Estimation of actual evapotranspiration by numerical modelling of water flow in the unsaturated zone

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Abstract The objective of this paper is to present a model to estimate actual evapotranspiration ET_A from reference evapotranspiration ET_0 by numerical modelling of water flow in the unsaturated zone of the soil. Water flow is described by the highly non-linear Richards equation with a sink term representing the root water uptake. Evaporation is included in the model as a Neumann boundary condition at the soil surface. Richards equation is solved in a one-dimensional domain using a mixed finite element method. The values of ET_A are obtained by applying a water stress factor to ET_0 to account for soil moisture changes during simulation period. The proposed numerical model is used to estimate ET_A in an experimental plot located in a flatland area in Buenos Aires (Argentina). Numerical results show that the proposed model is a useful tool for evaluating evapotranspiration under different scenarios.

Keywords: Actual evapotranspiration, unsaturated zone, numerical modelling

1. Introduction

Quantification of actual evapotranspiration (ET_A) is fundamental to obtain reliable estimates of groundwater recharge and to determine crop water requirements to maintain efficient irrigate agriculture (Zhang and Schilling 2006; Katerji and Rana 2006). Unfortunately, the estimation of ET_A is one of the most difficult tasks in hydrogeology and soil sciences due to complex interactions amongst the components of land-plant-atmosphere system. To facilitate the analysis of the evapotranspiration process it is convenient to introduce the concepts of reference evapotranspiration ET_0 and crop evapotranspiration under standard conditions ET_C (Allen et al. 1998).

Reference evapotranspiration ET_0 is the evapotranspiration rate from a hypothetical reference surface under optimum soil water conditions. ET_0 is a climatic parameter that expresses the evaporation power of the atmosphere independently of vegetation characteristics and soil factors. To estimate ET_0, the Food and Agriculture Organization of the United Nations (FAO) has proposed the Penman-Monteith equation (Monteith 1965) adapted to a green grass of uniform height, actively growing and adequately watered (reference surface) (Allen et al. 1998).

The crop evapotranspiration under standard conditions ET_C refers to the evapotranspiration of a specific crop from well-watered fields that achieve full production under the given climatic conditions. ET_C is determined by the following relationship:

$$ET_C = k_C \, ET_0 \tag{1}$$

where k_C is the crop coefficient which is experimentally determined. It is important to remark that ET_C is called "potential evapotranspiration" by many researchers but this denomination is discouraged due to ambiguities in its definition (Allen et al. 1998).

The actual evapotranspiration ET_A can be defined as the evapotranspiration from a crop grown under management and environmental conditions that differ from the standard conditions. ET_A can be expressed as follows

$$ET_A = k_S \, k_C \, ET_0 \tag{2}$$

where k_S is a water stress coefficient which describes the effect of water stress on crop evapotranspiration and this coefficient ranges from 0 to 1.

In this paper we estimate ET_A from ET_0 by computing the water stress coefficient k_S. To obtain k_S we model soil evaporation, root water uptake by plants and water flow in the saturated and unsaturated zones of the soil. Groundwater flow is described by the highly non-linear Richards equation in conjunction with the van Genuchten constitutive model and appropriate boundary conditions. Evaporation from soil surface is modelled as a Neumann boundary condition and root uptake is simulated by adding a sink term in Richards equation. Both the top boundary condition and the sink term are assumed to be functions of ET_0 and the Leaf Area Index (*LAI*).

In order to evaluate the performance of the proposed method, we estimate ET_A in an experimental plot located in Buenos Aires (Argentina) for the period 2003-2004. The FAO Penman-Monteith equation with daily resolution is used to compute ET_0. The available meteorological data include air temperature, wind speed, sun shine hours, relative humidity and precipitation. *LAI* values and soil properties are obtained form the literature. Numerical results show that the numerical model is a useful tool for evaluating and predicting evapotranspiration under non-standard conditions.

2. Numerical modelling of water flow in the unsaturated zone

Water flow in the unsaturated zone is predominantly vertical and can be simulated as one-dimensional flow using the Richards equation. The simulation domain is assumed to extend from the soil surface (z_{top}) to an arbitrary point in the saturated zone (z_{bot}). In the mixed form, Richards equation reads (Richards 1931; Celia et al. 1990):

$$\frac{\partial\theta(h)}{\partial t} - \frac{\partial}{\partial z}\left(K(h)\frac{\partial}{\partial z}\left(h+z\right)\right) = S(h), \quad z \in (z_{bot}, z_{top}) \tag{3}$$

where θ is the water content, h is the pressure head, K the hydraulic conductivity, S is a sink term representing the root water uptake by plants, z is the vertical coordinate (positive upward) and t is the time.

