

MULTIPLE CROSSOVER PER COUPLE AND FITNESS PROPORTIONAL COUPLE SELECTION IN GENETIC ALGORITHMS

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Abstract

Contrasting with conventional approaches to crossover, Multiple Crossover Per Couple (MCPC) is an alternative, recently proposed [1], approach under which more than one crossover operation for each mating pair is allowed. In genetic algorithms, *Proportional Selection* (PS) is a popular method to select individuals for mating based on their fitness values.

The Fitness Proportional Couple Selection (FPCS) approach, is a new selection method which creates an intermediate population of couples from where, subsequently, couples are selected for crossing-over based on couple fitness.

This paper proposes the combined use of MCPC and FPCS. Outstanding performance was achieved by joining both methods when optimising hard testing multimodal and unimodal functions. Some of these results and their comparison against results from conventional approaches are shown.

Keywords: genetic algorithms, selection, crossover, function optimisation.

¹The Research Group is supported by the Universidad Nacional de San Luis and the CONICET (Science and Technology Research Council).

1. INTRODUCTION

Crossover usefulness has been extensively discussed by a number of researchers (e.g.; [3], [4], [5], [6]). The conventional approach to crossover, involves to apply the operator only once on the selected pair of parents to create, precisely, two children ([7], [8], [9], [10]). From now on we call such procedure, the Single Crossover Per Couple (SCPC) approach. In order to deeply explore the recombination possibilities of previously found solutions, in previous research we conducted several experiments in which more than one crossover operation for each mating pair was allowed.

MCPC was applied to optimize classic testing functions and harder (nonlinear, nonseparable) functions [13]. The goodness of this approach prevailed under all tests and revealed that for both sets of functions, when MCPC is applied with 2, 3 and 4 crossovers per couple, results as good as under SCPC can be expected with an additional benefit in processing time. This was obtained through the ability showed by MCPC of exploiting the recombination of good, formerly found, solutions. Those experiments, reported elsewhere [2], also showed that due to the inherent increase in selection pressure introduced by the method, a risk of premature convergence appeared reaching near optimal (suboptimal) solutions.

In order to overcome this problem we devised an strategy to combine the exploitation introduced by MCPC with the exploration of new solutions in the problem space. The latter ability was achieved by introducing a new selection operator; the Fitness Proportional Couple Selection (FPCS).

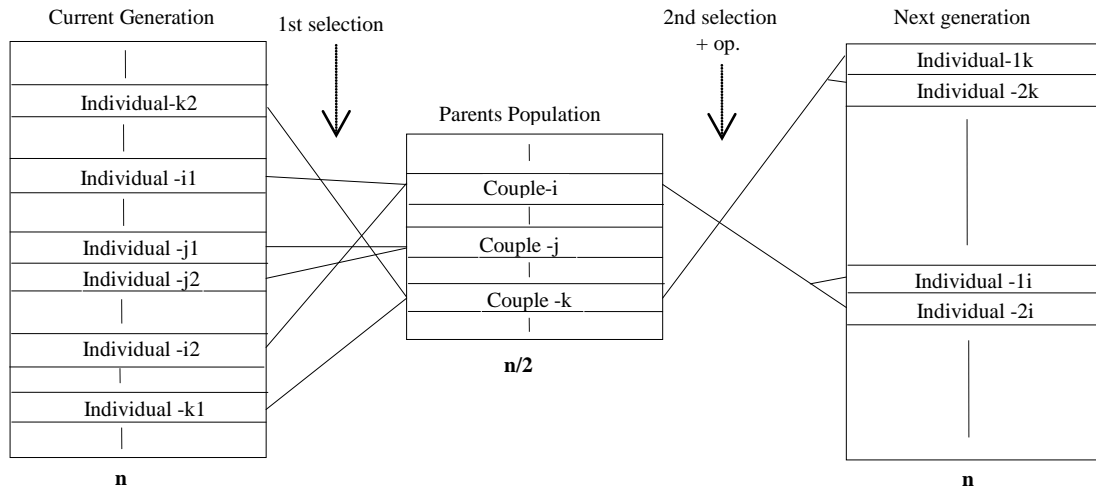
FPCS, divides the selection process into two steps; the first at individual level and the second at couple level. The method builds an intermediate population of parents which are individually selected from the old population by proportional selection, then the couples are selected according to the couple fitness. In this paper we show the results when greater fitness is associated to those couples exhibiting a greater fitness difference between their constituents. In this way, to benefit exploration, individuals likely to be distant have a greater chance to create more children than those that are residing in the neighbourhood.

