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Banking in Small Open Economies with Aggregate Liquidity Shocks

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Abstract

I extend the traditional Diamond Dybvig framework with aggregate liquidity shocks to small open economies. Currency board may imply perfect risk sharing (with perfect credit markets), contrary to Chang and Velasco 's findings (2000). With interim-date borrowing constraints and fixed exchange rates, Wallace' s (1990) partial suspension of convertibility of deposits is obtained. A banking system with an international lender may implement both allocations without runs. Flexible exchange rates with local-currency denominated deposits improves risk sharing relative to fixed exchange rates when borrowing constraints are present.

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1. Introduction

This paper presents an extension of the traditional Bryant (1981) and Diamond and Dybvig (1983) framework, embedded in a small open economy with aggregate preference shocks. In a three period economy, each consumer may become impatient (early consumer) or patient (late consumer). In the model, the number of impatient consumers in the interim period may be either high or low. As a benchmark, I first check that the socially efficient allocation with an (unlimited) international credit line from an *international lender* implies perfect risk sharing, meaning that a non-random and constant per-capita consumption for all agents. It is also obvious that the availability of international credit always allows the implementation of the optimal contract by a competitive banking system. This implementation is free of bank runs given that the international institution also acts as a lender of last resort with unlimited funds.

Since credit lines in reality are far from being unlimited as in the first case, I introduce borrowing constraints in the social planner's problem. I assume that total available international funds are bounded above. In this context it is shown that the constrained optimal contract implies partial suspension of convertibility of deposits. An inefficient bank run equilibrium reappears, provided that no extra credit is available for banks with a sufficiently illiquid long term asset when facing a panic. However, if an international lender of last resort is able to provide extra funds at an interest rate which is bounded above by a ratio of long term to short term deposits, then the run equilibrium disappears. This last result shows the importance of institutions such as the IMF as a liquidity-based-bank-run preventing device in borrowing-constrained small open economies. This issue has been one of the central topics of the discussion about the international financial architecture since the occurrence of the Asian Crisis in 1997.

This model allows an alternative interpretation of the rescue packages sent by international institutions to Argentina during the Mexican Crisis in 1995. The theoretical model have interesting implications in terms of policy recommendations in the context of the discussion about the role of international liquidity providers in preventing crises. The paper also provides an argument favoring fixed exchange rates with local lenders of last resort to complement the international funding in order to satisfy transitory liquidity needs in local currency.

This paper also adds an interesting analysis on the role of the exchange rate policy in the stability of the banking system. This work extends the discussion presented in Chang and Velasco (2000) about how flexible exchange rates make the financial system less fragile to the case of aggregate liquidity shocks. This paper shows that with flexible exchange rates it is possible to implement better allocations than with fixed exchange rates when the banking system is borrowing-constrained. Hence, fixed exchange rates or even currency boards could only implement a better allocation when no borrowing constraints are added. The partial - suspension - of - convertibility result is also replaced by a depreciation of the nominal exchange rate. Thus, the exchange rate policy is used to manage aggregate liquidity shocks concerning international currency.

This result also has important policy implications. In the context of the recent crisis in Argentina, one of the most important points of discussion was the dedollarization of the financial system and a subsequent change in the exchange rate regime. This paper shows as a corollary that a banking system with liabilities in local currency combined with flexible exchange rates is able to offer better ex-ante contracts to its depositors than with fixed exchange rates. This model predicts then that a depreciation of the local currency may be the result of an increase in the liquidity needs of the population. In other words, transitory illiquidity may imply the need of a depreciation or devaluation of local currency.

Section 2 discusses the literature on bank runs in closed and small open economies. Section 3 presents the economy. Section 4 analyzed the benchmark case in which the social planner has unrestricted credit in period 1. Section 5 adds borrowing constraints to the planner's problem of the economy in section 3. Section 6 analyzes the case of flexible exchange rates. Section 7 discusses some policy implications. Finally section 8 presents concluding remarks and points of future research.

2. Related Literature

As mentioned before, the model is built on the Bryant (1980) and Diamond and Dybvig (1983) tradition [1]. The main feature in the current paper is the presence of two currencies instead of the typical unique type of money in the literature. However the two papers share with the standard literature the potential existence of two equilibria, one involving runs.

The two main antecessors are the papers by Chang and Velasco (1998a and 2000). They construct a Bryant - Diamond - Dybvig model in a small open economy. In the first paper (1998b) the long term investment is financed partially by international borrowing (to be paid in the last period). Unlike Chang and Velasco (2000), in my model *both* types of consumers (impatient and patient) derive utility from local (real) currency holdings. The consumption for impatient agents is financed by short term international funds to be paid at the end of the economy.

Aggregate uncertainty is modelled as in Wallace (1988 and 1990). The amount

[1] For a survey on the bank runs literature see Freixas and Rochet (1997, chapter 7).

of short run withdrawals is stochastic. In the first paper Wallace (1988) shows that the two run-preventing regulatory regimes studied by Diamond and Dybvig cannot be implemented, due to the non-observability of proportion of impatient consumers. In the second paper Wallace (1990) presents a special case in which the banking system 's manager can learn the proportion through the order in which consumers withdraw in the interim period. He shows that partial suspension of convertibility in deposit contracts characterizes the planner's optimal allocation. I use this special device for the case of two currencies. More recently, Green and Lin (2000) have shown that with a finite number of depositors (and then, with *aggregate uncertainty* in terms of the proportion of each type of agents) there is a unique equilibrium involving no runs. However an important assumption in this work is the fact that depositors know their position in line, which is absent in Wallace (1990). Actually, Peck and Shell (2001) have recently shown that without this assumption, runs could be part of the optimal contract.

In terms of the empirical literature that motivates this work, the papers by Chang and Velasco's (1998b) and Corsetti, Pesenti and Roubini 's (1998) constitute competing interpretations on the causes of the recent Asian Crisis. The point of view adopted in this paper is close to what Chang and Velasco (1998b) call an *international - illiquidity - based* crisis. In a sense, what constitutes a crisis in my model can be viewed as a liquidity shock. However, as we see below, the interpretation of a *crisis* from the model is twofold. It can be viewed either as part of the fundamentals or as a self-fulfilling crisis. This discussion is better developed in section 6.

3. The Economy.

The economy lasts for three periods: $t = 0, 1, 2$. There are two currencies, called *home* currency, or *pesos*, and *foreign* currency or *dollars*. I also use the term *money* interchangeably with the term currency. There is only one consumption good, which is the numeraire. The economy is small and open. Hence the price in dollars of this good is assumed to be one, identifying then the consumption good directly with the foreign currency. The two usual technologies in the literature are assumed here. There is first a storage short run technology for the good that returns one unit in period $t + 1$ for each unit of the currency invested in period t (with $t = 0, 1$). On the other hand there is a long-term investment opportunity. For each unit of the good invested in this technology at date 0 it returns $R > 1$ units of the same type of money in period 2, but only $r \in (0, 1)$ in period 1.

There is a Lebesgue measure one of ex-ante identical consumers. At the beginning of period 1 each person receives an idiosyncratic preference shock. This determines whether the consumer survives until period 2 or dies at period 1. The ex-ante probability of dying in period 1 is π. The person who survives until date 1 is called *impatient*, otherwise she is *patient.* This probability is stochastic and unknown ex-ante. In period 0 the proportion of impatient is a random variable. For simplicity I adopt the device presented by Wallace (1990). Assume that π can be either $p\alpha + (1-p)$ with probability q_1 and $p\alpha$ with probability q_2. The complement is the set of patient agents. This is common knowledge. Any person is within the first group with probability p and within the second group with probability $(1-p)$. If a person is impatient her utility function is $u(c_1) + v(m_1)$, while if she is patient it is $u(c_2) + v(m_2)$. Here c_t denotes consumption of dollars at date t, while m_t is the consumption of pesos. The function u is C^2, strictly increasing and strictly concave. The function v is C^2 and strictly concave. However it possesses a satiation level $\bar{m} > 0$. This is taken from Chang and Velasco (2000), and it reflects the idea that liquidity in local currency has (limited) uses in local transactions. The difference with that paper is that not only patient agents but also the impatient ones derive utility from local-currency-real holdings. In some sense utility functions are symmetric unlike preferences in the cited paper.

The ex-ante utility is as follows.

$$p\left[\alpha\left[u\left(c_1^1\right) + v\left(m_1^1\right)\right] + (1-\alpha)\left(\sum_{s=1}^{2} q_s\left(u\left(c_2^1(s)\right) + v\left(m_2^1(s)\right)\right)\right)\right] \tag{3.1}$$
$$+(1-p)\left(q_1\left(u\left(c_1^2(1)\right) + v\left(m_1^2(1)\right)\right) + q_2\left(u\left(c_2^2(2)\right) + v\left(m_2^2(2)\right)\right)\right)$$

where $c_t^j(s)$ denotes consumption of dollars in period t, state s and position-in-line j, and similarly for $m_t^j(s)$. Here $s = 1$ denotes the state in which all the people in the second group are impatient, and $s = 2$ corresponds to the state in which all of them are patient. Similarly, $j = 1$ denotes the (individual) state in which the consumer is in the first group, while $j = 2$ denotes the state in which the consumer is in the second group. Note that consumption of impatient consumers within the first group does not depend on s, i.e., c_1^1 and m_1^1 are both independent of s. This is because I assume that the planner does *not* observe the aggregate state s. She must infer it from the number of impatient trying to withdraw at date 1. If only αp impatient consumers show up, the planner infers (correctly) that the proportion of impatient people is αp and then the state is $s = 1$. If the proportion exceeds αp, this must be clearly equal to $(\alpha p + (1-p))$ (provided that nobody lies). Hence whenever the planner observes that there are more impatient consumers than αp, then she infers that the state is $s = 2$. However, in any case, people who were lucky showing up first (among the first αp) should receive the same consumption in period 1 since the planner cannot know the state at that stage, due to the sequential service constraint.

4. Benchmark: The planner's problem with unlimited funds at date 1.

The planner borrows an amount of $\bar{d}$ units of the consumption good at date 0. Then she distributed this between the short run technology and the long run asset. Let y be the amount of the good invested in the short run technology. Let x denote the amount invested in the long run technology. At the beginning of period 1 the preference shock is realized. The planner does not know directly whether the proportion of impatient agents is high or low. She must learn this through the actual amount of agents withdrawing at date 1. The planner is able to differentiate patient consumers from impatient ones, though. Impatient consumers are ordered through a queue. In state 2 all impatient consumers receive the same amount of consumption c_1^1, as well as the satiation level of local currency, $\bar{m}$. Pesos are assumed to be issued by the planner at zero cost. In state 1, the first $p\alpha$ impatient consumers also get c_1^1, but the rest of impatient agents gets a (potentially different) amount $c_1^2(1)$, since the planner must learn through the queue whether the true state is 1 or 2. In each case the planner potentially borrows from abroad an additional amount of $\lambda(s)$ dollars to be repaid at date 2. In this last period, the planner pays off an amount of $c_2^j(s)$ dollars to those patient consumers in state s, as well as the amount $\bar{m}$ of pesos. This is done after repaying the total debt, at a gross rate of 1.

