Effective Prover for Minimal Inconsistency Logic

Adolfo Gustavo Serra Seca Neto and Marcelo Finger

Computer Science Department
Institute of Mathematics and Statistics
University of São Paulo
[adolfo,mfinger]@ime.usp.br

Summary. In this paper we present an effective prover for **mbC**, a minimal inconsistency logic. The **mbC** logic is a paraconsistent logic of the family of logics of formal inconsistency. Paraconsistent logics have several philosophical motivations as well as many applications in Artificial Intelligence such as in belief revision, inconsistent knowledge reasoning, and logic programming. We have implemented the KEMS prover for **mbC**, a theorem prover based on the **KE** tableau method for **mbC**. We show here that the proof system on which this prover is based is sound, complete and analytic. To evaluate the KEMS prover for **mbC**, we devised four families of **mbC**-valid formulas and we present here the first benchmark results using these families.

1 Introduction

In this paper we present new theoretical and practical results concerning paraconsistent logics. On the theoretical side, we have devised a **KE** tableau method for **mbC**, a minimal inconsistency logic, and proved that this proof system is correct, complete and analytic. And on the practical side, we have implemented a theorem prover based on the **mbC KE** proof system and proposed a set of benchmarks for evaluating **mbC** provers.

Paraconsistent logics are tools for reasoning under conditions which do not presuppose consistency [3]. These logics have several philosophical motivations as well as many applications in Artificial Intelligence such as in belief revision [12], inconsistent knowledge reasoning [8], and logic programming [1].

The relevance of reasoning in the presence of inconsistent information can be seen in the following example[1]. Suppose we are working with classical logic and we have a theory (which is a set of formulas) Γ such that $\Gamma \vdash A$ (i.e. from Γ we can deduce A) and also $\Gamma \vdash \neg A$. That is, this theory allows us to reach two contradictory conclusions. Suppose also that $\Gamma \vdash B$. In classical logic, from $\Gamma \vdash A$ and $\Gamma \vdash \neg A$ we can derive $\Gamma \vdash C$ for any formula C. In particular, $\Gamma \vdash \neg B$.

[1] We assume familiarity with the syntax and semantics of propositional classical logic.

In classical logic, a contradictory theory is also trivial, therefore useless. Paraconsistent logics separate these concepts: a contradictory theory needs not to be trivial. Therefore, in a paraconsistent logic such as **mbC**, one can have $\Gamma \vdash_{\mathbf{mbC}} A$, $\Gamma \vdash_{\mathbf{mbC}} \neg A$ and $\Gamma \vdash_{\mathbf{mbC}} B$ without necessarily having $\Gamma \vdash_{\mathbf{mbC}} \neg B$. Therefore, in paraconsistent logics one can have an inconsistent theory and still draw interesting conclusions from it.

There have been some implementations of paraconsistent formalisms [1, 4], but we do not know of any implementation of a special class of paraconsistency logics: logics of formal inconsistency (**LFI**s) [3]. This class internalizes the notions of consistency and inconsistency at the object-language level. We have extended the KEMS prover [11], originally developed for classical propositional logic, to deal with **LFI**s. The first version of this extension implements a tableau prover for **mbC**, one of the simplest representatives of this class of logics. The KEMS prover for **mbC** is implemented in Java and AspectJ. Java is a well established object-oriented programming language and AspectJ is the major representative of a new programming paradigm: aspect-oriented programming. Its source code available for download in [10].

The KEMS prover is a **KE**-based Multi-Strategy theorem prover. The **KE** system, a tableau method developed by Marco Mondadori and Marcello D'Agostino [7], was presented as an improvement, in the computational efficiency sense, over the Analytic Tableau method [13]. A tableau system for **mbC** had already been presented in [3], but this system is more similar to analytic tableaux than to **KE**: it has five branching rules, which can lead to an inefficent implementation. And although this system is sound and complete it is not analytic. Therefore, to implement the KEMS prover for **mbC** we devised an **mbC KE** system and obtained a sound, complete and analytic tableau proof system with only one branching rule.

