Hodrick-Prescott Filter in Practice

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ABSTRACT

Hodrick- Prescott filter has been the favourite empirical technique among researchers studying "cycles". Software facilities and the optimality criterion, from which the filter can be derived, can explain its wide use. However, different shortcomings and drawbacks have been pointed out in the literature, as alteration of variability and persistence and detecting spurious cycles and correlations. This paper discusses these critics from an empirical point of view trying to clarify what the filter can and cannot do. In particular, a less mechanical use for descriptive analysis is proposed: testing how the estimated cyclical component behaves and using autocorrelation adjusted standard errors to evaluate cross correlations to differentiate the "genuine" from "spurious" case. Simulation results to test these bivariate correlations when there is a "genuine" relationship are presented. Some examples of descriptive analysis for macro aggregates (real activity, trade flows and money) of Argentina are reported to show that not always the filter is appropriate and simple tools could be used to appreciate how the filtered series result and to evaluate cross correlations.

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Hodrick-Prescott Filter in Practice

Almost twenty years after its first presentation in the literature, Hodrick- Prescott (HP)[1] filter is still the favourite empirical technique among researchers who attempt to separate cyclical behaviour from the long run path of economic series. Applied to both "true" and "artificial" data, filtered series have been studied mainly to discover "stylised facts" in business cycles by observing and comparing univariate and cross moments: variability, autocorrelation, bivariate correlation, etc.

In spite of its wide use, not "mechanical" HP filtering has been exceptional given nowadays software facilities and invoking as justification the optimality criterion from which the filter can be derived. At the same time, a large literature has pointed out several "problems" of applying the "popular" filter, as alteration of variability and persistence and detecting spurious cycles and correlations, among the most important ones. The purpose of this paper is to discuss the filter from an empirical point of view trying to clarify what it can and cannot do and suggest some guidelines for evaluation. Next section describes the filter. Section 3 reviews some critical literature. Section 4 reinterprets them to derive evaluation criteria. Section 5 presents simulation results to evaluate bivariate correlations of filtered series. Section 6 discusses the filter in econometric models. Section 7 shows some examples of descriptive analysis for macro aggregates (real activity, trade flows and money) of Argentina. Section 8 concludes.

[1] Hodrick and Prescott (1981), reprinted in Hodrick and Prescott (1997).

2-**The HP filter**

The conceptual framework presented by Hodrick and Prescott can be summarised as follows,

$$y_t = g_t + c_t \qquad (1)$$

a given series y_t is the sum of growth component g_t and cyclical component c_t.

The growth component is determined from solving the next problem,

$$\min_{\{g_t\}_{t=-1}^{T}} \left\{ \sum_{t=1}^{T} c_t^2 + \lambda \sum_{t=1}^{T} [(g_t - g_{t-1}) - (g_{t-1} - g_{t-2})]^2 \right\} \qquad (2)$$

where the cyclical components are deviations from the long run path (expected to be near zero on average over long time period) and smoothness of the growth component is measured by the sum of squares of its second difference:

$$\Delta^2 g_t = (1-L)^2 g_t = [(g_t - g_{t-1}) - (g_{t-1} - g_{t-2})]$$

where L denotes the lag operator, L $x_t = x_{t-1}$

The parameter λ is a positive number which penalises variability in the growth component: the larger its value, the smoother g_t . In the limit as λ approaches infinity, the first difference $\Delta g_t = (g_t - g_{t-1})$ tends to a constant and the solution of the problem to a least square fit of a linear trend. In this original framework a prior value of the smoothing parameter is obtained by assuming a probability model in which:

$$c_t \sim IN (0 , \sigma^2_c) \qquad (3a)$$

$$\Delta^2 \quad g_t \quad = \varepsilon_t \qquad \sim \ IN\,(0\,,\ \sigma^2_g\,) \tag{3b}$$

The expected value of g_t given observations is the solution of the problem in equation (2) when $\lambda^{1/2} = \sigma_c / \sigma_g$.

Thus the authors suggest for quarterly data: $\lambda^{1/2}$ = [(5/ (1/8)] and λ =1600. However, they recognise the restriction imposed by these assumptions. Sensitivity analysis of results to such "λ" is explored which confirms it is a reasonable value for the case studied.

Three aspects merits to be remarked in this formulation: a) given equation (1), no irregular component is assumed in the decomposition of the series, which is therefore subsumed as part of the cyclical component; b) the minimisation problem, equation (2), and as consequence of a), supposes c_t as residual of the growth estimation (growth and cycle both unobservable) and c) the value of λ is not determined, in principle, by optimisation but it is matter of choice of empirical investigators, in general only based on "prior beliefs". These issues are later discussed.