The problem for the planner is then to maximize 3.1 subject to the constraints

$$x + y \leqq \bar{d} \tag{4.1}$$

$$p\alpha c_1^1 + (1-p)\, c_1^2(1) \leqq y + \lambda(1) \tag{4.2}$$

$$p\alpha c_1^1 \leqq y + \lambda(2) \tag{4.3}$$

$$p(1-\alpha)\, c_2^1(1) + \left(\bar{d} + \lambda(1)\right) \leqq Rx \tag{4.4}$$

$$p(1-\alpha)\, c_2^1(2) + (1-p)\, c_2^2(2) + \left(\bar{d} + \lambda(2)\right) \leqq Rx \tag{4.5}$$

for every t, s and j. The (sufficient) first order conditions give the following result.

Proposition 4.1. *The planner's solution implies that:*

$$c_1^1 = c_1^2(1) = c_2^j(2) = c_2(1)$$

That is, there is perfect risk sharing at the solution of the planner's problem. Moreover the optimal consumption of dollars is equal to

$$\bar{c} = (R-1)\,\bar{d}$$

Clearly, the optimal amount of pesos is equal to its satiation level, $\bar{m}$.

Proof. See Appendix. ■

In this economy clearly perfect capital markets in period 1 are enough to ensure perfect risk sharing for all agents. The presence of perfect credit markets at date 1 implies market completeness. This is the underlying argument of the last proposition, which contrasts clearly with the optimality-of-partial-suspension result by Wallace (1990). In his article he showed that partial suspension is optimal in a one - currency economy with fixed endowments. Proposition 4.1 confirms the fact that Wallace's result is a consequence of some type of market incompleteness.

4.1. Implementation of the planner's solution: the case of a currency board.

This subsection shows how to implement the optimal allocation considered above through financial institutions. Consider a mutual fund bank owned by all consumers that pools resources and offers contracts to consumers. There is also a Central Bank that supplies local currency holdings. In this subsection it is assumed that this institution acts as a *currency board*. This means that the Central Bank commits itself to buy and sell dollars for pesos at the fixed rate of 1. At date 0, the private bank borrows $\bar{d}$ dollars from abroad and invests this amount in the long run and the short run asset. The commercial bank issues deposit contracts to consumers, specifying the amount of pesos to be withdrawn in the corresponding period.

At the beginning of date 1, the state s is realized. Neither the commercial bank nor the Central Bank observe this realization. Impatient consumers show up at the private bank to withdraw pesos. In this case I do not assume sequential service constraints of any kind [2]. Instead the commercial bank first gets all con-

[2] There are ways to assume sequential service constraints here. The commercial bank would learn s through the amount of (impatient) consumers who show up to withdraw. The financial intermediary then does not know the true s until the first αp agents withdraw at date 1. If then more consumers show up, then the bank interprets that $s = 2$. Otherwise the bank knows that $s = 1$. At the beginning of date 1 the commercial bank borrows from an international lender an amount to satisfy exactly the pesos to be withdrawn by impatient agents. More concretely, every time a single consumer shows up at date 1, the commercial bank borrows the amount of dollars to be paid to that agent. One possible story is that the bank operator calls the offices of the lender abroad every time the operator receives a request for withdrawal at date 1. Alternatively, the operator may manage an account with the international lender and the Central Bank so that, every time an impatient consumer shows up at date 1, the operator executes the process through the account, getting automatically an electronic transfer from abroad, and immediately transferring these dollars to the Central Bank in exchange for pesos. However, results in this section do not depend on the absence of sequential service constraints.

sumers together and then the bank borrows the corresponding amount of dollars from abroad, selling them for pesos at the Central Bank and repaying then to consumers. After the withdrawing session is finished consumers use pesos for purposes not modeled here. This gives utility $v(m_1)$. Then agents exchange pesos for dollars at the Central Bank (at an exchange rate equal to one). Impatient consumers consume dollars and disappear.

At the beginning of date 2 the bank must repay total debt using a portion of the receipts from the long run investment returns. After repaying debt, the remaining dollars are sold to the Central Bank in exchange for pesos (at an exchange rate equal to one). Then the financial intermediary repeats the same process as in period 1 with the remaining patient consumers. These receive pesos to be used before being sold to the Central Bank. Finally, agents consume the dollars sold by the Central Bank and the economy disappears.

The commercial bank then solves the same problem as the planner does with the following extra - constraints.

$$\max \quad \left\{u\left(c_1^1\right)+v\left(m_1^1\right);u\left(c_1^2(1)\right)+v\left(m_1^2(1)\right)\right\}$$
$$\leqq \max\left[\left(u\left(c_2^2(2)\right)+v\left(m_2^2(2)\right)\right),\max_s\left[\left(u\left(c_2^1(s)\right)+v\left(m_2^1(s)\right)\right)\right]\right]$$

$$\begin{aligned}
p\alpha m_1^1+(1-p)\,m_1^2(1) &\leqq p\alpha c_1^1+(1-p)\,c_1^2(1)\\
m_1^1 &\leqq c_1^1\\
m_2^1(1) &\leqq c_2^1(1)\\
p(1-\alpha)\,m_2^1(2)+(1-p)\,m_2^2(2) &\leqq p(1-\alpha)\,c_2^1(2)+(1-p)\,c_2^2(2)
\end{aligned}$$

The first inequality corresponds to the incentive compatibility constraint. The others state that the amount of pesos used in every period and state cannot exceed the amount of pesos withdrawn from the commercial bank. This amount is equal to the amount of dollars consumed since those pesos are sold at the Central Bank after being used.

It is not difficult to show the conditions under which the planner's allocation presented above can be implemented in this currency board regime. The following proposition shows when this is possible.

Proposition 4.2. *Under the stated assumptions the planner's solution can be offered in a decentralized banking system equilibrium with a currency board. If* $\bar{m} \leqq \bar{c}$, *that is*

$$R-1 \geq \frac{\bar{m}}{\bar{d}}$$

then the first best is implemented with a banking contract that specifies a withdrawal of $\bar{c}$ *pesos per consumer. Otherwise implementation under these conditions* *is possible with a contract specifying a withdrawal of $\bar{m}$ pesos with a return clause of $m - c$ pesos to the Central Bank before selling of $\bar{c}$ pesos takes place.*

The proof of this result is in the appendix. It is not difficult to see the intuition. If the optimal consumption of dollars is greater than the satiation level of pesos, with the first specified contract all consumers use only a portion of the local currency withdrawn from banks and this amount is exactly the satiation level, $\bar{m}$. After using this agents sell all pesos to the Central Bank, equal to the first best amount of dollars $\bar{c}$. On the contrary, if the inequality is the reverse, the banking system within a currency board regime can offer a contract specifying an initial withdrawal of $\bar{m}$ pesos to be used. Then a clause of a $\bar{m} - \bar{c}$ pesos return to the Central Bank must be included so that total per-capita amount of pesos to be sold is only $\bar{c}$ pesos. The amount $\bar{m} - \bar{c}$ is financed by the international lender, given absence of constraints at this stage.

In the last proposition we assumed that in period 2 the commercial bank sells dollars after repaying debt to the Central Bank and there is no credit given by the international lender at date 2. If we relax this, then the condition stated in the last proposition is not necessary. Suppose that $R - 1$ is strictly greater than $\frac{\bar{m}}{d}$. Consider the following timing of actions. Beginning date 1, commercial banks sell all dollars to be paid to impatient consumers. Those dollars come from foreign borrowing and revenues from the short run investment. The total amount of dollars are enough to cover exactly the demand for dollars *and* pesos. The idea is that the commercial bank borrows exactly $\bar{m}$ dollars multiplied by the proportion of consumers withdrawing at date 1, which are automatically sold to the Central bank at a rate of 1 in exchanged for $\bar{m}$ pesos. Hence every impatient consumer gets exactly $\bar{m}$ pesos from the bank. Consumers use these pesos and then each agent returns $\bar{m} - \bar{c}$ pesos to commercial banks. Banks sell these pesos again in exchange for dollars, which are returned immediately to the international lender (assuming that the gross rate of interest is equal to 1). The remaining amount of pesos in the hands of impatient consumers is sold to the Central Bank in exchange for the dollars bought by the monetary authority to the commercial banks. Impatient agents consume those dollars at the end of period 1.

In period 2 a similar timing of event is observed. At the beginning of this period commercial banks obtain the results of the long term investment. The intermediaries pulls apart an amount equal to the dollars owed to the international lender due to debt contracted in period 1. After this, given the assumption above, commercial borrow again a per capita amount of dollars equal to $\bar{m} - \bar{c}$ from the international lender. These are sold to the Central Bank at a rate of 1 in exchange for pesos. Commercial banks pay $\bar{m}$ to each patient consumer. After using them consumers return $\bar{m} - \bar{c}$ pesos to commercial banks, who sell these pesos to the Central Bank in exchange for dollars and then return these dollars to the international lender. Patient consumers finally sell the remaining pesos to the Central Bank in exchange for $\bar{c}$ dollars and finally consume them.

Then the following proposition arises.

Proposition 4.3. *A banking system in a currency - board regime with an international lender of last resort with unlimited credit in periods 1 and 2 is always able to implement the optimal consumption allocation.*

The proof of this result is a straightforward extension of that of the last proposition and thus omitted. The intuition is immediate. If the first best allocation satisfies $\bar{m} \leqq \bar{c}$ the international lender of last resort actually operates as in the proposition 4.1. If the reverse inequality holds, then the international lender provides $\bar{m} > \bar{c}$ dollars in period 1, state s, and $\bar{m} - \left(Rx - \bar{d}\right)$ in period 2. In both cases the satiation level of local currency $\bar{m}$ is ensured to all agents. Given this, the optimal level of borrowing by the commercial banks ensure the first best consumption allocation of dollars, $\bar{c}$.

4.2. No-runs with infinitely available credit with unit gross rate.

Assume that the international institution commits to lend at zero net interest rates any amount of dollars above $\lambda(1)$. The next result shows that this commitment is enough to eliminate the run equilibrium in both exchange rate systems.

Proposition 4.4. *Suppose that the international lender lends any amount of dollars to the banking system at zero net interest rates. Hence the run equilibrium is eliminated, independently of the exchange rate system. (In the case of the fixed exchange regime the Central Bank may still act temporarily as a local lender of last resort).*

The proof is in the Appendix. The intuition is standard. The fact that $R > 1$ implies that the long term investment is always able to honor all debt (including that corresponding to the patient consumer allocation who withdraw early). Hence no bank can fail (in the fixed exchange rate regime, the Central Bank always have enough dollars to be sold in exchange for pesos). Since the equilibrium consumption allocation of dollars satisfies the incentive compatibility constraint then all patient consumers prefer to wait (in the absence of bank failure). Thus the international lender of last resort acts as a coordinating device that ensures that the unique equilibrium is the one without runs. Hence this institution is able to implement the first best allocation as an equilibrium (for the situations in which proposition 4.2 holds).

5. The planner's problem with borrowing constraints.

The last section presents an economy with perfect international credit markets. This clearly contradicts evidence. This is seriously a problem since most of the recent crises were somehow caused by borrowing constraints. This section adds restrictions in the borrowing of the social planner (or the commercial banks in the banking system) in period 1. These constraints are somehow justified by the way institutions such as the IMF lend in practice to developing countries. In particular, official documents from the IMF (2000) confirm that members face *quotas* of credit funding, based on the member 's relative size. These quotas make plausible that even a potential planner may face credit constraints that impedes perfect risk sharing. I then extend the analysis above considering a constrained optimum problem.

