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A Simple Theoretical Framework for the Analysis of Liability Dollarization

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Abstract

This paper presents a simple model of debt contracts in order to analyze the conditions under which domestic residents would choose to denominate debts in "dollars". In the model, borrowers are producers of non-traded goods, and subject to shocks on prices. The real exchange rate varies in response to real shocks. There is a domestic unit of account; prices in terms of that unit can be shocked by a (presumably policy - induced) disturbance. Debt obligations can be denominated in either traded goods (dollarized contracts) or local currency. When real and nominal shocks are possitively correlated, dollarized contracts tend to be preferable to (non-contingent) nominal contracts when nominal shocks are large and real shocks are small.

1 Introduction

The practices concerning the denomination of financial contracts between residents of a given country vary widely from case to case. In some instances, nominal contracting is the norm, even for assets with long maturities. In other countries, agents routinely utilize indexed units in a large set of transactions. Some economies are characterized by a large-scale use of foreign currency denominations. Clearly, the denomination of assets may

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have strong effects on macroeconomic performance. In particular, shocks which can lead to slight disturbances to the contractual system if liabilities are denominated in a certain unit can induce a major breakdown with another unit of denomination. “Dollarized” financial systems are vulnerable to large movements in the real exchange rate. There is ample evidence to this effect in recent episodes. The shortcomings of liability dollarization have been much discussed, for example, in the context of the Argentine crisis. At the same time, dollarized contracts emerge as a “market outcome”. It seems important to understand what leads agents to choose such arrangements.

In any case, the denomination of contracts influences the costs and benefits of alternative courses of monetary policies: dollarization can create “fear of floating” (Calvo and Reinhart (2002)). Conversely, the choice of contractual units can be expected to depend on the anticipated behavior of monetary authorities. The interaction between the features of monetary policies and the types of contracts which are prevalent in the economy is certainly an interesting and relevant matter, but we will not deal with it here. The analysis concentrates on the decisions of private agents, taking as given the determination of “outside” shocks and policy surprises.

The notion of “original sin” has received much attention in the literature (see, for example, Haussmann and Panizza (2003)). The argument refers to the inability of many governments to borrow in international markets by issuing debt denominated in their own currency. The phenomenon seems naturally linked to the incentives that governments may have to reduce the real value of their liabilities (or those of domestic residents) against non-residents (see Chamón (2001) and Tirole (2002))[1]. We are concerned with a related but different issue, namely, the dollarization of financial transactions due to the size and variance of nominal and real shocks. This problem may also be applied to the domestic capital market, which is what Haussmann and Panizza (2003) call “domestic original sin”. These authors state that a few countries suffer from a pure domestic original sin, that is, only five countries possess a *high* proportion of foreign - currency - denominated domestic debt. However, if we take a broader measurement definition of domestic original sin (including not only currency but also maturity mismatches) then the amount of countries is much larger.

We represent in a very simple way the incentives of individuals to write

[1] In a different framework, Jeanne (1999a, 1999b) also studies this problem in models with either moral hazard or adverse selection.

contracts of one type or other. The economy produces two goods. The output of traded goods is determined exogenously. In the simplest version of the economy, the future price of non tradeables is assumed to be (exogenously) stochastic. First, these prices are scaled by a random variable that indicates in a very elementary way the potential for "monetary shocks". Producers of non-traded goods must borrow resources in order to finance their production projects from lenders who own the traded good endowments. Borrowers consume both goods, according to Cobb-Douglas preferences. Their indirect utilities depend linearly on the value of realized income in the future. Lenders only consume tradeables. Default is assumed to entail social losses. There are two states for the nominal exchange rate and two states for the real exchange rates.

We have been able to obtain the following results. With "dollar" (T - good) denominated debt contracts default may occur when the real exchange rate takes a large enough value. In particular, when these are positively correlated, default occurs when the relative price of non - tradeables (in terms of tradeables) is low if in this scenario the real exchange rate is even larger than the minimum value to induce default with dollar contracts. Hence, if there is default with high real exchange rates with nominal contracts there must also be default with dollar contracts. There may also be default with low real exchange rates under nominal contracts if the volatility of the nominal exchange rate is high enough.

In terms of borrower's preferences for any of the two contract types, we were able to arrive to the following results considering the case when the real and the nominal exchange rate are positive correlated. If no default occurs under each contract, the borrower prefers the nominal one. The reason is that the nominal debt contract offers some extra hedge that the dollar contract does not generate. When the choice is restricted to the nominal contract with no default and the dollar contract with default with high real exchange rates, then it is possible that the borrower may still prefer the dollar contract over the nominal one, provided that the default cost is low enough and the borrower's payment with the high real exchange rate is close enough to the borrower's output. Also, if the choice is between a nominal contract that leads to default with low real exchange rates and a dollar contract that implies default with high real exchange rates, then the borrower chooses the dollar contract as long as the volatility of the real exchange rate is not *too* large. This confirms the obvious statement that not only nominal volatility but also real volatility matters for this kind of choice.

There has been some recent literature regarding the relationship between monetary policy and liability dollarization, in particular, Ize and Levy - Yeyati (2003), Ize and Parrado (2002), Jeanne (2003) and Broda and Levy-Yeyati (2003). The two first papers obtain the theoretical result that the proportion of dollar - denominated debt is increasing in the inflation volatility and decreasing in the real exchange rate volatility. Jeanne shows that when the probability of a future devaluation is large enough then debt in dollar is large and default occurs as long as the devaluation is large. Our results have some analogy to these. The Broda and Levy-Yeyati model studies the optimal deposit denomination problem by a bank, which resembles the borrower's choice problem in our model. However, their main result is driven by a negative externality from dollar-contract depositors against peso-contract depositors, i.e., in equilibrium deposit dollarization is too high relative to the efficient level. Our model does not rely on externalities. On the contrary, the equilibrium allocations obtained in this paper can also be interpreted as the result of a social planner's problem facing the same informational and contractual constraints as the agents in the model.

The rest of the paper is as follows. Section 2 presents the simplest model on the basis of an analysis of cases of default in individual experiments. Section 3 presents results that contemplate the preferences of borrowers. Section 4 presents the main conclusions.

2 A simple analysis: instances of default in an individual experiment with risk-neutral lenders

This section discusses the conditions under which nominal and tradeable-goods denominated debts are subject to default, taking as given the relative price of non-tradeables and assuming that credit is supplied by agents who have linear preferences on traded goods. Also (and this is a hypothesis that is maintained throughout the paper), the distribution of nominal variables (represented by the nominal exchange rate) is supposed to be exogenous with respect to the decisions of agents regarding the standard of denomination: this ignores the potential feedbacks of the contractual pattern on the incentives of monetary policies. The set of assumptions allows to establish some simple and intuitive propositions on the "vulnerability" of contracts to shocks.

2.1 Tradeable-goods denominated contracts

Domestic producers of non-traded goods finance projects that require k units of tradeables to generate a (deterministic) quantity of non-tradeables Ak. The price of non-tradeables can take on two possible values, p_{NL} and p_{NH}, and the probability of p_{NL} is denoted by π. In case of default, the output of the firm is appropriated by the lenders, and there is a fixed cost, of ψ traded goods, paid up to a third party whose decisions are not modelled in this exercise with an individual investor. The expected gross return in terms of tradeables demanded by lenders to supply credit is normalized to 1. The mean value (in terms of traded goods) of the non-tradeable output generated by the investment project is assumed to be larger than the required investment. Then:

2.1.1 No default

This requires:

$$Ap_{NH} > Ap_{NL} > 1$$

Obviously, the "dollar" value of output in the "bad state" must be sufficient to service the debt at the "no default" interest rate. This condition refers to the levels of the values of output; for a given mean, it will be more easily satisfied the smaller is the range in which the real exhange rate moves.

2.2 Default in state L

If the borrower defaults in state L, and not in H, clearly $Ap_{NL} < 1$, since otherwise the parties would choose a contract with $R = 1$ and there will be no default in any state. Now, the interest rate is given by the arbitrage condition $(1 - \pi) R + \pi [Ap_{NL} - \phi] = 1$ (where $\phi = \frac{\psi}{k}$). This gives the following expression for the interest rate

$$R = \frac{1 - \pi Ap_{NL} + \pi\phi}{1 - \pi}$$

In addition, the value of output in the "good state" must allow repayment at that interest rate:

$$Ap_{NH} - 1 > \pi \left[A \left(p_{NH} - p_{NL} \right) + \phi \right]$$

That is, the value of output in state H must be higher enough than 1 to cover the anticipated shortfall in the "bad state", including the default cost. Another (simpler) way of expressing that condition is that the expected "dollar" value of the project must exceed the cost of credit, including the expected default costs:

$$\pi Ap_{NL} + (1 - \pi) Ap_{NH} > 1 + \pi\phi$$

2.2.1 No credit

If $\psi > 0$, the previous condition need not be satisfied even though the expected value of non-tradeable production per unit of investment is larger than 1. In that case, there may be no credit denominated in tradeables.

2.3 Nominal contracts with correlated nominal and real exchange rates

The nominal exchange rate is denominated by λ, and can take two values, λ_L and λ_H. Instead of considering the general case, we begin by assuming that high real exchange rates coincide with a high nominal exchange rate. If q_{ij} is the probability that λ has value λ_i and p_N is p_{Nj} , then assume that q_{LL} $= q_{HH} = 0$, $q_{LH} = 1 - \pi$ and $q_{HL} = \pi$. Then we may consider the following subcases.

2.3.1 No default

In this case, the nominal interest rate R_N is such that $(1 - \pi) \frac{R_N}{\lambda_L} + \pi \frac{R_N}{\lambda_H}$ $= 1$, so that,

$$R_N = \frac{\lambda_L \lambda_H}{\pi \lambda_L + (1 - \pi) \lambda_H}$$

Clearly, the no-default conditions are:

$$Ap_{NL} > \frac{R_N}{\lambda_H} \quad \text{and} \quad Ap_{NH} > \frac{R_N}{\lambda_L}$$

It can be seen, in particular, that those conditions imply the (natural) requirement that the expected value of output in terms of traded goods exceed 1, but it is not necessary that $Ap_{NL} > 1$. Given that the nominal exchange rate would be higher in the state with low real prices of non-tradeables, the condition on p_{NL} is weaker than in the case of "dollarized" contracts. For similar reasons, the condition on p_{NH} is stronger.

If $Ap_{NL} < 1$, implying that there is default with traded-good-denominated contracts, the no-default conditions for nominal contracts can be written as:

$$\frac{1-(1-\pi)\,Ap_{NH}}{\pi Ap_{NH}} < \frac{\lambda_L}{\lambda_H} < \frac{(1-\pi)\,Ap_{NL}}{1-\pi Ap_{NL}}$$

These inequalities bound the ratio between the nominal exchange rates in both states (that is, the variability of the nominal variable) as a function of the real prices of non-tradeables. Nominal contracts can "avoid" default when "dollar" denominated contracts are subject to default if changes in the nominal rate counteract the effects of "real" shocks on the prices of non-tradeables and, at the same time, the variability of the nominal exchange rate is not "too large".

It can be seen that, provided that the values of non-tradeable output satisfy the expected value condition, $\pi Ap_{NL}+ (1-\pi)Ap_{NH} > 1$, there is no default with nominal contracts if $\lambda_L p_{NH} = \lambda_H P_{NL}$, that is, if "monetary policy stabilizes" the nominal price of non-traded goods. That condition reproduces the situation that would be generated with contracts with payments indexed with the prices of non-traded goods (given that there is no uncertainty on the volume of output).

