Multi-Criteria Decision Model based on AHP and Linguistic Information

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Abstract.

Multi-Criteria Decision Analysis (MCDA) is a usual activity among organisations and decisions related to people's activities. Due to the complexity of considering multiple criteria, to select an alternative is a non-trivial task. From operative levels to managerial ones, MCDA is implemented by using several (formal and informal) techniques. Two useful techniques that help to make a decision are the Analytic Hierarchy Process (AHP) and MCDA models based on Linguistic Information (LI). This work describes a MCDA framework that combines the mentioned techniques in order to provide more confidence in the decision making process. To test the proposed model, framework was used to select the adequate network configuration to improve quality of service (QoS). Finally, the framework's outputs were compared to real experts' opinions obtaining satisfactory results.

1. Introduction

Currently, decision making is a very complex process since it involves recognition, analysis and evaluation of diverse aspects. For this reason, the use of Decision Support Systems (DSS) [1, 2] is very desirable in order to obtain more confidence and to reduce the uncertainty. There are many types of DSS and they are used from personal to managerial and enterprise purposes [3, 4, 5]. DSS can be used to assist in individual or group decisions [6, 7, 8] and they implement diverse techniques [9, 10] on stand-alone and web-based architectures [11, 12, 13].

A decision problem involves selecting between several alternatives, in general two or more, based on multiple criteria. Although there are many Multi Criteria Decision Analysis (MCDA) methods, all of them have common components [14]: a finite set of alternatives, at least two criteria and a decision maker.

MCDA can be classified into outranking methods [15, 16], Multi Attribute Utility Theory methods [17, 18] and non-classical methods, all of which are based on mathematical foundations and use many environment representations such as hierarchical structures, fuzzy expressions, linguistic terminology, etc. Decision maker's judgments reflect his/her preferences among the criteria and are used to compute the most adequate alternative. This recommended alternative can be viewed as a unique element or the top of in the alternative ranking.

Clearly, DSS outputs should be as representative of the user's preferences as possible. In this paper a DSS framework is presented and evaluated. It implements two MCDA methods: Analytic Hierarchy Process (AHP) [17] and a Linguistic Information model (LI) [19, 20]. AHP represents overall decision problem by using a hierarchical structure, where the main goal is the root, the criteria constitute the second level, and finally, the alternatives are disposed in the third level. Decision Maker uses a fundamental scale to express their preferences between criteria and alternatives (related to each criterion). Finally, the ranking of alternatives is computed integrating all preferences. This ranking is a list of alternatives ordered according to their ability/suitability to solve the problem.

In LI model, the information is not expressed by means of numerical values, but rather in a qualitative one, expressing imprecise knowledge and using natural language words. There are many approaches to manage linguistic assessments [19] that imply computing usage with words methods to obtain results in MCDA [20, 21]. In LI the results could be showed in a linguistic way [22], but here this outcome will be showed in utility ranking to evaluate obtained results in both methods.

This work is focused on the implementation of the two presented MCDA methods and the evaluation of their results. The article is structured as follows: Section 2 briefly presents the basic concepts of AHP and LI models and their computational fundamentals. Then, in Section 3 the proposed framework is described. In Section 4 three decision making scenarios in networking are described in detail. Results are summarized and analyzed in Section 5. Finally, conclusions are exposed in Section 6.

2. Decision making techniques

2.1. Analytic Hierarchy Process (AHP)

AHP is a multi-criteria decision support tool developed by Thomas Saaty [17]. It helps decision makers to choose between alternative solutions based on criteria and alternatives analysis, using pair-wise comparisons. These comparisons determine the priorities of a set of elements (criteria or alternatives) and are made by means of a value scale. One scale is the Fundamental Scale of Saaty, composed of the values 1 to 9 and their multiplicative reciprocals. Each value states the importance degree of an element over another. In order to use AHP, the problem has to be defined in a hierarchical structure. The goal, the criteria, the sub-criteria and the alternatives are set hierarchically from top to bottom.

Element comparisons produce matrices which must be complete and consistent [23]. Actually, since it is hard to obtain a fully consistent matrix, this restriction has got a tolerance degree. Saaty defined a method to determine the consistency within a matrix, using the Consistency Ratio (CR). Also, he stated that a matrix with a CR less or equal than 0,1 is acceptable for a matrix to be used in AHP. The CR is obtained with the following formula:

$$CR = CI/RI \quad (1)$$

where CI is the Consistency Index and RI the Random Index. The formula below shows how to get the CI:

$$CI = (\lambda_{max} - n)/(n - 1) \quad (2)$$

where λ_{max} is the maximum eigenvalue of the matrix, and n its order.

RI is the average CI of 500 random matrices with the same order and there is a RI for each matrix order.

After all the matrices are complete and consistent the alternatives priority ranking can be computed. The first step is to obtain the priority ranking of the elements in each matrix. The criteria comparison ranking yields the criteria vector. Additionally, there is a ranking of the alternatives under each criterion, which are all gathered in a matrix. After that, a matrix product is made with the criteria vector and the alternatives-criterion matrix to produce the final ranking. For further details refer to [17].

