrapolates them in time [16]. It is a very consistent relative positioning and its accuracy is of about 1 meter. The system's objective is to study waste from the initializing process to isolate the most important systematic errors that introduce the corresponding equations to each satellite. It is applicable for a reduced time of 40 minutes approximately; since later the systematic error changes, in this case a new error must be calculated again. In regions where differential corrections aren't available and it is paid for the service, like in South American, African and Australian, this system become more interesting.
- e.* Besides, there exist raise systems that increase accuracy to sub-metric levels. Those based on satellites SBAS (Satellite-Based Augmentation System), based on ground GBAS (Ground-Based Augmentation System) and based on aircraft ABAS (Aircraft-Based Augmentation System) [1]. Most of these implementations are used in different applications and some of them are available for users without special permissions. Even then, costs are high due to the need of certain devices with special characteristics or some infrastructure in agreement with the accuracy level aimed at.

Errors produced by the GPS system affect in the same way the receivers located near each other in a limited radius [1]. This implies that errors are strongly correlated among near receivers. Thus, if the error produced in one receiver is known, it can be spread towards the rest in order to make them correct their position. This principle is only applicable to receivers that are exactly the same; since, if different, their specifications change so the signal processed by one individual is not the same to that processed by another one. All GPS differential methods use the same concept [17]. DGPS requires a base station with a GPS receiver in a precise known position. The base compares its known position with that calculated by the satellite signal. The estimated difference in the base is applicable then to the mobile GPS receiver as a differential correction with the premise that any two receivers relatively near experiment similar errors [6].

In this article it is carried out a detailed analysis of the way to calculate direction and magnitude error of a standard GPS in order to build a DGPS system. In Section 2 techniques to calculate errors are analyzed. In Section 3 algorithms and techniques used are described. In Section 4, the last one, conclusions and future work are presented. Acknowledgment is mentioned at the end of document, before references.

## 2 Techniques for error analysis

The experiment carried out is based on the principle of the adopted methodology by the DGPS but with a low cost standard GPS receiver. In order to get measurements, three Garmin 18X USB GPS receivers are used connected to two notebooks. The base station is composed of a notebook and two of the three GPS receivers; the mobile for the notebook and the remaining GPS receiver. The link between the base station and the mobile one is a wireless connection from point to point link.

In this context, in the base system measurements from the GPS receivers are obtained and after a certain period of time, which is necessary for the system stabilization, two positions are estimated. The positions' estimation is carried out with a Kalman filter, since an estimation problem with so many noisy redundant data is a natural application for the Kalman filter; this allows using some of the redundant information to remove the effects from the error sources. The Kalman filter is used to eliminate the white Gaussian noise [18]. Receivers are placed at a known distance (4.53 meters) between themselves (relative positioning). At the end of this stage, a cloud of points is obtained with the positions delivered by the receivers. In Fig. 1 it is observed the cloud of points from both receivers as well as each of the estimated points with different shape and color that correspond to the GPS 1 and GPS 2 receivers. The estimated point from GPS 1 is selected as anchor point of the whole experiment. From this, all necessary calculations are carried out with the objective of finding the GPS system error.

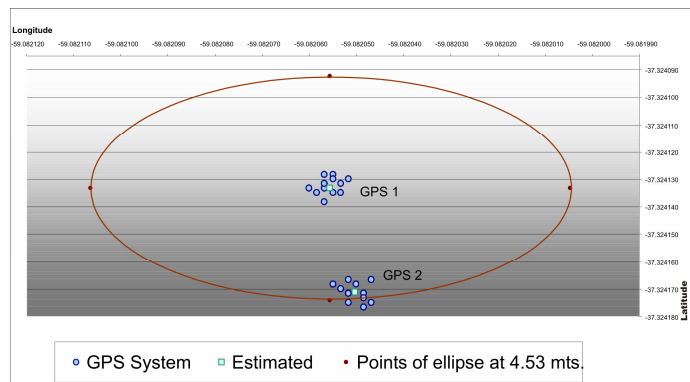


Figure 1. Estimated points for GPS 1 and GPS 2.