2.3.2 Default in state of high real exchange rate

In this case, there is default when $p_N = p_{NL}$. A condition for this to hold is

$$Ap_{NL} < \frac{\lambda_L}{\pi\lambda_L + (1-\pi)\,\lambda_H}$$

which is a stronger requirement than that which establishes default in the state p_{NL} with traded-goods contracts. Now, the nominal interest rate would be given by the condition $(1-\pi)\,\frac{R_N}{\lambda_L} + \pi\,(Ap_{NL} - \phi) = 1$ or,

$$R_N = \frac{\lambda_L}{1-\pi}\,[1 - \pi Ap_{NL} + \pi\phi]$$

This implies that the solvency condition in state p_{NH} has the same expression than in the case with traded-goods contracts:

$$Ap_{NH} - 1 > \pi \left[A \left(p_{NH} - p_{NL} \right) + \phi \right]$$

It can be seen that, in order to have default in the state p_{NL} with nominal contracts, there must also be default with traded-goods denominated contracts: the rise in the nominal exchange rate does not compensate for the fall in the "dollar" value of output. In fact, here, the nominal contract is equivalent to a "dollar" contract, since in both cases the traded-goods value of payments in the p_{NH} state must be equal, so that it covers the expected value of "dollar losses" resulting from the "liquidation" of the project in the state of high real exchange rate.

2.3.3 Default in the state with low real exchange rate

Nominal contracts may imply default in states where there would never be default with traded-goods contracts: this would be the case when the nominal exchange rate is "so high" in the state p_{NL} that this drives the nominal interest rate to a point where the nominal value of output in the state p_{NH} is insufficient to service the debt. Thus, the condition for default in state p_{NH} is:

$$Ap_{NH} < \frac{\lambda_H}{\pi \lambda_L + (1 - \pi) \lambda_H}$$

That is, $\frac{\lambda_H}{\lambda_L}$ has to be high enough so as to compensate for the fact that $Ap_{NH} > 1$. This corresponds to a situation with a large "nominal variability", which would tend to make nominal contracts less attractive.

In order for the revenue in state p_{NL} to allow repayment of the debt, it must be that: $Ap_{Nl} > 1 - (1 - \pi) \left[A \left(p_{NH} - p_{NL} \right) - \phi \right]$ or:

$$\pi Ap_{NL} + (1 - \pi) Ap_{NH} > 1 + (1 - \pi) \phi$$

Interestingly, this does not necessarily imply $Ap_{NL} > 1$, if ϕ is sufficiently small. Nominal impulses can then "inverse" the states of default, and make the firm solvent in state p_{NL} (which may be a state of default with T-contracts) and subject to default in state p_{NH} (which is the "good state" with T-contracts). The gross interest rate must now satisfy $\pi \frac{R_N}{\lambda_H} + (1 - \pi) \left(Ap_{NH} - \phi \right) = 1$

$$R_N = \left(\frac{\lambda_H}{\pi} \right) \left(1 + (1 - \pi) \phi - (1 - \pi) Ap_{NH} \right)$$

2.3.4 No credit

If the conditions for "solvency in at least one state" are not met, there will not be nominal contracting. In this case of correlated high real exchange rate- high nominal exchange rate, since the conditions for the firm to be insolvent in the p_{NL} state with nominal contracts imply those for insolvency in that state for T-contracts, it follows that, if there is no possibility of credit in nominal terms, "dollar" credits are also not feasible. Of course, the result does not mean that nominal contracting will "dominate" T-contracts: such contracts may imply less chances of default if "nominal variability" is large.

2.4 Nominal contracts with negatively correlated real and nominal exchange rates

This case (maybe not a "realistic" one) is that in which the movements in the nominal exchange rate amplify the price movements induced by real shocks: the currency revalues nominally when the real exchange rate is high (and, therefore, when the price of non-traded goods would tend to fall under a fixed exchange rate). Here, the assumption is: $q_{LL} = \pi$, $q_{HH} = 1 - \pi, q_{LH} = q_{HL} = 0$.

2.4.1 No default

The conditions are now:

$$Ap_{NL} > \frac{\lambda_H}{\pi\lambda_H + (1-\pi)\,\lambda_L} \quad \text{and} \quad Ap_{NH} > \frac{\lambda_L}{\pi\lambda_H + (1-\pi)\,\lambda_L}$$

The second inequality is always satisfied since, by assumption, $Ap_{NH} > 1$. The first one requires $Ap_{NL} > 1$. In this case, there can only be no default (in any state) under nominal contracting if there is never default with "dollar contracts". Also, with nominal contracts, there will never be default in state p_{NH} if there is no default in state p_{NL}.

2.4.2 Default in the state with high real exchange rate

The conditions for this case to be consistent with equilibrium are:

$$Ap_{NL} < \frac{\lambda_H}{\pi\lambda_H + (1-\pi)\,\lambda_L}$$

and

$$\pi A p_{NL} + (1-\pi) A p_{NH} > 1 + \pi\phi$$

This implies that, every time that there is default in state p_{NH} under T-contracting, there will also be default under nominal contracting. Here, there is no case in which nominal contracts "avoid" defaults that occur under "dollar contracts".

2.4.3 No credit

There can be situations where nominal credit is impossible (i.e. there would be default in any state) while there is no default at all with T-contracts. This would hold if

$$1 < A p_{NL} < \frac{\lambda_H}{\pi\lambda_H + (1-\pi)\lambda_L}$$

and

$$1 < \pi A p_{NL} + (1-\pi) A p_{NH} < 1 + \pi\phi$$

3 Ranking of contracts according to the preference of borrowers

Given the multiplicity of cases considered above, we focus only on some of them. In particular, we will take the assumption that the real exchange rate and the nominal exchange rate are positively correlated, that is, $q_{LL} = q_{HH} = 0$, $q_{LH} = 1-\pi$ and $q_{HL} = \pi$. We think that this may be the most relevant assumption given that usually nominal and rel devaluations happen together. Within this case we consider three subcases. The first one compares the nominal and T - good contract when there is no default under each unit of account. The second subcase compares both contracts when there is no default with the nominal one and when there is default when p_{NL}. The third subcase focuses on the situation with default when p_{NH} if using the T good contract and with default at p_{NL} if using the nominal contract. Given that borrowers possess Cobb-Douglas preferences, we know that the indirect utility at a state λ, p_N is given by $\frac{I(p_N,\lambda)}{\lambda p_N^{1-\alpha}}\Phi$, where $\Phi \equiv$

$\alpha^\alpha (1-\alpha)^{1-\alpha}$, $I(\lambda, p_N)$ is the nominal income received by the borrower in state (λ, p_N), and where α constitutes the weight of the utility function on tradeable goods. We now consider the expected value of this indirect utility function under each subcase.

3.1 No default under both contracts

Clearly we have here that the value of the borrower's nominal income and the indirect utility function in each state (λ, p_N) are given by:

	$T-$Good contract	Nominal Contract
$I(\lambda, p_N)$	$\lambda [Ap_N - 1]$	$\lambda A p_N - \frac{\lambda_L \lambda_H}{\pi\lambda_L + (1-\pi)\lambda_H}$
Indirect Utility	$\Phi \left[A p_N^\alpha - p_N^{\alpha-1} \right]$	$\Phi \left[A p_N^\alpha - \frac{\lambda_L \lambda_H}{\pi\lambda_L + (1-\pi)\lambda_H} \frac{p_N^{\alpha-1}}{\lambda} \right]$

Hence the expected indirect utility is in each case

$$\Phi \left[AE\left[p_N^\alpha\right] - E\left[p_N^{\alpha-1}\right] \right] \quad \text{with the } T \text{ good contract}$$
$$\Phi \left[AE\left[p_N^\alpha\right] - \pi \frac{\lambda_L}{\pi\lambda_L + (1-\pi)\lambda_H} p_{NL}^{\alpha-1} - (1-\pi) \frac{\lambda_H}{\pi\lambda_L + (1-\pi)\lambda_H} p_{NH}^{\alpha-1} \right] \quad \text{with nominal}$$

Thus, the relevant comparison is:

$$\pi p_{NL}^{\alpha-1} + (1-\pi) p_{NH}^{\alpha-1} \lesseqgtr \pi \frac{\lambda_L}{\pi\lambda_L + (1-\pi)\lambda_H} p_{NL}^{\alpha-1} + (1-\pi) \frac{\lambda_H}{\pi\lambda_L + (1-\pi)\lambda_H} p_{NH}^{\alpha-1}$$

The next result shows the following:

Proposition 1 *Under the assumption $q_{LL} = q_{HH} = 0$, $q_{LH} = 1-\pi$ and $q_{HL} = \pi$, if there is no default under any contract, then the borrower prefers (ex-ante) the nominal contract to the "dollar" contract.*

Proof. It suffices to show that

$$\pi p_{NL}^{\alpha-1} + (1-\pi) p_{NH}^{\alpha-1} > \pi \frac{\lambda_L}{\pi\lambda_L + (1-\pi)\lambda_H} p_{NL}^{\alpha-1} + (1-\pi) \frac{\lambda_H}{\pi\lambda_L + (1-\pi)\lambda_H} p_{NH}^{\alpha-1}$$

Now, we know that the latter is true if and only if

$$\begin{aligned} & \pi p_{NL}^{\alpha-1}\left[\pi\lambda_L+(1-\pi)\lambda_H\right]+(1-\pi)\,p_{NH}^{\alpha-1}\left[\pi\lambda_L+(1-\pi)\lambda_H\right] \\ > \;& \lambda_L\pi p_{NL}^{\alpha-1}+\lambda_H(1-\pi)\,p_{NH}^{\alpha-1} \end{aligned}$$

which is equivalent to:

$$\pi p_{NL}^{\alpha-1}\left[\pi\lambda_L+(1-\pi)\lambda_H-\lambda_L\right]>(1-\pi)\,p_{NH}^{\alpha-1}\left[\lambda_H-\left[\pi\lambda_L+(1-\pi)\lambda_H\right]\right]$$

$\Leftrightarrow$

$$\pi p_{NL}^{\alpha-1}(1-\pi)\left[\lambda_H-\lambda_L\right]>(1-\pi)\,p_{NH}^{\alpha-1}\pi\left[\lambda_H-\lambda_L\right]$$

and, given $\lambda_H>\lambda_L$, and $\pi\in(0,1)$, this is equivalent to

$$p_{NL}^{\alpha-1}>p_{NH}^{\alpha-1}$$

and since $p_{NH}^{\alpha-1}>0$ then this last inequality is equivalent to

$$\frac{p_{NH}^{1-\alpha}}{p_{NL}^{1-\alpha}}>1$$

But this is true since $p_{NH}>p_{NL}$. Hence we have shown that the expected indirect utility from the nominal contract is strictly greater than that with T - good contract. ■

The intuition is the following. If both contracts are available and none of them implies default, the nature of the correlation between the nominal and real exchange rates (between λ and p_N) gives rise to the following effect. With T good contracts, the interest rate does not allow the borrower to absorb the shock on λ as much as with nominal shocks. Indeed, with T good contracts, the negative term is just the expression $\frac{\pi}{p_{NL}^{1-\alpha}}+\frac{(1-\pi)}{p_{NH}^{1-\alpha}}$. Instead, with nominal contracts the negative term is $\frac{\pi}{p_{NL}^{1-\alpha}}\left[\frac{\lambda_L}{\pi\lambda_L+(1-\pi)\lambda_H}\right]+\frac{(1-\pi)}{p_{NH}^{1-\alpha}}\left[\frac{\lambda_H}{\pi\lambda_L+(1-\pi)\lambda_H}\right]$. Thus, in this second expression, when the denominator is a *low* number ($p_{NL}^{1-\alpha}$) the denominator is not equal to one (as in the dolar contract) but a number less than one. The same occurs when the denominator is a large number. In this case the numerator is also larger than one. Hence the nominal interest rate works as a *hedge* which is not the case with a dollar contract.