2.2. Linguistic Foundations

The use of linguistic information is suitable when there is uncertainty in the context, when experts' knowledge is too imprecise to justify the use of precise numbers, and when there is a certain tolerance to the imprecision as is the study case proposed in this work. Here, a multiplicity of services, applications and networking users coexisting in the same scenario are necessary to organize. This situation, as well as many others, involves the uses of tools to handle uncertainty of information. Thus, the use of Fuzzy Linguistic Approach (FLA) to model and manage this kind of information and some of its extensions such as Computing with Words (CW) will be useful to solve this kind of problems. The FLA represents qualitative aspects such as linguistic values by means of linguistic variables. This approach is adequate in some situations, for example, when attempting to qualify phenomena related to human perception. Also, it uses words in natural language and it has been applied with very good results in different fields [24, 25, 26].

The semantics of the terms are given by fuzzy numbers defined in the $[0,1]$ interval, which are described by membership functions. Another important aspect to analyse is the "granularity of uncertainty", i.e., the level of discrimination among different counts of uncertainty.

The use of the FLA implies processes of computing with words. This framework represents the linguistic information with the linguistic 2-tuple representation model [27]. The 2-tuple linguistic model is based on the symbolic method and takes the concept of Symbolic Translation as the base of its representation. The Symbolic Translation of a linguistic term $s_i \in \mathrm{S} = \{s_0, \dots, s_g\}$ is a numerical value assessed in $[-0.5, 0.5)$ that supports the "difference of information" between an amount of information $\beta \in [0, g]$ and the closest value in $\{0, \dots, g\}$ that indicates the index of the closest linguistic term in s_i, being $[0, g]$ the interval of granularity of S.

From this concept the 2-tuple linguistic representation model, $(s_i, \alpha_i), s_i \in \mathrm{S}$ y $\alpha_i \in [-0.5, 0.5)$, defines a set of functions between linguistic 2-tuples and numerical values.

From numerical values to 2-tuple, let be $\mathrm{S} = \{s_0, \dots, s_g\}$ a linguistic term set and $\beta \in [0, g]$ a value supporting the result of a symbolic aggregation operation, then the 2-tuple that expresses the equivalent information to β is obtained with the following function:

$$\Delta: [0, g] \to \mathrm{S} \times [-0.5, 0.5)$$

$$\Delta(\beta) = (s_i, \alpha), \begin{cases} s_i, i = \mathrm{round}(\beta) \\ \alpha = \beta - i, \alpha \in [-0.5, 0.5) \end{cases} \quad (3)$$

where $\mathrm{round}(\cdot)$ is the usual round operation, s_i has the closest index label to "β" and "α" is the value of the symbolic translation.

It is noteworthy to point out that Δ is a one to one mapping [27] and $\Delta^{-1}: \mathrm{S} \times [-0.5, 0.5) \to [0, g]$ is defined as $\Delta^{-1}(s_i, \alpha) = i + \alpha$. Thus, a 2-tuple is identified by means of a numeric value in the interval $[0, g]$. Besides, the transformation of a linguistic term into a linguistic 2-tuples consists of adding value 0 as symbolic translation: $s_i \in \mathrm{S} \Rightarrow (s_i, 0)$. This model has a linguistic computational technique associated. For further detailed description see [28].

An important aspect of the MCDM is the aggregation process in order to obtain a unique final result to each alternative. To do that, it uses aggregation operators that allow to accomplish a global value from individual values. This framework uses Weighted Mean Aggregation Operator (WM) over 2-tuple linguistic representation model that are defined as follows [27]:

Definition 1. WM: Let $\{(r_1, \alpha_1), \dots, (r_n, \alpha_n)\} \in \mathrm{S}$ be a vector of linguistic 2-tuples, and w be a weighting vector, $w = \{w_1, \dots, w_n\} \in [0,1]$, such that $\sum_{i=1}^{n} w_i = 1$. The 2-tuple aggregation operator associated with w is the function $\overline{\mathrm{G}}^w: \mathrm{S}^n \to \mathrm{S}$ defined by:

$$\begin{aligned} \overline{\mathrm{G}}^w[(r_1, \alpha_1), \dots, (r_n, \alpha_n)] &= \Delta\left(\sum_{i=1}^{n} w_i \Delta^{-1}(r_i, \alpha_i)\right) \\ &= \Delta\left(\sum_{i=1}^{n} w_i \beta_i\right) \end{aligned} \quad (4)$$

A rational assumption about the resolution of decision making process could be associating more weight to the criteria which have more importance, thus, w is based on the criteria importance. Here, the weighted vector is computed by AHP process and it is obtained by the matrix of comparison between criteria through preference relationship.

3. Multi-Criteria Decision Framework

The developed and implemented framework uses two techniques in order to acquire experts' judgments and summarize them. Based on those judgments, the summarizing process leads to a ranking which shows the suitability of each alternative to solve the problem.