More formally, assume that the social planner maximizes (3.1) subject to the constraints (4.2), (4.4), (4.5) and the following equations.

$$x + y \leqq d \tag{5.1}$$

$$d + \lambda(s) \leqq \bar{d}, \quad s = 1, 2 \tag{5.2}$$

where now d is the decision variable and $\bar{d}$ denotes now the (exogenous) total credit availability at date 1. Here it is assumed that the planner does not observe s at the beginning of period 1. Instead the impatient consumers form a line in front of the pay-off window of the planner, and this learns whether s is 1 or 2 through the amount of impatient agents who show up at date 1. In particular, if the planner only ends up receiving αp agents in period 1, then she interprets that $s = 1$. If however after paying to αp agents there is at least an extra consumer showing up for payment, then she learns that $s = 2$. The probability that, in state 1, each consumer is included in the first αp agents is equal to p. In this case, for every impatient agent who shows up in period 1 for payment the planner may borrow a certain amount of dollars from the international lender to pay off the consumer. Hence the international lender also learns the state s exactly as the social planner does. Relaxations of this assumption seem important but it involves more complicated issues on contract design. This is left for future research.

5.1. Characterization of the constrained optimum.

Due to the borrowing restrictions at date 1, the optimal consumption allocation under these constraints may not be deterministic as in the unrestricted credit case. In particular, it is possible that the consumption allocation of dollars may not be the same for the impatient consumers who are paid first and those who are paid secondly. The next proposition shows that first - in - line depositors get a higher level of consumption than the those in the second group.

Proposition 5.1. *The second best allocation implies that* $c_1^1 > c_1^2(1)$. *In this allocation all agents consume the satiation level of pesos,* $\bar{m}$.

The proof is presented in the appendix. Hence, partial suspension of convertibility (in the sence of Wallace, 1990) reappears here, even though (imperfect) credit markets are available for the planner in the interim period. This proposition implies that it is enough to impose an exogenous constraint to the planner (borrower) involving periods 0 and 1 loans to generate this result. In a sense, a binding borrowing constraint is associated with high liquidity needs in the economy. In this situation, the proposition would predict that banks should partially suspend the convertibility of certificates of deposit in an event of aggregate *illiquidity*, which is signaled by facing binding borrowing constraints.

In the proof of the last proposition it is shown that impatient agents's consumption can be financed in different ways. However, it is characterized by strictly positive borrowing in period 1, state 1, and also strictly positive investment in the short run asset and/or strictly positive borrowing at date 1, state 2. Hence it can be assumed without loss of generality that the consumption of the first αp consumers is entirely financed by the liquid investment and, in state 1, the rest of payments is financed through borrowing.

5.2. Implementation of the constrained efficient allocation

The allocation obtained above can be implemented through a similar banking system to the one presented in section 4. However I introduce an additional informational assumption for banks. In period 0 banks are formed as mutual funds. Each bank is operated on behalf of consumers. Banks borrow from a (private) international lender an amount d of dollars and they invest it in both the liquid and the illiquid asset. The idea is that commercial banks (as defined before) face the borrowing constraint (5.2) at date 1. Therefore financial intermediaries can only borrow up to $\bar{d}$ dollars in total.

The rest of the implementation depends on the assumption about $\bar{m}$ and the optimal consumption of dollars. Assume $\bar{m} \leqq c_1^2(1)$ first. Consider then the following timing of actions. At the beginning of period 1, state s is realized but not observed by anybody in this economy. Each consumer does observe her own type privately. Commercial banks, who held an amount of liquid reserves in dollars, sell them to the Central Bank in exchange for pesos. Then impatient consumers show up at the bank in a random order. If $s = 2$, only a proportion of αp consumers withdraw from the banks. Each one gets c_1^1 pesos, financed by the selling of dollars to the Central Bank. If $s = 1$, and after the first αp consumers withdraw, more consumers will show up at the bank. Then the financial institution immediately borrows $\lambda(1)$ dollars from the same lender as in period 0 and sell them to the Central Bank in exchange for pesos. Hence the rest of the impatient consumers (measure of $1-p$) get $c_1^2(1)$ pesos. Given that $\bar{m} \leqq c_1^2(1)$, each of the impatient consumers use $\bar{m}$ pesos (a portion of what each withdrew from the bank) giving her utility of $v(\bar{m})$. After this, consumers sell all pesos to the Central Bank in exchange for dollars and consume them. In period 2, commercial banks obtain the receipts from the illiquid investment. First they return all debt to be honored to the international lender. The remaining dollars are sold to the Central Bank, which are just enough to get pesos (at an exchange rate of 1) and honor all patient consumer's deposits. Each of the patient consumers gets $c_2(s)$ pesos in state s. Given that $c_1^1 \leqq c_2(s)$ for all s, then patient agents have enough pesos to satisfy the optimal satiation level $\bar{m}$. At the end of period 2, patient agents sell all pesos in exchange for dollars (at a one-to-one basis) and then consume all dollars. Hence, under the condition $\bar{m} \leqq c_1^2(1)$, a banking system within a currency board regime implements the credit - constrained optimal allocation. The next proposition summarizes this.

Proposition 5.2. *Whenever $\bar{m} \leqq c_1^2(1)$ the second best allocation is implemented as an equilibrium of a banking system similar to that of section 4.2 within a currency bard regime.*

The formal proof is omitted, since it follows similar lines as in the proof of proposition 4.2. If the condition above is *not* satisfied, a local lender of last resort, providing transitory liquidity in pesos for at least a group of impatient agents, may be needed to implement the second best allocation. Consider the following timing of actions. Actions at date 0 are identical to the system described above in this section. At the beginning of period 1, commercial banks sell all dollars from the liquid investment in exchange for pesos. If $s=2$, the Central Bank also lends to the commercial bank a per-capita amount of local currency equal to $[\max\{0; \bar{m}-c_1^1\}]\,\alpha p$, so that each impatient consumer withdraws from the commercial bank exactly $\bar{m}$ pesos. Each of these consumers is committed to return $\max\{0; \bar{m}-c_1^1\}$ pesos to the commercial bank before selling pesos to the Central Bank. If $s=1$, then private banks detect this realization of the state as long as an additional impatient consumer shows up at date 1 after the first αp agents withdrew. In this case, each commercial bank immediately borrows $\lambda(1)$ dollars from the international lender and then sell them to the Central Bank in exchange for $\lambda(1)$ pesos. In addition, the Central Bank lends an additional amount of $[\bar{m}-c_1^2(1)]\ (1-p)$ pesos to the commercial bank, so that each of these impatient consumers also withdraw $\bar{m}$ pesos. Each of these consumers is also committed to return to the private bank exactly $[\bar{m}-c_1^2(1)]$ pesos before selling pesos to the Central Bank. All these pesos are used during period 1, giving utility $v(\bar{m})$. Consumers then return to the private banks the committed amount of pesos. Commercial banks then return these pesos lent by the monetary authority, destroying those pesos. The remaining stock of local currency is sold by the impatient consumers to the Central Bank in exchange for dollars at the exchange rate of one. Impatient agents consume those dollars at the end of period 1.

In period 2 a similar timing of event is observed. At the beginning of this period commercial banks get the results of the long term investment. The intermediaries return all dollars owed to the international lender. The remaining is sold to the Central Bank in exchange for pesos. Commercial banks potentially borrow from the Central Bank so that each patient consumer withdraws $\bar{m}$ pesos. The pesos to be lent by the Central Bank in state s is equal to $\max\{0; \bar{m} - c_2(s)\}$ multiplied by the total Lebesgue measure of patient consumers in state s. Each consumer has the obligation to return the amount $\max\{0; \bar{m} - c_2(s)\}$ to the private bank before selling any pesos to the Central Bank. Consumers then use $\bar{m}$ pesos. Then patient agents return the committed amount of pesos to the commercial banks, and these repay the debt to the Central Bank. Patient consumers finally sell the remaining pesos to the Central Bank in exchange for dollars at the exchange rate of one and finally consume them.

This argument allows to show that:

Proposition 5.3. *Suppose $\bar{m} > c_1^2(1)$. Then implementation of the constrained optimum demands a fixed exchange rate regime with a local lender of last resort.*

The proof is in the appendix. As long as the satiation level of pesos satisfies both inequalities above, impatient consumers first - in - line get enough pesos as in the second best solution with a currency board. This is because commercial banks can sell a sufficient amount of dollars to the local Central Bank in order to satisfy pesos consumption with what deposit certificates give to agents. If the above inequality does not hold, the Central Bank must lend the difference to the commercial banks in at least one state and period. This loan is repaid immediately before consumers sell pesos for dollars at the Central Bank.

5.3. Bank Runs and Lenders of Last Resort

With credit constraints, it is natural to re-think how inefficient bank runs arise in equilibrium. Clearly, as it is the case in the literature the conditions for which bank runs constitute another equilibrium is linked to the return of early liquidation of the long run asset, r, as well as to the consumption allocation at the constrained optimum. (The proof is found in the appendix).

Proposition 5.4. *Assume that*

$$r < R\left(\frac{c_1^2(1)}{c_2(1)}\right)$$

Then the contract that implements the constrained optimum (whether it is done under the currency board or the fixed exchange rate regime) is subject to runs. That is, there is another equilibrium in which either commercial banks (in the currency board case) or the Central Bank (in the fixed exchange rate regime case) fail in period 1. Otherwise no failure takes place.

This is an extension of the result by Chang and Velasco (2000). Given the condition above the commercial banks need to liquidate all of the long term investment in the interim period. Recall that if more than a measure of αp consumers withdraw at date 1 then financial intermediaries and the Central Bank infer that the true state is $s = 1$. But then each consumer should get $c_1^2(1)$. However the inequality implies that total date-1 assets are lower than total dollars demanded in period 1, implying the failure of either the intermediary (in the currency board regime) or the monetary authority (in the fixed exchange rate regime, due to its role as local lender in pesos).

Thus, it is clear that the existence of an international lender of last resort constitutes a run-preventing device. Suppose that this international institution provides liquidity in dollars in period 1 above the debt constraint imposed by the private international institution, $\bar{d}$ dollars. Then, it is easy to show the following result (the proof is in the appendix).

Proposition 5.5. *Assume that an international lender of last resort is able to lend any amount whenever a threat of panic arises in the banking system. Then, as long as the gross interest rate is less than or equal to* $c_2(1) \;/\; c_1^1(2)$ *the run equilibrium is eliminated.*

The main conclusion is that this international lending institution eliminates the run equilibrium allowing for implementation of the second best allocation without panics, as long as the interest rate charged by the international lender is *not too high.* Therefore the international lender of last resort is always sufficient to prevent liquidity-based bank runs in a two-currency economy, where the local currency is used only intra-period.