3.2 No default with nominal contract, default with T - good contracts with high real exchange rates

Given that the nominal contract is preferred with no default with either kind of contract, it would be intuitive to think that the nominal contract will also be preferred if there is default with dollar contracts in some state. However there are cases (if the cost of default is very small and the payment that the borrower has to make with the nominal contract in the high real exchange rate state is close enough to his output) when this intuitive proposition fails to hold.

The nominal income and indirect utility function in each state for the T good contract is:

Value of p_N	p_{NH}	p_{NL}
Nominal Income	$\lambda\left[Ap_{NH}-\left(\frac{1-\pi Ap_{NL}+\pi\phi}{1-\pi}\right)\right]$	0
Indirect Utility	$\Phi\left[Ap_{NH}^{\alpha}-\left(\frac{1-\pi Ap_{NL}+\pi\phi}{1-\pi}\right)p_{NH}^{\alpha-1}\right]$	0

Hence the ex-ante utility when using a T - good contract in this subcase is:

$$(1-\pi)\,\Phi\left[Ap_{NH}^{\alpha}-\left(\frac{1-\pi Ap_{NL}+\pi\phi}{1-\pi}\right)p_{NH}^{\alpha-1}\right]$$

while for the nominal contract it is the same expression as before. Hence it is easy to show that the borrower prefers T - good contract to the nominal contract if and only if

$$\frac{(1-\pi)}{p_{NH}^{1-\alpha}}\left[\frac{\lambda_H}{\pi\lambda_L+(1-\pi)\,\lambda_H}-\left(\frac{1-\pi Ap_{NL}+\pi\phi}{1-\pi}\right)\right]>\frac{\pi}{p_{NL}^{1-\alpha}}\left[Ap_{NL}-\frac{\lambda_L}{\pi\lambda_L+(1-\pi)\,\lambda_H}\right]$$

From here it is trivial to get the following two corollaries.

Corollary 2 *If ϕ is close enough to Ap_{NL} then the borrower prefers the nominal contract.*

Proof. If $\phi=Ap_{NL}$ then $\frac{\lambda_H}{\pi\lambda_L+(1-\pi)\lambda_H}-\left(\frac{1-\pi Ap_{NL}+\pi\phi}{1-\pi}\right)=\frac{\lambda_H}{\pi\lambda_L+(1-\pi)\lambda_H}-\frac{1}{1-\pi}$. But since $\frac{\lambda_H(1-\pi)}{\pi\lambda_L+(1-\pi)\lambda_H}<1$ then $\frac{\lambda_H}{\pi\lambda_L+(1-\pi)\lambda_H}-\frac{1}{1-\pi}$ is clearly negative. However the right hand side is strictly positive, so the inequality above does not hold when $\phi=Ap_{NL}$. The corollary follows trivially by continuity. ■

Corollary 3 *If ϕ is small enough, and as Ap_{NL} is as close to $\frac{\lambda_L}{\pi\lambda_L+(1-\pi)\lambda_H}$ as desired, then the borrower prefers the dollar contract to the nominal one.*

Proof. This is not difficult. The condition of $Ap_{NL} > \frac{R_N}{\lambda_H} = \frac{\lambda_L}{\pi\lambda_L+(1-\pi)\lambda_H}$, gives that $\frac{\lambda_H}{\pi\lambda_L+(1-\pi)\lambda_H} - \left(\frac{1-\pi Ap_{NL}}{1-\pi}\right) > 0$. Thus, for ϕ small enough then $\frac{\lambda_H}{\pi\lambda_L+(1-\pi)\lambda_H} - \left(\frac{1-\pi Ap_{NL}+\pi\phi}{1-\pi}\right) > 0$. Also, if Ap_{NL} is sufficiently close to $\frac{\lambda_L}{\pi\lambda_L+(1-\pi)\lambda_H}$ then $\left[Ap_{NL} - \frac{\lambda_L}{\pi\lambda_L+(1-\pi)\lambda_H}\right]$ is as close to 0 as desired. Then the inequality above is satisfied under these conditions. ■

The intuition here is simple. When Ap_{NL} is very close to $\frac{\lambda_L}{\pi\lambda_L+(1-\pi)\lambda_H}$ the borrower gets very little income when $p_N = p_{NL}$. On the other hand, when ϕ is small enough, the borrower gets a higher income under a T good contract since, although there is default when p_{NL}, at p_{NH} the borrower pays a smaller interest rate than with a nominal contract. Given that the cost of default is small, the insurance against default given by the nominal contract turns out to be too expensive.

3.3 Default with nominal contract when low real exchange rate and default with dollar contracts when high real exchange rate.

This subcase implies the following pattern of nominal income for each different contract

	(λ_L, p_{NH})	(λ_H, p_{NL})
Nominal	0	$\lambda Ap_{NL} - \left(\frac{\lambda_H}{\pi}\right)(1+(1-\pi)(\phi - Ap_{NH}))$
$T-$ good	$\lambda\left[Ap_{NH} - \left(\frac{1-\pi Ap_{NL}+\pi\phi}{1-\pi}\right)\right]$	0

Therefore, the ex-ante expected utility for the borrower is respectively:

- Nominal

$$\begin{aligned} & \Phi\left[\pi Ap_{NL}^{\alpha} - \frac{(1+(1-\pi)(\phi - Ap_{NH}))}{p_{NL}^{1-\alpha}}\right] \\ = \quad & \Phi\left[\pi Ap_{NL}^{\alpha} + (1-\pi)Ap_{NL}^{\alpha}\left(\frac{p_{NH}}{p_{NL}}\right) - \left(\frac{1+(1-\pi)\phi}{p_{NL}^{1-\alpha}}\right)\right] \end{aligned}$$

- T-good

$$\begin{aligned}
&\Phi\left[(1-\pi)\,Ap_{NH}^{\alpha}-\left(\frac{1-\pi Ap_{NL}+\pi\phi}{p_{NH}^{1-\alpha}}\right)\right]\\
=\;&\Phi\left[(1-\pi)\,Ap_{NH}^{\alpha}+\pi Ap_{NH}^{\alpha}\left(\frac{p_{NL}}{p_{NH}}\right)-\left(\frac{1+\pi\phi}{p_{NH}^{1-\alpha}}\right)\right]
\end{aligned}$$

Recall that here we need to have the following conditions:

$$\begin{aligned}
Ap_{NL} &< 1<1+\pi\phi, \quad \pi Ap_{NL}+(1-\pi)\,Ap_{NH}>1+\pi\phi\\
Ap_{NH} &< \frac{\lambda_H}{\pi\lambda_L+(1-\pi)\,\lambda_H}\\
\pi Ap_{NL}+(1-\pi)\,Ap_{NH} &> 1+(1-\pi)\,\phi
\end{aligned}$$

Hence it is clear that the T - good contract is preferred if and only if

$$\begin{aligned}
&A\left[p_{NH}^{\alpha}\left(1-\pi+\pi\left(\frac{p_{NL}}{p_{NH}}\right)\right)-p_{NL}^{\alpha}\left(\pi+(1-\pi)\left(\frac{p_{NH}}{p_{NL}}\right)\right)\right]\\
>\;&\left(\frac{1+\pi\phi}{p_{NH}^{1-\alpha}}\right)-\left(\frac{1+(1-\pi)\,\phi}{p_{NL}^{1-\alpha}}\right)
\end{aligned}$$

or

$$A\left[\pi p_{NL}+(1-\pi)\,p_{NH}\right]\left[p_{NH}^{\alpha-1}-p_{NL}^{\alpha-1}\right]>\left(\frac{1+\pi\phi}{p_{NH}^{1-\alpha}}\right)-\left(\frac{1+(1-\pi)\,\phi}{p_{NL}^{1-\alpha}}\right)$$

This is impossible if $\pi>\frac{1}{2}$ since then $1+\pi\phi>1+(1-\pi)\,\phi$ and this implies that

$$\begin{aligned}
\left(\frac{1+\pi\phi}{p_{NH}^{1-\alpha}}\right)-\left(\frac{1+(1-\pi)\,\phi}{p_{NL}^{1-\alpha}}\right) &> (1+\pi\phi)\left[p_{NH}^{\alpha-1}-p_{NL}^{\alpha-1}\right]\\
&> A\left[\pi p_{NL}+(1-\pi)\,p_{NH}\right]\left[p_{NH}^{\alpha-1}-p_{NL}^{\alpha-1}\right]
\end{aligned}$$

since $\left[p_{NH}^{\alpha-1}-p_{NL}^{\alpha-1}\right]<0$. Therefore a necessary condition for this to hold is $\pi<\frac{1}{2}$ (although not sufficient). In fact it is clear that.

Proposition 4 *When $\pi<\frac{1}{2}$ then the T good contract is preferred if and only if*

$$\left[\frac{p_{NH}}{p_{NL}}\right]^{1-\alpha}<\frac{A\left[\pi p_{NL}+(1-\pi)\,p_{NH}\right]-(1+\pi\phi)}{A\left[\pi p_{NL}+(1-\pi)\,p_{NH}\right]-(1+(1-\pi)\,\phi)}$$

The proof is a trivial algebraic manipulation of the inequality above and it is thus ommitted. This states that, for a borrower to find more profitable to choose the *dollar* contract in this case (when there is default with high real exchange rate than the nominal contract with default in the low real exchange rate), the volatility of p_N must be bounded above, provided that the low real exchange rate scenario is more likely than the high real exchange rate state.

3.3.1 Default with high real exchange rates with nominal contracts

In this case we know that the conditions over the values of λ and p_N are such htat

$$Ap_{NL} < \frac{\lambda_L}{\pi\lambda_L + (1-\pi)\lambda_H}$$

which implies that there is default with dollar contract. Note however that the nominal interest rate in this case is equal to $R_N = \frac{\lambda_L}{1-\pi}\left[1 - \pi Ap_{NL} + \pi\phi\right]$, whereas with T - good contracts it is equal to $\frac{1}{1-\pi}\left[1 - \pi Ap_{NL} + \pi\phi\right]$. Therefore, the ex-ante utility obtained by the borrower under any contract is the same. Thus the borrower is indifferent between the nominal and the dollar contract.

4 Indexed contracts and noisy prices in partial equilibrium.

In policy as well as academic discussions regarding debt design and the original sin problem, some researchers have suggested the need of developing financial instruments indexed to some nominal price index,w such as the CPI or similar. This framework can be used to analyze the desirability of such indexed debt contracts. The main problem is that, specially in high inflation situations, it is hard to index to the *current* inflation rate. That is, at the beginning of month t, only inflation between $t-2$ and $t-1$ can be known. Hence contracts may index to this inflation rate. However, it may be the case that the inflation between $t-1$ and t is very different from the former inflation rate. Hence indexation may lead to distorted payoffs, in the sense that this type of adjustment may imply real effects on debtors's debt repayments. This difficulty (which could only be avoided with very stable inflation rates) may reduce the incentives to contract through indexed nominal interest rates. We investigate this using our simple framework.

From the model above, suppose now that, when λp_N is realized, borrowers observe this realization completely accurately. The rest of the economy, including lenders, only observe a noisy realization instead, equal to $\lambda p_N + x$, where x constitutes a random variable, independent of both λ and p_N. In our simple scheme, we further assume that x can only take values in the set $\{-\varepsilon, 0, \varepsilon\}$, with $\varepsilon \geq 0$. Note that the case of $\varepsilon = 0$ is the case where lenders observe exactly λp_N. Assume furthermore that $\Pr(x = -\varepsilon) = \Pr(x = \varepsilon) = \frac{1}{3}$, so clearly $E(x) = 0$. The idea is that this noise variable does not introduce any bias (in an expected sense).