The whole process of assisting experts to make a decision starts with defining a decision support project. While creating a project, a name and an objective are required. It is also essential to select the models which will be used to gather and summarize the judgments. Afterwards, it is time to select experts, and to define criteria and alternatives. At least one expert must be added to the project to proceed. Additionally, by using the LI model it is required to select a linguistic term set for each expert. Each criterion and each alternative require a name and a description. Finally, when all experts, criteria and alternatives are created, the project is made available to the corresponding experts and is ready to collect their judgments. Fig. 1 shows this process and its architecture.

While using AHP, the expert is asked to complete and check the consistency of all comparison matrices needed. He or she can start with any of them, the criteria comparison matrix or alternatives comparison ones. All judgments of the matrix must be completed by selecting the relative importance between the two elements compared. After that, the consistency check is done and the matrix is able to be used in AHP. If consistency is not acceptable, the expert is asked for changing his or her judgments. Suggestions are provided to help changing judgments using the Saaty's correction method [17].

If the expert is evaluating with Linguistic Information, they must evaluate how suitable each criterion is within the alternative in order to solve the problem. I.e. if a criterion has a poor performance, then the expert will assess such criterion with a poor value according to certain linguistic scale and their point of view. On the other hand, good performances will be assessed as high suitable criterion. The expert has to value each criterion performance in each alternative.

When all AHP matrices of the expert are complete and consistent, the calculation of the alternative ranking according with the data of those matrices is allowed. Contrarily to AHP model, with the LI model it is not necessary to check consistency, as it is to provide complete assessments in order to accomplish final results.

The specific calculations follow exactly the mathematic formulae shown in Section 2, and the final results are normalized and expressed in percentage probabilities for each alternative. Those percentages indicate how suitable an alternative is in order to solve the problem. As well as showing this ranking for each expert included in the project, the developed framework allows to aggregate the results of multiple experts (within the same project) in one ranking.

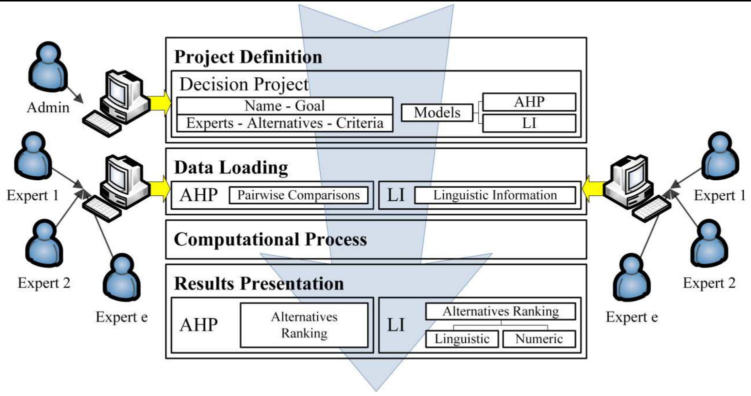

Figure 1.System architecture.

4. Test Scenarios

A networking problem in academic environment is analyzed in order to test the proposed framework. Generally, a university network has several kinds of users, uses many network protocol families, offers real-time services and different traffic types to share the bandwidth on a link in a non-controlled fashion. There are required resource management mechanisms at the gateway which implement Traffic Control tools and prioritize critical services.

The basic idea is to select the best configuration, or alternative, suited to certain scenario. Each scenario consists of different usages of network services in the University. The alternatives are defined with a finite set of types of traffic (ToT) or criteria. And each criterion could be a network service, a user or a group of users that could be considered separately.

The analyzed scenarios are the following:

- **Scenario 1.** Several courses situation. There are different courses assigned to other institutions and the informatics laboratories are busy. E-learning classes are being used and VoIP proofs are being developed.
- **Scenario 2.** End of semester. At this moment, the students, professors and researchers do many web queries, the faculty has classrooms filled and there are two simultaneous videoconferences. Administrative employees need to use a system for salary payment.
- **Scenario 3.** Very Crowded Hours. It is a merge of the above scenarios where the University is crowded and the students are using different internet services such as social networks, messaging, streaming, etc. Also, research groups have to complete online forms and different offices need to run software update processes.

The resources demands in these situations generate congestion to external networks and the internet links. It is clear that there must be an optimal control of those resources to guarantee the critical traffic at the academic institution. Thereafter, six criteria are considered, among which the most important ToTs are:

- **C_1.** Transactional Traffic. File Transfer Protocol, Mail services.
- **C_2.** Administrative Systems. Application servers, remote databases access.
- **C_3.** Real time Traffic. On-line audio and video, Voice over IP and videoconferences.
- **C_4.** Web content. Web browsing, home banking, news, webmail.
- **C_5.** Social networks and messaging. Social networks and messaging programs like Facebook, Skype, Youtube.
- **C_6.** Laboratories and Researchers. Users groups of informatics laboratories and researchers

Besides, four possible configurations which have been previously made are:

- **A_1.** Equitable. Among the services considered important for this alternative are administrative systems and research groups with not real-time traffic. ToTs such as file transfers and mail services are more benefited than real-time traffic. Therefore, in times of congestion, this configuration is adapted to basic and routine activities of the staff working regularly at the faculty. Social networks and web browsing have not assigned priority and they are the least important.
- **A_2.** Multimedia and Communications. This configuration requires low jitter and low latency for seamless communication in real time services. On the

other hand, another ToT shares the remaining bandwidth but prioritizing email communications, messaging and social networking. In this configuration the web browsing consumption has the lowest priority over the traffic types listed above. However, administrative traffic does not have the highest priority in this configuration estimating to be enough for habitual tasks. Users groups of laboratories are not included in this prioritization scheme.