The following sections describe the new selection method, the experiments performed on a set of diverse multimodal and unimodal testing functions and, finally, a comparative analysis of results is shown.

2. THE FITNESS PROPORTIONAL COUPLE SELECTION METHOD

The basic idea of FPCS is to create an intermediate population of couples, from the current population of individuals, and subsequently select for mating those pairs showing higher fitness. The criteria to assign the fitness of a couple was based on their dissimilarity. The method can be sketched as follows;

- Initially a number of individuals are selected by proportional selection to build a population of parents.
- A couple fitness value, computed as the absolute value of the difference of fitness between their components, is assigned to each mating pair.
- Couples are selected for reproduction by proportional selection (according to couple fitness). The process of producing offspring is controlled, for each mating pair, in order not to exceed the population size.



1st Selection: proportional selection to the individual fitness.

2nd selection + op: proportional selection to the couple fitness plus classic genetic operators.

Couple fitness = $|mate_1.fitness - mate_2.fitness|$.

n: population size.

Fig. 1. Couple Selection Process Scheme

Figure 1 shows the general FPCS scheme. As an example, suppose the following scenario; the j^{th} couple is made by two individuals of high comparable fitness and the i^{th} couple is formed by a medium and a low fitness individual, then there is a greater chance for the i^{th} couple to be selected. This criterion attempts to maintain the genetic diversity to avoid stagnation but assuming a lower convergence rate.

3. TEST FUNCTIONS

To analyse the behaviour of the new approach and following the Whitley et al. proposal [13], we selected a set compounded by nonlinear, unimodal, multimodal and nonseparable testing functions [11], [12], [14], [15]. They were:

- **f₁:** Griewangk's Function F8

$$f_1(x_i) = 1 + \sum_{i=1}^5 \frac{x_i^2}{4000} - \prod_{i=1}^5 \left(\cos\left(\frac{x_i}{\sqrt{i}}\right) \right),$$

$$x_i = -600:600, i = 1:5;$$

minimum global value: 0.0

- **f₂:** Griewangk's Function F8 modified

$$f_2(x_i) = \sum_{k=1}^3 \sum_{j=k+1}^4 1 + \left(\frac{1}{2}(x_k + x_j)\right)^2 - \cos\left(\frac{1}{2}(x_k + x_j)\right)^2,$$

$$x_i = -600:600, i = 1:4;$$

minimum global value: 0.0

- **f₃:** De Jong's Function 1 modified

$$f_3(x_i) = \sum_{k=1}^3 \sum_{j=k+1}^4 \left(\frac{1}{2}(x_k + x_j)\right)^2,$$

$$x_i = -5.12:5.12, i = 1:3;$$

minimum global value: 0.0

- **f₄:** Branins's Rcos Function

$$f_4(x_1, x_2) = \left(x_2 - \frac{5.1}{4 \cdot \pi^2} \cdot x_1^2 + \frac{5}{\pi} \cdot x_1 - 6 \right)^2 + 10 \cdot \left(1 - \frac{1}{8 \cdot \pi} \right) \cdot \cos(x_1) + 10,$$

$x_1 = -5:10$, $x_2 = 0:15$;
 minimum global value: 0.397887

- **f₅**: Easom's Function

$$f_5(x_1, x_2) = - \left(\cos(x_1) \cdot \cos(x_2) \cdot e^{-\left((x_1 - \pi)^2 + (x_2 - \pi)^2 \right)} \right),$$

$x_i = -100:100$, $i = 1:2$;
 minimum global value: -1

- **f₆**: Rastrigin's Function

$$f_6(x_i) = 10 \cdot n + \sum_{i=1}^n (x_i^2 - 10 \cdot \cos(2 \cdot \pi \cdot x_i)),$$