Debt contracts now are indexed to the public variable $\lambda p_N + x$. This means that the gross interest rate is proportional to $\lambda p_N + x$, more precisely, equal to $r^*(\lambda p_N + x)$, where r^* is a positive scalar, endogenously determined in our model. As long as borrowers do not defauly, they obtain a nominal income equal to $A\lambda p_N \bar{k} - r^*(\lambda p_N + x)\bar{k} = [\lambda p_N (A - r^*) - r^* x]\bar{k}$. Clearly, the borrower's nominal income in default is 0. A lender obtains $r^*(\lambda p_N + x)\bar{k}$ if no default occurs, and $\lambda A p_N \bar{k}$ if default happens.

Hence we can write down the first order condition for the lender:

$$\frac{1}{3}\sum_x \left\{ \sum_{i,j \in \Delta_x^{ind}} q_{ij}\left[p_{Nj} + \frac{x}{\lambda_i}\right] r^* + \sum_{i,j \in D_x^{ind}} q_{ij}\left[A p_{Nj} - \phi\right] \right\} = 1$$

where Δ_x^{ind} is the set of indices i, j such that no default occurs at x, and D_x^{ind} denotes the set where default occurs, given x. Solving for r^* we obtain that

$$r^* = \frac{1 - \frac{1}{3}\sum_x \sum_{i,j \in D_x^{ind}} q_{ij}\left[A p_{Nj} - \phi\right]}{\frac{1}{3}\sum_x \sum_{i,j \in \Delta_x^{ind}} q_{ij}\left[p_{Nj} + \frac{x}{\lambda_i}\right]} = \frac{3 - \sum_x \sum_{i,j \in D_x^{ind}} q_{ij}\left[A p_{Nj} - \phi\right]}{\sum_x \sum_{i,j \in \Delta_x^{ind}} q_{ij}\left[p_{Nj} + \frac{x}{\lambda_i}\right]}$$

As before, clearly

$$\begin{aligned} \Delta_x^{ind} &= \{i, j : \lambda_i p_{Nj}(A - r^*) \geq r^* x\} \\ D_x^{ind} &= \{i, j : \lambda_i p_{Nj}(A - r^*) < r^* x\} \end{aligned}$$

We now pursue our analysis taking the special cases considered above.

4.1 Direct correlation between nominal and real exchange rates.

This is the case of $q_{HH} = q_{LL} = 0$, and $q_{HL} = \pi$, $q_{LH} = 1 - \pi$. We consider two cases: no default and default when $x = \varepsilon$ only (for any λp_N). This last situation would emerge when the contract is indexed to a *past* inflation rate which is sufficiently higher than the current inflation rate faced by the borrower, inducing a default.

4.1.1 No default

This happens when ε is small enough so that for all realizations λp_N clearly D_{ε}^{ind}. If this condition holds, clearly there will be no default if x is either 0 or $-\varepsilon$. Therefore we need

$$\varepsilon \leq \lambda_H p_{NL} \left[\frac{A}{r^*} - 1 \right]; \ \varepsilon \leq \lambda_L p_{NH} \left[\frac{A}{r^*} - 1 \right]$$

or

$$\varepsilon \leq \min \{\lambda_H p_{NL}; \lambda_L p_{NH}\} \left[\frac{A}{r^*} - 1 \right]$$

If this is true then

$$\begin{aligned} r^*_{nodef} &= \frac{1}{\frac{\pi}{3}\left[3p_{NL} + \sum_x \left(\frac{x}{\lambda_H}\right)\right] + \frac{1-\pi}{3}\left[3p_{NH} + \sum_x \left(\frac{x}{\lambda_L}\right)\right]} \\ &= \frac{1}{\pi p_{NL} + (1-\pi) p_{NH}} \end{aligned}$$

Therefore the condition over ε can be written as

$$\varepsilon \leq \min \{\lambda_H p_{NL}; \lambda_L p_{NH}\} \left[A \left[\pi p_{NL} + (1-\pi) p_{NH} \right] - 1 \right]$$

The borrower's expected utility can be also obtained:

$$V^{b,ind}_{nodef} = \Lambda \left[\pi p^{\alpha}_{NL} + (1-\pi) p^{\alpha}_{NH}\right] \left[A - r^*\right] = \Lambda \left[\pi p^{\alpha}_{NL} + (1-\pi) p^{\alpha}_{NH}\right] \left[A - \frac{1}{\pi p_{NL} + (1-\pi) p_{NH}}\right]$$

where $\Lambda \equiv \alpha^{\alpha} (1-\alpha)^{1-\alpha}$

The next question is the following. Suppose that borrowers can actually choose between an indexed contract and a T-good denominated contract. Hence we can show that:

Proposition 5 *If no default occurs under both nominal indexed and dollar denominated contracts then borrowers choose the nominal indexed contract.*

The proof is in the appendix. This result is not surprising. Given that no default occurs under both contracts, the indexed debt possesses better risk sharing properties than the dollar contract. The next case assumes a value of ε so that default occurs for any λp_N.

4.1.2 Default when ε is large

Suppose now that as long as x is not ε no default occurs, but when $x = \varepsilon$ borrowers always default. Hence this implies

$$\varepsilon > \lambda_H p_{NL} \left[\frac{A}{r^*} - 1 \right]; \; \varepsilon > \lambda_L p_{NH} \left[\frac{A}{r^*} - 1 \right]$$

so

$$\varepsilon > \max \left\{ \lambda_H p_{NL}; \lambda_L p_{NH} \right\} \left[\frac{A}{r^*} - 1 \right]$$

Given that default occurs for $x = \varepsilon$, for any λp_N, then r^* must satisfy:

$$\begin{aligned} r^* &= \frac{1 + \frac{1}{3} \left[\phi - A \left(\pi p_{NL} + (1-\pi) p_{NH} \right) \right]}{\frac{1}{3} \left[2 \left(\pi p_{NL} + (1-\pi) p_{NH} \right) - \varepsilon \left(\frac{\pi}{\lambda_H} + \frac{1-\pi}{\lambda_L} \right) \right]} \\ &= \frac{3 + \left[\phi - A \left(\pi p_{NL} + (1-\pi) p_{NH} \right) \right]}{2 \left(\pi p_{NL} + (1-\pi) p_{NH} \right) - \varepsilon \left(\frac{\pi}{\lambda_H} + \frac{1-\pi}{\lambda_L} \right)} \end{aligned}$$

Therefore the condition on ε can be written as

$$\varepsilon > \max \left\{ \lambda_H p_{NL}; \lambda_L p_{NH} \right\} \left[\frac{A \left[2 \left(\pi p_{NL} + (1-\pi) p_{NH} \right) - \varepsilon \left(\frac{\pi}{\lambda_H} + \frac{1-\pi}{\lambda_L} \right) \right]}{3 + \left[\phi - A \left(\pi p_{NL} + (1-\pi) p_{NH} \right) \right]} - 1 \right]$$

Note that the left hand side is zero at ε^* that satisfies

$$2 \left(\pi p_{NL} + (1-\pi) p_{NH} \right) - \varepsilon^* \left(\frac{\pi}{\lambda_H p_{NL}^{1-\alpha}} + \frac{1-\pi}{\lambda_L p_{NH}^{1-\alpha}} \right) = \frac{3 + \left[\phi - A \left(\pi p_{NL} + (1-\pi) p_{NH} \right) \right]}{A}$$

$$\varepsilon^* = \frac{2\left(\pi p_{NL} + (1-\pi) p_{NH}\right) - \left(\frac{3+\phi - A(\pi p_{NL} + (1-\pi) p_{NH})}{A}\right)}{\left(\frac{\pi}{\lambda_H} + \frac{1-\pi}{\lambda_L}\right)}$$

Given that the right hand side is decreasing in ε, then the condition above holds at least for all $\varepsilon > \varepsilon^*$ (as well as for $\varepsilon < \varepsilon^*$ but sufficiently close to ε^*). Now, we also want that the parameters are such that the indexed contract leading to no default is not available. Hence

$$\varepsilon > \lambda_H p_{NL} \left[A\left(\pi p_{NL} + (1-\pi) p_{NH}\right) - 1\right]; \ \varepsilon > \lambda_L p_{NH} \left[A\left(\pi p_{NL} + (1-\pi) p_{NH}\right) - 1\right]$$

so

$$\varepsilon > \max\left\{\lambda_H p_{NL}; \lambda_L p_{NH}\right\} \left[A\left(\pi p_{NL} + (1-\pi) p_{NH}\right) - 1\right]$$

Therefore

$$\begin{aligned} \varepsilon \quad > \quad & \max\left\{\lambda_H p_{NL}; \lambda_L p_{NH}\right\} \max\left\{\left[\frac{A\left[2\left(\pi p_{NL} + (1-\pi) p_{NH}\right) - \varepsilon\left(\frac{\pi}{\lambda_H} + \frac{1-\pi}{\lambda_L}\right)\right]}{3 + \left[\phi - A\left(\pi p_{NL} + (1-\pi) p_{NH}\right)\right]} - 1\right]\right. \\ & \left.\left[A\left(\pi p_{NL} + (1-\pi) p_{NH}\right) - 1\right]\right\} \end{aligned}$$

Suppose that this holds. Then the utility for the borrower is

$$\begin{aligned} V^{b,ind}_{def,\varepsilon} \quad = \quad & \frac{\Lambda}{3} 2\left[\pi p^{\alpha}_{NL} + (1-\pi)^{\alpha} p_{NH}\right] \left[A - \left(\frac{3 + \phi - A\left(\pi p_{NL} + (1-\pi) p_{NH}\right)}{2\left(\pi p_{NL} + (1-\pi) p_{NH}\right) - \varepsilon\left(\frac{\pi}{\lambda_H} + \frac{1-\pi}{\lambda_L}\right)}\right)\right] \\ & + \frac{\Lambda}{3} \varepsilon \left[\frac{\pi}{\lambda_H p^{1-\alpha}_{NL}} + \frac{1-\pi}{\lambda_L p^{1-\alpha}_{NH}}\right] \left(\frac{3 + \phi - A\left(\pi p_{NL} + (1-\pi) p_{NH}\right)}{2\left(\pi p_{NL} + (1-\pi) p_{NH}\right) - \varepsilon\left(\frac{\pi}{\lambda_H} + \frac{1-\pi}{\lambda_L}\right)}\right) \end{aligned}$$

Comparison with a T-good contract without default. The next question is whether a borrower finds more profitable to use an indexed contract that induces default at ε or a T-good contract that induces no default. For this, the utility for the borrower of this last type of debt contract is

$$V^{b,T}_{nodef} = \Lambda\left[A\left[\pi p^{\alpha}_{NL} + (1-\pi) p^{\alpha}_{NH}\right] - \left[\pi p^{\alpha-1}_{NL} + (1-\pi) p^{\alpha-1}_{NH}\right]\right]$$