- **A_3.** Data Transfer. It promotes transactional and email services bandwidth in order to transfer more data in the minimum possible time. Furthermore, high availability and guaranteed delivery is selected for administrative systems. Web traffic has lower priority than the former. Then, the remaining priority is assigned to the traffic used by research groups, real-time communications and social networks.
- **A_4.** Browsing. It involves low delay and high bandwidth for all web browsers services. It obtains a quick response visible to the user for this service. Also, administration systems and research groups are balanced in priority with intermediate bandwidths. The other types of traffic (real-time traffic and transactional traffic) have less bandwidth and priority than the ToT mentioned above.

5. Surveys and Results

Following the problem definition, the data gathering process is performed. To do that, eight experts who work in networking field and work as university teachers are chosen, and each of them is assigned one scenario to analyze. This assignment is shown in Table 1.

Table 1. Allocation of experts to scenario.

Scenario	1			2			3	
Expert	E_1	E_2	E_3	E_4	E_5	E_6	E_7	E_8

In order to achieve LI Decision Making results, two linguistic term sets are defined, allocated to experts to express their opinions. These sets, S^7 and S^9 , have granularity 7 and 9 allowing two knowledge degrees whose syntax and semantics are explained to each expert. S^9 is allocated to experts E_4 and E_7, . The remaining ones have seven terms.

Then, each expert completed the survey according to the following steps:

a) Reading their corresponding scenarios, in detailed mode, as well as the alternatives and criteria.
b) Carry out the survey using the framework AHP module.
c) Complete the survey using the framework LI module.
d) Fill in an information sheet pointing out the final ranking according to their knowledge and their point of view indicating the suitability of each alternative for the analyzed scenario, from the best to the worst. This information is compared with the results obtained from the decision models (AHP and LI) and the reliability and accuracy of the framework are checked.

The main purpose while testing the system is to evaluate its results according to experts' judgments. These results should be compared with the ranking proposed by experts without the decision support system. Thus, the reliability of the system can be determined using both proposed decision models. In order to do that, next subsections show the outcomes obtained in two methods implemented in this framework, especially analyzing matches or mismatches and why these results are obtained according to a set of metrics or conditions:

a) First element matching: if check, in both rankings the first alternative is the same and the winning alternative is found by the framework.
b) Top two elements matching: same as above but taking two elements into account.
c) All elements matching: the full ranking obtained by the framework is the same as the expert's one.
d) Two alternatives swapped: this means that two alternatives keep on same position in both rankings and the other two are in swapped orders. The swapped alternatives can be any two in the ranking.
e) Three alternatives mismatching: only one alternative is in the same position in both rankings and the other ones are swapped.
f) All alternatives mismatching: there are no alternatives in the same position in both rankings.

5.1. General Results

After the experts had given their judgments, the results were summarized in the following tables. These tables briefly show the results using each technique and the previous mentioned metrics.

In Table 2, the results of experts' rankings are shown using the above mentioned metrics for AHP. It is shown that in most situations the winning alternative is the same in both rankings. Also, many of them have two of the runner-up alternatives swapped.

Table 2. AHP rankings' metrics.

	Scenario	1			2			3	
	Experts	E_1	E_2	E_3	E_4	E_5	E_6	E_7	E_8
Measurements	First alternative matching	✓	✓	✓	✓	✓	✓		
	Top two alternatives matching					✓	✓		
	All alternatives matching						✓		
	Two alternatives swapped	✓	✓	✓	✓	✓			
	Three alternatives mismatching								
	All alternatives mismatching							✓	✓

Table 3 shows the final results for each expert involved in the decision process using LI domain. Here, it shows that in most assessments the winning alternative is the same for both rankings. Only expert E_7 does not match.The same happens in top two elements matching analysis, where E_4, E_7 and E_8 do not match; but only two assessments match for all alternatives in expert's ranking and system's ranking.

Table 3. LI rankings' metrics.