A useful insight of the HP filter can be derived from its representation on time domain as presented in King and Rebelo (1993) who consider the case of "infinite sample" ignoring "applied" questions of endpoints treatment (see Hodrick and Prescott, 1981,1997). In this case growth component can be expressed as,

$$g_t \quad = \sum_{j=-\infty}^{\infty} w_j \quad y_{t-j} \quad = G(L)\ y_t \tag{4}$$

that is, g_t is a two side weighted moving average of the original series y_t and therefore,

$$c_t \quad = [1- G(L)\ ]\ y_t \ = C(L) \quad y_t \tag{5}$$

the cyclical component is also a moving average of the series. $G(L)$ and $C(L)$ are "linear filters".

Since the information set of this optimisation problem is the whole sample, the first order condition , from (2) given (1),

$$0 = -2(y_t - g_t) + 2\lambda[(g_t - g_{t-1}) - (g_{t-1} - g_{t-2})]$$
$$-4\lambda[(g_{t+1} - g_t) - (g_t - g_{t-1})]$$
$$+2\lambda[(g_{t+2} - g_{t+1}) - (g_{t+1} - g_t)]$$

which can be written as

$$F(L)\; g_t = y_t \qquad (6)$$

$F(L)$ is the lag polynomial,

$$\begin{aligned} F(L) &= \lambda L^{-2} - 4\lambda L^{-1} + (6\lambda+1) - 4\lambda L + \lambda L^2 \\ &= [\lambda (1-L)^2 (1-L^{-1})^2 + 1] \\ &= [\lambda \Delta^{4*} + 1] \end{aligned} \qquad (7)$$

where * indicates "centred" or " a forward second difference of the backward second difference",

$$\begin{aligned} \Delta^{4*} &= L^{-2} - 4L^{-1} + 6 - 4L + L^2 \\ &= [(1-L)^2 (1-L^{-1})^2] \end{aligned}$$

Thus,

$$F(L)^{-1} = G(L)$$

and

$$C(L) = [F(L) - 1] \,.\, F(L)^{-1}$$

$$= \lambda\Delta^{4*} \quad / \quad [\ \lambda\Delta^{4*} + 1\ ] \qquad (8)$$

Hence, King and Rebelo indicate that this cyclical filter "is capable of rendering stationary any integrated process up to fourth order, since there are four differences in the numerator".

Notwithstanding the above derivation of HP filter (minimising a cost function which penalises both departure of actual series from growth and changes in the rate of growth), there is another – less formal- interpretation of the filter: the long run component, the "trend", is what an analyst would draw by hand through the plot of the data (see, Kydland and Prescott, 1990)[2]. Next section concentrates on its critics.

3- A review of critics.

Different papers have analysed shortcomings and drawbacks of the filter. A good summary of them is offered by Ravn and Uhlig (1997): the filter might generate most of the cycles, the filter is only "optimal" (minimum –square- error) in special cases and may produce extreme second order properties of detrended data. They, however, suggest that "none of these shortcomings and undesirable properties are particularly compelling: the HP filter has withstood the test of the time and the fire of discussion remarkably well (op.cit., p 1.)[3]. Some of this critical literature is next discussed trying to precise how "compelling" such critics are, in the following sections.

Singleton (1988) pointed out that pre-filtering has important effects on the dynamic interrelation among series (assuming a VAR representation), in particular he found inconsistent

[2] It is also virtually identical to a "natural cubic spline" for a given λ (see Doornik and Hendry, 1996)
[3] Then they dedicate to analyse how to adjust λ.

estimates of parameters. He proposed to study secular and cyclical frequencies simultaneously. Section 6 considers these issues.

More recent literature concentrates on other features of the HP filter. King and Rebelo (1993) showed cases in which persistence, variability and comovement of (simulated and actual) economic series are altered after filtering in comparison with those of raw data. They also found that HP filter is optimal -in the sense of minimising the mean square error- for a limited class of ARIMA models, which "are unlikely to be even approximately true in practice" (op.cit. p. 230).

More influential has been Cogley and Nason (1995) that concentrated their critics on the possibility of obtaining "spurious cycles " when filtering "difference stationary data"(like a random-walk representation). Harvey and Jaeger (1993) extended the analysis to show the possibility of "spurious sample cross correlation" between spurious cycles. Then they put a warning to the "uncritical use of mechanical detrending" (op. cit., p.231). These authors also interpreted HP filter in terms of "structural time series models"(Harvey, 1989) which would correspond to a special (restricted) case of them.

Canova (1998) compared HP with other detrending techniques concluding that "stylised facts" are highly dependent on the alternative methods in practice.

As this type of filter (and the decomposition it assumes) has a long history, the controversy about "filtering" is neither new. Hodrick and Prescott dated the filter in 1923 and similar approaches even in the last century. At the same time the decomposition of economic series is mainly based of 1919 work of Persons, based on the idea of different causal forces of cyclical and trend components. [4]

[4] See Singleton (1988) for a discussion of this work an its critics supported by the famous debate between Burns and Mitchell and Koopmans.

On the other side, Harvey and Jaeger considered the spurious cyclical behaviour from applying HP filter as "a classic example" of the Yule-Slutsky effect (op.cit., p.234).