To evaluate our prover correctness and performance, we needed some families of **mbC** problems. As we do not know any family of valid formulas elaborated specially for **mbC** or any paraconsistent logic, we devised four families of **mbC**-valid problems for evaluating **mbC** provers. These families are not classically valid, since all of them use the non-classical consistency connective. With these families we obtained the first benchmark results for the KEMS **mbC** implementation.

1.1 Outline

In section 2 we present the **mbC** logic. The **mbC KE** system is exhibited in section 3. There we also prove its analyticity, soundness and completeness. In section 4 we show the problem families we devised to evaluate **mbC** provers and in section 5 we present the results obtained with the KEMS prover for **mbC** using these families as benchmarks. Finally, in section 6 we draw some conclusions and point to future work.

2 The mbC Logic

The **mbC** logic is a member of the family of logics of formal inconsistency [3]. Logics of formal inconsistency are a class of paraconsistent logics that internalize the notions of consistency and inconsistency at the object-language level. Paraconsistent logics are tools for reasoning under conditions which do not presuppose consistency [3]. Formal characterizations of paraconsistent logics and logics of formal inconsistency can be found, respectively, in [9] and [3].

The logic **mbC** is the weakest[2] **LFI** based on classical logic presented in [3]. It uses the same set of connectives as propositional classical logic (the binary connectives $\wedge, \vee, \rightarrow$, and the unary connective $\neg$), plus a new one: the unary *consistency* ($\circ$) connective. The intended reading of $\circ A$ is 'A is consistent', that is, if $\circ A$ is true, A and $\neg A$ are not both true. In **mbC**, $\circ A$ is logically independent from $\neg(A \wedge \neg A)$, that is, $\circ$ is a primitive unary connective, not an abbreviation depending on conjunction and negation, as it happens in da Costa's C_n hierarchy of paraconsistent logics [5]. Its axiomatization is shown below:

Axiom schemas

$A \rightarrow (B \rightarrow A)$
$(A \rightarrow B) \rightarrow ((A \rightarrow (B \rightarrow C)) \rightarrow (A \rightarrow C))$
$A \rightarrow (B \rightarrow (A \wedge B))$
$(A \wedge B) \rightarrow A$
$(A \wedge B) \rightarrow B$
$A \rightarrow (A \vee B)$
$B \rightarrow (A \vee B)$
$(A \rightarrow C) \rightarrow ((B \rightarrow C) \rightarrow ((A \vee B) \rightarrow C))$
$A \vee (A \rightarrow B)$
$A \vee \neg A$
$\circ A \rightarrow (A \rightarrow (\neg A \rightarrow B))$

Inference rule

(Modus Ponens) $\dfrac{A, A \rightarrow B}{B}$

Now we present the formal definition of satisfiable and valid formulas in **mbC** [3]. Let $\mathbf{2} \stackrel{\text{def}}{=} \{0, 1\}$ be the set of truth-values, where 1 denotes the 'true' value and 0 denotes the 'false' value. An **mbC**-*valuation* is any function $v : For \longrightarrow \mathbf{2}$ subject to the following clauses:

$v(A \wedge B) = 1$ iff $v(A) = 1$ and $v(B) = 1$;
$v(A \vee B) = 1$ iff $v(A) = 1$ or $v(B) = 1$;
$v(A \rightarrow B) = 1$ iff $v(A) = 0$ or $v(B) = 1$;
$v(\neg A) = 0$ implies $v(A) = 1$;
$v(\circ A) = 1$ implies $v(A) = 0$ or $v(\neg A) = 0$.

A formula X is said to be *satisfiable* if truth-values can be assigned to its propositional variables in a way that makes the formula true, i.e. if there is at least one valuation such that $v(X) = 1$. A formula is a *valid* if all possible valuations make the formula true. For instance, the formula $\neg(A \wedge \neg A \wedge \circ A)$ is a valid in **mbC**, while $\neg(A \wedge \neg A)$ is satisfiable.

[2] It is the weakest because all other **LFI**s presented in [3] prove more theorems.