	Scenario	1			2			3	
	Experts	E_1	E_2	E_3	E_4	E_5	E_6	E_7	E_8
Measurements	First alternative matching	✓	✓	✓	✓	✓	✓		✓
	Top two alternatives matching		✓	✓		✓	✓		
	All alternatives matching		✓			✓			
	Two alternatives swapped	✓		✓	✓		✓		
	Three alternatives mismatching							✓	✓
	All alternatives mismatching								

The following tables and figures show in depth the ranking comparisons of each scenario/expert. For AHP, each figure shows the criteria preference (pie graph at top) and the influence of each criterion in alternatives final utility (bar graph at bottom). These values are obtained while multiplying the alternatives-criteria rankings matrix by the criterion ranking vector. For LI, the system outcomes are shown in bar graphs where each bar represents the importance of the alternatives in the final ranking and each segment of the bar represents the weight of each criterion multiplied by its linguistic assessments for the current alternative.

5.2. Scenario 1: Several Courses Situation

To depict the obtained results, the important criteria influence in both methods is described. Fig. 2 shows the results using AHP and its parts (a), (b) and (c) show the results of experts E_1, E_2, and E_3 respectively. Then, Fig. 3 shows the same for the LI model.

Table 4. Scenario 1: Experts' results comparison.

Expert's Ranking	System's Ranking	
	AHP	LI
A_4	A_4: 44,77%	A_4: 51,32%
A_2	**A_3: 24,05%**	**A_3: 22,72%**
A_3	**A_2: 19,62%**	**A_2: 18,78%**
A_1	A_1: 11,57%	A_1: 07,18%

(a) for E_1

Expert's Ranking	System's Ranking	
	AHP	LI
A_4	A_4: 36,67%	A_4: 47,76%
A_2	**A_3: 26,96%**	A_2: 28,79%
A_3	**A_2: 20,22%**	A_3: 16,54%
A_1	A_1: 16,15%	A_1: 06,90%

(b) for E_2

Expert's Ranking	System's Ranking	
	AHP	LI
A_4	A_4: 33,92%	A_4: 35,50%
A_2	**A_1: 26,88%**	A_2: 27,32%
A_3	A_3: 25,28%	**A_1: 24,20%**
A_1	**A_2: 13,91%**	**A_3: 12,98%**

(c) for E_3

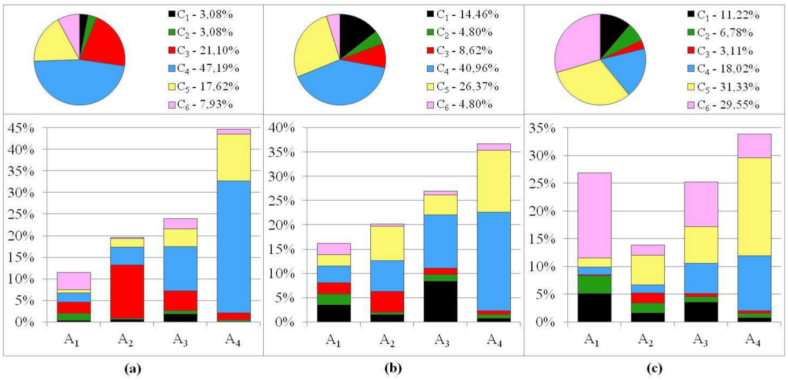

Figure. 2. Scenario 1: Results for AHP.

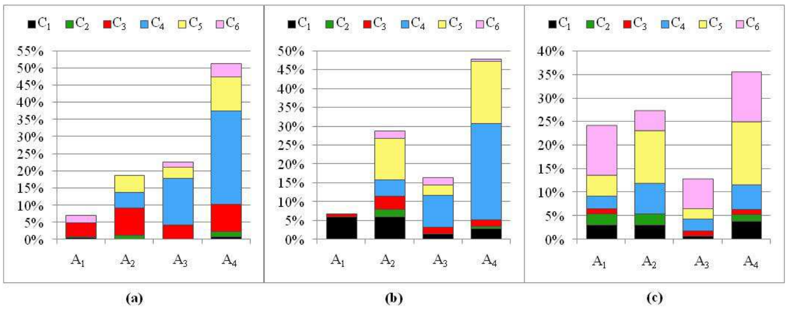

Figure. 3. Scenario 1: Results for LI.

Fig. 2 (a) shows how determining C_4 is in the final ranking of E_1. Since A_3 is better than A_2 considering C_4 and C_3 and its varying dominance degree in such alternatives. Fig. 2 (b) shows the same behavior for E_2 with criteria C_4 and C_5 in A_4, A_3 and A_2. In addition, it shows that there is just a little difference in the ranked percentages for the swapped alternatives.

Moreover, Fig. 2 (c) shows the dominance degree of C_6 and C_5 with its influence in the final results. Remarkably, Table 4 shows for expert E_3 a higher difference between the swapped alternatives where AHP difference is 12,97% (A_1: 26,88% - A_2: 13,91%).

In regards to IL results, Fig. 3 (a) the system's ranking top alternative is A_4 and the last one is A_1 what matches with expert's ranking but A_2 and A_3 are swapped. This is because of the influence of C_4 and C_5. Fig. 3 (b) illustrates that all alternatives in system ranking match with the ranking given by expert E_2. Fig. 3 (c) shows that top two alternatives match with expert ranking and the last two are swapped. Here, the combination of C_4 and C_5 is decisive to set top alternatives. However, A_3 and A_1 are swapped with a high difference of 11,22% (see Table 4) because of low dominance of C_4 and C_5.