With both estimated positions, it proceeds to calculate the distance between them. If the estimated distance is different from the actual one (more/less a threshold) it is

detected that there is a positioning error. In the case of Fig. 2, the estimated distance is lesser than the actual one. The actual distance is represented by the green point at the intersection of the straight line and the circumference (Intersection point 1).

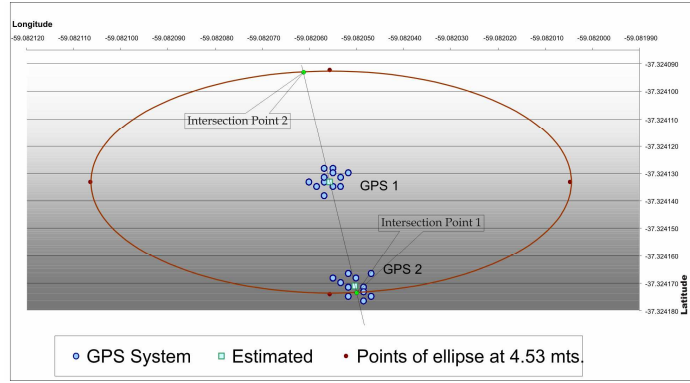


Figure 2. Intersection of straight line and circumference.

Besides, Fig. 2 shows a circumference with a radius equal to the actual distance measured with a tape measure from GPS 1 to GPS 2 with center in the estimated point for the GPS 1. A circumference is chosen, as GPS 2 can be at that distance but in any point of the circumference (it is observed a shape of ellipse because of a latitude grade has a different measurement in meters that has a grade in longitude).

This is the working principle that the GPS system uses to get the receiver's position [1].

After calculating the distance of the estimated points and contrasting it to the actual one, a positioning error is deduced. Once it is known that there is an error, its compute its magnitude and direction. On the one hand, the two estimated points are learnt with which the straight line is drawn and which bonds them. Equation 1 belongs to the straight line that crosses these two points

$$\frac{(y - y1)}{(y2 - y1)} = \frac{(x - x1)}{(x2 - x1)}. \quad (1)$$

where the  $x$  represents the component of Longitude and the  $y$  that of Latitude.  $x1$ ,  $y1$ ,  $x2$ , and  $y2$  are the components of point 1 and 2 respectively.

On the other hand, it is known that the GPS 2 is in some point of the circumference with center in the GPS 1 and of radius the distance that is defined at the moment of positioning the two receivers. Equation 2 belongs to the circumference with center in  $(x1, y1)$  of radius  $r$ .

$$(x - x1)^2 + (y - y1)^2 = r^2. \quad (2)$$

As it was mentioned before, the GPS 2 receiver must be in some point of the circumference with center in the estimated point for GPS 1 and with the same

distance with which GPS 2 receiver was placed. In the case presented, it is observed in Fig. 2 that the estimated point for GPS 2 is nearer than the actual distance at which the receiver is placed on ground. Knowing about the equations that define the straight line crossing both estimated points and the radius circumference equation equal to the GPS 1-GPS 2 distance with center in GPS 1, it is proceeded to approximate, by means of the intersection of the straight line and the circumference. This intersection is presented as a polynomial of second degree. It is mathematically solved and a, b and c coefficients are obtained (equations 4, 5 and 6). Equation 3 only presents an auxiliary estimate in order not to repeat it in the other operations and to make the rest of the equations more legible. With these coefficients cleared by means of Bascara (equation 7) the roots are found (two because of being of second degree) from the polynomial. From the two roots found, one is chosen and the intersection points are calculated.

$$divisor = y_2^2 - 2y_2y_1 + y_1^2. \quad (3)$$

$$a = 1 + \frac{(x_2^2 - 2x_2x_1 + x_1^2)}{divisor}. \quad (4)$$

$$b = -2y_1 + \frac{(-2x_2^2y_1 + 4x_2x_1y_1 - 2x_1^2y_1)}{divisor}. \quad (5)$$

$$c = y_1^2 - r^2 + \frac{(x_2^2y_1^2 - 2x_2x_1y_1^2 + x_1^2y_1^2)}{divisor}. \quad (6)$$

$$x = \frac{-b \pm \sqrt{(b^2 - 4ac)}}{2a}. \quad (7)$$

It is worth mentioning that in order to minimize the estimates and be able to normalize, everything is transferred to a same unit (meters) and it is displaced to (0,0) in order to later obtain the difference between estimated points 1 and 2.