Slutsky in 1937 (see Sargent (1979) for an exposition) considered the effect of starting with a white noise μ_t, taking a two period moving sum n-times and then first differences m-time,

$$(1 + L)^n \quad (1-L)^m \quad \mu_t \; = \; z_t \tag{9}$$

then he showed that z_t has cyclical behaviour as $n \rightarrow \infty$ [5]. Given equations (4) and (5) such possibility cannot be excluded a priori when applying HP filters.

Recalling that the problem of "filtering" is closely related to seasonal adjustment in empirical works, next section will reinterpret critics as part of a progressive approach in order to derive some guideline to evaluate the resulting decomposition.

4- Reinterpreting critics from a practical point of view

Suppose that an investigator is ready to apply HP filter in order to separate growth from cycle of economic series taking advantage from "easy use" software facilities. What can be learnt from the previous discussion? First of all, " a more critical and less mechanical" use of the filter is required.

Since modelling of unobservable components "g_t" and "c_t" from "y_t" is the issue, the additive (or log additive) decomposition, equation (1) should be assumed as the univariate representation and therefore, Persons' views on different driven forces of components should be

[5] The spectrum of z_t has a sharp peak at frequency $\pi/2$. Similar results are found for the "long swings" arising from Kuznets transformation.

shared. This also implies that the seasonal component -if present- has been somehow removed (whose effects could merit additional discussions) and the irregular has been absorbed by "c_t".

Moreover, since both components are determined from a given "y_t", the separation supposes that the fraction "$1/ [\lambda \Delta^{4*} + 1] \; y_t$" corresponds to "$g_t$" and "$\lambda\Delta^{4*} / [\lambda\Delta^{4*} + 1] \; y_t$" corresponds to "$c_t$" (from equations (7) and (8)). The "weights" are the same for all series except for λ, which reflects the trade off between minima series departures from long-run and minima departures of last growth rate (equation (2)) . Are these terms those that matter for the cost function? Does it make sense to peg the rate of growth or the level of the long run component to their past values (as in the case of "exponential smoothing", see King and Rebelo, 1993)? Are such terms the only ones or cross terms should also be included in the relevant function to minimise? Although these questions are difficult to answer a priori, a good practice would indicate to check if what is obtained by "filtering" is what is expected to be.

As previously seen the terms in the cost function is weighted by λ, which is the only parameter under "control". Unless the researcher performed a maximum likelihood approach to estimate simultaneously λ, its value should be "guessed"[6]. The default value has been set at 1600 for quarterly data accordingly to the basic probability model of HP summarised in section 2, which depends on the assumption about the ratio of variance between cyclical and growth rate white noises (see also King and Rebelo, 1993, p.224). In the Hodrick- Prescott ´s paper λ takes values from 400 to ∞ (perfect smoothing) for the sensitivity analysis of the filtered data. However, the range for this periodicity might be considerably wider if a different representation is assumed, as in Nelson and Plosser (1982, see also Canova 1994) which closest value is about 1.

Other frequencies are still more controversial. From the default value of 1600 for quarterly data, linear or quadratic adjustments have been used in applied works (say λ= 400 or

100 for annual data). Recently, a power adjustment of 4 has been proposed since the transfer function is in this way invariant to the sampling frequency (λ = 6.25 for annual data) (Ravn and Uhlig, 1997)

Given such a range of values, again, empirical work should evaluate whether or not estimated growth and cycle reject their conjectured behaviour. Minima criteria are: all long-run components (low frequencies) should be part of "g_t" whereas other components of shorter periodicity (higher frequencies) should be left to "c_t" . However, it is expected to be not too “noisy" (not too much weight on the highest frequencies). Although Hodrick and Prescott present unit root tests of the cyclical components, it is not common to see such tests, spectra or just correlograms. In other cases, neither a visual inspection is offered to evaluate “how well” the investigator “draw by hand” the trend, using a specific λ.

The probability model from which Hodrick and Prescott derived a prior for λ has some interpretation problem as it has been used as a “paradigm” in this literature even though these authors recognised the limitations of assuming such a representation.(op. cit., p.4). As shown in section 2, equations (6) and (7),

$$(\lambda\Delta^{4\,*}+1)\, g_t = y_t$$

$$\lambda\Delta^{4\,*} g_t = y_t - g_t = c_t$$

$$\lambda\, [1-L^{-1}]^2 \quad [1-L]^2\; g_t = c_t$$

and given the data generating process (DGP) assumed for the long run component, (equation (3b))

$$\lambda\, [\, 1-L^{-1}]^2 \quad \varepsilon_t = c_t$$

$$\lambda\; [\, \varepsilon_t \; -2\; \varepsilon_{t+1} \; + \; \varepsilon_{t+2} \;] = c_t$$

(10)

[6] Harvey and Jaeger found usually too low values of λ when applying maximun likelihood.

Therefore c_t is not white noise as assumed in the DGP (equation (3a)) but λ times a non-invertible MA(2) whose roots are, therefore, outside the range of those showing cyclical behaviour. Note that the information in t+1 and t+2 is known since the optimisation is over the whole sample t = -1... T (equation (2)).