5.3. Scenario 2: End of Semester

Fig. 4 (a) illustrates that expert E_4 preferred A_3 over A_2 where C_1, C_4 and C_6 are more influenced in this result in spite of the fact that there is little difference between them. Expert E_5 has the two least important alternatives swapped, A_2 and A_4, by just a slight difference. Fig. 4 (b) shows that criteria C_4 and C_6 make A_4 more important than A_2. Despite that they are not the main criteria for the alternatives' ranking. E_6's rankings match perfectly.

According to LI results, Fig. 5 (a) shows the obtained E_4 ranking where the most influential criterion is C_4. Furthermore, A_3 with a higher value in C_4 wins A_2 swapping positions since C_3 and C_6 are equally important. Expert E_5 alternative ranking matches perfectly with system ranking (Table 5). For E_6, the most dominant criteria are C_2 and C_3, which influence to get top two alternatives, A_1 and A_3. The other two alternatives, A_2 and A_4, are swapped with a little difference between them of about 0,3% due to the fact that A_4 is better valued in C_2 (see Table 5).

Table 5. Scenario 2: Experts' results comparison.

Expert's Ranking	System's Ranking	
	AHP	LI
A_4	A_4: 32,30%	A_4: 37,84%
A_2	**A_3: 26,60%**	**A_3: 22,97%**
A_3	**A_2: 22,88%**	**A_2: 19,85%**
A_1	A_1: 18,22%	A_1: 19,34%

(a) for E_4

Expert's Ranking	System's Ranking	
	AHP	LI
A_1	A_1: 37,16%	A_1: 34,17%
A_3	A_3: 22,25%	A_3: 30,84%
A_2	**A_4: 21,25%**	A_2: 17,89%
A_4	**A_2: 19,34%**	A_4: 17,10%

(b) for E_5

Expert's Ranking	System's Ranking	
	AHP	LI
A_1	A_1: 31,32%	A_1: 39,57%
A_3	A_3: 25,76%	A_3: 25,65%
A_2	A_2: 24,07%	**A_4: 17,54%**
A_4	A_4: 18,86%	**A_2: 17,24%**

(c) for E_6

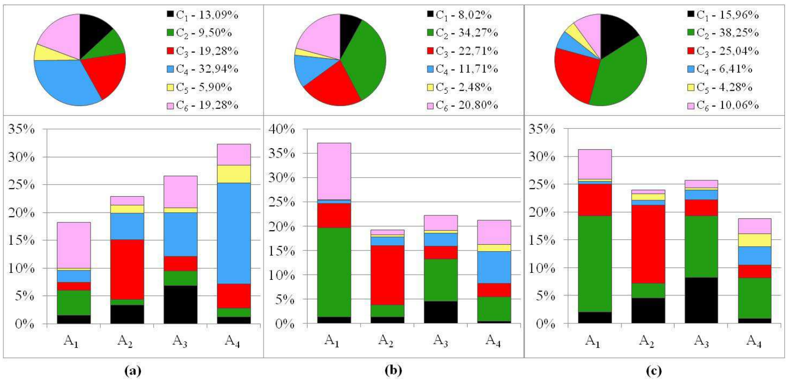

Figure 4. Scenario 2: Results for AHP.

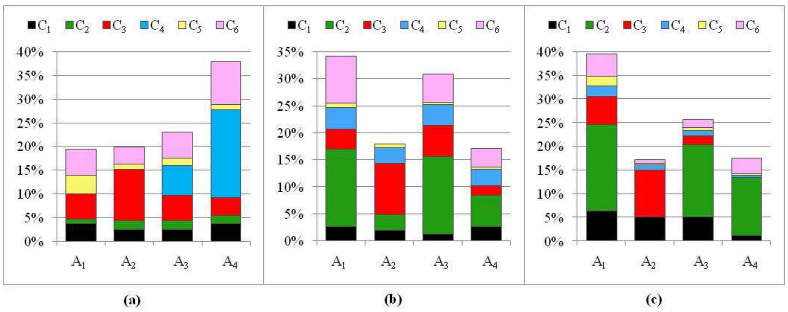

Figure 5. Scenario 2: Results for LI.

5.4. Scenario 3: Very Crowded Hours

Expert E_7's alternatives ranking is swapped in pairs: A_2 with A_3 and A_1 with A_4 (see Table 6 and Fig. 6 (a)). For the first pairs of alternatives, the determining criterion is C_3, and for the second one is C_4. Even though the difference in percentages within the final ranking of each pair is not high, the main problem is the mismatching first alternatives in the rankings. In expert E_8 judgments (see Fig. 6 (b)) all alternatives mismatch among both rankings and some differences are a little higher in percentages, as well as in absolute positions within the ranking. In this case, the determining criteria are C_3 and C_4.