$x_i = -5.12:5.12$, $n = 5$;
 minimum global value: 0.0

4. EXPERIMENTAL TESTS

Many series, with randomised initial population, of 6 runs each one, were performed on every function, in such a way that run 1 performed only one crossover per couple (SCPC) and run i ($2 \leq i \leq 6$) executed exactly i crossovers, that means $2i$ children, per couple (MCPC). In every case each new created offspring was inserted into the next generation, until the population size was reached. We make reference here only to four of those experiments:

Experiment	Crossover Probability	Mutation Probability	# of generations
E1	.50	.005	2500
E2	.50	.005	3000
E3	.50	.005	4000
E4	.50	.005	5000

Table 1: Parameters of the MCPC GA used in diverse experiments.

The experiments were based on a simple, but not canonical, Genetic Algorithm, [14], [15] with binary coded chromosomes, elitism, bit swap mutation, one point crossover and population size fixed to 70 individuals. In the case of the Easom's function the probabilities were set to 0.65 and 0.05 for crossover and mutation respectively. Due to the increase in selective pressure introduced by MCPC it was observed that the reduction of running time was paid by being more distant from the known or estimated optimum value. Hence, the range of this error is an important issue to consider.

The following relevant performance variables were examined:

Ebest = $((opt_val - best\ value)/opt_val)100$

It is the percentile error of the best found individual when compared with opt_val^2 . It gives us a measure of how far are we from that opt_val .

Epop (Ep) = $((opt_val - mean\ pop\ value)/opt_val)100$

It is the percentile error of the population mean fitness when compared with opt_val . It tell us how far the mean fitness is from that opt_val .

Gbest (Gb) : Indicates the generation where the best valued individual (retained by elitism) was found.

² opt_val is the known, or estimated, optimum value

Itime (It) = $((T_m - T_o)/T_o)100$, where T_o = running time using *proportional selection*
 T_m = running time using FPCS.

Running time increment. It is the percentile of time increase when compared with classic selection.

All the values analysed were mean values obtained from the many series for each fixed number of crossovers completed, on each function.

5. A GENERAL OUTLOOK ON RESULTS

After accomplishing the experimental runs, the following statistical analysis were done:

- I. A general overview for MCPC values contrasted with the corresponding SCPC values, under FPCS. Mean values for the first three of the above mentioned performance variables were studied. This was done for all functions and experiments.
- II. A comparative analysis between the effects of the new FPCS method and *proportional selection* for all of the above mentioned variables, on every function over all the experiments, contrasting results under SCPC and MCPC.

Table 2 and figure 2, show results from statistical analysis I (mean values over all experiments), corresponding to the parameters set indicated in section 4. In what follows, the discussion should be followed by looking also to the appendix on fitness landscapes.

#cross	f1	f2	f3	f4	f5	f6
1	0.06175	0.00140	0.00000	0.00216	0.64013	0.03238
2	0.06448	0.00140	0.00000	0.00216	1.26583	0.00005
3	0.05557	0.00140	0.00000	0.00216	0.01228	0.07557
4	0.05754	0.00140	0.00000	0.00216	1.26507	0.12970
5	0.07048	0.00140	0.00000	0.00216	1.26713	0.13129
6	0.06285	0.00140	0.00000	0.00216	0.01505	0.16078