Hence the T-good contract is preferrable if and only if

$$\begin{aligned}
& A\left[\pi p_{NL}^{\alpha}+(1-\pi)\,p_{NH}^{\alpha}\right]-\left[\pi p_{NL}^{\alpha-1}+(1-\pi)\,p_{NH}^{\alpha-1}\right] \\
\geq\ & \frac{2}{3}\left[\pi p_{NL}^{\alpha}+(1-\pi)\,p_{NH}^{\alpha}\right]\left[A-\left(\frac{3+\phi-A\left(\pi p_{NL}+(1-\pi)\,p_{NH}\right)}{2\left(\pi p_{NL}+(1-\pi)\,p_{NH}\right)-\varepsilon\left(\frac{\pi}{\lambda_H}+\frac{1-\pi}{\lambda_L}\right)}\right)\right] \\
& \frac{\varepsilon}{3}\left[\frac{\pi}{\lambda_H p_{NL}^{1-\alpha}}+\frac{1-\pi}{\lambda_L p_{NH}^{1-\alpha}}\right]\left(\frac{3+\phi-A\left(\pi p_{NL}+(1-\pi)\,p_{NH}\right)}{2\left(\pi p_{NL}+(1-\pi)\,p_{NH}\right)-\varepsilon\left(\frac{\pi}{\lambda_H}+\frac{1-\pi}{\lambda_L}\right)}\right)
\end{aligned}$$

Rearranging:

$$\frac{A}{3}\geq\frac{\pi p_{NL}^{\alpha-1}+(1-\pi)\,p_{NH}^{\alpha-1}+\frac{1}{3}\left[\frac{(3+\phi)\left[\varepsilon\left(\frac{\pi}{\lambda_H p_{NL}^{1-\alpha}}+\frac{1-\pi}{\lambda_L p_{NH}^{1-\alpha}}\right)-2\left[\pi p_{NL}^{\alpha}+(1-\pi)p_{NH}^{\alpha}\right]\right]}{2(\pi p_{NL}+(1-\pi)p_{NH})-\varepsilon\left(\frac{\pi}{\lambda_H}+\frac{1-\pi}{\lambda_L}\right)}\right]}{\pi p_{NL}^{\alpha}+(1-\pi)\,p_{NH}^{\alpha}+\left[\frac{(\pi p_{NL}+(1-\pi)p_{NH})\left[\varepsilon\left(\frac{\pi}{\lambda_H p_{NL}^{1-\alpha}}+\frac{1-\pi}{\lambda_L p_{NH}^{1-\alpha}}\right)-2\left[\pi p_{NL}^{\alpha}+(1-\pi)p_{NH}^{\alpha}\right]\right]}{2(\pi p_{NL}+(1-\pi)p_{NH})-\varepsilon\left(\frac{\pi}{\lambda_H}+\frac{1-\pi}{\lambda_L}\right)}\right]}$$

This is true in particular for A large enough. On the other hand, we had that

$$\begin{aligned}
\varepsilon\ >\ & \max\left\{\lambda_H p_{NL};\lambda_L p_{NH}\right\}\max\left\{\left[\frac{A\left[2\left(\pi p_{NL}+(1-\pi)\,p_{NH}\right)-\varepsilon\left(\frac{\pi}{\lambda_H}+\frac{1-\pi}{\lambda_L}\right)\right]}{3+\left[\phi-A\left(\pi p_{NL}+(1-\pi)\,p_{NH}\right)\right]}-1\right]\right. \\
& \left.\left[A\left(\pi p_{NL}+(1-\pi)\,p_{NH}\right)-1\right]\right\}
\end{aligned}$$

All these three inequalities involving A, ϕ and ε imply that the borrower prefers the $T-$ good contract (with no default).

Note in particular that, for ϕ is large enough, it is not necessarily true that the borrower prefers the $T-$good contract. The reason is that even though the indexed interest rate is an increasing function of ϕ, the ex-ante utility for the borrower of this contract is not necessarily decreasing in the interest rate. This is because there exists a state of nature in which $x=-\varepsilon$, and so the borrower pays less than the *true* value of Ap_N. This cost-savings are larger the larger is the interest rate. Hence, only when

$$\varepsilon\left(\frac{\pi}{\lambda_H p_{NL}^{1-\alpha}}+\frac{1-\pi}{\lambda_L p_{NH}^{1-\alpha}}\right)-2\left[\pi p_{NL}^{\alpha}+(1-\pi)\,p_{NH}^{\alpha}\right]<0$$

or

$$\varepsilon < \frac{2\left[\pi p_{NL}^{\alpha} + (1-\pi)\, p_{NH}^{\alpha}\right]}{\left(\frac{\pi}{\lambda_H p_{NL}^{1-\alpha}} + \frac{1-\pi}{\lambda_L p_{NH}^{1-\alpha}}\right)}$$

it is true that for ϕ large enough (and A fixed) the borrower prefers the T-good contract wityh no default to the indexed contract with default at $x = \varepsilon$.

Case of default with T-good contracts Suppose now that the comparison is between an indexed contract with default at $x = \varepsilon$ and a T-good contract that induces default at $\lambda_H p_{NL}$. In such a case the indirect utility of this T-good contract is

$$V_{T-def}^{B} = \Lambda p_{NH}^{\alpha}\left[(1-\pi)\, A - \frac{(1 - \pi A p_{NL} + \pi\phi)}{p_{NH}}\right]$$

Hence this T-good contract is preferred to the indexed contract if and only if

$$\begin{aligned}
& p_{NH}^{\alpha}\left[(1-\pi)\, A - \frac{(1 - \pi A p_{NL} + \pi\phi)}{p_{NH}}\right] \\
\geq\ & \frac{2}{3}\left[\pi p_{NL}^{\alpha} + (1-\pi)\, p_{NH}^{\alpha}\right]\left[A - \left(\frac{3 + \phi - A\left(\pi p_{NL} + (1-\pi)\, p_{NH}\right)}{2\left(\pi p_{NL} + (1-\pi)\, p_{NH}\right) - \varepsilon\left(\frac{\pi}{\lambda_H} + \frac{1-\pi}{\lambda_L}\right)}\right)\right] \\
& + \frac{\varepsilon}{3}\left[\frac{\pi}{\lambda_H p_{NL}^{1-\alpha}} + \frac{1-\pi}{\lambda_L p_{NH}^{1-\alpha}}\right]\left(\frac{3 + \phi - A\left(\pi p_{NL} + (1-\pi)\, p_{NH}\right)}{2\left(\pi p_{NL} + (1-\pi)\, p_{NH}\right) - \varepsilon\left(\frac{\pi}{\lambda_H} + \frac{1-\pi}{\lambda_L}\right)}\right)
\end{aligned}$$

and again:

$$\begin{aligned}
\varepsilon\ &>\ \max\left\{\lambda_H p_{NL}; \lambda_L p_{NH}\right\} \max\left\{\left[\frac{A\left[2\left(\pi p_{NL} + (1-\pi)\, p_{NH}\right) - \varepsilon\left(\frac{\pi}{\lambda_H} + \frac{1-\pi}{\lambda_L}\right)\right]}{3 + \left[\phi - A\left(\pi p_{NL} + (1-\pi)\, p_{NH}\right)\right]} - 1\right]\right. \\
& \left[A\left(\pi p_{NL} + (1-\pi)\, p_{NH}\right) - 1\right]\}
\end{aligned}$$

Again, both inequalities imply restrictions on the parameters. SEE IF I CAN CHARACTERIZE THIS MORE SHARPLY.

5 Generalization to the case of endogenous relative prices.

Consider again an economy that lasts for two periods, $t = 0, 1$, and with two types of consumers, lenders and borrowers. Unlike the model presented above, *both* types of agents consume two commodities, a tradeable (denoted by T) good and a non - tradeable (N) good. Lenders receive a sure endowment of T goods equal to $\bar{k} > 0$ at date 0 and a random date - 1 endowment of T goods equal to $\widetilde{y}_{1T}$. Entrepreneurs have no endowment but own a technology that produces N goods in period 1 out of T goods invested in period 0. Assume that the scale of the investment is fixed. Borrowers may invest 0 tradeable goods in period 0 (and get 0 non tradeable goods in period 1) or either they invest a fixed amount $\bar{k} > 0$ of T goods at date 0 and receive $A\bar{k}$ units of N goods in period 1, where $A > 1$. In order to invest entrepreneurs borrow $\bar{k}$ units of T goods from lenders at date 0. For this to happen the entrepreneur and the lender sign a debt contract promising a gross interest rate equal to R. In period 1 entrepreneurs sell part of the non tradeable goods produced in that same period to honor the debt. However the enforcement of this repayment is partial. It is assumed that the entrepreneur may default on its debt contract. However, there exists a third party with the power of (potentially) punishing the borrower if this defaults. The paper will assume two alternative scenarios about the punishment technology. We also assume for simplicity that $\widetilde{y}_{1T}$ can be expressed as $\widetilde{z}\bar{k}$.

Lenders are assumed to have the following preferences:

$$U^L = \bar{k} - k + E_0^L \left[u \left(c_T^L, c_N^L \right) \right]$$

where $k \in \left[0, \bar{k}\right]$ is the amount of T goods lent to entrepreneurs, c_j^L the amount of j goods ($j = T$,N) consumed by lenders at date 1 and E_0^L is the expectation operator according to the lender's date 0 beliefs. Entrepreneurs are assumed to have preferences represented by an ex-ante expected utility function

$$E_0^B \left[u \left(c_T^B, c_N^B \right) \right]$$

where c_j^B is the amount of j goods ($j = T$,N) consumed by borrowers at date 1 and E_0^B is the expectation operator according to the entrepreneur's date 0 beliefs. The function u is assumed to be identical across agents and to have a Cobb-Douglas form:

$$u\left(c_T^h, c_N^h\right) = \left(c_T^h\right)^{\alpha}\left(c_N^h\right)^{1-\alpha}$$

with $0 < \alpha < 1$ and $h = L, B$.

As before, the absolute date 1 price of T goods is assumed to be equal to $\widetilde{\lambda}$, while the date 1 absolute price of N goods is equal to $\widetilde{\lambda} p_N$, where p_N denotes again the relative price of non tradeables in terms of tradeables and $\widetilde{\lambda}$ being the nominal shock in period 1. We assume that

Assumption λ is a strictly positive random variable with support in the set $\Lambda = \{\lambda_L, \lambda_H\}$, where $0 < \lambda_L < \lambda_H$. The random variable z has support in the set $Z = \{z_L, z_H\}$ with $0 < z_L < z_H$. The joint distribution is given by $q(z, \lambda)$, where $0 \leqq q(z, \lambda) \leqq 1$ and $\sum_{z,\lambda} q(z, \lambda) = 1$. Let $S \equiv \Lambda \times Z$.

6 The general model under symmetric information.

After getting the equilibrium price, we turn to the analysis of the equilibrium contracts. The first subsection assumes that the transfers that borrowers must repay ex - post are denominated in non tradeables. Throughout this section the state of nature (λ, z) is common knowledge. Lenders also know that, in case of a default by the borrowers, the third party will the total amount of output obtained by the entrepreneurs and give this to the lenders, where $l = T, N$, depending upon the denomination of the debt contract. Hence, the default decision depends on the values of z and λ.

We assume again the existence of a fixed social cost $\psi > 0$ of default. Recall that ψ represents the amount of tradeables that lenders loose when borrowers decide not to repay their debt according to the contract. This represents all kinds of inefficiencies such as litigation costs. This cost represents the income of the third party in charge of enforcing this law, called the *judge*. For the sake of simplicity assume a continuum of these judges, so that the Lebesgue measure of judges is equal to the Lebesgue measure of lenders and that of borrowers. The judge also has the same Cobb - Douglas utility function as lenders and borrowers. This assumption will ensure that the aggregate demand will not depend on the parameter ψ.