5.4. An example: linear quadratic case

To illustrate these conditions I present a numerical example based on linear quadratic utility functions. Assume that

$$u(c) = -\frac{\gamma}{2}c^2 + \beta c + \delta$$

where all coefficients are strictly positive and $\beta > R\bar{d}$. This is so to assume that, on the relevant domain, $u'(c) > 0$. Assume also that

$$v(m) = -\frac{1}{2}m^2 + \pi m + \nu$$

Then:

$$\bar{m} = \pi$$

Given the utility function u above we have that the first order conditions can be reduced to the following linear system.

$$\begin{aligned}
c_1^1 - q_1 c_1^2(1) - q_2 c_2(2) &= 0 \\
-\gamma c_1^1 + \gamma R q_1 c_2(1) + \gamma R q_2 c_2(2) &= \beta(R-1) \\
R\alpha p c_1^1 + R(1-p) c_1^2(1) + p(1-\alpha) c_2(1) &= (R-1)\bar{d} \\
R\alpha p c_1^1 + (R-1)(1-p) c_1^2(1) + (p(1-\alpha) + 1 - p) c_2(2) &= (R-1)\bar{d}
\end{aligned}$$

which can be written in the following way

$$\begin{bmatrix} 1 & -q_1 & 0 & -q_2 \\ -\gamma & 0 & \gamma R q_1 & \gamma R q_1 \\ R\alpha p & R(1-p) & p(1-\alpha) & 0 \\ R\alpha p & (R-1)(1-p) & 0 & (p(1-\alpha)+1-p) \end{bmatrix} \begin{bmatrix} c_1^1 \\ c_1^2(1) \\ c_2(1) \\ c_2(2) \end{bmatrix} = \begin{bmatrix} 0 \\ \beta(R-1) \\ \bar{d}(R-1) \\ \bar{d}(R-1) \end{bmatrix}$$

The proof of this is directly derived from the first order conditions and left to the reader. As the explicit solution does not give a specially intuitive condition, I prefer to report the solutions to these problems for numerical examples. Assume the following values for the parameters.

p	α	q_1	R	β	γ
0.25	0.20	0.5	1.5	10	1

The following table shows the second best allocations for three values of $\bar{d}$.

allocation\value of $\bar{d}$	**4**	**5**	**6**
$\mathbf{c}_1^1$	1.1742	1.5419	1.9097
$\mathbf{c}_1^2(1)$	0.5548	0.9484	1.3419
$\mathbf{c}_2(1)$	6.4387	6.5871	6.7355
$\mathbf{c}_2(2)$	1.7935	2.1355	2.4774

The next table shows values of π that allows implementation by a commercial banking system with a currency board regime for each value of $\bar{d}$, given in fact by the second row $c_1^2(1)$. It also gives the upper bound for r so that illiquidity is verified. It finally has upper bounds for the interest rate charged by the international lender of last resort in order to prevent runs.

variables\value of d	**4**	**5**	**6**
π	0.5548	0.9484	1.3419
r	0.1292	0.2159	0.2988
int. rate	11.605	6.9455	5.0194

Note that the maximum interest rate that can be charged by the international lender of last resort behaves as if it is decreasing in $\bar{d}$. A possible interpretation states that, as the availability of credit in the second best problem increases, the disparity between consumption assigned to (second group) impatient agents and the patient consumers at date 2 decreases. But the upper bound of the interest rate on the credit line provided by the international liquidity provider is lower the closer are these two consumption values. Basically, the greater $\bar{d}$, the more generous is the payment to the impatient consumers in period 1 and the greater is the need of credit in this same period to prevent panics. Hence the interest rate must be less tight to implement this procedure.

6. Constrained optimum and Flexible Exchange Rates

In the last section, implementation of the constrained optimal allocation implies that, when $\bar{m} > c_1^2(1)$, impatient agents who withdrew pesos must return them to the Chang and Velasco (2000) demonstrated that, in the absence of aggregate uncertainty, a $flexible$ exchange rate regime is able to implement the optimal allocation without runs. This policy can be interpreted in fact as a threat of devaluation, since in their equilibrium the exchange rate remains fixed at 1. Hence, their model does not generate any equilibrium exchange rate depreciation. The model presented in last section can be adapted so that the implementation of a constrained optimum implies exchange rate devaluations.

Suppose an economy as in section 5. Consider the planner problem first. Assume now that the planner can observe the aggregate state s at the beginning of period 1. In this case, there is clearly only one consumption variable for that state. That is, $c_1(s)$ denotes consumption of dollars for the agent given that she is impatient in period 1 and given the state s. Hence, ex-ante preferences are now defined as follows.

$$q_1 \{[p\alpha + (1-p)] [u(c_1(1)) + v(m_1(1))] + p(1-\alpha) [u(c_2(1)) + v(m_2(1))]\}$$
$$+q_2 \{p\alpha [u(c_1(2)) + v(m_1(2))] + [p(1-\alpha) + (1-p)] [u(c_2(2)) + v(m_2(2))]\}$$

where $c_t(s)$ denotes the consumption of dollars by the agent at date $t = 1, 2$, aggregate state s, and $m_t(s)$ denotes the amount of pesos consumed at date t, state s. The problem for the planner now is to maximize this utility function subject to the same constraints as the planner in section 5 (I maintain the borrowing constraint). It is clear that the following proposition must be true (The proof is in the appendix).

Proposition 6.1. *The solution to the planner 's problem in the case of a publicly observed aggregate state s is incentive - compatible for each consumer and implies a higher ex-ante expected utility relative to that with the informational restrictions.*

Therefore, this allocation is Pareto - improving relative to that studied in last section. It is still less preferable than the one in section 4, given that $c_1(1) < c_1(2)$ at the solution. That is, optimal consumption at date 1 is stochastic.

6.1. Implementation: Flexible (Contingent) Exchange Rate Policy

This subsection embeds the same banking system described in section 5 into a flexible exchange rate regime. This exchange rate policy is similar to that used in Chang and Velasco (2000). Consider the banking system analyzed in section 5. This means that commercial banks cannot directly learn the realization of s at the beginning of date 1. With fixed exchange rates, commercial banks cannot implement the allocation in the last subsection, since the optimal allocation implies a different consumption allocation for all impatient agents in period 1, according to the realization of the aggregate state. However, a similar banking system can work here if coped with a *flexible* exchange rate.

Consider the following bank. It borrows d dollars from abroad at date 0 and invests in both the illiquid long term and the liquid short term assets. Each consumer has the right to withdraw $\bar{m}$ pesos in period 1, in any state and max $\{\bar{m}, c_2(s)\}$ at date 2, state s. At the beginning of date 1, the realization of s is not observed by anyone in the economy. The realization of types are observed only by each individual consumer. Then the Central Bank issues $(p\alpha + 1 - p)\,\bar{m}$ pesos to the commercial banks. Each commercial bank pays $\bar{m}$ pesos to the impatient consumers withdrawing in the interim period. If the aggregate state is $s = 2$, then the commercial bank pays returns to the Central Bank a total of $(1 - p)\,\bar{m}$ pesos after honoring the deposits of the $p\alpha$ impatient consumers. Each impatient agent uses the $\bar{m}$ pesos (getting utility $v(\bar{m})$). Then these consumers sell $\bar{m}$ pesos at the Central Bank in exchange for dollars. The Central Bank has learned the realization of s through the amount of withdrawals at the commercial banks.

Therefore the banking authority fixes the exchange rate between pesos and dollars in the following way

$$e_1(s) \equiv \frac{\bar{m}}{c_1(s)} \tag{6.1}$$

This implies that each impatient consumer gets exactly $c_1(s)$ dollars to consume at date 1. Finally, the same timing is observed at date 2. At the beginning of this period, the Central Bank issues max $\{\bar{m}, c_2(s)\}$ pesos multiplied by the proportion of patient consumers in state s. These pesos are delivered to the commercial banks. Each private bank pays this amount of pesos to each patient consumer withdrawing in period 2. After this, the private bank gets the revenue from the long term asset, returns foreign debt dollars and sell the remaining dollars to the Central Bank. Meanwhile, patient consumers use $\bar{m}$ pesos giving utility $v(\bar{m})$. Then, consumers sell the pesos to the Central Bank at the exchange rate

$$e_2(s) \equiv \max\left\{\frac{\bar{m}}{c_2(s)}; 1\right\} \tag{6.2}$$

It can be easily seen that each patient agent gets $c_2(s)$ dollars to be consumed at the end of period 2. Therefore, this financial system implements the optimal allocation. The next proposition summarizes this result.

Proposition 6.2. *A banking system as described in section 5 can implement the optimal allocation if the Central Bank fixes the exchange rates as in equations 6.1 in period 1 and 6.2 in period 2.*

Thus, exchange rates do depend on the state and time. In other words, the exchange rate policy that implements the optimal allocation is contingent. The important point is that, even with foreign credit restrictions and lack of information about the realization of the aggregate shock, this banking system is able to implement a better allocation than that of section 5. However, it seems that a flexible exchange rate is important for this implementation to occur.

6.2. Exchange Rate Policy as a Bank-Run Preventing Device

Chang and Velasco (2000) showed that a similar exchange rate policy prevents bank runs. In this paper, the same conclusion can be easily obtained after some adaptations of that same result.

Proposition 6.3. *Under contingent (perfectly credible) exchange rate policies, the unique equilibrium corresponds to the optimal allocation. Hence bank runs cannot occur in equilibrium.*

Proof. The Central Bank issues at the beginning of date 1 first an amount of $(p\alpha + 1 - p)\,\bar{m}$ pesos to be delivered to commercial banks. Suppose that $s = 1$. If, after the first $p\alpha + 1 - p$ people withdraw $\bar{m}$ pesos more consumers show up (this happens in the event of a run), then the commercial bank asks for an extra $p\,(1-\alpha)\,\bar{m}$ pesos to be delivered to all consumers who want pesos in that period. Suppose that only $p\,(1-\alpha) - \pi$ extra consumers withdraw. Hence the commercial bank returns the difference to the Central Bank and also (in exchange for the pesos payed to consumers) delivers $(p\alpha + 1 - p)\,c_1\,(1)$ dollars, coming from liquid assets and foreign borrowing only (there is no early liquidation of the long term asset). Hence, if total amount of circulating pesos is $[(p\alpha + 1 - p) + p\,(1-\alpha)$ $-\pi]\ \bar{m}$ pesos, the Central Bank sets simply the exchange rate equal to

$$\widehat{e}_1\,(1) \equiv \frac{[(p\alpha + 1 - p) + p\,(1-\alpha) - \pi]\bar{m}}{(p\alpha + 1 - p)\,c_1\,(1)}$$

Each consumer gets in period 1 a total of $\frac{(p\alpha+1-p)}{[(p\alpha+1-p)+p(1-\alpha)-\pi]}c_1\,(1)$ dollars, which is clearly less than $c_1\,(1)$. Since the optimal allocation implies that $c_2\,(s) \geq c_1\,(1)$, then all patients will not find optimal to withdraw at date 1.