Another view of the same question is obtained when deriving growth (equations (6) and (7)) from the DGP assumed (equations (3a) and (3b)),

$$(\lambda\Delta^{4\,*}+1)\, g_t = y_t = (\varepsilon_t \;/\; \Delta^2) + c_t$$

$$\Delta^2 g_t = \varepsilon_t + (1-L^2)\, c_t - \lambda\,(1-L^{-1})^2\,(1-L)^4\, g_t \tag{11}$$

then $\Delta^2 g_t$ cannot be white noise as supposed in the DGP.

Similar considerations apply to the structural representation whenever it is equivalent to $g_t \approx$ ARIMA (0,2,1) and $c_t \approx$ ARMA (2,1), subject to restrictions (the AR part corresponding to complex roots (Harvey and Jaeger, 1993 ,p. 234) and a difference stationary (as that analysed by Cogley and Nason(1995)) or second difference stationary representation of y_t. In the first case, equation (11) can be generalised and $\Delta^2 g_t$ does not result as MA(1). For the latter, assuming

$$\Delta^2 y_t = \eta_t \qquad\qquad \eta_t \sim \text{IID}\,(0,\ \sigma^2_\eta)$$

and

$$c_t = \{\lambda\Delta^{4*} \;/\; [\lambda\Delta^{4*}+1]\}\, y_t = \lambda\Delta^{4*}\,\eta_t \;/\; [\lambda\Delta^{4*}+1]\,\Delta^2$$

or

$$c_t = \lambda\,(1-L^{-1})^2\,\eta_t + \lambda\,(1-L^{-1})^2\,(1-L)^2\, c_t \tag{12}$$

and therefore, c_t does not appear as a “typical” cycle within the class of ARMA models.

Nelson and Plosser (1982) suggested this kind of problem when expressed “HP strategy implicitly imposes a components model on the data without investigating what restrictions are implied (a difficult task in their model) and whether those restrictions are consistent with the data”, p.158

Therefore, researchers on the HP filter should have in mind a DGP which differs from those that can be expressed in terms of the family of the ARIMA class, since much of the debate can be put in terms of the conjectured DGP. Then, the task is to look for tools to test that the results obtained do not reject the conjectures. Evaluating the behaviour of estimated components -as above discussed- would be one part of the question. The other is the possibility of spurious cross correlation between spurious cycles.

Harvey and Jaeger (1993) made a simulation exercise assuming as DGP independent random walks and first differences random walks and showed that spurious correlation between spurious cycles may not be negligible. They evaluated cross correlation of these independent processes using asymptotic standard errors (SE) (Brockwell and David ,1987, p.400) and recommend reporting SE in addition to point estimates of cross–correlations (p.245). While their simulation concentrates on rejecting the null (ρ_{xy}=0) when it is true by construction, the other side of the test should be performed: not rejecting the null when it is false, but the DGP should be different of random walk or difference random walk, otherwise it makes no sense the HP filtering. Next section presents some results of a simulation exercise using U.S GNP series, for which “the HP filter is tailor-made for extracting the business cycle component” (Harvey and Jaeger(1993), p. 236).

5- Simulation results: testing "genuine" cross correlations.

Following Harvey and Jaeger (1993) sample cross correlations could be evaluated taking into account their asymptotic distribution

$$r_{xy}(h) \sim AN\left(0, T^{-1}\left(1 + 2\sum_{j=1}^{\infty} \rho_x(j).\rho_y(j)\right)\right)$$

where $r_{xy}(h)$ is the sample cross correlation at lag h between two series with sample T and $\rho_x(j)$, $\rho_y(j)$ are the autocorrelation of stationary processes x_t and y_t at lag j. In this way the probability of finding large spurious correlation between independent spurious cycles could be taken into account.

In order to consider how it could perform in the case of evaluating a "genuine" correlation between two series with "typical" cyclical behaviour (for which the HP filter would be most appropriate) three series were generated assuming that the cyclical component of the US GNP[7] (now X_t) contributes 80%, 50%, 20% and 10% to the variance of the artificial series. Thus, normal random numbers were added as errors to obtain y_{80t}, y_{50t}, y_{20t}. Appendix 1 shows cross plots and autocorrelations for each series for the sample 59(1)-98(3). Table 1 reports simulated sample cross correlations and the autocorrelation adjusted SE times the limit of the 95% confidence interval for $\rho_{xy}(0) = 0$ where ρ_{xy} denotes population cross correlation coefficient between two independent stationary series. The empirical (large sample) approximations to the SE were made considering the sample autocorrelation $r_x(j)$ and $r_y(j)$ with j=1 to J, J= 4/T[8].

[7] The series is taken from the National Bureau of Economic Research.

[8] Note that it is usual to make similar approximations to evaluate univariate autocorrelations (see Nelson,1973).