The proposed Neumann type boundary condition at the soil surface (z_{top}) is:

$$-K(h)\frac{\partial}{\partial z}\left(h+z\right) = P - E(h) \tag{4}$$

where P is the rainfall intensity and E is the soil evaporation intensity which depends on the soil water content near surface. According with (4) the soil evaporation process is assumed to be limited to the soil surface.

The Dirichlet type boundary condition prescribed at the bottom edge of the domain z_{bot} is:

$$h = z_{wt} - z_{bot} \tag{5}$$

where z_{wt} denotes the position of the water table.

Soil hydraulic functions $\theta(h)$ and $K(h)$ are described using the van Genuchten constitutive model (van Genuchten 1980). This model is widely used by the hydrological community and reads as:

$$\begin{gathered}\theta(h) = \begin{cases} (\theta_s - \theta_r)\left[1+(\alpha\,|h|)^n\right]^{-1+1/n} + \theta_r & h<0 \\ \theta_s & h \geq 0 \end{cases} \\ K(h) = \begin{cases} K_s \dfrac{\left[1-(\alpha\,|h|)^{n-1}(1+(\alpha\,|h|)^n)^{-1+1/n}\right]^2}{\left[1+(\alpha\,|h|)^n\right]^{1/2-1/2n}} & h<0 \\ K_s & h \geq 0 \end{cases}\end{gathered} \tag{6}$$

where θ_r and θ_s are the residual and saturated water contents, respectively; α and n are empirical fitting parameters and K_S is the saturated hydraulic conductivity.

The proposed mathematical models for soil evaporation $E(h)$ and sink term $S(h)$ depend on the reference soil evaporation E_0 and the reference transpiration T_0, respectively. The partition between E_0 and T_0 is performed using a Beer-Lambert law based on the leaf area index LAI (Huygen et al. 1997):

$$\begin{aligned} E_0 &= ET_0 \exp(-a_{bl} LAI) \\ T_0 &= ET_0 (1-\exp(-a_{bl} LAI)) \end{aligned} \tag{7}$$

where α_{bl} accounts for the interception of the radiation by vegetation. A classical value for α_{bl} is 0.5 and this value will be used in the present analysis (Varado et al. 2006).

The mathematical models for E and S are expressed as follows:

$$\begin{aligned} E(h) &= \beta_E(h)\, k_C\, E_0 \\ S(h) &= g(z)\beta_T(h)\, k_C\, T_0 \end{aligned} \tag{8}$$

where $\beta_E(h)$ and $\beta_T(h)$ are functions that describe respectively the effect of water stress on soil evaporation and crop transpiration, and $g(z)$ is the root density function. The water stress functions $\beta_E(h)$ and $\beta_T(h)$ range from 0 to 1 and in this study are assumed to have the following expressions:

$$\beta_E(h) = \begin{cases} 0 & h < h_{E1} \\ \dfrac{h-h_{E1}}{h_{E2}-h_{E1}} & h_{E1} \le h \le h_{E2} \\ 1 & h > h_{E2} \end{cases} \qquad \beta_T(h) = \begin{cases} 0 & h < h_{T1} \\ \dfrac{h-h_{T1}}{h_{T2}-h_{T1}} & h_{T1} \le h \le h_{T2} \\ 1 & h > h_{T2} \end{cases} \tag{9}$$

where h_{E1} and h_{T1} are cut-off values for actual evaporation and transpiration, and h_{E2} and h_{T2} are cut-off values for evaporation and transpiration under standard conditions.

The root density function $g(z)$ is assumed to be constant:

$$g(z) = \frac{1}{z_{top} - z_{root}}, \qquad z \in (z_{root}, z_{top}) \tag{10}$$

being z_{root} the maximum root depth.

Richards equation (3) with boundaries condiditions (4)-(5) is solved using a hybridized mixed finite element method for space discretization combined with a backward Euler scheme in time (Guarracino 2001). Non-linear terms of Richards equation are linearized using the modified Picard iteration scheme proposed by Celia et al. (1990). The algorithm obtained with this approximation produces perfectly mass conservative numerical solutions and is computationally efficient (Cesanelli 2007).