Suppose that the true state is $s = 2$. The argument is similar to the one above. Suppose that the proportion of consumers withdrawing at date 1 is strictly greater than $p\alpha$. Let $p\alpha + \eta$ denote the actual proportion of agents withdrawing $\bar{m}$ pesos in period 1. Total amount of pesos issued at that date is then $(p\alpha + \eta)\,\bar{m}$. Again, commercial banks deliver to the Central Bank a total amount of $(p\alpha + \eta)\,c_1\,(1)$. This is so because the proportion of consumers withdrawing at date 1 is strictly higher than $p\alpha$. Banks then wrongly perceive that the state is $s = 1$. Hence they liquidate the short term asset and borrow from abroad so that banks get $(p\alpha + 1 - p)\,c_1\,(1)$ dollars to be delivered to the Central Bank. In this case, then the exchange rate is equal to

$$\begin{aligned} \widehat{e}_1\,(1) &\equiv \frac{[p\alpha + \eta]\bar{m}}{(p\alpha + \eta)\,c_1\,(1)} \\ &= \frac{\bar{m}}{c_1\,(1)} \end{aligned}$$

Thus, each patient consumer gets $c_1\,(1)$ dollars at date 1 if they run, which is (weakly) dominated by $c_2\,(s)$. Hence, no patient consumer chooses to withdraw at date 1 in state 2. This shows that the optimal allocation constitutes the unique equilibrium of this system. ■

Note that the argument uses the fact that the exchange rate policy *will* be applied in period 1 with certainty. If the Central Bank cannot commit to this policy ex-ante, then the same argument may fail, depending upon the incentives to deviate by the monetary authority. Kawamura (2001) actually shows that, in absence of aggregate uncertainty, the devaluation threat introduced by Chang and Velasco (2000) may not be credible, in the sense that a benevolent Central Bank may want to deviate from that policy. However the same result has not been extended to the aggregate uncertainty case.

7. Empirical and Policy Implications

The results in this paper sheds some light on the discussion about contingent credit lines for financial systems. Propositions 4.1 and 5.1 state that international institutions may provide funds with high liquidity needs in the short run, so that withdrawals do not have to be suspended. This is helpful to interpret the banking distress situation in Argentina, in 1995, after the Mexican crisis. The model suggests that the help from the International Monetary Fund, the Inter American Development Bank and the World Bank was mainly directed to provide funds due to a fundamental liquidity shock faced by the banking system of this country (besides the banking system restructuring process, see Camdessus, 1995). It is interesting to compare this view to the traditional self-fulfilling run interpretation of such a crisis. The banking crisis in Argentina in 1995 may be viewed as a high realization of the liquidity shock. This interpretation seems to be more consistent with the model presented here than with the usual (inefficient) equilibrium interpretation. As Chang and Velasco (1998a) and others have shown, in a traditional Diamond - Dybvig model without aggregate uncertainty a lender of last resort always prevents runs. Hence runs cannot occur in these equilibria. This is inconsistent with the evidence. Proposition 5.1 specially implies that, in state 1 (when the proportion of impatient consumers is high) banks must reduce payments at some point. It also implies that the borrowing constraint is binding. This can be interpreted as a situation in which the government negotiates an increase in loans (which is not necessary with lower liquidity needs). This negotiation actually happened in 1995 (see the document IMF News, 1995). The actual increase in Disbursements from 1994 to 1995 to Argentina was larger than 2.5 times (from 611.95 millions of ADR's to 1,558.966 millions).

On the other hand, proposition 5.3 states that an international lender may need to be coupled with a suitable local lender of last resort to implement the optimal allocation, except when the demand for local currency is not *too high*. In a sense this result suggests that both institutions could complement, not substitute, each other. However such a local lender cannot have a loose behavior. The main danger of a local lender in this situation is to worsen the foreign reserves situation having too many customers with local currency holdings selling them to the Central Bank. Thus the purpose of the local lender must be limited only to cover local currency liquidity needs. In a different model, Antinolfi, Huybens and Keister (2000) emphasize the need of restrictions imposed on the local lender. In particular, they show that a costless local lender (as it is in my model) implies a continuum of hyperinflationary equilibria. This clearly calls for either high interest rates or for upper bounds on local currency supply to rule this situation out.

It is worth emphasizing that this paper does not try to answer general questions on social optimality of a currency board compared to other exchange rate regimes. The scope of this work is more modest. However this paper predicts that, for economies with *low* level of demand for local currencies, a currency board is consistent with *full* insurance against liquidity shocks. As long as either the return on long term investments is high enough (and certain) or the availability of credit in dollars is high enough, then a currency board is able to provide perfect risk sharing to the customers of a banking system with simple peso-denominated deposit contracts. More in general this implies that countries with a very low relative demand for local currency (relative to foreign currency) may be more propense to use currency boards. Unfortunately I have found no evidence on this issue yet. However this prediction can be contrasted using indirect indicators such as foreign - currency - denominated deposit contracts. A more direct way to contrast the prediction is to look at the units of account used in several main transactions. Casual evidence in Argentina suggests that most important types of transactions (such as real estate and durables) are made using the American dollar as the unit of account. I leave this problem for future research.

The analysis in section 6 implies that, when foreign credit restrictions are binding, then the optimal allocation may call for flexible exchange rates. This statement *must* be qualified. Results in last section only refer to the virtues of flexibility of exchange rates in terms of absorbing liquidity shocks. However, there is a well-known large body of literature about the credibility issues that flexible exchange rates with loose fiscal behavior imply in emerging economies. In a sense, section 6 is not intended to defend flexible exchange rates *per-se*. On the contrary, it confirms the necessity of rules (although these are contingent) to make the policy credible (recall that the exchange rate policy in section 6 is credible due to perfect commitment by the Central Bank).

This analysis also seems helpful to reflect on the recent recipees on the recent Argentine crisis. Several policy suggestions were given as a way out of the currency board regime. Among others, perhaps the Haussman's proposal (2001) was the best known in the academic area, although this proposal has been published in a newspaper. In any case, Haussman proposed a de-dollarization of liabilities in Argentina as an essential pre-requisite to floating exchange rates. In terms of the model in section 6, this proposal seems to make sense for banks (under the caveats discussed in the paragraph above). The model predicts that flexible rates with peso - denominated bank liabilities implies a better insurance against aggregate liquidity shocks than either a fixed exchange rate, a currency board or even a dollarized[3] banking system.

A special remark is that the local lender is active only when the transitory illiquidity in local currency arises. This assumes a large degree of commitment by the institution acting as a local lender (usually the Central Bank). This also has policy implications. Implementing such mechanism in this way implies the creation of institutions or legal systems that prevents *irresponsible* behavior by the acting local lender (*creating* liquidity when there is no need of it). In the case of the first best allocation, we know that the limit of this amount of credit is equal to the difference between the per-capita desired amount of pesos and the net return on the long term investment, multiplied by the credit available in the first period, in per capita terms. This is a measurable limit for credit in pesos by the local monetary authority. Hence the setting up of a local lender of last resort demands the creation of very solid laws and institutions to avoid local lender misbehavior. Another possible way is to have international institutions such as the IMF monitoring the functioning of such local lenders.

On the other hand, as long as the funds from this credit line are used to help *transitorily illiquid* financial systems, then proposition 5.4 specifies that these credit lines, *regardless of the exchange rate regime*, always prevents runs. Also, proposition 5.4 suggests upper bounds for the interest rate that the international lender of last resort must charge in order to make repayment feasible. Once more the main problem here is to measure deposits with different horizons so that interest rates on these credit lines still allow its preventing role. I do not suggest to take these ratios literally, but they constitute a major guide for interest rate negotiations.

From the paragraphs above it is clear that implementing such institutions is not easy. Monitoring costs (in the sense of keeping track of deposits) and the problem of measuring the liquidity needs in each currency are difficult. This does not mean that they are infeasible in practice, but the implicit informational assumptions give a warning in terms of how to implement them. In any case all these regulatory regimes implied by the results deal with *liquidity* problems. It does not say anything in terms of solvency issues. The main challenge in practice is to discover whether certain financial distress phenomena were caused by liquidity or solvency problems. This still remains an open question for the policy makers.

[3] The dollarization case was not discussed since it is a trivial extension of Wallace 's (1990) work.

8. Concluding remarks and possible extensions

This paper has presented an extension of the Diamond - Dyvbig framework to a banking system with two currencies. It was also assumed that the proportion of impatient consumers is unknown (ex-ante). The first point is that a banking system within a currency board regime implements a first best allocation as an equilibrium, although this implementation depends on the type of deposit contract. Then, a local lender of last resort may be useful for implementation.

A second important message is that this model is able to predict a reduction in deposit payments when the liquidity shock is high. The model also predicts a binding borrowing constraint. These two features resembles events in which local banking systems received funds from international institutions when facing some situation of *distress*. The model also emphasizes the importance of the interest rate level on those funds. These cannot be too high so that repayment is ensured. The upper bound is related to the ratio of long term over short term deposits. This has implications on the design process of contingent credit lines.

A possible direction for future work is the construction of a version of this model in a world integrated economy with two tradable currencies, following also similar ideas as in Allen and Gale (2000). There are several issues that can be addressed with this framework. Perhaps one of the most discussed issues is the incentive to constitute the reserves for the international lender of last resort. In the paper I have presented such problem could not be studied since the economy was of the small open type. A world integrated economy with well-defined participants could help to see when each country is willing to deposit funds in an international institution.

Fundamental shocks can be introduced, making either the short term rate (as in Chang and Velasco, 1998a) or the long investment return (as in Allen and Gale, 1998) stochastic. This would allow to study solvency - based runs and the role of the lenders of last resort to prevent such runs, if these are not optimal. Nevertheless, problems of asymmetric information could worsen here. The reason is that, when returns are risky, adverse selection may not allow for availability of an international lender of last resort. This issue should be studied in combination with a world-integrated environment.

A related topic to the solvency problem is the explicit separation between managers and depositors. By study a version of this banking model in which managers do not have the same objective as the depositors the moral hazard considerations mentioned above could be seriously addressed. That is, moral hazard considerations are to be studied in settings where those objectives are discordant, since it is obvious that when they are the same hidden action problems cannot arise. There are several alternatives for modelling this. There is a vast literature on incomplete contracts in banking (see Dewatripont and Tirole, 1993 and 1994). Chang and Velasco (1998a) also present a model in which the banking sector is monopolistic. Any of these frameworks could be helpful to study moral hazard and lenders of last resort. Finally, issues on insurance schemes (in the spirit of Druck, 2000, for example) can also be considered in the international setting.

A. Proofs

A.1. Proof of Proposition 4.1

The first order conditions are as follows:

$$\begin{aligned} R\left[\phi_2(1)+\phi_2(2)\right] &= \phi_0 && \text{(A.1)}\\ \left[\phi_1(1)+\phi_1(2)\right] &\leqq \phi_0 && \text{(A.2)}\\ \phi_1(1) &\leqq \phi_2(1) && \text{(A.3)}\\ \phi_1(2) &\leqq \phi_2(2) && \text{(A.4)} \end{aligned}$$

$$\begin{aligned} u'\left(c_1^1\right) &= \phi_1(1)+\phi_1(2) && \text{(A.5)}\\ q_1 u'\left(c_1^2(1)\right) &= \phi_1(1) && \text{(A.6)}\\ q_1 u'\left(c_2^1(1)\right) &= \phi_2(1) && \text{(A.7)}\\ q_2 u'\left(c_2^j(2)\right) &= \phi_2(2) && \text{(A.8)} \end{aligned}$$

where $j = 1, 2$ in the last inequality. This last equality implies that $c_2^1(2) = c_2^2(2) \equiv c_2(2)$. The first four expressions correspond to the first derivative of the Lagrangian with respect to x, y, $\lambda(1)$ and $\lambda(2)$ respectively. The last four expressions are the first order conditions with respect to c_1^1, $c_1^2(1)$, $c_2^1(1)$ and $c_2(2)$.