6.1 The equilibrium relative price of non-tradeables.

Before entering in the discussion about the debt contract that arises in equilibrium, it is easy to show that the equilibrium price p_N^* does not depend on the particular contract that lenders and borrowers sign ex - ante. Let $\tau(\lambda, z, p_N)$ be the transfer of resources (denominates in tradeables) from borrowers to lenders given τ, z and p_N. Clearly the Cobb-Douglas assumption about preferences for L, B and J imply that the aggregate demand for tradeables in period 1 is equal to

$$\begin{aligned}
c_T^B(p_N, \lambda) + c_T^L(p_N, \lambda) + c_T^J(p_N, \lambda) &= \frac{\alpha I_1^B(\lambda, p_N, z)}{2\lambda} + \frac{\alpha I_1^L(\lambda, p_N, z)}{2\lambda} + \frac{\alpha I_1^J(\lambda, p_N, z)}{2\lambda} \\
&= \left(\frac{\alpha}{2\lambda}\right)\left(I_1^B(\lambda, p_N, z) + I_1^L(\lambda, p_N, z) + I_1^J(\lambda, p_N, z)\right)
\end{aligned}$$

where $I_1^h(\lambda, p_N, z)$ is the period 1 ex-post income of $h = L, B, J$. We have in general that

$$\begin{aligned}
I_1^B(\lambda, p_N, z, k) &= \begin{cases} A\lambda p_N \bar{k} - \tau(\lambda, p_N, z) & \text{if no default} \\ A\lambda p_N \bar{k} - \tau(\lambda, p_N, z) - \lambda(1-\chi)\psi & \text{if default} \end{cases} \\
I_1^L(\lambda, p_N, z) &= \begin{cases} \lambda z \bar{k} + \tau(\lambda, p_N, z) & \text{if no default} \\ \lambda z \bar{k} + \tau(\lambda, p_N, z) - \lambda\chi\psi & \text{if default} \end{cases} \\
I_1^J(\lambda, p_N, z) &= \begin{cases} 0 & \text{if no default} \\ \psi & \text{if default} \end{cases}
\end{aligned}$$

This is because whatever is paid by debtors needs to go to lenders. Therefore $I_1^B(\lambda, p_N, z) + I_1^L(\lambda, p_N, z) + I_1^J(\lambda, p_N, z) = A\lambda p_N \bar{k} + \lambda z \bar{k}$ where $1_{\{D\}}$ is an indicator function that takes the value 1 if (z, λ) are such that there is default and 0 otherwise. Therefore in equilibrium we must have $\left(\frac{\alpha}{\lambda}\right)\left(A\lambda p_N \bar{k} + \lambda z\right)$ $= z\bar{k}$, since $\frac{z}{2}$ is the aggregate per capita supply of tradeables in period 1. From here it is clear that the value of p_N that satisfies this is

$$p_N^*(z) = \left(\frac{z}{A}\right)\left(\frac{1-\alpha}{\alpha}\right)$$

Hence we have proved the following

Proposition 6 *In any equilibrium the value of p_N is equal to $\left(\frac{z}{A}\right)\left(\frac{1-\alpha}{\alpha}\right)$.*

6.2 Tradeable - denominated debt contracts.

Assume now that contracts are denominated in tradeables. Suppose then that the contract specifies that for every unit of T good borrowed by the entrepreneur in period 0 she must return R_T units of T goods (if debt is honored) in period 1. Then the borrower may choose to default on the remaining debt contract. If default takes place, the third party (the judge) seizes a proportion θ of the non-tradeable good produced by the entrepreneur at date 1 and transfers it to the lender. The social cost of default is still equal to ψ units of T goods and split in a fraction χ for lenders and $(1-\chi)$ for borrowers. Default does not occur if $R_T\ \bar{k} \leqq \theta\ p_N(z)\ A\ \bar{k} + (1-\chi)\ \psi$. Otherwise there is default.

We introduce the following notation.

$$\widehat{\Delta} \equiv \left\{(\lambda, z) : R_T \bar{k} \leqq p_N(z) A \bar{k}\right\}; \quad \widehat{D} \equiv S \backslash \widehat{\Delta}$$

As before, we will assume that these sets are well - defined, that is, we will assume that an equilibrium exists. With this notation at hand, we can characterize the ex-post income of borrowers and lenders:

$$I_B(\lambda, z) = \begin{cases} \left[p_N(z) A - R_T\right] \lambda \bar{k} & \forall (\lambda, z) \in \widehat{\Delta} \\ 0 \quad \forall (\lambda, z) \in \widehat{D} \end{cases}$$

$$I_L(\lambda, z, k) = \begin{cases} \lambda z \bar{k} + \lambda R_T k & \forall (\lambda, z) \in \widehat{\Delta} \\ \lambda z \bar{k} + \lambda p_N(z) A k - \lambda \psi & \forall (\lambda, z) \in \widehat{D} \end{cases}$$

Given the Cobb - Douglas utility function for both lenders and borrowers the first order condition for the lender can be shown to be:

$$1 + \sum_{(\lambda,z)\in\widehat{D}} q(z, \lambda) \frac{\psi \alpha^{\alpha} (1-\alpha)^{1-\alpha}}{(p_N(z))^{1-\alpha} \bar{k}}$$

$$= \sum_{(\lambda,z)\in\widehat{\Delta}} q(\lambda, z) R_T \left(\frac{\alpha^{\alpha} (1-\alpha)^{1-\alpha}}{p_N(z)^{1-\alpha}}\right) + \sum_{(\lambda,z)\in\widehat{D}} q(\lambda, z) p_N(z)^{\alpha} A \left(\alpha^{\alpha} (1-\alpha)^{1-\alpha}\right)$$

On the other hand, the ex-ante indirect utility function of the borrower is U_T^B , equal to:

$$\left(\alpha^{\alpha} (1-\alpha)^{1-\alpha}\right) A \bar{k} \sum_{(\lambda,z)\in\widehat{\Delta}} q(\lambda, z) p_N(z)^{\alpha} - \bar{k} \sum_{(\lambda,z)\in\widehat{\Delta}} q(\lambda, z) R_T(\lambda, z) \left(\frac{\alpha^{\alpha} (1-\alpha)^{1-\alpha}}{p_N(z)^{1-\alpha}}\right)$$

Now, from the FOC of the lender we get

$$
\begin{aligned}
& -\bar{k} \sum_{(\lambda,z)\in\widehat{\Delta}} q(\lambda,z) R_T(\lambda,z) \left(\frac{\alpha^{\alpha}(1-\alpha)^{1-\alpha}}{p_N(z)^{1-\alpha}} \right) \\
= \; & \bar{k} \sum_{(\lambda,z)\in\widehat{D}} q(\lambda,z) p_N(z)^{\alpha} A \left(\alpha^{\alpha}(1-\alpha)^{1-\alpha} \right) - \bar{k} - \sum_{(\lambda,z)\in\widehat{D}} q(z,\lambda) \frac{\psi \alpha^{\alpha}(1-\alpha)^{1-\alpha}}{(p_N(z))^{1-\alpha}}
\end{aligned}
$$

Therefore replacing this in U_T^B we obtain:

$$
\begin{aligned}
U_T^B &= \left(\alpha^{\alpha}(1-\alpha)^{1-\alpha} \right) A\bar{k} \sum_{(\lambda,z)} q(\lambda,z) p_N(z)^{\alpha} - \bar{k} \\
&= \left(\alpha^{\alpha}(1-\alpha)^{1-\alpha} \right) A\bar{k} \sum_{(\lambda,z)} q(\lambda,z) \left[\left(\frac{z}{A} \frac{1-\alpha}{\alpha} \right) \right]^{\alpha} - \bar{k} - \sum_{(\lambda,z)\in\widehat{D}} q(z,\lambda) \frac{\psi \alpha^{\alpha}(1-\alpha)^{1-\alpha}}{(p_N(z))^{1-\alpha}} \\
&= (1-\alpha) A^{1-\alpha} \bar{k} \sum_{(\lambda,z)} q(\lambda,z) z^{\alpha} - \bar{k} - \psi (1-\alpha) A^{1-\alpha} \bar{k} \sum_{(\lambda,z)\in\widehat{D}} q(z,\lambda) z^{\alpha}
\end{aligned}
$$

For the lender it is obvious that:

$$
U_T^L = \bar{\bar{k}} + \alpha^{\alpha}(1-\alpha)^{1-\alpha} \bar{k} \sum_{(z,\lambda)} q(\lambda,z) \left(\frac{z}{(p_N(z))^{1-\alpha}} \right)
$$

We further characterize the two cases to consider here:

- **Case T1:** $\widehat{\Delta} = \{z_H, z_L\}$

This implies that there is never default under a non - contingent, T good denominated contract. This is the same as stating that $\lambda R_T \leqq \lambda \left(\left(\frac{1-\alpha}{\alpha} \right) z_L + (1-\chi) \left(\frac{\psi}{\bar{k}} \right) \right)$, and that $\widehat{D} = \emptyset$. Let $\pi \equiv q_{LL} + q_{HL}$, so $1-\pi = q_{LH} + q_{HH}$. Hence the equilibrium interest rate must be equal to:

$$
R_T = \frac{1}{A^{1-\alpha} \alpha \left(\frac{\pi}{z_L^{1-\alpha}} + \frac{1-\pi}{z_H^{1-\alpha}} \right)}
$$

So for this to be an equilibrium it must be the case that

$$
\frac{1}{A^{1-\alpha} \alpha \left(\frac{\pi}{z_L^{1-\alpha}} + \frac{1-\pi}{z_H^{1-\alpha}} \right)} \leqq \left(\frac{1-\alpha}{\alpha} \right) z_L
$$

Proposition 7 *For a sufficiently large z_L such that $\pi A^{1-\alpha} z_L^{\alpha} > 1$, then $\widehat{\Delta}$ $= \{z_H, z_L\}$ is consistent with equilibrium.*

The proof is in the appendix. This is a generalization of our simpler case in section 2. Note that the presence of Cobb-Douglas preferences implies the exponents of A and z_L.

- **Case T2:** $\widehat{\Delta} = \{z_H\}$

Therefore $\widehat{D} = \{z_L\}$. For this to be true we need of course that R_T be less than or equal to $z_H \left(\frac{1-\alpha}{\alpha}\right) \bar{k}$ and strictly greater than $z_L \left(\frac{1-\alpha}{\alpha}\right) \bar{k}$. From the first order conditions it can be shown that R_T is equal to:

$$R_T = \left(\frac{\frac{1}{A^{1-\alpha}} + \frac{\alpha\psi}{\bar{k}} \frac{\pi}{z_L^{1-\alpha}} - (1-\alpha)\pi z_L^{\alpha}}{\alpha(1-\pi)} \right) z_H^{1-\alpha}$$

Proposition 8 *There exists a sufficiently large value of z_H and a small z_L (that makes the denominator of R_T is strictly positive) such that this case is consistent with an equilibrium. The inequalities are:*

$$\left(\frac{\frac{1}{A^{1-\alpha}} + \frac{\alpha\psi}{\bar{k}} \frac{\pi}{z_L^{1-\alpha}} - (1-\alpha)\pi z_L^{\alpha}}{(1-\alpha)(1-\pi)} \right) \leq z_H^{\alpha}$$

$$\frac{1}{A^{1-\alpha}(1-\alpha)\left(\frac{\pi}{z_L^{1-\alpha}} + \frac{1-\pi}{z_H^{1-\alpha}}\right)} > z_L$$

This result generalizes what was obtained in the exogenous p_{Nj} model. Note that the first inequality implies

$$(1-\alpha)\left[\pi z_L^{\alpha} + (1-\pi) z_H^{\alpha}\right] \geq \frac{1}{A^{1-\alpha}} + \frac{\alpha\psi}{\bar{k}} \frac{\pi}{z_L^{1-\alpha}}$$

which is the modified version of the condition obtained in the simpler model. Note that the weight α as well as the term $A^{1-\alpha}$ now both appear as a consequence of the assumption of symmetric (Cobb-Douglas) preferences. The interpretation, however, does not change: the expected (weighted) T-good value of the project exceeds the cost of credit, including the expected default costs.