3 A KE System for mbC

The *Analytic Tableau* method is probably the most studied tableau method. It was presented in [13] as "an extremely elegant and efficient proof procedure for propositional logic". The **KE** System, a tableau method developed by Marco Mondadori and Marcello D'Agostino [7], was presented as an improvement, in the computational efficiency sense, over the Analytic Tableau method. It is a refutation system that, though close to the Analytic Tableau method, is not affected by the anomalies of cut-free systems [6].

In [3], a sound and complete tableau proof system for **mbC** is presented. It was obtained by using a method introduced in [2]. This method is a generic method that automatically generates a set of tableau rules for certain logics. For **mbC**, the rules obtained for its binary connectives are the same as that from classical analytic tableaux. The system also has a branching rule (called R_b) similar to **KE** PB rule, as well as rules for negation ($\neg$) and consistency ($\circ$). In total, this tableau system has 5 branching rules.

$$\frac{\begin{array}{l}\mathtt{T}\ A \vee B\\ \mathtt{F}\ A\end{array}}{\mathtt{T}\ B}\ (\mathtt{T} \vee 1) \qquad \frac{\begin{array}{l}\mathtt{T}\ A \vee B\\ \mathtt{F}\ B\end{array}}{\mathtt{T}\ A}\ (\mathtt{T} \vee 2) \qquad \frac{\mathtt{F}\ A \vee B}{\begin{array}{l}\mathtt{F}\ A\\ \mathtt{F}\ B\end{array}}\ (\mathtt{F}\vee)$$

$$\frac{\begin{array}{l}\mathtt{F}\ A \wedge B\\ \mathtt{T}\ A\end{array}}{\mathtt{F}\ B}\ (\mathtt{F} \wedge 1) \qquad \frac{\begin{array}{l}\mathtt{F}\ A \wedge B\\ \mathtt{T}\ B\end{array}}{\mathtt{F}\ A}\ (\mathtt{F} \wedge 2) \qquad \frac{\mathtt{T}\ A \wedge B}{\begin{array}{l}\mathtt{T}\ A\\ \mathtt{T}\ B\end{array}}\ (\mathtt{T}\wedge)$$

$$\frac{\begin{array}{l}\mathtt{T}\ A \rightarrow B\\ \mathtt{T}\ A\end{array}}{\mathtt{T}\ B}\ (\mathtt{T} \rightarrow 1) \qquad \frac{\begin{array}{l}\mathtt{T}\ A \rightarrow B\\ \mathtt{F}\ B\end{array}}{\mathtt{F}\ A}\ (\mathtt{T} \rightarrow 2) \qquad \frac{\mathtt{F}\ A \rightarrow B}{\begin{array}{l}\mathtt{T}\ A\\ \mathtt{F}\ B\end{array}}\ (\mathtt{F} \rightarrow)$$

$$\frac{\begin{array}{l}\mathtt{T}\ \neg A\\ \mathtt{T}\ \circ A\end{array}}{\mathtt{F}\ A}\ (\mathtt{T}\neg) \qquad \frac{\mathtt{F}\ \neg A}{\mathtt{T}\ A}\ (\mathtt{F}\neg)$$

$$\begin{array}{ccc} & \wedge & \\ \mathtt{T}\,A & & \mathtt{F}\,A\end{array}\ (\mathrm{PB})$$

Fig. 1. mbC KE tableau expansion rules

As explained in [6], branching rules lead to inefficiency. To obtain a more efficient proof system, we devised an original **mbC KE** system using signed formulas (see Figure 1). A *signed formula* is an expression $S\,X$ where $S \in \{\mathtt{T}, \mathtt{F}\}$ is called the *sign* and X is a propositional *formula*. The symbols $\mathtt{T}$ and $\mathtt{F}$, respectively representing the truth-values true and false, can be used as signs. The *conjugate* of a signed formula $\mathtt{T}\,A$ (or $\mathtt{F}\,A$) is $\mathtt{F}\,A$ (or $\mathtt{T}\,A$). The **mbC** ($\mathtt{T}\,\neg$) rule is a **LFI** version of classical propositional logic ($\mathtt{T}\,\neg$) [6]. It states clearly that in **mbC** we need $\mathtt{T}\,\neg A$ and $\mathtt{T}\,\circ A$ to obtain $\mathtt{F}\,A$. In classical logic, we can obtain $\mathtt{F}\,A$ directly from $\mathtt{T}\,\neg A$.