I first show that $y^* = 0$. Using contradiction, assume that $y^* > 0$. Then from A.1 and A.2 we have

$$\begin{aligned} \left[\phi_1(1)+\phi_1(2)\right] &= R\left[\phi_2(1)+\phi_2(2)\right]\\ &> \left[\phi_2(1)+\phi_2(2)\right] \end{aligned}$$

On the other hand, we have

$$\begin{aligned} \phi_1(1) &\leqq \phi_2(1)\\ \phi_1(2) &\leqq \phi_2(2) \end{aligned}$$

and therefore

$$\left[\phi_1(1)+\phi_1(2)\right] \leqq \left[\phi_2(1)+\phi_2(2)\right]$$

a contradiction. Hence $y^* > 0$.

Then this implies that $\phi_1(1) = \phi_2(1)$ and $\phi_1(2) = \phi_2(2)$. From the expressions A.7, A.8 and A.5 we get

$$\begin{aligned} u'\left(c_1^2(1)\right) &= u'\left(c_2^1(1)\right) \\ u'\left(c_1^1\right) &= q_1 u'\left(c_2^1(1)\right) + q_2 u'\left(c_2(2)\right) \end{aligned}$$

Obviously, from the first equation we get $c_1^2(1) = c_2^1(1)$. On thee other hand, from the constraints (holding with strict equality)

$$p\alpha c_1^1 + (1-p)\, c_1^2(1) = \lambda(1)$$

$$p\alpha c_1^1 = \lambda(2)$$

$$p(1-\alpha)\, c_2^1(1) + \left(\bar{d} + \lambda(1)\right) = R\bar{d}$$

$$p(1-\alpha)\, c_2^1(2) + (1-p)\, c_2^2(2) + \left(\bar{d} + \lambda(2)\right) = R\bar{d}$$

implies that

$$\begin{aligned} & p(1-\alpha)\, c_2^1(1) + p\alpha c_1^1 + (1-p)\, c_1^2(1) \\ =\ & (R-1)\, \bar{d} \\ =\ & [p(1-\alpha) + (1-p)]\, c_2^2(2) + p\alpha c_1^1 \end{aligned}$$

and therefore

$$p(1-\alpha)\, c_2^1(1) + (1-p)\, c_1^2(1) = [p(1-\alpha) + (1-p)]\, c_2^2(2)$$

But $c_1^2(1) = c_2^1(1)$. This implies that $c_1^2(1) = c_2^1(1) = c_2^2(2) \equiv \bar{c}$. But then, from

$$u'\left(c_1^1\right) = q_1 u'\left(c_2^1(1)\right) + q_2 u'\left(c_2(2)\right)$$

we get that $c_1^1 = \bar{c}$, showing that perfect risk sharing is the only solution to the planner's problem.

Then, from the constraint

$$p(1-\alpha)\, c_2^1(2) + (1-p)\, c_2^2(2) + \left(\bar{d} + \lambda(2)\right) = R\bar{d}$$

we have that

$$[p(1-\alpha) + (1-p)]\, \bar{c} + \lambda(2) = (R-1)\, \bar{d}$$

But

$$\lambda(2) = p\alpha\bar{c}$$

so that

$$[p(1-\alpha) + (1-p)]\, \bar{c} + p\alpha\bar{c} = (R-1)\, \bar{d}$$

and then

$$\bar{c} = (R-1)\, \bar{d}$$

This ends the proof. ■

A.2. Proof of Proposition 4.2

Ignoring the incentive compatibility constraints for a moment, the first order conditions of the commercial banking problem can be written as follows.

$$\begin{aligned} R\left[\phi_2(1)+\phi_2(2)\right] &= \phi_0 && \text{(A.9)}\\ \left[\phi_1(1)+\phi_1(2)\right] &\leqq \phi_0 && \text{(A.10)}\\ \phi_1(1) &\leqq \phi_2(1) && \text{(A.11)}\\ \phi_1(2) &\leqq \phi_2(2) && \text{(A.12)} \end{aligned}$$

$$\begin{aligned} u'\left(c_1^1\right) &= \phi_1(1)+\phi_1(2)-\psi_1(1)-\psi_1(2) && \text{(A.13)}\\ v'\left(m_1^1\right) &= \psi_1(1)+\psi_1(2) && \text{(A.14)}\\ q_1u'\left(c_1^2(1)\right) &= \phi_1(1)-\psi_1(1) && \text{(A.15)}\\ q_1v'\left(m_1^2(1)\right) &= \psi_1(1) && \text{(A.16)}\\ q_1u'\left(c_2^1(1)\right) &= \phi_2(1)-\psi_2(1) && \text{(A.17)}\\ q_1v'\left(m_2^1(1)\right) &= \psi_2(1) && \text{(A.18)}\\ q_2u'\left(c_2^j(2)\right) &= \phi_2(2)-\psi_2(2) && \text{(A.19)}\\ q_2v'\left(m_2^j(2)\right) &= \psi_2(2) && \text{(A.20)} \end{aligned}$$

where the multipliers $\psi_1(1)$, $\psi_1(2)$, $\psi_2(1)$ and $\psi_2(2)$ are the multipliers of the following restrictions

$$\begin{aligned} p\alpha m_1^1+(1-p)\,m_1^2(1) &\leqq p\alpha c_1^1+(1-p)\,c_1^2(1) && \text{(A.21)}\\ p\alpha m_1^1 &\leqq p\alpha c_1^1 && \text{(A.22)}\\ p(1-\alpha)\,m_2^1(1) &\leqq p(1-\alpha)\,c_2^1(1) && \text{(A.23)}\\ p(1-\alpha)\,m_2^1(2)+(1-p)\,m_2^2(2) &\leqq p(1-\alpha)\,c_2^1(2)+(1-p)\,c_2^2(2) && \text{(A.24)} \end{aligned}$$

Note first that at the solution of this problem it must be that $y^*=0$ here too. Then we must have again that:

$$\begin{aligned} \phi_1(1) &= \phi_2(1) && \text{(A.25)}\\ \phi_1(2) &= \phi_2(2) && \text{(A.26)} \end{aligned}$$

and also it still true that $c_2^1(2)=c_2^2(2)$ and now $m_2^1(2)=m_2^2(2)\equiv m_2(2)$. Then:

$$\begin{aligned} q_2\left[u'(c_2(2))+v'(m_2(2))\right] &= \phi_1(2) && \text{(A.27)}\\ \left[u'\left(c_1^2(1)\right)+v'\left(m_1^2(1)\right)\right] &= \left[u'\left(c_2^1(1)\right)+v'\left(m_2^1(1)\right)\right] && \text{(A.28)} \end{aligned}$$

And also it must happen that:

$$\begin{aligned} & \left[u'\left(c_1^1\right) + v'\left(m_1^1\right)\right] \\ = & \left[\phi_1(1) + \phi_1(2)\right] \\ \leqq & R\left[\phi_2(1) + \phi_2(2)\right] \\ = & R\left\{\left[u'\left(c_2^1(1)\right) + v'\left(m_2^1(1)\right)\right] + q_2\left[u'\left(c_2(2)\right) + v'\left(m_2(2)\right)\right]\right\} \end{aligned} \tag{A.29}$$

On the other hand, from the date 2 budget constraints we need to have

$$p(1-\alpha)c_2^1(1) + (1-p)c_1^2(1) = \left[p(1-\alpha) + (1-p)\right]c_2(2) \tag{A.30}$$

Consider first the case in which $\bar{m} \leqq \bar{c}$. This implies that the solution to the bank problem is identical to the first best allocation. This is so because at $\bar{m}$ we have $v'(\bar{m}) = 0$. In this case we should have then $\psi_1(1) = \psi_1(2) = \psi_2(1) = \psi_2(2) = 0$. But then the consumption allocation that satisfies the first order conditions of the bank problem are identical to those of the social planner. But the first best allocation satisfies with equality the incentive compatibility constraint. Then whenever $\bar{m} < \bar{c}$ the first best is the solution to the commercial bank optimization problem. By strict concavity of the objective function this solution is unique.

Suppose then that $\bar{c} < \bar{m}$. Now the first best cannot be a solution to the commercial bank problem because otherwise the constraints A.21, A.22, A.23 and A.24 are violated. This shows the first part of the proposition.

Next I show the existence of an equilibrium in which $c_1^1 = c_1^2(1)$ and $m_t(s) < \bar{m}$ whenever $\bar{c} < \bar{m}$. Guessing that all four constraints A.21, A.22, A.23 and A.24 are binding it must be that $\psi_t(s) > 0$. Guessing also that $c_1^1 = c_1^2(1)$ we have that:

$$\begin{aligned} & \left[u'\left(c_1^1\right) + v'\left(m_1^1\right)\right] \\ = & \left[\phi_1(1) + \phi_1(2)\right] \\ = & q_1\left[u'\left(c_1^2(1)\right) + v'\left(m_1^2(1)\right)\right] + \phi_1(2) \\ = & q_1\left[u'\left(c_1^1\right) + v'\left(m_1^1\right)\right] + \phi_1(2) \end{aligned} \tag{A.31}$$

Then

$$\begin{aligned} q_2\left[u'\left(c_1^1\right) + v'\left(m_1^1\right)\right] &= \phi_1(2) \\ &= q_2\left[u'\left(c_2(2)\right) + v'\left(m_2(2)\right)\right] \end{aligned} \tag{A.32}$$

and then

$$\left[u'\left(c_1^1\right) + v'\left(m_1^1\right)\right] = \left[u'\left(c_2(2)\right) + v'\left(m_2(2)\right)\right] \tag{A.33}$$

But we also had

$$\begin{aligned} u'\left(c_2^1(1)\right)+v'\left(m_2^1(1)\right) &= u'\left(c_1^2(1)\right)+v'\left(m_1^2(1)\right) \\ &= u'\left(c_1^1\right)+v'\left(m_1^1\right) \\ &= u'\left(c_2(2)\right)+v'\left(m_2(2)\right) \end{aligned}$$

But then it must be the case that $c_2^1(1)=c_2(2)$. Suppose otherwise. The first case is that $c_2^1(1)<c_2(2)$. Then $u'(c_2^1(1))>u'(c_2(2))$. But then $v'(m_2^1(1))<v'(m_2(2))$. This implies that $m_2^1(1)>m_2(2)$. But then $c_2^1(1)>c_2(2)$, a contradiction. The opposite inequality works in a similar way. Then it must be the case that $c_2^1(1)=c_2(2)$ and $m_2^1(1)=m_2(2)$. This also implies from the equality A.30 that

$$c_2^1(1)=c_2(2)=c_1^2(1) \tag{A.34}$$

but then $c_1^1=c_2^1(1)\equiv c^{cb}$ and then also $m_2^1(1)=m_2(2)=m_1^2(1)=m_1^1\equiv m^{cb}$. This implies perfect risk sharing but it is clearly less preferable than the first best since the level of local currency consumption m^{cb} is less than $\bar{m}$. ■

A.3. Proof of Proposition 5.1

Firstly, it is clear that the optimal amount of local currency is $\bar{m}$, since the cost of printing pesos is always zero. Next, the necessary and sufficient first-order conditions with respect of x, y and $\lambda(s)$ of the second best problem are the following.