TABLE 1
Simulated "sample" cross correlations and autocorrelation adjusted SE

	$r_{xy}(0)$	$(1.96 \,.\, T^{-1/2})(1 + 2 \sum_{j=1}^{j=J} r_x(j).\, r_y(j))^{1/2}$
y_{80t}	0.896	0.28
y_{50t}	0.699	0.24
y_{20t}	0.436	0.22
y_{10t}	0.309	0.16

The first column reports sample cross corelations between x_t (the "c_t" of the US GNP) and y_t (generated for explained variances of 80,50,20 and 10%); the second column shows autocorrelation adjusted SE times the limit (absolute value) of 95% confidence interval; J= 40 . Note that $(1.96 \,.\, T^{-1/2}) = 0.16$

In each case, even for smallest, cross-correlation can be empirically detected as significant (not inside the 95% confidence interval for $\rho_{xy}= 0$). As the exercise suggests, using these "autocorrelation adjusted" SE could help to evaluate cross correlations between cyclical components[9] having some protection from the "spurious correlation problem". Note that the same SE can be used for different h.

6. Why not to use filtered series when estimating econometric relationships.

Although almost nobody could disagree about using seasonally adjusted data to better understand economic series behaviour as part of a descriptive analysis, their use is more debatable for econometric modelling in a multivariate framework. Ericsson, Hendry and Tran (1994) summarises polar positions: Wallis (1974) considers the implications of estimation with seasonally adjusted data when the DGP relationship involves unadjusted data whereas Sims analysed the converse situation: estimation with unadjusted data when the DGP relationship involves the non-seasonal components. In each case the model is mis-specified (the dynamics

[9] They can be useful only if the cyclical components result as "stationary".

alters) and estimates are generally inconsistent. [10] The same considerations are relevant for filtered data: separate cycle and growth may be useful for descriptive analysis but their use for econometric relationships depends on the conjectured DGP. A similar reasoning can be made about the alteration of variability and persistence: filtered and raw data have different sample moments but which is the appropriate depends on the beliefs about the DGP (as in King and Rebelo,1993).

Whenever economic agents were supposed not to separate components a "Wallis effect" (see Hendry and Mizon, 1979) may alter econometric relationships as follows when used "filtered series",

$$\alpha(L)\, y_t = \beta(L)\, x_t + u_t \qquad u_t \sim IID(0, \sigma^2_u)$$

$$y_t^a = \delta(L)\, y_t$$

$$x_t^a = \gamma(L)\, x_t$$

where $\alpha(L)$ and $\beta(L)$ are polynomials in L and $\delta(L)$ and $\gamma(L)$ are linear filter, such as G(L) or C(L) (see equations (4) and (5). Then

$$\alpha(L)\, y_t^a = \beta(L)\, x_t^a + \beta(L)\,[\,\delta(L) - \gamma(L)\,]\; x_t + \delta(L)\, u_t$$

Using filtered series in dynamic econometric models implies -for different filters- an "omitted variable problem" (from the second term) and an "autocorrelation problem" (from the third term), both as part of the error term. This shows that a necessary condition to obtain consistent estimators is to adjust the series using the same λ (here

[10] Ericsson, Hendry and Tran compared both type of models. They found no differences for cointegrating relationships but alteration of dynamics and exogeneity status.

δ (L) = γ (L)). Such requirement may be critical if different parameters are appropriate for each series, according to their univariate behaviour.

However, even if this not the case, autocorrelation would still be present[11] and, therefore, inconsistent and inefficient estimates would arise in dynamic equations like one of a VAR system. All depends on the "beliefs" about the DGP: whether or not "unadjusted" data enter the model. But, this would be testable since the presence of autocorrelation (different from first order) when using filtered data would be an indication of the presence of a "Wallis effect".

There is another question related to "exogeneity" when using a filtered series as explanatory variable. Conditioning on (sequential data) "x_t" is here modified since "x_t^a", that is filtered "x_t", supposes an information set which includes future information within sample but not known at each t.

It is worth noting that, even when suitable evaluated cross correlations (taking into account adjusted SE) could be part of a "explorative" analysis, there is no guarantee of obtaining unbiased estimates of such linear relationships if more variables contribute to explain them. The bivariate correlations are also more likely to be unstable as Bardsen, Fisher and Nymoen (1995) showed for activity- inflation and real wages-unemployment using the U.K and Norwegian data.

On the other hand, regressions that use "HP filtered series" require not only the same λ when adjusting all the series but also a careful study of the residual autocorrelation in dynamic models to avoid inconsistencies. This testing could be useful to evaluate whether or not evidence rejects the conjectured model involving unadjusted variables.

[11] Unless the assumption about the original disturbance is not correct and the same filter for y_t makes this term white noise.

Finally, an alternative approach to pre-filtering series for multivariate dynamic econometric modelling is to leave the data "inform" about different filters. Seasonality, long-run, and cyclical behaviour can be jointly modelled following a "general to particular" approach (see Hendry, 1995). "Linear filters" - as G(L) or C(L) of equation (4) y (5)- can be embedded in "linear (dynamic) models" without "constraining" the lag weights (the "w_j"). However, these "data-based" filters would use only "past" information ($j > 0$) in the "conditioning" set.