3. Estimation of ET

The estimation of ET_A requires the computation of ET_0. In this study ET_0 values are obtained with the FAO Penman-Monteith equation using the standard method presented in Allen et al. (1998). The selection of the time step with which ET_0 is calculated depends on the purpose of the calculation, the accuracy required and the time step of the meteorological data available. For theoretical and practical reasons, a daily time step is adopted for ET_0 calculation.

In order to derive an expression of the water stress coefficient k_S, we analyze the daily water balance in the domain simulation. Water balance is obtained by integrating in time and space Richards equation:

$$\int_{t_i}^{t_{i+1}} \int_{z_{bot}}^{z_{top}} \left[\frac{\partial \theta(h)}{\partial t} - \frac{\partial}{\partial z}\left(K(h) \frac{\partial}{\partial z}\left(h+z\right) \right) - S(h) \right] dz\, dt = 0 \tag{11}$$

being t_i and t_{i+1} the beginning of day i and day $i+1$, respectively. Integrating by parts (11) and introducing the boundary conditions (4) and (5) we obtain the following expression for water balance (Cesanelli 2007):

$$\int_{z_{bot}}^{z_{top}} \theta(t_{i+1})dz - \int_{z_{bot}}^{z_{top}} \theta(t_i)dz - \int_{t_i}^{t_{i+1}} Pdt + \int_{t_i}^{t_{i+1}} E(h)dt - \int_{t_i}^{t_{i+1}} K(h)\tfrac{\partial}{\partial z}(h+z)\Big|_{z_{bot}} dt - \int_{t_i}^{t_{i+1}} \int_{z_{root}}^{z_{top}} S(h)dzdt = 0 \quad (12)$$

Terms of mass balance equation (12) represent, respectively, the water content in the soil profile at the beginning of the day $i+1$, the water content at the beginning of the day i, precipitation, soil evaporation, deep percolation and root uptake on day i. Equation (12) is used to test the ability of the numerical algorithm to conserve mass. Integrals in (12) are computed using the numerical solutions of Richards equation.

Actual evaporation on day i, E^i_A, can be computed from the fourth term of (12) using the proposed model for *E(h)* (8):

$$E^i_A = \left[\frac{1}{(t_{i+1} - t_i)} \int_{t_i}^{t_{i+1}} \beta_E(h)dt \right] k_C\, E^i_0 \quad (13)$$

where E^i_0 denotes reference evaporation on day i.

On the other hand, actual transpiration on day i, T^i_A, can be obtained from the sixth term of (12) using the proposed model for *S(h)* (8):

$$T^i_A = \left[\frac{1}{(t_{i+1} - t_i)} \int_{t_i}^{t_{i+1}} \int_{z_{bot}}^{z_{top}} g(z)\beta_T(h)dzdt \right] k_C\, T^i_0 \quad (14)$$

where T^i_0 denotes reference transpiration on day i.

Expressions (13) and (14) provides a method to compute separately actual evaporation and transpiration. The actual evapotranspiration on day i, ET^i_A, is the sum of E^i_A and T^i_A and can be calculated from (7), (13) and (14) as follows:

$$ET^i_A = k^i_S\, k_C\, ET^i_0 \quad (15)$$

where the water stress coefficient k^i_S has the following expression:

$$k^i_S = \frac{\exp(-a_{bl}LAI)}{(t_{i+1} - t_i)} \int_{t_i}^{t_{i+1}} \beta_E(h)dt + \frac{1 - \exp(-a_{bl}LAI)}{(t_{i+1} - t_i)} \int_{t_i}^{t_{i+1}} \int_{z_{bot}}^{z_{top}} g(z)\beta_T(h)dzdt\,. \quad (16)$$

The proposed method to estimate ET_A can be stated as follows:

1) Compute reference evapotranspiration ET_0 using the FAO Penman-Monteith equation with daily resolution.
2) Solve Richards equation using the proposed boundary conditions (4)-(5) and soil evaporation and root uptake models (8).
3) Compute daily water stress coefficients k^i_S using (16).
4) Compute actual evapotranspiration ET^i_A values using (15).