$$\begin{aligned} R\left[\phi_2(1)+\phi_2(2)\right] &= \phi_0 && \text{(A.35)} \\ \left[\phi_1(1)+\phi_1(2)\right] &\leqq \phi_0 && \text{(A.36)} \\ \phi_1(s) &\leqq \phi_2(s)+\tau(s) && \text{(A.37)} \end{aligned}$$

where $\tau(s)$ is the multiplier of the constraint $d+\lambda(s)\leqq\bar{d}$. The FOC corresponding to the consumption allocations are as follows.

$$\begin{aligned} u'\left(c_1^1\right) &= \phi_1(1)+\phi_1(2) && \text{(A.38)} \\ q_1u'\left(c_1^2(1)\right) &= \phi_1(1) && \text{(A.39)} \\ q_1u'\left(c_2^1(1)\right) &= \phi_2(1) && \text{(A.40)} \\ q_2u'\left(c_2^j(2)\right) &= \phi_2(2) && \text{(A.41)} \end{aligned}$$

which again implies that $c_2^1(2)=c_2^2(2)\equiv c_2(2)$. Finally, the FOC with respect to d is

$$\phi_0=\sum_{s=1}^{2}\tau(s)+\sum_{s=1}^{2}\phi_2(s) \tag{A.42}$$

This condition must hold since it must be the case that $d > 0$. Otherwise $x = 0$ but then consumption of patient agents is always zero. Hence this last FOC must hold with equality. Since the objective function is strictly concave, the solution to be characterized must be unique. I show now that this equilibrium is characterized by $\lambda(1) > 0$, $\lambda(2) \geq 0$ and $y \geq 0$. Under these conditions it must be the case that

$$\begin{aligned} R[\phi_2(1) + \phi_2(2)] &= \phi_0 && \text{(A.43)} \\ [\phi_1(1) + \phi_1(2)] &\leqq \phi_0 && \text{(A.44)} \\ \phi_1(s) &\leqq \phi_2(s) + \tau(s) && \text{(A.45)} \\ \phi_0 &= \sum_{s=1}^{2} \tau(s) + \sum_{s=1}^{2} \phi_2(s) && \text{(A.46)} \end{aligned}$$

and so

$$R[\phi_2(1) + \phi_2(2)] = \sum_{s=1}^{2} \tau(s) + \sum_{s=1}^{2} \phi_2(s) \tag{A.47}$$

This implies

$$\sum_{s=1}^{2} \tau(s) = (R-1)[\phi_2(1) + \phi_2(2)] > 0 \tag{A.48}$$

which means that for at least one s, $\tau(s) > 0$. On the other hand it must be the case that

$$R[\phi_2(1) + \phi_2(2)] = [\phi_1(1) + \phi_1(2)] \tag{A.49}$$

This is because either $y > 0$, or, if this is zero, then it must be the case that $\lambda(s) > 0$ for $s = 1, 2$. In both cases we arrive to this last equality. Hence we have:

$$u'\left(c_1^1\right) = R\left[q_1 u'(c_2(1)) + q_2 u'(c_2(2))\right] \tag{A.50}$$

On the other hand we have that if $\lambda(1) > 0$, then

$$\begin{aligned} q_1 u'\left(c_1^2(1)\right) &= q_1 u'\left(c_2^2(1)\right) + \tau(1) && \text{(A.51)} \\ &\geq q_1 u'\left(c_2^2(1)\right) \end{aligned}$$

which implies $c_1^2(1) \leqq c_2^2(1)$. Also, from the date 1 and 2 budget constraints:

$$p(1-\alpha)c_2(1) + (1-p)c_1^2(1) = [p(1-\alpha) + (1-p)]c_2(2) \tag{A.52}$$

since they hold with equality. Therefore it must be the case that

$$c_1^2(1) \leqq c_2(2) \leqq c_2^2(1) \tag{A.53}$$

However, the fact that the date 1 budget constraints hold with equality implies

$$\lambda(1) = (1-p)\, c_1^2(1) + \lambda(2) \tag{A.54}$$

but so $\lambda(1) > \lambda(2)$. On the other hand, it clear that

$$\begin{aligned} R\left[\phi_2(1) + \phi_2(2)\right] &= \sum_{s=1}^{2} \tau(s) + \sum_{s=1}^{2} \phi_2(s) = \left[\phi_1(1) + \phi_1(2)\right] \\ \phi_1(s) &\leqq \phi_2(s) + \tau(s), \quad s = 1, 2 \end{aligned}$$

But the first two equalities imply that for each s, $\phi_1(s) = \phi_2(s) + \tau(s)$. Suppose, instead, that for some s, $\phi_1(s) < \phi_2(s) + \tau(s)$. Then, summing over s we have $[\phi_1(1) + \phi_1(2)] < \sum_{s=1}^{2} \tau(s) + \sum_{s=1}^{2} \phi_2(s)$, which contradicts the second equality on the first line. Hence it must happen that $\phi_1(s) = \phi_2(s) + \tau(s)$ for every s. On the other hand, since $\lambda(1) > \lambda(2)$, clearly it is true that $\lambda(1) > 0$. Therefore it must be the case that $\tau(1) > 0$. If this were not the case, then $\tau(2)$ should be positive, since for at least one s, $\tau(s) > 0$. But then $\lambda(2) + d = \bar{d}$, but then $\lambda(1) + d > \bar{d}$, violating the constraint. Therefore $\tau(1) > 0$ and so $\lambda(1) + d = \bar{d}$. But then $\lambda(2) + d < \bar{d}$ and so $\tau(2) = 0$. This implies that:

$$\begin{aligned} u'\left(c_1^1\right) &= \left[\phi_1(1) + \phi_1(2)\right] \\ &= q_1 u'\left(c_1^2(1)\right) + \phi_1(2) \\ &= q_1 u'\left(c_1^2(1)\right) + \phi_2(2) \\ &= q_1 u'\left(c_1^2(1)\right) + q_2 u'\left(c_2(2)\right) \end{aligned} \tag{A.55}$$

and so

$$q_1\left[u'\left(c_1^1\right) - u'\left(c_1^2(1)\right)\right] = q_2\left[u'\left(c_2(2)\right) - u'\left(c_1^1\right)\right] \tag{A.56}$$

meaning that

$$sgn\left[c_1^1 - c_1^2(1)\right] = sgn\left[c_2(2) - c_1^1\right] \tag{A.57}$$

I show now that $sgn\left[c_1^1 - c_1^2(1)\right] > 0$, which proves partial suspension of convertibility. Suppose that this is not the case, that is, $sgn\left[c_1^1 - c_1^2(1)\right] < 0$. Then $c_2(2) < c_1^1$. But then $c_2(2) < c_1^2(1)$, contradicting the statement above. Therefore it must be that $c_1^1 \geq c_1^2(1)$. However, if $c_1^1 = c_1^2(1)$ then it must be true that

$$c_1^1 = c_1^2(1) = c_2(1) = c_2(2) \tag{A.58}$$

But this implies that $q_1 u'\left(c_1^2(1)\right) = q_1 u'\left(c_2^2(1)\right)$. But then, from

$$q_1 u'\left(c_1^2(1)\right) = q_1 u'\left(c_2^2(1)\right) + \tau(1) \tag{A.59}$$

then $\tau(1) = 0$, contradicting the result above. This implies that $c_1^1 > c_1^2(1)$ showing that partial suspension of convertibility of deposits must hold. This also implies that $c_2(2) > c_1^1$, showing that the optimal allocation is incentive compatible. This ends the proof of this proposition. ■

A.4. Proof of Proposition 5.3

Ignoring the incentive compatibility constraints, the problem can be written as the maximization of

$$p\left[\alpha\left[u\left(c_1^1\right)+v\left(m_1^1\right)\right]+(1-\alpha)\left(\sum_{s=1}^{2} q_s\left(u\left(c_2^1(s)\right)+v\left(m_2^1(s)\right)\right)\right)\right] \tag{A.60}$$
$$+(1-p)\left(q_1\left(u\left(c_1^2(1)\right)+v\left(m_1^2(1)\right)\right)+q_2\left(u\left(c_2^2(2)\right)+v\left(m_2^2(2)\right)\right)\right)$$

subject to

$$x+y \leqq d \tag{A.61}$$
$$p\alpha c_1^1+(1-p)\,c_1^2(1) \leqq \lambda(1)+y \tag{A.62}$$
$$p\alpha c_1^1 \leqq \lambda(2)+y \tag{A.63}$$
$$p(1-\alpha)\,c_2(1) \leqq Rx-\bar{d} \tag{A.64}$$
$$[p(1-\alpha)+(1-p)]\,c_2(2) \leqq Rx-\bar{d} \tag{A.65}$$
$$d+\lambda(s) \leqq \bar{d} \tag{A.66}$$

together with

$$\lambda(1) \leqq p\alpha w_1^1+(1-p)\,w_1^2(1) \tag{A.67}$$
$$\lambda(2) \leqq p\alpha w_1^1 \tag{A.68}$$
$$Rx-\bar{d} \leqq p(1-\alpha)\,w_2^1(1) \tag{A.69}$$
$$Rx-\bar{d} \leqq p(1-\alpha)\,w_2^1(2)+(1-p)\,w_2^2(2) \tag{A.70}$$
$$m_t^j(s) \leqq w_t^j(s) \tag{A.71}$$

The first order conditions with respect to x, y and $\lambda(s)$ are as follows.

$$R\left[\phi_2(1)+\phi_2(2)-\eta_2(1)-\eta_2(2)\right] = \phi_0 \tag{A.72}$$
$$\left[\phi_1(1)+\phi_1(2)\right] \leqq \phi_0 \tag{A.73}$$
$$\phi_1(1) \leqq \phi_2(1)+\eta_1(1) \tag{A.74}$$
$$\phi_1(2) \leqq \phi_2(2)+\eta_1(2) \tag{A.75}$$

where $\eta_1(1)$ is the multiplier of A.67, $\eta_1(2)$ that of A.68, $\eta_2(1)$ that of A.69 and $\eta_2(2)$ that of A.70. The rest of the first order conditions are as follows.

$$u'\left(c_1^1\right) = \phi_1(1)+\phi_1(2) \tag{A.76}$$
$$q_1 u'\left(c_1^2(1)\right) = \phi_1(1) \tag{A.77}$$
$$q_1 u'\left(c_2^1(1)\right) = \phi_2(1) \tag{A.78}$$
$$q_2 u'\left(c_2^j(2)\right) = \phi_2(2) \tag{A.79}$$

$$\begin{aligned}
p\alpha v'\left(m_1^1\right) &= \theta_1^1 && \text{(A.80)}\\
q_1\left(1-p\right)v'\left(m_1^2\left(1\right)\right) &= \theta_1^2\left(1\right) && \text{(A.81)}\\
q_1 p\alpha v'\left(m_2^1\left(1\right)\right) &= \theta_2\left(1\right) && \text{(A.82)}\\
q_2 v'\left(m_2\left(2\right)\right) &= \theta_2\left(2\right) && \text{(A.83)}
\end{aligned}$$

and

$$\begin{aligned}
\left[\eta_1^1+\eta_1^2\left(1\right)\right]p\alpha+\theta_1^1 &= 0 && \text{(A.84)}\\
\eta_1^2\left(1\right)p\alpha+\theta_1^2\left(1\right) &= 0 && \text{(A.85)}\\
p\left(1-\alpha\right)\eta_2\left(1\right)+\theta_2\left(1\right) &= 0 && \text{(A.86)}\\
\left[p\left(1-\alpha\right)+1-p\right]\eta_2\left(2\right)+\theta_2\left(2\right) &= 0 && \text{(A.87)}
\end{aligned}$$

where $\theta_t^j\left(s\right)$ is the multiplier of the constraints A.71. However, since all multipliers are non-negative, then the last four equations imply that $\eta_1^1=\eta_1^2\left(1\right)=\eta_2\left(1\right)=\eta_2\left(2\right)=0$. This implies that the optimal local currency holding is equal to $\bar{m}$, the satiation level. But then the first order conditions above are exactly those of the socially efficient allocation. Then the optimal dollar consumption allocation is the one given in the constrained optimum. This completes the proof. ■