3.1 Analyticity, Correctness and Completeness Proof for the mbC KE system

An **mbC KE** proof enjoys the *subformula property* if every signed formula in the proof tree is a subformula of some formula in the list of signed formulas to be proved. Let us call *analytic* the applications of PB which preserve the subformula property, and the *analytic restriction of* **mbC KE** the system obtained by restricting PB to analytic applications. Given a rule R of an expansion system **S,** we say that an application of R to a branch θ is *analytic* when it has the *subformula property,* i.e. if all the new signed formulas appended to the end of θ are subformulas of signed formulas occurring in θ. According to [6], a *rule* R is *analytic* if every application of it is analytic. It is easy to notice that all **mbC KE** rules except (PB) are analytic.

We prove here that the **mbC KE** system is analytic, sound and complete (some proofs were omitted due to lack of space). It is easy to show a procedure that transforms any proof in the original tableau system for **mbC** ([3]) in an **mbC KE** proof, thus proving that **mbC KE** system is also sound and complete. We will not do this here. Instead, we will demonstrate that even the analytic restriction of **mbC KE** is sound and complete. That is, when performing a proof we can restrict ourselves to analytic applications of PB, applications which do not violate the subformula property, without affecting completeness.

The proof will be as follows. First we will redefine the notion of downward saturatedness for **mbC**. Then we will prove that every downward saturated set is satisfiable. The **mbC KE** proof search procedure for a set of signed formulas S either provides one or more downward saturated sets that give a valuation satisfying S or finishes with no downward saturated set. Therefore, if an **mbC KE** tableau for a set of formulas S closes, then there is no downward saturated set that includes it, so S is unsatisfiable. However, if the tableau is open and completed, then any of its open branches can be represented as a downward saturated set and be used to provide a valuation that satisfies S. By construction, downward saturated sets for open branches are analytic, i.e. include only subformulas of S. Therefore, the **mbC KE** system is analytic. As a corollary, it is also sound and complete.

Definition 1. A set of signed formulas DS is *downward saturated* if

1. whenever a signed formula is in DS, its conjugate is *not* in DS;
2. when all premises of any **mbC KE** rule (except PB) are in DS, its conclusions are also in DS;
3. when the major premise of a **mbC KE** rule is in DS, either its auxiliary premise or its conjugate is in DS.

For **mbC KE**, item (3) above is valid for every rule except ($\mathtt{T}\,\neg$). In this case, if $\mathtt{T}\,\neg X$ is in DS, either $\mathtt{T}\,\circ X$ or $\mathtt{F}\,\circ X$ is in DS only if $\circ X$ is a subformula of some other formula in DS.

We extend valuations to signed formulas in an obvious way: $v(\mathtt{T}\,A) = v(A)$ and $v(\mathtt{F}\,A) = 1 - v(A)$. A set of signed formulas L is satisfiable if it is not empty and there is a valuation such that for every formula $\mathcal{S}X \in L$, $v(\mathcal{S}X) = 1$. Otherwise, it is unsatisfiable.

Lemma 1. (Hintikka's Lemma) Every downward saturated set is satisfiable.