Table 2. Ebest values for each test function

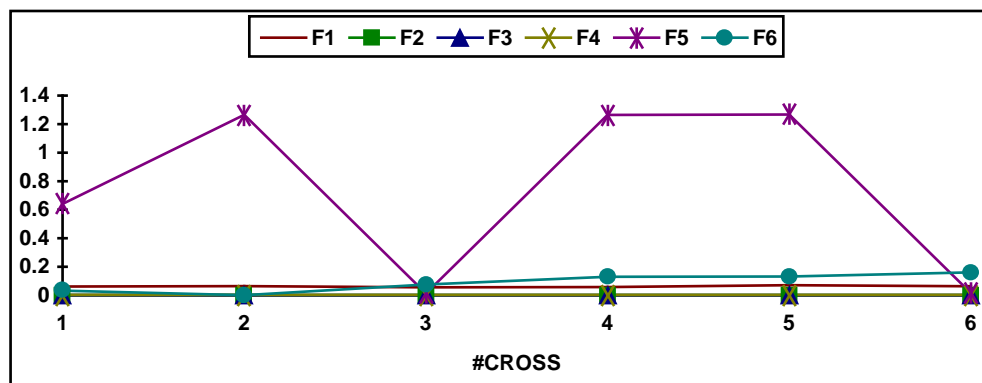


Fig. 2. Ebest values for each test function

Observing the above table and figure it can be concluded that better result values are achieved when a number of 2 to 4 crossovers is allowed per selected couple. Running the experiments, functions **f2**, **f3** and **f4** showed to be easy to optimise for the simple GA under FPCS while **f1** (highly multimodal), **f5** (unimodal and highly deceptive for a simple GA) and **f6** (highly multimodal) showed to be the more difficult ones to optimise.

It worths to remark the behavior of the searching for the difficult Easom function where best results are obtained by assigning a considerable number (6 and 12) of offspring per

selected couple. This result shows the effectiveness of the exploitation of previous good results when the search process was lucky and free of stagnation (some individuals fall into the hole). Consequently it is recommended to introduce some further criteria to guide the search.

Now some details on the results of the experiments over the most interesting functions (*f1* and *f5*) is shown in the following discussion. Only some discussion on *f6*, and not detailed information, is also included because similar results were obtained with *f1*.

5.1 MCPC VERSUS SCPC UNDER FPCS

FUNCTION F1: Typical Griewank's function F8

#cross	E1	E2	E3	E4
1	0.077717	0.062612	0.058653	0.048016
2	0.072343	0.065930	0.066127	0.053532
3	0.065673	0.053601	0.059071	0.043947
4	0.074566	0.061527	0.049691	0.044358
5	0.065718	0.083368	0.068639	0.064204
6	0.080176	0.061837	0.049286	0.060086

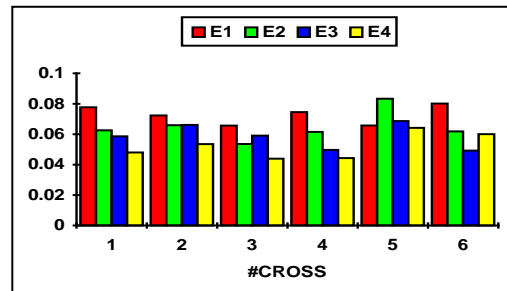


Fig. 3. Ebest values for function *f1*

As shown in figure 3 when the number of generations is incremented, results are improved. In many instances, for the values shown in boldface MCPC defeats SCPC. It happens, particularly, when the number of crossovers allowed is 3 or 4. The best value for *Ebest* was obtained for 3 crossovers and 5000 generations.

#cross	E1	E2	E3	E4
1	2202	2463	3252	3617
2	2208	2668	3018	3692
3	2108	2433	3303	3385
4	2079	2360	2958	3254
5	1967	2617	3028	3146
6	2200	2545	2805	3580

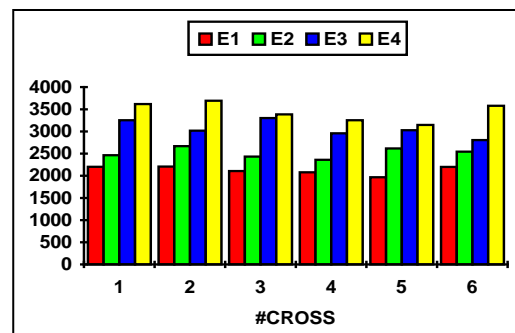


Fig. 4. Gbest values for function *f1*

Figure 4 indicates that when the number of generations is incremented the best individual is found at a later stage of the evolutionary process. This ensure a slower but a better convergence to the optimum as it is confirmed by the values of *Ebest* shown above.