After having obtained the two roots, only one is taken into account. The nearest to the estimated point of GPS 2 in some of its components is chosen, in this case in Latitude, since the other one is meaningless due to being too far away (on the other side of the circumference).

$$result_{latitude} = rootCloser(y_2, roots.root_1, roots.root_2);$$

Finally, the other component (Longitude) is obtained depending on the Latitude found in the previous point as seen in equation 8.

$$result_{longitude} = x_1 + \left( (x_2 - x_1) * \frac{(result_{latitude} - y_1)}{y_2 - y_1} \right). \quad (8)$$

The corrected point (green) is already there, as shown in Fig. 3, being this exactly in the intersection of the straight line and the circumference, as it has been planned.

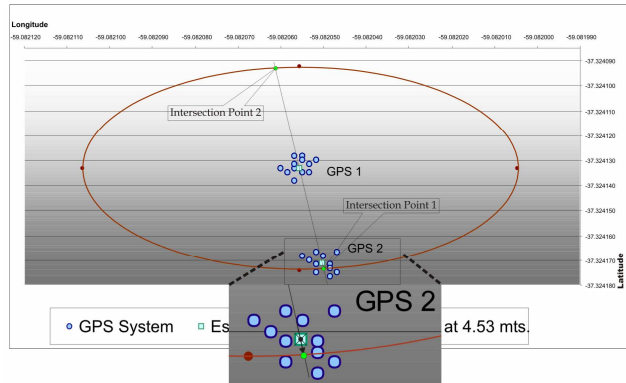


Figure 3. Corrected point (intersection point 1).

With these operations the error magnitude is known but it is also necessary to get the direction of the found error. Error direction can be given in any of the four senses and their combination. Four base cases are detected from which other four are derived like is observed in Table I:

TABLE I. DETERMINATION OF ERROR DIRECTION.

estimated	
latitude	longitude
greater	greater
greater	lesser
greater	equal
lesser	greater
lesser	lesser
lesser	equal
equal	greater
equal	lesser
equal	equal
latitude	longitude
actual	

The way to read the table is top-down. For example, in the second case, estimated latitude is greater than actual latitude, but estimated longitude is lesser than actual. In the case where actual and estimated components are equal, like in ninth row of the Table I, there is no correction.

Therefore, in order to get the error direction it is proceeded to verify which side of the corrected point the estimated point is. If the straight line from estimated point 1 to 2 is horizontal or vertical, some of the components are null, in the horizontal case,

Latitude is eliminated and in the vertical the Longitude; that is, there are no corrections in these components since they are not required.

Finally, in Fig. 4 it is observed the resulting vector for the study case introduced.

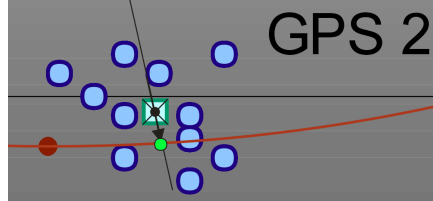


Figure 4. Resulting vector.

### 3 Algorithms and used techniques

For the analysis of data, combinations of different techniques and algorithms are used in order to find a better result. In a first processing stage, applied mathematics covers:

- Static Kalman: is a set of mathematic equations that provide an efficient recursive solution of the method of least squares. This solution allows calculating a linear, unbiased and optimum estimator of the state of a process in each moment of time ( $t$ ) with base on the available information at the moment  $t-1$ , and update, with the available information at the moment  $t$ , the estimator value.
- Dynamic Kalman: is that system in which the  $x$  variable value to be estimated has a value that changes throughout the time ( $x_{i+1} \neq x_i$ ), but these states have some known relationship with the instant  $i$  and  $i+1$ . For example, if an object position is measured, it can be predicted that the position will be:

$$x_{i+1} = x_i + \Delta t * v_i$$

where  $\Delta t$  is the passed time and  $v_i$  the speed at instant  $i$ . Position can be obtained by a GPS, for instance, and speed with an additional measurement element such as the accelerometer.