7. Some examples for macro-aggregates of Argentina

In this section the evaluation previously suggested for descriptive-explorative analysis is illustrated with argentine macro-aggregates: a subset of series[12] (GDP, consumption, investment, trade flows and M1) considered by Kydland and Zarazaga (1997). Firstly, the univariate behaviour of the cyclical components "c_t" is studied by observing autocorrelations and performing usual unit-root tests. Then, cross correlations are evaluated using the SE which allow statistically differentiation from the "spurious correlation" case[13].

Appendix 2 reports Dickey-Fuller statistics and autocorrelations (tables 2.1 and 2.2). For all the series these statistics (or their augmented versions when necessary) reject the null of a unit root (at traditional levels) for the cyclical component except in the case of investment. For this series, different λ were tried (from 400 to 6400) but the null cannot be rejected in any case. Visual inspection of the respective autocorrelations confirms that estimated cycles look like "stationary series" being also far from a "noisy" behaviour.

Table 2.3 and 2.4 in the Appendix 2 show the relation between the cyclical component of GDP ant those of the other macro-aggregates. They are very similar to the obtained by Kydland and Zarazaga, both volatility and correlations for all the series. In particular cross

[12] The series are taken from the Statistical Appendix of the Economic Ministry and the Central Bank of Argentina.

correlations look quite high. However, when "genuine" correlation (see Table 2.4) are evaluated using the adjusted SE, the case of M1 cannot be detected as significant (the 95% confidence for a zero cross-correlation includes the computed value). Then, in the case of money, correlation cannot be empirically distinguished from the spurious case. The rest of the evaluated series remains showing significant correlations[14].

8. Conclusions

Different shortcomings and drawbacks of the Hodrick- Prescott filter have been pointed out in the literature which at the same time do not appear to have had great effects on its wide use in empirical research. This paper discusses these critics trying to see what can be learnt from them. First, researchers should be aware of the decomposition of the series that the filter assumes. Then, a less mechanical use is proposed by testing how the estimated cyclical component behaves and using autocorrelation adjusted standard errors to evaluate cross correlations to differentiate the "genuine" from "spurious" case.

Although the role of the filter as part of a descriptive analysis cannot be denied (as it cannot be the use of seasonally adjusted series), econometric dynamic modelling of filtered series is more problematic if the data generating process involves unfiltered series.

Examples of descriptive analysis for macro–aggregates of Argentina show that not always the filter, mechanically applied, is appropriate. Simple tools could be informative about how the filtered series result and to evaluate significant cross correlations.

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[13] For simplicity only "time domain" tools are presented.
[14] Given the reported Dickey-Fuller statistic, the case of investment cannot be evaluated.

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Appendix 1: Simulation results

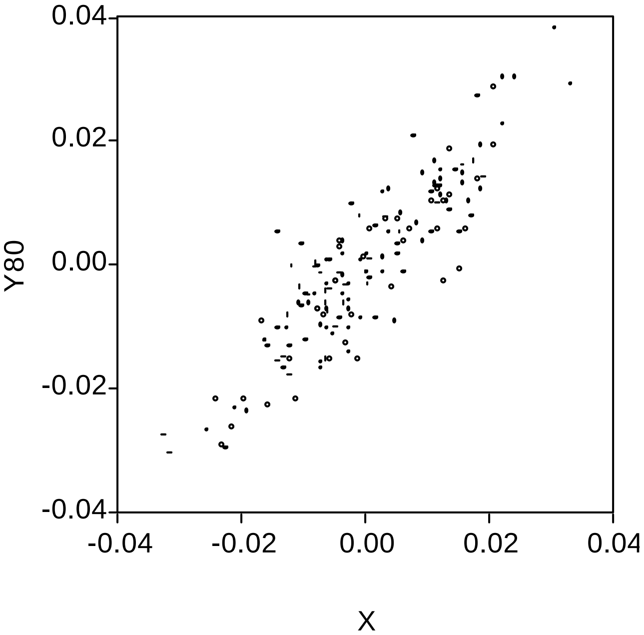
$Y_{it} = X_t + u_{it}$ where: i = 80, 50, 20, 10, $u_{it} \sim IN(0,\sigma^2_{ui})$ and X_t = USGNP