Regarding LI results, Table 6 (a) shows the worst case for matching alternatives where three top alternatives mismatch. According to Fig. 6 (a), C_3 is the most important criterion. For this reason A_2 is allocated first in system ranking. In addition, C_2 and C_6 allow to place A_1 in the second position and A_3 in the third.

Therefore, the only matching alternative is A_4. Table 7 (b) illustrates that the system ranking only matches at the top alternative. In this case, C_3 and C_4 are the most important criteria that define the final ranking. In addition, C_3 and C_4 determine the top position, A_1, while C_3 determines A_2 to change positions with A_3 with a difference of 3,14% (see Table 6 (b)).

Table 6. Scenario 3: Experts' results comparison.

Expert's Ranking	System's Ranking AHP	System's Ranking LI
A_3	**A_2: 30,86%**	**A_2: 30,79%**
A_2	**A_3: 25,53%**	**A_1: 25,91%**
A_1	**A_4: 23,56%**	**A_3: 25,12%**
A_4	**A_1: 20,04%**	A_4: 18,18%

(a) for E_7

Expert's Ranking	System's Ranking AHP	System's Ranking LI
A_1	**A_3: 29,21%**	A_1: 29,11%
A_3	**A_2: 27,26%**	**A_2: 25,16%**
A_4	**A_1: 22,32%**	**A_3: 23,71%**
A_2	**A_4: 21,21%**	**A_4: 22,02%**

(b) for E_8

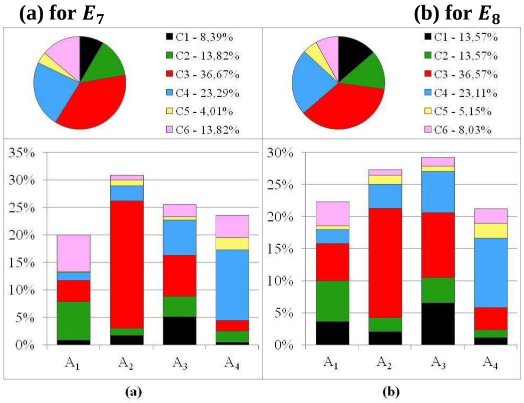

Figure 6. Scenario 3: Results for AHP.

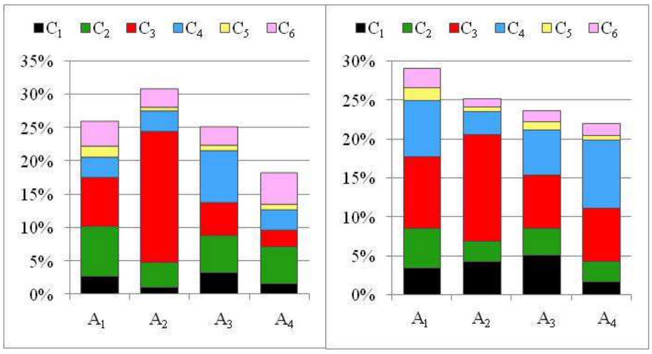

Figure 7. Scenario 3: Results for LI.

5.5. Results Aggregation

Since the framework provides two rankings to the user, one per method, it could be interesting to aggregate both in a single value and contrast them with the expert's ranking. Therefore, this aggregation is made expert by expert with AHP and LI rankings.

After testing several techniques to aggregate rankings such as averaging based approaches and Ordered Weighted Averaging (OWA) aggregation operators [29, 30], the weighted addition is chosen to show the results. The selection of this operator is performed taking into account the order importance in both methods. The weights vector w_i is obtained by [31]:

$$w_i = \frac{e^{n-i}}{\sum_{j=1}^{n} e^{n-i}} \quad (5)$$

where i is the index or order in the ranking, which stands for values 1 to 4 and n is the number of elements, which is 4 in this case.

As a result, the top elements are much more significant than the last ones. Thus, the weights vector is $w_i = (0.643; 0.236; 0.087; 0.0321)$ and the importance for the alternative A_i is computed as follows:

$$Importance(A_i) = w_{Index(A_i)}^{(\text{AHP})} \cdot r_{A_i}^{(\text{AHP})} + w_{Index(A_i)}^{(\text{LI})} \cdot r_{A_i}^{(\text{LI})} \quad (6)$$

where $w_{\text{Index}(A_i)}^{(\text{AHP})}$ is the corresponding weight to the index of A_i in the AHP ranking, $r_{A_i}^{(\text{AHP})}$ is the result value for A_i in AHP, $w_{Index(A_i)}^{(\text{LI})}$ is the corresponding weight to the index of A_i in the LI ranking and $r_{A_i}^{(\text{LI})}$ is the result value for A_i in LI. Table 7 shows the normalized results.

As shown in Table7, the combination of two calculated rankings by using the two different methods AHP and LI into an unique one, only improves two cases making them closer to the experts' rankings (see Table 7 (b) and (f)). In Table 4 (a), the two rankings have swapped alternatives (A_3 and A_2) compared with expert's ranking. Therefore, the resultant aggregated system ranking has the same swapped alternatives. The aggregation produces an inaccurate final system ranking with alternatives A_4 and A_2 (see Table 4 (b) and Table 7 (e)) as a consequence of AHP mismatch. For those cases where two rankings are far away from expert's ranking (see Table 6 (a) and (b)), the results aggregation process cannot correct anything if there are incorrect inputs.