Proof. For any downward saturated set DS, we can easily construct a valuation v such that for every signed formula $\mathcal{S}X$ in the set, $v(\mathcal{S}X) = 1$. How can we guarantee this is in fact a valuation? First, we know that there is no pair $\mathtt{T}\,X$ and $\mathtt{F}\,X$ in DS. Second, **mbC KE** rules preserve valuations. That is, if $v(\mathcal{S}X_i) = 1$ for every premise $\mathcal{S}X_i$, then $v(\mathcal{S}C_j) = 1$ for all conclusions C_j. And if $v(\mathcal{S}X_1) = 1$ and $v(\mathcal{S}X_2) = 0$, where X_1 and X_2 are, respectively, major and minor premises of an **mbC KE** rule, then $v(\mathcal{S}'X_2) = 1$, where $\mathcal{S}'X_2$ is the conjugate of $\mathcal{S}X_2$. For instance, suppose $\mathtt{T}\,A \wedge B \in DS$, then $v(\mathtt{T}\,A \wedge B) = 1$. In accord with the definition of downward saturated sets, $\{\mathtt{T}\,A, \mathtt{T}\,B\} \subseteq DS$. And by the definition of valuation, $v(\mathtt{T}\,A \wedge B) = 1$ implies $v(\mathtt{T}\,A) = v(\mathtt{T}\,B) = 1$. □

Theorem 1. DS' is a set of signed formulas. DS' is satisfiable *if and only if* there exists a downward saturated set DS" such that DS' $\subseteq$ DS".

Corollary 1. DS' is a unsatisfiable set of formulas if and only if there is no downward saturated set DS" such that DS" $\subseteq$ DS'.

Theorem 2. The **mbC KE** system is analytic.

Proof. The **mbC KE** proof search procedure for a set of signed formulas S either provides one or more downward saturated sets that give a valuation satisfying S or finishes with no downward saturated set. If an **mbC KE** tableau for a set of formulas S closes, then there is no downward saturated set that includes it, so S is unsatisfiable. If the tableau is open and completed, then any of its open branches can be represented as a downward saturated set and be used to provide a valuation that satisfies S. By construction, downward saturated sets for open branches are analytic, i.e. include only subformulas of S. Therefore, the **mbC KE** system is analytic. □

Corollary 2. The **mbC KE** system is sound and complete.

4 Problem Families

We present below the problem families we devised to evaluate **mbC** theorem provers. We had two objectives in mind. First, to obtain families of **mbC**-valid problems whose **mbC KE** proofs were as complex as possible. And second, to devise problems which required the use of many, if not all, **mbC KE** rules. These families are not classically valid, since all of them have formulas with the non-classical consistency connective.

4.1 First family

Here we present the first family (Φ^1) of valid sequents for **mbC**. In this family all **mbC** connectives are used. It is easy to obtain polynomial **mbC KE** proofs for this family of problems. The sequent to be proved for the n-th instance of this family (Φ^1_n) is:

$$\bigwedge_{i=1}^{n}(\neg A_i), \bigwedge_{i=1}^{n}((\circ A_i) \to A_i), [\bigvee_{i=1}^{n}(\circ A_i)] \vee (\neg A_n \to C) \vdash C \tag{1}$$

The explanation for this family is as follows. Suppose we are working with a database that allows inconsistent information representation. A_i means that someone expressed an opinion A about an individual i and $\neg A_i$ means that someone expressed an opinion $\neg A$ about this same individual. For instance, if A means that a person is nice, $\neg A_3$ means that at least one person finds 3 is not nice, and A_4 means that at least one person finds 4 nice. Then $\circ A_i$ means that either all people think i is nice, or all people think i is not nice, or there is no opinion A recorded about i. $\circ A_i \to A_i$ means that if all opinions about a person are the same, then that opinion is A.

For a subset of individuals numbered from 1 to n, we have $\neg A_i$ and $\circ A_i \to A_i$ for all of them. From the fact that either $\neg A_n \to C$ or for one of them we have $\circ A_i$, we can conclude C.

4.2 Second Family

The second family of problems for **mbC** (Φ^2) is a variation over the first family whose proofs are exponential in size. The sequent to be proved for the n-th instance of this family (Φ^2_n) is:

$$\bigwedge_{i=1}^{n}(\neg A_i),\ [\textstyle\bigwedge_{i=1}^{n}[(\circ A_i) \to ([\bigvee_{j=i+1}^{n} \circ A_j] \vee ((\neg A_n) \to C))]],$$
$$[\bigvee_{i=1}^{m}(\circ A_i)] \vee (\neg A_n \to C) \vdash C$$

This family is a modification of the first family where instead of a conjunction of $\circ A_i \to A_i$, we have a conjunction of $\circ A_i \to ([\bigvee_{j=i+1}^{n} \circ A_j] \vee ((\neg A_n) \to C))$ meaning that for every person numbered 1 to n, if all opinions about a person are the same, then either all opinions about some other person with a higher index are the same or $(\neg A_n) \to C$ is true.