#cross	E1	E2	E3	E4
1	25.3028	24.5810	27.0611	26.4806
2	25.5282	26.1561	24.9862	22.9136
3	26.1285	25.0881	24.9649	26.5468
4	27.3112	26.4273	25.2620	26.7178
5	25.3753	23.3847	26.4615	25.4626
6	25.7223	27.5452	25.9428	25.7632

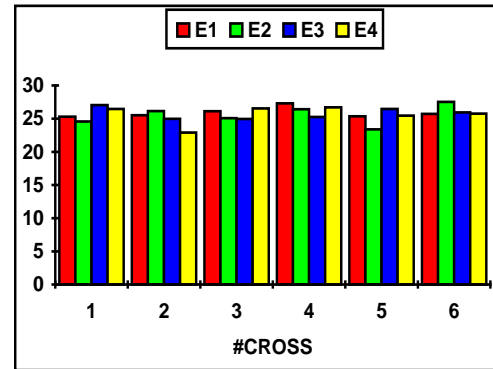


Fig. 5. Epop values for function *f1*

Running under FPCS, *Epop* values are relatively high for the *f1* function. This was expected because we tried tentatively to reduce the selective pressure introduced by MCPC by assigning highest fitness to dissimilar couples. Consequently the difficulty of a simple GA to develop niches with a considerable number of outperformers is increased and the risk of stagnation reduced in multimodal functions. This is remarkably observed in harder functions such as *f1* and *f6*, where results are similar.

FUNCTION *f5*: Easom's Function

#cross	E1	E2	E3	E4
1	0.018987	0.014531	2.514529	0.012455
2	0.015569	2.515568	0.018682	2.513492
3	0.012456	0.015569	0.010700	0.010379
4	2.512528	2.517642	0.012454	0.017643
5	5.016604	0.018683	0.017644	0.015569
6	0.013493	0.017659	0.018682	0.010380

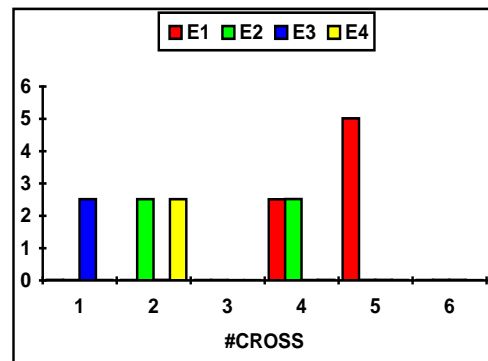


Fig. 6. Ebest values for function *f5*

For the *f5*, very hard unimodal function, it can be observed that mean values of *Ebest* are in general better for 3 and 6 crossovers per couple. For these number of crossovers all the percentile error values are less than 0.02% over all experiments. It does not happens under SCPC.

#cross	E1	E2	E3	E4
1	987	1339	1487	2280
2	1176	1543	1870	1607
3	1375	1371	1532	2035
4	1066	1254	1584	1820
5	1223	1067	1529	1890
6	1140	1356	1870	1900

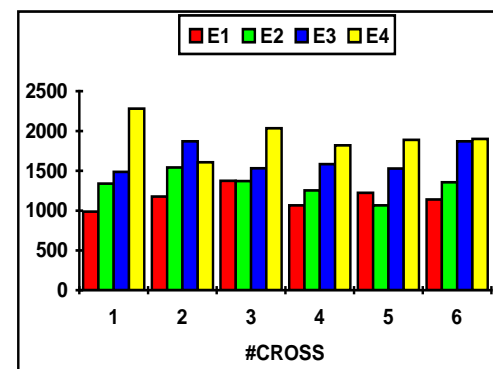


Fig. 7. Gbest values for function *f5*

The above table and figure show that when the number of generations is increased then the best individual is found later. This is an indication of the slower convergence rate to the optimum for this function.

#cross	E1	E2	E3	E4
1	40.6430	40.8518	41.4920	40.8122
2	40.8842	41.1716	40.5584	40.9890
3	40.3601	40.0976	40.0865	40.6230
4	41.0940	40.9782	40.4342	40.2207
5	42.2002	40.7074	40.7598	39.5367
6	40.8346	40.7405	40.4883	40.9275