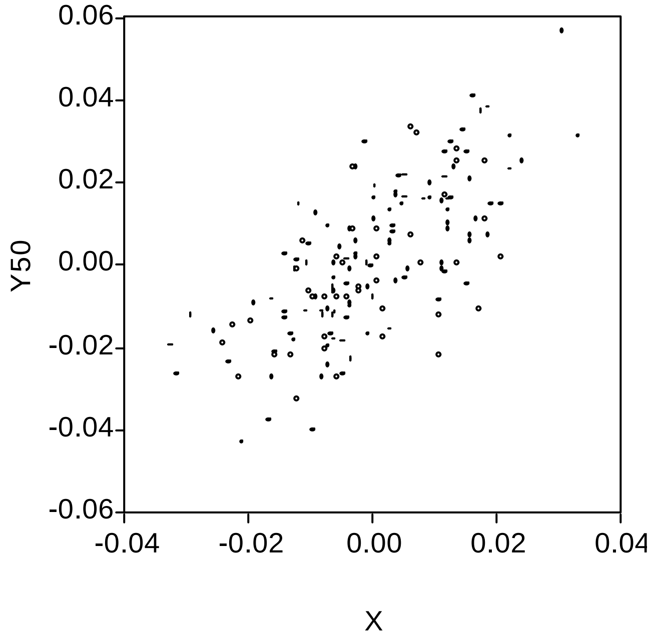
$$\sigma_{ui}^2 = 0.25\sigma_x^2 \; for R^2 = 0.80$$

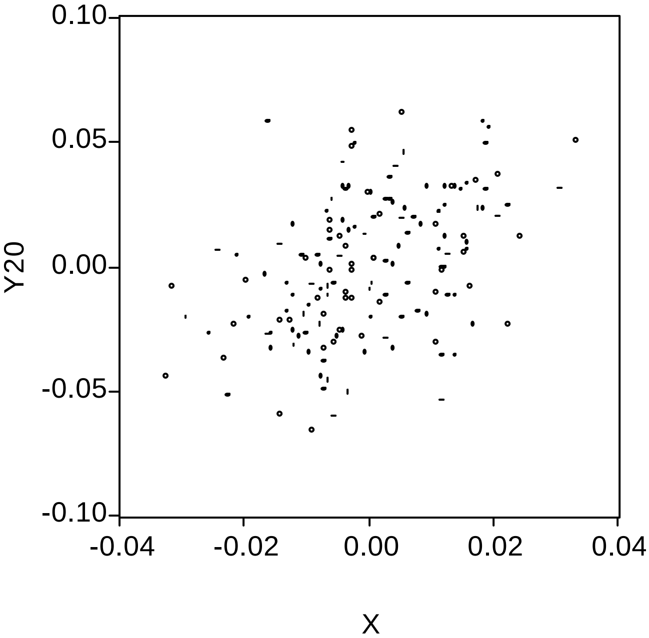
$$\sigma_{ui}^2 = \sigma_x^2 \; for R^2 = 0.50$$