4. Results

In this section the proposed method is used to estimate ET_A on a patch located in La Plata, Buenos Aires. The climate of the area is warm and humid with a rainfall average of about 1020 mm per year. The study site is characterized by a loamy sand soil, a shallow water table and low surface runoff. The land-surface vegetation is a green grass that completely covers the soil. The similarity between the site vegetation and the reference surface for which the FAO Penman-Monteith was derived allows to consider a crop coefficient $k_C=1$. The *LAI* value is assumed to be 2, which correspond to a natural pasture (Peter-Lidard et al. 2001)

The meteorological data measured at the Facultad de Cs. Astronómicas y Geofísicas weather station include air temperature, wind speed, sun shine hours, relative humidity and precipitation. Figure 1 shows daily values of temperature and rainfall for the period 2003-2004, where day number 1 corresponds to the 1st January 2003. The mean air temperature is about 17 ˚C with monthly values ranging from 10 ˚C to 24 ˚C. Daily values of ET_0 were computed based on the available meteorological data and the FAO Penman-Monteith equation.

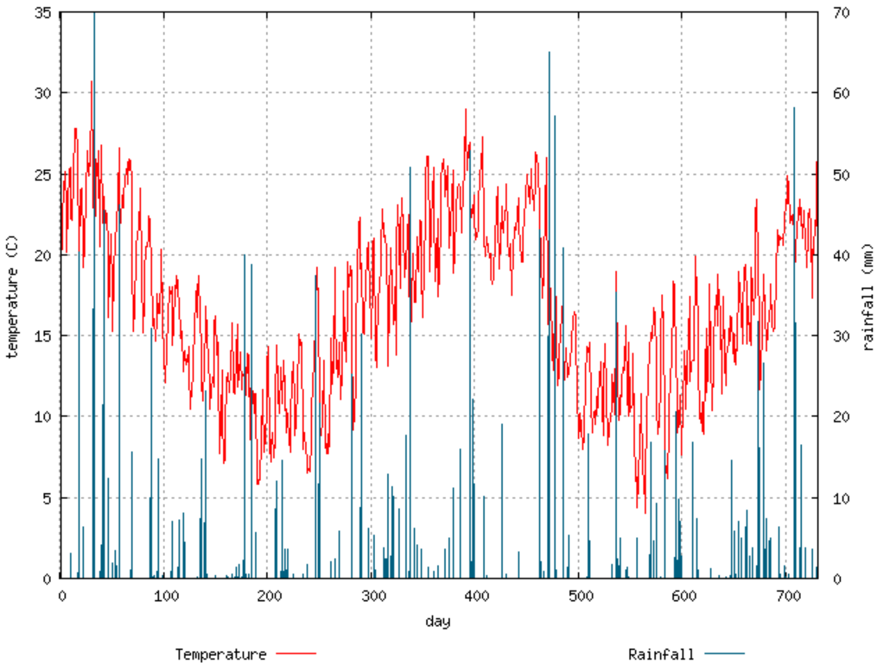

Fig. 1: Daily values of temperature and rainfall.

The water table was estimated to be 150 cm below land surface for the period considered. The soil texture is characterized by the following van Genuchten parameters obtained by Carsel and Parrish (1988) for a loamy sand: θ_s = 0.410, θ_r = 0.065, α = 0.075 cm^{-1}, n = 1.89, K_s = 106.10 cm/day. Cut-off values for water stress functions (9) are assumed to be $h_{E1}= h_{T1}$=-400 cm and $h_{E2}= h_{T2}$=-80 cm. The maximum root depth is 100 cm and the soil profile considered for numerical simulation is 400 cm wide.

Based on parameter values presented above, Richards equation with the proposed boundary conditions is solved. Using the approximate values of pressure head h we compute separately actual soil evaporation E_A and actual transpiration T_A using expressions (13) and (14), respectively.

Figure 2 shows daily E_A and E_0 values for the study period. Actual and reference values are similar during raining days when water content at soil surface is optimum. E_A values decrease between two successive rain events in proportion to the amount of water available near soil surface. Note that differences between E_A and E_0 show considerable variability.

Reference and actual transpiration values are shown in Figure 3. Values of T_A and T_0 are almost identical during winter seasons indicating optimum soil water conditions in the root zone. The main difference between T_A and T_0 take place during summers when the root uptake is high and the water available in the root zone can not respond to the transpiration demand.