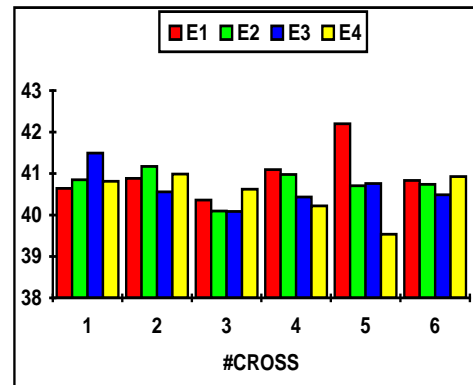


Fig. 8. Epop values for function f_5

Epop values for the f_5 function are the highest of the testing set when running under FPCS. This effect can be explained because there is a very low probability for individuals to find the hole where the optimum value can be met; a needle in a haystack problem. Also here the simple GA is unable to grow a niche with many outstanding individuals. Conversely many of them remains in the plate zone of the fitness landscape and this, in fact, affects the mean fitness of the population.

Now we pass to discuss some results on statistical analysis II; the comparison between FPCS and PS when applied to MCPC and SCPC.

5.2 FPCS VERSUS PS

FUNCTION f_1 : Typical Griewank's function F8

#cross	PS	FPCS
1	0.10829	0.06175
2	0.10870	0.06448
3	0.11620	0.05557
4	0.12142	0.05754
5	0.11889	0.07048
6	0.11515	0.06285

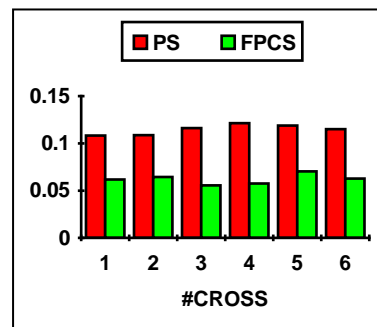


Fig. 9. Ebest for f_1

The table and figure for *Ebest* in f_1 shows a significant reduction of the error which is about of 50% on each number of crossovers. The values shown above are outcomes from over all experiments. This important results is replicated and even improved for the Rastrigin (f_6) function. Consequently these evidence supports the hypothesis that the new approach is highly recommended for optimising hard multimodal functions.

#cross	PS	FPCS
1	1458	2884
2	1744	2896
3	1953	2807
4	1993	2663
5	1958	2690
6	1973	2782

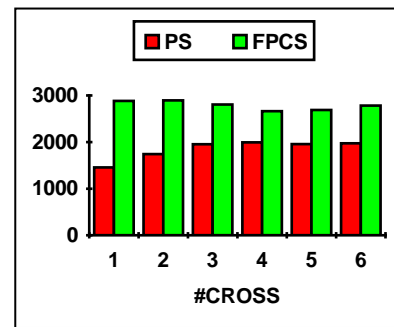


Fig. 10. Gbest for *f1*

Throughout the experiments and for any number of crossovers under FPCS the best value is found later than under PS. This is the expected consequence of choosing dissimilar couples to avoid stagnation.

#cross	PS	FPCS
1	9.544	25.856
2	9.810	24.896
3	10.457	25.682
4	9.612	26.430
5	11.347	25.171
6	11.001	26.243

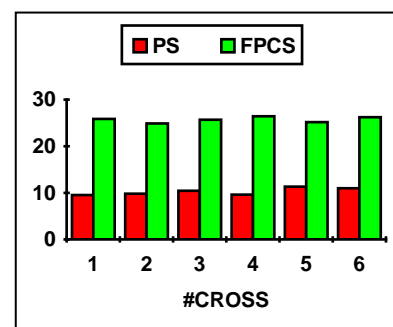


Fig. 11. Epop for *f1*

Epop values are higher for the *f1* function under FPCS than under PS. Under FPCS, to avoid the risk of stagnation, we assign highest fitness to dissimilar couples. As a consequence the population remains disseminated on the fitness landscape and few outperformers are created. This natural consequence can be notably observed in both multimodal functions, *f1* and *f6*.

#cross	FPCS
1	1.22
2	2.69
3	3.02
4	3.36
5	3.51
6	3.56

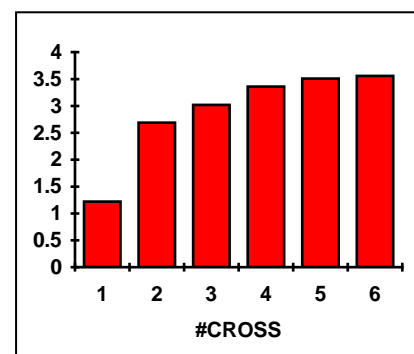


Fig. 12. Itime for *f1*

Increments of processing time under FPCS for *f1* varies from 1 to 4 %. These values are similar for all functions.