A.5. Proof of proposition 5.4

The condition

$$r<R\left(\frac{c_1^2\left(1\right)}{c_2\left(1\right)}\right) \tag{A.88}$$

is equivalent to

$$r<\frac{p\left(1-\alpha\right)c_1^2\left(1\right)}{\left(\frac{p(1-\alpha)c_2(1)}{R}\right)} \tag{A.89}$$

which is true if and only if

$$r<\frac{p\left(1-\alpha\right)c_1^2\left(1\right)}{\left(\frac{p(1-\alpha)c_2(1)+\bar{d}-\bar{d}}{R}\right)} \tag{A.90}$$

However, the second best implies that

$$\bar{d}=\lambda\left(1\right)+d \tag{A.91}$$

and so

$$r<\frac{p\left(1-\alpha\right)c_1^2\left(1\right)}{\left(\frac{p(1-\alpha)c_2(1)+(\lambda(1)+d)-(\lambda(1)+d)}{R}\right)} \tag{A.92}$$

and also at the solution of the second best problem:

$$Rx - d - \lambda(1) = p(1-\alpha) c_2(1) \tag{A.93}$$

so that the last inequality is equivalent to

$$r\left(x - \frac{\lambda(1) + d}{R}\right) < p(1-\alpha) c_1^2(1) \tag{A.94}$$

But then the same argument as in proposition **??** is applied here. Suppose all patient consumers believe that the others withdraw from the commercial banks at date 1. The intermediaries pay c_1^1 to the first αp and $c_1^2(1)$ consumers, financing both by borrowing $\lambda(1)$ dollars from abroad and or with the total amount of short run investment y (if this is positive). Note again that although the true state may be $s = 2$, if $\alpha p + 1 - p$ agents show up then the commercial bank thinks that the true state is $s = 1$. If more consumers show up the intermediaries must still pay $c_1^2(1)$pesos to each one, which will be exchanged for dollars at the Central Bank. But the amount of resources left is equal to $r\left(\bar{x} - \frac{d+\lambda(1)}{R}\right)$. This happens because the intermediary liquidates $\left(\bar{x} - \frac{d+\lambda(1)}{R}\right)$ units of the long term investment at date 1 in order to satisfy withdrawals. Since this is strictly less than payments needed to be done if all the rest of agents withdraw then the bank fails (in the sense that not all consumers are satisfied, although debt at date 2 is perfectly honored).

All agents know that this happens if everybody runs against the bank. Then if every individual consumer thinks that the rest of the population runs, then it is optimal to withdraw early. This is because the expected utility of withdrawing early is strictly greater than $u(0) + v(0)$. But then in this case the commercial bank fails.

Suppose that the opposite inequality holds. I show now that there is no run. Suppose then that

$$r \geq R\left(\frac{c_1^2(1)}{c_2(1)}\right) \tag{A.95}$$

Therefore, by the same argument as before,

$$r\left(x - \frac{\lambda(1) + d}{R}\right) \geq p(1-\alpha) c_1^2(1) \tag{A.96}$$

Consider now the following situation. Suppose that a total measure of $p\alpha + 1 - p + \widehat{\pi}$ consumers try to withdraw from commercial banks at date 1, with $\widehat{\pi} < p(1-\alpha)$. Again the first $p\alpha$ consumers get c_1^1 while the rest of agents get $c_1^2(1)$. In order to satisfy withdrawals by $\widehat{\pi}$ consumers commercial banks must liquidate a portion of the long term investment. Let $\widehat{l}$ be the total liquidation of this investment. This must satisfy

$$\widehat{l} = \frac{\widehat{\pi} c_1^2(1)}{r} \tag{A.97}$$

In period 2 total per-capita resources are given by:

$$\begin{aligned} & Rx - \bar{d} - R\widehat{l} && \text{(A.98)} \\ = \ & p(1-\alpha)c_2(1) - R\frac{\widehat{\pi} c_1^2(1)}{r} \end{aligned}$$

But since $r \geq R\left(\frac{c_1^2(1)}{c_2(1)}\right)$, then $\frac{R}{r}c_1^2(1) \leqq c_2(1)$ and therefore

$$\begin{aligned} p(1-\alpha)c_2(1) - R\frac{\widehat{\pi} c_1^2(1)}{r} & \geq p(1-\alpha)c_2(1) - \widehat{\pi} c_2(1) && \text{(A.99)} \\ & = [p(1-\alpha) - \widehat{\pi}]\, c_2(1) \end{aligned}$$

This means that commercial banks have enough resources so that patient consumers waiting until period 2 consume $c_2(1)$ dollars and $\bar{m}$ pesos (provided that a local lender of last resort may cover extra local currency needs if $\bar{m} > c_2(1)$). Hence banks do not fail and patient consumers do not find optimal to run. This is because, by running they get at most c_1^1 dollars (if included first in line) that can be stored until til period 2. Since they are patient consumers, they consume a null amount of local currency. Hence each patient consumer is strictly better off by waiting until date 2. This concludes the proof. ■

A.6. Proof of Proposition 5.5

The only change in the proof of proposition 4.4 is in period 2 resources. If the interest rate is equal to ρ then the amount of dollars available at date 2 is

$$Rx - \left(\bar{d} + \lambda(1)\right) - \rho\bar{\pi}c_1^2(1) \tag{A.100}$$

By the same arguments as before this is equal to

$$p(1-\alpha)c_2(1) - \rho\hat{\pi}c_1^2(1) \tag{A.101}$$

Since $\rho \leqq c_2(1) \,/\, c_1^2(1)$ then the last expression is at least equal to $c_2(1) \geq c_1^2(1)$. This implies again that for any patient consumer it is best to wait until period 2. This ends the proof. ■

A.7. Proof of proposition 6.1

The first order condition that characterizes the optimal plan in this case are the following.

$$q(s)\,u'(c_t(s)) = \phi_t(s)$$

with $t = 1, 2$, and $s = 1, 2$,

$$\begin{aligned}
\phi_0 &= R\left[\phi_2(1) + \phi_2(2)\right] \\
\phi_0 &= \sum_{s=1}^{2} \left(\phi_2(s) + \tau(s)\right) \\
\phi_1(1) + \phi_1(2) &\leqq \phi_0 \\
\phi_1(s) &\leqq \phi_2(s) + \tau(s)\,, \quad s = 1, 2
\end{aligned}$$

where the first expression is the first derivative of the Lagrangian with respect to $c_t(s)$, the second two equalities are the derivatives of the Lagrangian with respect to x and d (both should be positive to ensure that $c_2(s) > 0$) and the two weak inequalities are the first derivatives of the Lagrangian with respect to y and $\lambda(s)$ respectively. It is clear again that either $y > 0$ or, if $y = 0$, then $\lambda(s) > 0$ for both s. Therefore

$$R\left[\phi_2(1) + \phi_2(2)\right] = \sum_{s=1}^{2} \left(\phi_2(s) + \tau(s)\right) = \phi_1(1) + \phi_1(2)$$

This implies that $\phi_1(s) = \phi_2(s) + \tau(s)$ for each s. Otherwise the second equality would be violated. The first equality implies that at least for some s, $\tau(s) > 0$, since $R > 1$. Therefore, for both s, $u'(c_2(s)) \leqq u'(c_1(s))$, with at least for some s, the inequality is strict. Hence it is true that $c_2(s) \geq c_1(s)$ for both s, and with strict inequality for at least one s. Hence the optimal solution is incentive compatible for patient consumers.

To show that this allocation is better from the ex-ante point of view for agents, consider the following. Take the optimal consumption allocation from the planner's problem in section 5. Consider the following alternative consumption allocation

$$\begin{aligned}
\widehat{c}_1(1) &\equiv \left(\frac{p\alpha}{p\alpha + 1 - p}\right) c_1^1 + \left(\frac{1-p}{p\alpha + 1 - p}\right) c_1^2(1) \\
\widehat{c}_1(2) &\equiv c_1^1 \\
\widehat{c}_2(s) &\equiv c_2(s)
\end{aligned}$$

It is clear that the *hat* allocation, together with the investment and borrowing plan $(x, y, d, \lambda(1), \lambda(2))$ from the solution satisfy all constraints. This is because

$$
\begin{aligned}
& [p\alpha + 1 - p]\, \widehat{c}_1(1) \\
= \; & [p\alpha + 1 - p] \left(\frac{p\alpha}{p\alpha + 1 - p} \right) c_1^1 + \left(\frac{1-p}{p\alpha + 1 - p} \right) c_1^2(1) \Big] \\
= \; & p\alpha c_1^1 + (1-p)\, c_2^1(1) \\
= \; & y + \lambda(1)
\end{aligned}
$$

and clearly, $\widehat{c}_1(2)$ and $\widehat{c}_2(s)$ also satisfy the other constraints by construction. However, this *hat* allocation gives a strictly higher ex-ante utility. This is because of the following. Original preferences are represented by equation 3.1. Given that the solution implies that $m_t(s) = \bar{m}$ for all t and s, then the relevant part of the utility function (the part that only includes utility for dollars) can be written as

$$
\begin{aligned}
& q_1 \left[p\alpha u\left(c_1^1\right) + (1-p)\, u\left(c_1^2(1)\right) + p(1-\alpha)\, u\left(c_2(1)\right) \right] \\
& + q_2 \left[p\alpha u\left(c_1^1\right) + \left(p(1-\alpha) + 1 - p\right) u\left(c_2(2)\right) \right]
\end{aligned}
$$

But since u is strictly concave

$$
u\left(\widehat{c}_1(1)\right) > \left(\frac{p\alpha}{p\alpha + 1 - p} \right) u\left(c_1^1\right) + \left(\frac{1-p}{p\alpha + 1 - p} \right) u\left(c_1^2(1)\right)
$$

and hence

$$
p\alpha u\left(c_1^1\right) + (1-p)\, u\left(c_1^2(1)\right) < [p\alpha + 1 - p]\, u\left(\widehat{c}_1(1)\right)
$$

Therefore ex-ante utility under the *hat* allocation is strictly higher than under the optimal solution with non public revelation of s (solution of the planner's problem of section 5). Therefore the solution to the planner's problem in section 6 must yield a strictly higher utility than the solution to the planner's problem in section 5. This ends the proof.

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