6.3 Characterization with nominal, non - contingent, debt contracts

Assume that the borrower signs a contract with a lender specifying that, for every unit of T good received at date 0, the borrower pays $R(\lambda, z)$ of nominal units in period 1. If the borrower does not honor her debt, the third party forces the borrower to transfer a fraction θ of the non tradeables to the lenders. Then, the borrowers's date 1 (ex-post) income (measured in tradeables) is equal to

$$\lambda p_N(z) A\bar{k} - \min\left\{R(\lambda, z)\bar{k}, A\lambda p_N \bar{k}\right\}$$

Therefore perfect repayment is observed if and only if $R(\lambda, z)\bar{k} \leqq A\lambda p_N(z)\bar{k}$. We can actually write down the entrepreneur's ex-post income at date 1:

$$I_1^B(\lambda, z) = \begin{cases} \bar{k}\left[A\lambda p_N(z) - R(\lambda, z)\right], & \text{if } R(\lambda, z)\bar{k} \leqq A\lambda p_N A\bar{k} \\ 0, \quad \text{otherwise} \end{cases}$$

Given the Cobb-Douglas type of utility function the (ex-post) indirect utility function is equal to

$$\frac{I_1^B(\lambda, z)}{\lambda} \frac{\left[\alpha^\alpha (1-\alpha)^{1-\alpha}\right]}{p_N(z)^{1-\alpha}}$$

For lenders, the analysis is similar. Assume that these consumers can lend any amount k to the borrowers. Therefore we can write down the lender's ex-post income in period 1:

$$I_1^L(\lambda, z, k) = \begin{cases} \lambda z\bar{k} + R(\lambda, z) k, & \text{if } R(\lambda, z) \leqq A\lambda p_N(z) \\ \lambda z\bar{k} + \lambda p_N(z) Ak - \lambda\psi, & \text{otherwise} \end{cases}$$

Given the Cobb-Douglas form the ex-post indirect utility function for lenders is just

$$\frac{I_1^L(\lambda, z, k)}{\lambda} \frac{\left[\alpha^\alpha (1-\alpha)^{1-\alpha}\right]}{p_N(z)^{1-\alpha}}$$

The lender will choose $k \geq 0$ so as to maximize over k in $\left[0, \bar{k}\right]$

$$\bar{\bar{k}} - k + E_0\left\{\frac{I_1^L(\lambda, z, k)}{\lambda} \frac{\left[\alpha^\alpha (1-\alpha)^{1-\alpha}\right]}{p_N(z)^{1-\alpha}}\right\}$$

subject to

$$\bar{\bar{k}} - k + E_0 \left\{ \frac{I_1^L(\lambda, z, k)}{\lambda} \frac{\left[\alpha^\alpha (1-\alpha)^{1-\alpha}\right]}{p_N(z)^{1-\alpha}} \right\} \geq \bar{k} + E_0 \left\{ z\bar{k} \frac{\left[\alpha^\alpha (1-\alpha)^{1-\alpha}\right]}{p_N(z)^{1-\alpha}} \right\}$$

and note that $I_1^L(\lambda, p_N, k)$ is always an affine function of k. Hence, the level of k is undetermined from the lender's side, which instead gives the interest rate which is consistent with equilibrium. Thus, we will consider any equilibrium with $k = \bar{k}$. The lender also considers $p_N(z)$ as exogenous, so the individual k does not affect p_N for the individual lender.

Let us introduce some further notation. We first assume that in equilibrium, p_N is a function of (λ, z), which will be true as shown below. Let Δ be the set of (λ, z) such that $R(\lambda, z)\bar{k} \leqq \theta A \lambda p_N A \bar{k} + \lambda (1-\chi)\psi$ (that is, entrepreneurs do not default on debt). Let D be the set complement of Δ, that is, D is the set of pairs (z, λ) such that borrowers default. With this notation at hand, the solution to the lender's problem (in an interior solution) implies

$$\begin{aligned}
&\alpha^\alpha (1-\alpha)^{1-\alpha} \sum_{(\lambda,z)\in\Delta} q(z,\lambda) \left(\frac{R(\lambda, z)}{\lambda (p_N(z))^{1-\alpha}} \right) \\
&+\alpha^\alpha (1-\alpha)^{1-\alpha} \sum_{(\lambda,z)\in D} q(z,\lambda)\, p_N(z)^\alpha A = 1 + \sum_{(\lambda,z)\in D} q(z,\lambda) \frac{\psi \alpha^\alpha (1-\alpha)^{1-\alpha}}{(p_N(z))^{1-\alpha}\bar{k}}
\end{aligned}$$

This is because the objective function is linear in k. It s easy to show that any interior k as a solution implies a value of R which violates the constraint. If R satisfies this equality then the constraint also holds with equality, and so k must be equal to $\bar{k}$. For now assume the existence of Δ, D so that we can compute the ex-ante utility levels for lenders and borrowers. For lenders it is trivial. Given the last first order condition it is clear that U^L is always equal to

$$U^L_{nom} = \bar{\bar{k}} + \alpha^\alpha (1-\alpha)^{1-\alpha} \bar{k} \sum_{(\lambda,z)} q(\lambda, z) \left(\frac{z}{(p_N(z))^{1-\alpha}} \right)$$

For borrowers the ex-ante expected utility is:

$$U^B_{nom} = \alpha^\alpha (1-\alpha)^{1-\alpha} \bar{k} A \left\{ \sum_{(\lambda,z)\in\Delta} q(\lambda,z) p_N(z)^\alpha + (1-\theta) \sum_{(\lambda,z)\in D} q(\lambda,z) p_N(z)^\alpha \right\}$$
$$-\alpha^\alpha (1-\alpha)^{1-\alpha} \bar{k} \sum_{(\lambda,z)\in\Delta} q(\lambda,z) \frac{R(\lambda,z)}{\lambda} p_N(z)^{\alpha-1}$$

But from the first order condition of the lender we know that

$$\begin{aligned} & \alpha^\alpha (1-\alpha)^{1-\alpha} \sum_{(\lambda,z)\in\Delta} q(z,\lambda) \frac{R(\lambda,z)}{\lambda p_N(z)^{1-\alpha}} \\ = \; & 1 + \sum_{(\lambda,z)\in D} q(z,\lambda) \frac{\psi\alpha^\alpha (1-\alpha)^{1-\alpha}}{(p_N(z))^{1-\alpha} \bar{k}} - \alpha^\alpha (1-\alpha)^{1-\alpha} A \sum_{(\lambda,z)\in D} q(z,\lambda) p_N(z)^\alpha \end{aligned}$$

Therefore we replace this in U^B to get:

$$\begin{aligned} & U^B_{nom} \\ = \; & \alpha^\alpha (1-\alpha)^{1-\alpha} \bar{k} A \sum_{(\lambda,z)\in\Delta} q(\lambda,z) p_N(z)^\alpha + \alpha^\alpha (1-\alpha)^{1-\alpha} \bar{k} A \sum_{(\lambda,z)\in D} q(\lambda,z) p_N(z)^\alpha \\ & -\bar{k} - \alpha^\alpha (1-\alpha)^{1-\alpha} \psi \sum_{(\lambda,z)\in D} \frac{q(\lambda,z)}{(p_N(z))^{1-\alpha}} \\ = \; & \alpha^\alpha (1-\alpha)^{1-\alpha} \bar{k} A \sum_{(\lambda,z)} q(\lambda,z) p_N(z)^\alpha - \bar{k} - \alpha^\alpha (1-\alpha)^{1-\alpha} \psi \sum_{(\lambda,z)\in D} \frac{q(\lambda,z)}{(p_N(z))^{1-\alpha}} \\ = \; & (1-\alpha) A^{1-\alpha} \bar{k} \sum_{(\lambda,z)} q(\lambda,z) z^\alpha - \bar{k} - (1-\alpha) A^{1-\alpha} \bar{k} \psi \sum_{(\lambda,z)\in D} \frac{q(\lambda,z)}{z^{1-\alpha}} \end{aligned}$$

Therefore, under non - tradeable denominated debt contracts the level of the equilibrium indirect utility for entrepreneurs depends on which states the entrepreneur finds optimal to default on her debt.

6.3.1 The case of positive correlation between real exchange rates and nominal exchange rates.

As in the simple model of the first part, we assume that $q_{LL} = q_{HH} = 0$, and $q_{HL} = \pi$, $q_{LH} = 1-\pi$, with $\pi \in (0,1)$. We assume as usual that both λ_H

and z_H are always high enough so that (λ_H, z_H) is always in Δ. Consider the following cases.

- **Case N1:** $D = \emptyset$.

This implies:

$$R_{nom} = \frac{1}{A^{1-\alpha}\alpha\left(\frac{1-\pi}{\lambda_L z_H^{1-\alpha}} + \frac{\pi}{\lambda_H z_L^{1-\alpha}}\right)} = \frac{\lambda_H \lambda_L}{A^{1-\alpha}\alpha\left(\frac{\pi}{z_L^{1-\alpha}}\lambda_L + \frac{1-\pi}{z_H^{1-\alpha}}\lambda_H\right)}$$

For this to be an equilibrium we need that $z_L\left(\frac{1-\alpha}{\alpha}\right) \geq \frac{R_{nom}}{\lambda_H}$ and $z_H\left(\frac{1-\alpha}{\alpha}\right) \geq \frac{R_{nom}}{\lambda_L}$. The first inequality is equivalent to:

$$z_L \geq \frac{\lambda_L}{A^{1-\alpha}\alpha\left(\frac{\pi}{z_L^{1-\alpha}}\lambda_L + \frac{1-\pi}{z_H^{1-\alpha}}\lambda_H\right)} = \frac{1}{A^{1-\alpha}\alpha\left(\frac{\pi}{z_L^{1-\alpha}} + \frac{1-\pi}{z_H^{1-\alpha}}\frac{\lambda_H}{\lambda_L}\right)}$$

while the second inequality is the same as:

$$z_H \geq \frac{\lambda_H}{A^{1-\alpha}\alpha\left(\frac{\pi}{z_L^{1-\alpha}}\lambda_L + \frac{1-\pi}{z_H^{1-\alpha}}\lambda_H\right)} = \frac{1}{A^{1-\alpha}\alpha\left(\frac{\pi}{z_L^{1-\alpha}}\frac{\lambda_L}{\lambda_H} + \frac{1-\pi}{z_H^{1-\alpha}}\right)}$$

Proposition 9 *This case is consistent with equilibrium z_L is high enough. Given these constraints, any value of λ_j's (satisfying $\lambda_H > \lambda_L$) and $z_H > z_L$ are consistent with equilibrium.*

This case states that sufficient conditions to never observe default is that the value of z_L be high enough. It is not difficult to show that the condition on z_L is weaker in the case of nominal contracts than that of "dollarized" contracts. Again, similar arguments imply that the condition on z_H is stronger in the case of nominal contracts relative to that of "dollarized" ones[2]. This is just a generalization of the results obtained in the partial equilibrium model.

These are somehow too strong conditions. As in the partial equilibrium model, we do not need to have z_L large enough so that there is no default under the dollar contract and the nominal contract. The following proposition shows this.

[2]The proof of these claims are available upon request.