FUNCTION *f5*: Easom's Function

#cross	PS	FPCS
1	0.39515	0.64013
2	0.03856	1.26583
3	0.06548	0.01228
4	0.31116	1.26507
5	0.10053	1.26713
6	0.04772	0.01505

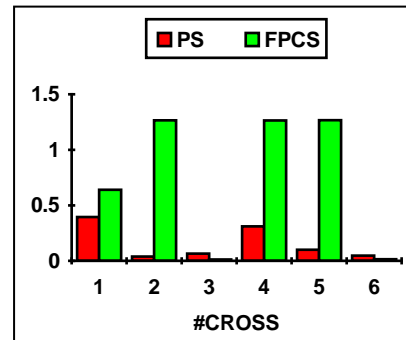


Fig. 13. Ebest values for function f_5

For function f_5 Ebest values are in general better under PS than under FPCS, but the best values are recorded under FPCS with 3 and 6 number of crossovers.

#cross	PS	FPCS
1	1253	1523
2	1161	1549
3	1251	1578
4	1443	1431
5	1325	1427
6	1279	1567

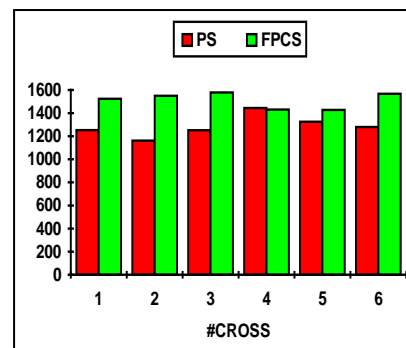


Fig. 14. Gbest values for function f_5

Again under FPCS the best valued individual is found later than under PS, but differences with PS are not so high as with multimodal functions.

# cross	PS.	FPCS
1	48.666	40.950
2	48.657	40.901
3	48.263	40.292
4	48.025	40.682
5	48.484	40.801
6	48.720	40.748

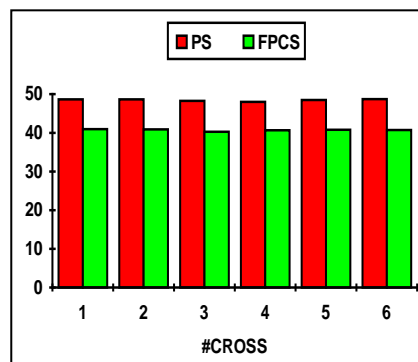


Fig. 15. Epop values for function f_5

Epop values for the f_5 function are the highest of the testing set under both selective approaches. For this unimodal function it is unlikely, by using a simple GA, to find the way to the basin and most of the individuals remains in the plane. Consequently it is difficult to find, initially, individuals with much different fitness and the FPCS strategy behaves not so efficiently as expected. Another ways of providing genetic diversity, such as selecting different parameters sets, must be devised.

6. CONCLUSIONS

The paper discusses a hybrid approach; combining MCPC and FPCS methods for crossover and selection. The rationale behind this approach relies in the attempt to diminish the selective pressure introduced by MCPC, seeking for higher quality of solutions. In this work we attempted indirectly to improve genetic diversity by assigning higher fitness to dissimilar couples expecting a consequently greater distance between parents in the fitness landscape.

The approach is quite successful for classic testing functions f2, f3 and f4 and for highly multimodal hard functions such as f1 and f6, but results are not as good when a deceptive hard unimodal function as f5 is undertaken. As results are promising, and in order to find possible extensions of the hybrid approach, the investigation will be addressed to further refinements including varied parameters sets and guided convergence for unimodal hard functions.

7. ACKNOWLEDGEMENTS

We acknowledge the cooperation of the project group for providing new ideas and constructive criticisms. Also to the Universidad Nacional de San Luis and the CONICET from which we receive continuous support.

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