- Kalman with adjustment of error standard deviation ( $\sigma_R$ ): the deviation is modified and checked in order to see which adjust better. This measure is calculated as the square root of variance, which is at the same time the sum of the squares of each error (Table II) as shown in equation 9. It is worth mentioning that from Table II the only error that is not taken into account is that of signal P(Y) arrival; since work is carried out without the precision code.

$$\sigma_R = \sqrt{3^2 + 5^2 + 2.5^2 + 2^2 + 1^2 + 0.5^2} m = 6,7 m . \quad (9)$$

TABLE II. GPS ERROR SOURCE.

Source	Effect
Arrival of signal C/A	± 3 m.
Arrival of signal P(Y)	± 0.3 m.
Ionosphere	± 5 m.
Ephemeris	± 2,5 m.
Satellite clock error	± 2 m.
Multipath	± 1 m.
Troposphere	± 0.5 m.
Numerical errors	± 1 m.

Now, error standard deviation ( $\sigma_{rc}$ ) in the receiver's position is estimated, but having into account additionally the PDOP (Position Dilution of Precision) and the numerical error; therefore, the PDOP is added to the calculated deviation from typical errors, since for each measurement taken, this varies according to the instant geometry of satellites. The result of standard deviation used for the Kalman filter is equation 10.

$$\sigma_{rc} = \sqrt{PDOP^2 * \sigma_R^2 + \sigma_{num}^2} = \sqrt{PDOP^2 * 6.7^2 + 1^2} m. \quad (10)$$

From this comes the fact of applying Kalman with adjustment of standard deviation, since it fluctuates for each piece of information coming from the receivers in each moment as geometry of satellites varies.

- Points average: one of the media limitations is that it is affected by extreme values, very high values tend to increase it while very low values tend to reduce it, this implies that it may stop being representative of the population. It is analyzed but not implemented in the solution.
- Fuzzy logic: it is used to determine the error degree that a position has. Rules that determine the error a position presents are related to analyzing some parameters like: PDOP, SNR (Signal-to-Noise Ratio) and difference of tracked satellites. The fuzzy system output weights the Kalman filter gain, providing more weight to more precise positions and the other way round.
- Filters allow discarding measurements with much noise or error that influence over the final result of an estimation of a position. Thus, measurements having many errors do not slant the final estimation towards a position far away from the actual one. The application of these filters can be made as measurements are not very far away in time and it is supposed that the Vehicle in which the mobile receiver is placed does not move at high speed; this implies that values do not change radically. High/low step filters are used in an analogical way to the electronic filter.

Since the tool is thought to operate in different places of heterogeneous characteristics, relations and configurations are used in order to be able to customize the use according to needs. Relations and configurations used are the following:

- Degree/Meters Relation in Latitude: given the asymmetry, in different places on Earth, the distance that a Latitude degree measures varies.
- Degree/Meters Relation in Longitude: ditto to Latitude.



- Cold Start Time: a start time is considered so the system can be stabilized. In this time, samples of the device are taken and, only at its end, estimation is carried out. The objective is to reduce or soften systematic and random errors of the GPS system.
- Receivers' distance threshold: it can be determined, in an accurate way, the whole interval of the distance measurement between the base devices. As positioning is relative and its distance is known, it can be added a  $\pm$  value, since there exists a possibility that an element of distance measurement be not accurate enough. Besides, it reduces the computational load because of not having to process data if distance is within the allowed threshold.

## 4 Conclusion and future works

The technique developed allows to obtain magnitude and direction error of the GPS system. This correction is used by another receiver to correct its own position and thus increase the positional accuracy with the aim of measuring the most precise distances.

The experiments carried out with different sets of data provide positions that are used to measure distances and error fluctuates in  $\pm 1$  meter the 95% of measurements and in some cases in  $\pm 0.20$  meters. In addition, with the techniques used it is possible to prevent peaks of errors measures over 15 meters.

The principle is based in mathematical, geometrical functions and filters.

As future work, the aim is to increase the accuracy until reaching a maximum error of positioning of  $\pm 0.10$  meters. This increase can include the use of another additional signal. Besides, further measurements will be carried out in order to analyze data and to deduce its behavior. Additional techniques like bayesian networks, neuronal networks, genetic algorithms and clustering will be analyzed in order to improve the accuracy obtained until the moment.

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