Table 7. Methods' aggregated results comparison.

Expert's Ranking	System's Ranking
A_4	A_4: 80,46%
A_2	**A_3: 14,41%**
A_3	**A_2: 04,35%**
A_1	A_1: 00,78%

(a) for E_1 in S_1

Expert's Ranking	System's Ranking
A_4	A_4: 76,02%
A_2	A_2: 12,00%
A_3	A_3: 10,95%
A_1	A_1: 01,03%

(b) for E_2 in S_1

Expert's Ranking	System's Ranking
A_4	A_4: 71,28%
A_2	**A_1: 13,52%**
A_3	**A_2: 11,03%**
A_1	**A_3: 04,18%**

(c) for E_3 in S_1

Expert's Ranking	System's Ranking
A_4	A_4: 73,04%
A_2	**A_3: 18,99%**
A_3	**A_2: 06,02%**
A_1	A_1: 01,95%

(d) for E_4 in S_2

Expert's Ranking	System's Ranking
A_1	A_1: 72,81%
A_3	A_3: 19,94%
A_2	**A_4: 03,80%**
A_4	**A_2: 03,45%**

(e) for E_5 in S_2

Expert's Ranking	System's Ranking
A_1	A_1: 72,91%
A_3	A_3: 19,45%
A_2	A_2: 04,23%
A_4	A_4: 03,41%

(f) for E_6 in S_2

Expert's Ranking	System's Ranking
A_3	**A_2: 69,22%**
A_2	**A_3: 14,36%**
A_1	A_1: 11,82%
A_4	A_4: 04,60%

(g) for E_7 in S_3

Expert's Ranking	System's Ranking
A_1	**A_3: 37,70%**
A_3	**A_1: 37,37%**
A_4	**A_2: 22,43%**
A_2	**A_4: 02,50%**

(h) for E_8 in S_3

Finally, Table 8 shows the general results using the defined metrics. It can also be said that aggregated rankings are acceptable when compared with experts' rankings.

Table 8. Aggregated rankings' metrics.

	Scenario	**1**			**2**			**3**	
	Experts	E_1	E_2	E_3	E_4	E_5	E_6	E_7	E_8
Measurements	First alternative matching	✓	✓	✓	✓	✓	✓		
	Top two alternatives matching		✓			✓	✓		
	All alternatives matching		✓				✓		
	Two alternatives swapped	✓			✓	✓		✓	
	Three alternatives mismatching			✓					
	All alternatives mismatching								✓

6. Conclusions

The system produces, in a satisfactory way with the tested situations, many similar rankings to the experts and the techniques. Besides, the first alternative is kept in almost all experiments, so the top decision for each situation is usually hit. In both techniques, there are swapped alternatives despite the existence of small difference with the expert ranking for each expert.

In AHP, the experts weighted alternatives and criteria in such a way that the final rankings are slightly different in some cases. These situations arise when the criteria importance emphasizes some alternatives that are far different preferred under certain criterion. Additionally, in a few situations there was a difference resulting of the contributions of many criteria. As it was mentioned, the first alternative chosen by the expert is the main concern/goal, and it was achieved in most cases.

In LI method, each expert gives an assessment of the performance degree for each criterion of each alternative by using a linguistic term set. In most of the cases, the system calculated ranking does not match completely with the experts' given ranking. However, there is an agreement of the top alternatives between the system ranking and expert´s ranking. The results with LI model are good in most cases and there exists a small difference when there are swapped alternatives. Therefore, it can be said that in most of the cases the goal alternative was achieved by the system.

In order to test the results of both DSS methods an aggregation operator is used and a single value for each alternative is obtained. This value reflects the expert´s opinion taking into account two points of views of the DSS techniques used. Although some results were not greatly improved, the aggregated rank is better than each DSS technique.

Experts can also contrast their decision against the system ranking. Furthermore, the ranking of alternatives with intermediate results would help the experts in knowing how their judgments impact on each alternative. These could possibly reveal something that experts do not take into account in their final decision.

This Framework was optimized to obtain reliable and accurate results and simplify the data gathering in both models. Thus, it minimizes the expert adaptation process to the system.

Presently, it is being worked on to extend the Framework functionalities optimizing the data gathering interface and including other aggregation operators.

Acknowledgments

This work has been supported by the project "***Design of Techniques for uncertainty handling in multiple experts decision support systems***" UTN-1315 (National Technological University). We are especially thankful to the engineers who tested the proposed model: Ricardo Calcagno, Javier Dusek, Roly Scappini, Andrés Fantín, Diego Bolatti, Leandro Flashka, Carlos Cuevas and Fabián Insaurralde.

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Received: December 2013. Accepted: February 2014.