$$\sigma_{ui}^2 = 4\sigma_x^2 \; for R^2 = 0.20$$

$$\sigma_{ui}^2 = 9\sigma_x^2 \; for R^2 = 0.10$$







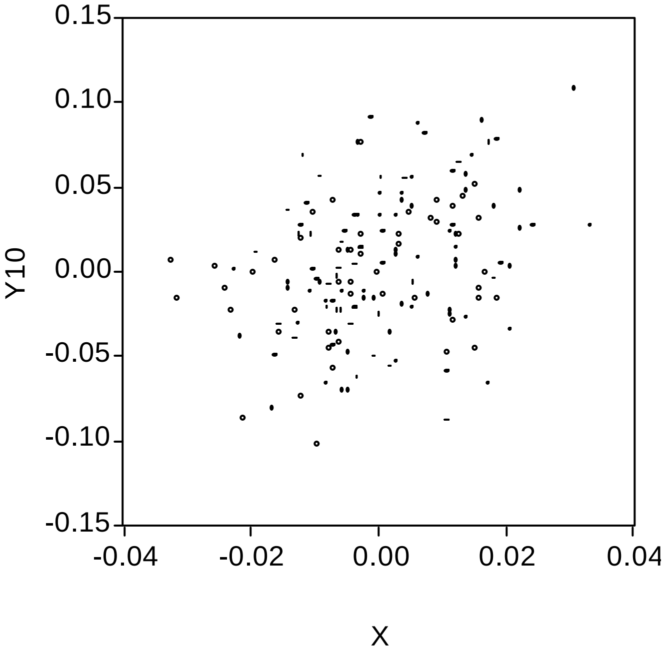


Table 1.2
Autocorrelations

T=159 j=T/4=40

Lag	X	Y10	Y20	Y50	Y80
1	0.778	0.007	0.154	0.319	0.610
2	0.512	-0.154	0.114	0.110	0.380
3	0.256	0.072	0.057	0.153	0.188
4	0.035	0.052	0.028	0.018	0.020
5	-0.163	-0.010	0.056	-0.048	-0.132
6	-0.260	-0.031	0.019	-0.091	-0.231
7	-0.299	-0.020	-0.147	-0.125	-0.248
8	-0.300	0.134	-0.264	-0.036	-0.256
9	-0.240	-0.024	-0.111	-0.147	-0.202
10	-0.180	-0.211	-0.147	-0.267	-0.106
11	-0.166	-0.012	-0.068	-0.126	-0.055
12	-0.210	0.015	-0.192	-0.093	-0.128
13	-0.209	-0.031	-0.043	-0.119	-0.109
14	-0.181	-0.159	-0.012	-0.174	-0.066
15	-0.137	0.066	0.087	-0.008	-0.058
16	-0.050	0.037	0.123	0.065	-0.108
17	-0.002	-0.010	0.156	0.028	-0.052
18	0.044	-0.100	-0.018	0.000	-0.043
19	0.054	0.017	0.023	0.054	-0.038
20	0.091	0.081	0.144	0.077	-0.029
21	0.067	-0.076	0.059	-0.033	0.046
22	0.030	-0.046	-0.067	-0.007	0.070
23	-0.005	-0.005	0.008	0.003	0.035
24	0.009	0.137	-0.085	0.084	0.097
25	0.006	0.082	0.048	0.090	0.059
26	-0.019	0.079	0.114	0.107	0.014
27	-0.013	-0.059	-0.036	0.007	0.039
28	-0.010	0.008	-0.062	0.019	-0.009
29	-0.023	-0.087	0.029	-0.044	-0.018
30	-0.038	-0.106	0.135	-0.083	0.005
31	0.001	0.014	0.040	-0.004	-0.004
32	-0.020	0.066	-0.017	0.002	0.018
33	-0.062	-0.084	-0.029	-0.100	-0.028
34	-0.052	0.058	0.036	-0.013	-0.016
35	-0.050	-0.087	-0.023	-0.142	-0.026
36	-0.053	-0.172	0.049	-0.156	0.000
37	-0.055	0.066	-0.142	0.012	-0.001
38	-0.023	-0.137	-0.128	-0.119	0.011
39	-0.008	0.018	-0.107	-0.036	-0.026
40	0.023	-0.010	0.040	-0.025	-0.026

Appendix 2: Argentine macro-aggregates

Table 2.1
Unit –Root Tests

Serie	ADF(j)
GDP	ADF(1)=-2.998*
Total Consumption	ADF(1)=-3.229*
Exports	ADF(0)=-5.164**
Imports	ADF(1)=-3.116*
M1	ADF(3)=-3.224*
Investment	ADF(1)=-2.34

All cases include the constant and j indicates the lags of the Augmented Dickey-Fuller test
*indicates significance at 5 per cent
** indicates significance at 1 per cent

Table 2.2
Autocorrelations

T=68/4=17

Lag	GDP	Total Consumption	Exports	Imports	M1	Investment
1	0.783	0.804	0.413	0.876	0.910	0.821
2	0.562	0.555	0.154	0.669	0.790	0.68
3	0.384	0.37	0.163	0.447	0.613	0.485
4	0.136	0.145	-0.039	0.213	0.424	0.247
5	-0.098	-0.038	0.046	0.009	0.199	0.025
6	-0.155	-0.096	0.073	-0.151	0.012	-0.131
7	-0.263	-0.223	-0.240	-0.281	-0.192	-0.284
8	-0.411	-0.361	-0.285	-0.382	-0.351	-0.418
9	-0.403	-0.375	-0.185	-0.409	-0.490	-0.451
10	-0.356	-0.359	-0.203	-0.422	-0.562	-0.482
11	-0.341	-0.369	-0.232	-0.452	-0.624	-0.499
12	-0.305	-0.358	-0.262	-0.461	-0.617	-0.486
13	-0.217	-0.317	-0.270	-0.433	-0.593	-0.427
14	-0.210	-0.294	-0.187	-0.379	-0.508	-0.396
15	-0.165	-0.231	-0.088	-0.328	-0.412	-0.322
16	-0.106	-0.174	-0.127	-0.268	-0.275	-0.239
17	-0.068	-0.131	-0.061	-0.215	-0.165	-0.140

Relations between macro-aggregates and GDP

Table 2.3
Volatilities and Correlations

Series	Absolute Volatility	Relative Volatility	Contemporaneous correlation
GDP	0.044		
Total Consumption	0.052	1.182	0.962 (Procyclical)
Exports	0.075	1.705	-0.602 (Countercyclical)
Imports	0.182	4.136	0.804 (Procyclical)
M1	0.646	14.681	-0.391 (Countercyclical)
Investment	0.129	2.932	0.936 (Procyclical)

Absolute volatility corresponds to the standard deviation of the series; Relative volatility represents the ratio between the absolute volatility of the variable of reference and the absolute volatility of GDP and Contemporaneous correlation measures the direction and closeness of the linear relationship between the variable of reference and the GDP.

Table 2.4
Sample Cross Correlations and Adjusted SE

	$r_{xy}(0)$	$(1.96.\ T^{-1/2})(1 + 2 \sum_{j=1}^{j=J} r_x(j).\ r_y(j))^{1/2}$
Total Consumption	0.962	0.541
Exports	-0.602	0.429
Imports	0.804	0.584
M1	-0.391	0.619

The first column reports sample cross correlations between x_t (the "c_t" of the GDP) and y_t (macro-aggregates) the second column shows autocorrelation adjusted SE times the limit (absolute value) of 95% confidence interval; J=17.