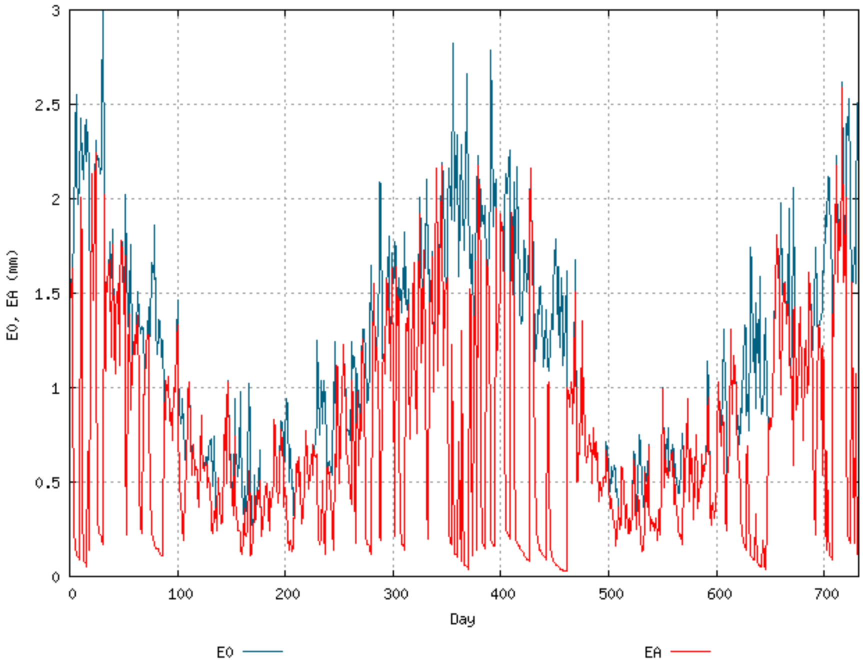

Fig. 2: Daily values of E_A and E_0.

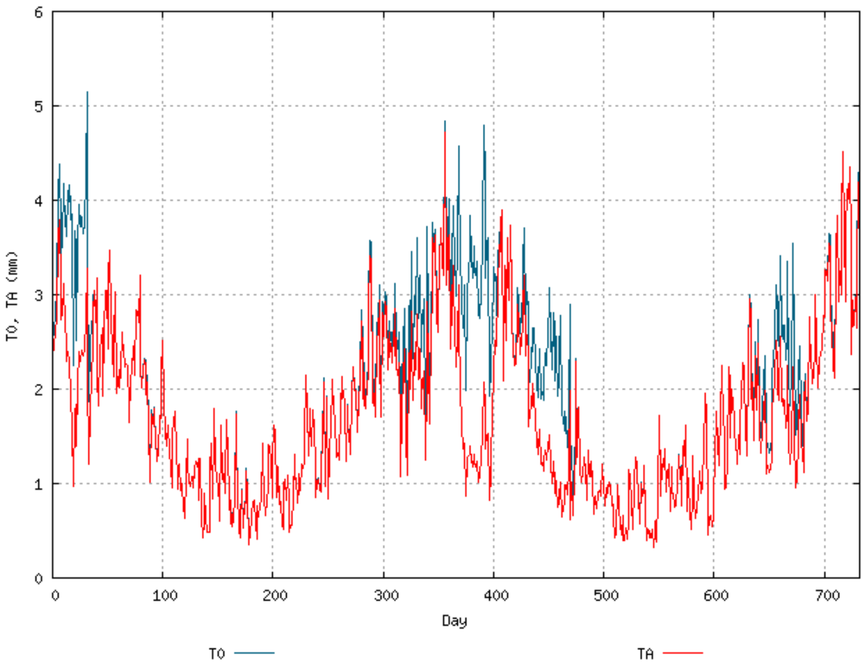

Fig. 3: Daily values of T_A and T_0.

The water stress coefficients k_S computed using (16) is shown in Figure 4. k_S values are useful to identify periods where ET_A occurred under standard conditions (k_S =1) and under non-standard conditions (k_S <1). Figure 5 shows daily ET_A and ET_0 obtained using (15). The largest differences between ET_A and ET_0 take place during summers in agreement with the largest differences between T_A and T_0. Finally, from the analysis of Figures 2, 3 and 5 we can conclude that ET_A is mainly determined by T_A in the study site.