4.3 Third Family

With the third family of problems we intended to develop a family whose instances required the application of all **mbC KE** rules. To devise the third family (Φ^3), we have made some changes to the second family trying to make it more difficult to prove. The n-th instance of this family (Φ^3_n) is the following sequent:

$$U_l \wedge U_r,$$
$$\bigwedge_{i=1}^{n}(\neg A_i),$$
$$\bigwedge_{i=1}^{n}[(\circ A_i) \to ((((\neg A_n) \wedge U_l) \to C) \vee \bigvee_{j=i+1}^{n} \circ A_j)],$$
$$(\bigvee_{i=1}^{n} \circ A_i) \vee ((U_r \wedge (\neg A_n)) \to C)$$
$$\vdash C' \to (C'' \vee C)$$

4.4 Fourth Family

This is the only of these families where negation appears only in the conclusion. The n-th instance of this family (Φ_n^4) is the following sequent:

$$\bigwedge_{i=1}^{n}(A_i), \bigwedge_{j=1}^{n}((A_j \vee B_j) \to (\circ A_{j+1})), [\bigwedge_{k=2}^{n}(\circ A_k)] \to A_{n+1} \vdash \neg\neg A_{n+1}$$

Note: if $n \leq 2$, $[\bigwedge_{i=2}^{n}(\circ A_i)]$ in $[\bigwedge_{i=2}^{n}(\circ A_i)] \to A_{n+1}$ is replaced by the $\top$ formula.

This family formulas can be explained as follows. We have two formulas to represent two types of opinion: A and B. First we assume A_i for every i from 1 to n. Then we suppose for all j from 1 to n that $(A_j \vee B_j)$ implies $\circ A_{j+1}$. And finally we assume that for every k from 2 to n the conjunction of A_k's implies A_{n+1}. It is easy to see that from these assumptions we can deduce A_{n+1}. So we can also deduce its double negation: $\neg\neg A_{n+1}$.

5 Evaluation

Theorem provers are usually compared by using benchmarks. We have extended KEMS prover [11] to prove **mbC** theorems and evaluated it using as benchmarks the problem families presented in section 4. In Table 1 we show some of the results obtained. The tests were run on a personal computer with an Athlon 1100Mhz processor, 384Mb of memory, running a Linux operating system with a 2.26 kernel.

Problem	Time spent (s)	Problem size	Proof size	Tree height
Φ_4^1	0.06	47	197	4
Φ_7^1	0.046	80	491	7
Φ_{10}^1	0.08	113	911	10
Φ_4^2	0.071	77	570	7
Φ_7^2	1.54	164	7350	13
Φ_{10}^2	21.964	278	116037	19
Φ_4^3	0.058	94	706	6
Φ_7^3	1.097	187	5432	9
Φ_{10}^3	17.595	307	52540	12
Φ_4^4	0.007	47	181	3
Φ_7^4	0.013	83	433	3
Φ_{10}^4	0.023	119	793	3

Table 1. KEMS results for **mbC**

From these results it is clear that the second and third families are much more difficult to prove than the other two. And interestingly enough it was easier to prove the third than the second family.

6 Conclusion

We have presented an effective prover for **mbC**: a minimal inconsistency logic. The **mbC KE** system it implements was proven to be sound, complete and analytic. Besides that, it has only one branching rule. We devised some families of valid problems to evaluate our prover correctness and performance. These families can be used to evaluate any **mbC** theorem prover. The KEMS prover for **mbC** obtained the first benchmark results for these problem families.

In the future we intend to design different KEMS strategies for **mbC**. For instance, we want to implement a strategy that uses some derived rules not presented here. After that, we want to extend the KEMS prover to deal with C_1, the first logic in da Costa's C_n hierarchy of paraconsistent logics [5].

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