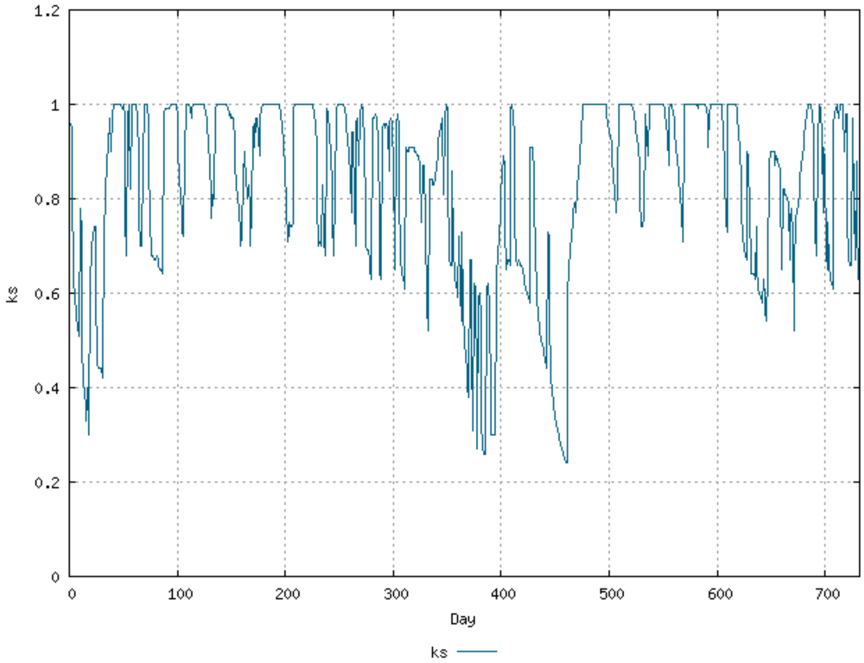

Fig. 4: Daily values of water stress coefficients k_S.

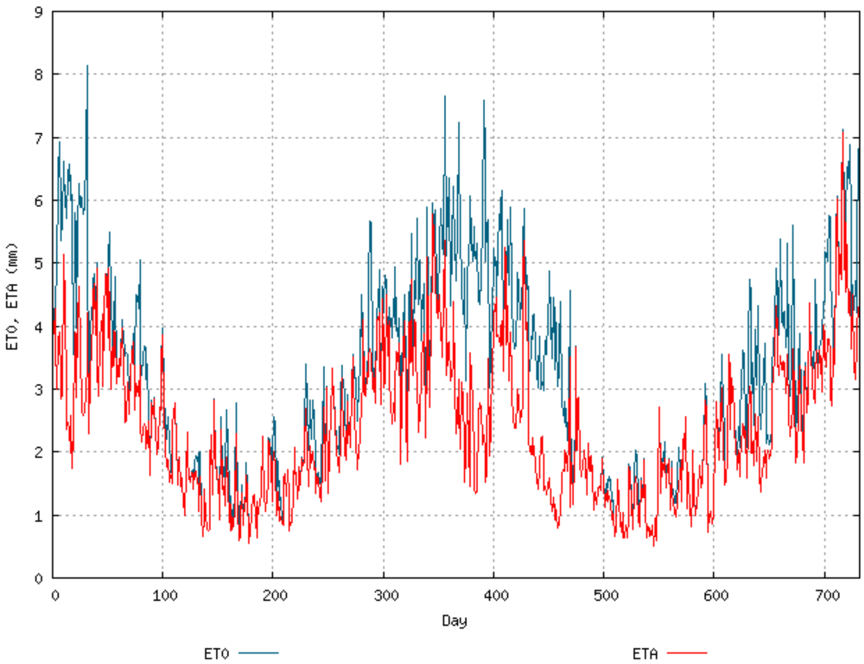

Fig. 5: Daily values of ET_A and ET_0.

5. Conclusions

A method to estimate actual evapotranspiration ET_A from reference evapotranspiration ET_0 by numerical modelling of water flow in the unsaturated zone of the soil is presented. The proposed method is an effort to understand and quantify evapotranspiration processes and it has been used to calculate ET_A on daily scale for the period 2003-2004 in a flatland area in Buenos Aires (Argentina). Numerical results show that the numerical model improves the understanding of the most important physical processes and becomes a useful tool for evaluating and predicting evapotranspiration under different scenarios and for studying problems like the influence of crop rotation on flood events.

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