Proposition 10 *If z_L is not large enough so that $\widehat{D} = \emptyset$, then it is still true that $D = \emptyset$ if the following holds.*

$$\left(\frac{z_H^{1-\alpha}}{1-\pi}\right)\left(\frac{1}{(1-\alpha)A^{1-\alpha}} - \pi z_L^{\alpha}\right) \leq \frac{z_L}{z_H}\frac{\lambda_H}{\lambda_L} \leq \frac{\left(\frac{\pi}{z_L^{1-\alpha}}\right)}{\frac{1}{(1-\alpha)A^{1-\alpha}} - (1-\pi)\, z_H^{\alpha}}$$

Again, this generalizes a similar condition obtained when p_N was assumed to be exogenous. The main difference is the presence of $\frac{z_L}{z_H}$ in the inequality as well as $\frac{\lambda_H}{\lambda_L}$.

- **Case N2:** $\Delta = \{(\lambda_L, z_H)\}$

Here $D = \{(\lambda_H, z_L)\}$. If this were the case then

$$R_{nom}^{(\lambda_L, z_H)} = \left(\frac{\frac{1}{A^{1-\alpha}} + \frac{\alpha\psi}{k}\frac{\pi}{z_L^{1-\alpha}} - (1-\alpha)\,\pi z_L^{\alpha}}{\alpha}\right)\left(\frac{z_H^{1-\alpha}}{1-\pi}\right)\lambda_L$$

For this to be consistent with an equilibrium we need to get

$$\begin{aligned} z_L\left(\frac{1-\alpha}{\alpha}\right)\lambda_H &< R_{nom} \\ R_{nom}^{(\lambda_L, z_H)} &\leqq \lambda_L z_H\left(\frac{1-\alpha}{\alpha}\right) \end{aligned}$$

The first inequality states that the value of the output obtained by the borrower is not enough to repay his debt, inducing default at state (λ_H, z_L). The second inequality means that, given this fact, the borrower does not default on the other state. These two inequalities yield the following result.

Proposition 11 *For this case to be consistent with an equilibrium it is sufficient to have a value of z_H and of λ_L high enough and z_L low enough so that*

$$z_L < \frac{\lambda_L}{A^{1-\alpha}(1-\alpha)\left[\frac{\pi}{z_L^{1-\alpha}}\lambda_L + \frac{1-\pi}{z_H^{1-\alpha}}\lambda_H\right]}$$

$$z_H^{\alpha} \geq \frac{1}{(1-\pi)(1-\alpha)}\left[\frac{1}{A^{1-\alpha}} + \pi z_L^{\alpha}\left(\frac{\alpha\phi}{z_L} - (1-\alpha)\right)\right]$$

where $\phi \equiv \frac{\psi}{k}$.

This result is also a generalization of the results obtained in section 2.3.2. Note again that the second inequality is also present in the case of tradeable good contracts with default in z_L. The first inequality is stronger than the first inequality under which there is default with dollar contracts in z_L. Hence, we have again that if there is default with nominal contracts in the state (λ_H, z_L) then there must be default in the same state with T-good contracts.

- **Case N3:** $\Delta = \{(\lambda_H, z_L)\}$

Here $D = \{(\lambda_L, z_H)\}$. This case states that the default decision occurs when the real exchange rate is low. If this were true then

$$R_{nom}^{(\lambda_H, z_L)} = \left(\frac{\frac{1}{A^{1-\alpha}} + \frac{\alpha\psi}{\bar{k}} \left(\frac{1-\pi}{z_H^{1-\alpha}} \right) - (1-\alpha)(1-\pi) z_H^{\alpha}}{\alpha\pi} \right) \lambda_H z_L^{1-\alpha}$$

If this were an equilibrium we need that $R_{nom}^{(\lambda_H, z_L)} \leqq \lambda_H z_L \left(\frac{1-\alpha}{\alpha}\right)$ and $R_{nom} > \lambda_L \ z_H \left(\frac{1-\alpha}{\alpha}\right)$.

Proposition 12 *Suppose that z_H is small enough so that $z_H < \frac{1}{A^{\frac{1-\alpha}{\alpha}} [(1-\alpha)(1-\pi)]^{\frac{1}{\alpha}}}$. Then there is default with nominal contracts in the state (λ_L, z_H) if $\left(\frac{\lambda_H}{\lambda_L}\right)$ is large enough and if*

$$z_L^{\alpha} (1-\alpha) \pi \geq \left[\frac{1}{A^{1-\alpha}} + \frac{\alpha\psi (1-\pi)}{\bar{k} z_H^{1-\alpha}} - (1-\alpha)(1-\pi) z_H^{\alpha} \right]$$

(given that the right hand side is non-negative).

Hence this case implies a nominal shock very volatile and again that the expected value in tradeables of the output is large enough to compensate the cost of credit including the default cost, although wih the extra caveat that z_H cannot be too large. This generalizes again the similar case studied in the simpler model. Note that the second inequality again implies that the expected value of z^{α} (multiplied by the constant $1-\alpha$) must be bounded below. As the reader can see, this case is somehow special, since the conditions on z_L and z_H are also rather special.

6.4 Dominance of T-good-denominated or nominal non-contingent debt contracts

We can now compute the difference between the indirect utilities with nominal contracts and with contracts denominated in T goods for the borrowers (for lenders the difference is always 0).

$$U^B - U_T^B = -\psi\alpha^\alpha (1-\alpha)^{1-\alpha} \left(\sum_{(\lambda,z)\in D} \frac{q(\lambda,z)}{(p_N(z))^{1-\alpha}} - \sum_{z\in \widehat{D}} \frac{q(\lambda,z)}{(p_N(z))^{1-\alpha}} \right)$$

It is clearly the case that $sgn\left(U^B - U_T^B\right)$ must have the opposite sign of $\sum_{(\lambda,z)\in D} \frac{q(\lambda,z)}{(p_N(z))^{1-\alpha}} - \sum_{(\lambda,z)\in \widehat{D}} \frac{q(\lambda,z)}{(p_N(z))^{1-\alpha}}$. Given that the functional form p_N is the same in D and $\widehat{D}$, and given that $q(\lambda,z) \in (0,1)$ and $p_N > 0$ in equilibrium then the sign of this last term is determined by whether $\#D$ is strictly greater or strictly less than $\#\widehat{D}$. This is an important difference with respect to the first model. The symmetry of preferences allows to characterize the dominance much more easily, by just looking at the states in which default occurs and computing that difference.

We study this dominance case by case. Suppose first that there is no default under both contracts. Hence the difference is automatizally 0. This is completely different from the same case in the first model. The reason of this is again symmetry of preferences. This basically vanished the hedging effect present in the simpler model. This is because the lender now also cares about non-tradeables, that is, the price λz. This is reflected in the interest rate, which was absent in the first model.

Clearly, if there is no default with the T-good contract and default with the nominal one (in the cases consistent with equilibrium) clearly the borrower chooses the T-good contract. Henceforth, the rest of this subsection will assume that the T-good contract induces default in (λ_H, z_L). (The case of no default implies that $\widehat{D}$ so the nominal contract is always (at least weakly) dominated.

- **Case N1.**

We already stated that there exists a set of values of the exogenous variables where this is consistent with equilibrium. Since there is no default with the nominal contract, clearly $U^B - U_T^B = -\psi\alpha^\alpha (1-\alpha)^{1-\alpha} \left[\frac{\pi}{(p_N(z_L))^{1-\alpha}} - 0\right] =$

$-\frac{A^{1-\alpha}\pi\alpha}{(z_L)^{1-\alpha}} < 0$. Therefore here the nominal contract dominates the $T - good$ contract.

- **Case N2**

Here we have that there is default with the nominal contract in the state (λ_H, z_L). Hence $U^B - U^B_T = 0$, so the borrower is indifferent between the two cases.

- **Case N3**

Here there is default under nominal contracts in the state (λ_L, z_H), while default occurs with T golod contracts in the state (λ_H, z_L). We first need to show that no mutually contradictory inequalities arise here. This situation is characterized by the following set of inequalities:

$$\left(\frac{\frac{1}{A^{1-\alpha}} + \frac{\alpha\psi}{k}\frac{\pi}{z_L^{1-\alpha}} - (1-\alpha)\pi z_L^\alpha}{(1-\alpha)(1-\pi)}\right) \leq z_H^\alpha; \quad \frac{1}{A^{1-\alpha}(1-\alpha)\left(\frac{\pi}{z_L^{1-\alpha}} + \frac{1-\pi}{z_H^{1-\alpha}}\right)} > z_L$$

$$z_L^\alpha (1-\alpha)\pi \geq \left[\frac{1}{A^{1-\alpha}} + \frac{\alpha\psi(1-\pi)}{\bar{k}z_H^{1-\alpha}} - (1-\alpha)(1-\pi) z_H^\alpha\right]; \quad z_H^\alpha < \frac{1}{A^{1-\alpha}\left[(1-\alpha)(1-\pi)\right]}$$

plus the fact that $\left(\frac{\lambda_H}{\lambda_L}\right)$ must be large enough. These four inequalities can be reduced to:

$$\left(\frac{\frac{1}{A^{1-\alpha}} + \frac{\alpha\psi}{k}\frac{\pi}{z_L^{1-\alpha}} - (1-\alpha)\pi z_L^\alpha}{(1-\alpha)(1-\pi)}\right) \leq z_H^\alpha < \frac{1}{A^{1-\alpha}\left[(1-\alpha)(1-\pi)\right]}$$

$$\frac{\left[\frac{1}{A^{1-\alpha}} + \frac{\alpha\psi(1-\pi)}{\bar{k}z_H^{1-\alpha}} - (1-\alpha)(1-\pi) z_H^\alpha\right]^{\frac{1}{\alpha}}}{\left[(1-\alpha)\pi\right]^{\frac{1}{\alpha}}} \leq z_L < \frac{1}{A^{1-\alpha}(1-\alpha)\left(\frac{\pi}{z_L^{1-\alpha}} + \frac{1-\pi}{z_H^{1-\alpha}}\right)}$$

The values of (z_L, z_H) consistent with these inequalities (and $z_L < z_H$) are rather special, and not always exist. However, when ψ is sufficiently small then it is possible to find values of z_L and z_H satisfying all relevant inequalities. If these conditions are met, then the difference $U^B - U_T^B$ is equal to:

$$-\psi\alpha A^{1-\alpha}\left[\frac{1-\pi}{z_H^{1-\alpha}} - \frac{\pi}{z_L^{1-\alpha}}\right]$$

This is strictly negative if and only if

$$\frac{z_H}{z_L} > \left(\frac{1-\pi}{\pi}\right)^{\frac{1}{1-\alpha}}$$

so in this case (under the conditions of case 2) the T - good contract dominates the nominal contract. Note that since $\frac{z_H}{z_L} > 1$ this implies that $\pi < \frac{1}{2}$.[3]

6.5 The case of non-tradeable (nominal indexed) denominated contracts

Suppose now a contract that specifies an interest rate (contigent on z) expressed in units of N goods. This is equivalent to have a nominal contract whose payments are contingent to the value of $\lambda p_N(z)$. The contract specifies that, if no default occurs, the value of pesos to be paid is equal to $\lambda p_N(z) r$, where r must be specified. Note that r must be less than A for at least one value of z, otherwise the entrepreneur prefers not to borrow any amount of T good in period 0. Hence the following assumption is made to guarantee this:

Assumption The values of (z_L, z_H) satisfies

$$\pi z_L^\alpha + (1-\pi) z_H^\alpha \geq \frac{1}{A^{1-\alpha}(1-\alpha)}$$

In this case the following proposition can be shown.

Proposition 13 *The assumption above is sufficient to obtain an r such that the contract specified above implies D being empty. The value of r is given by*

$$r = \frac{A^\alpha}{(1-\alpha)\left[\pi z_L^\alpha + (1-\pi) z_H^\alpha\right]}$$

[3]The reader can check that the following parameter values are consistent with this case: $A = \frac{3}{2}$, $\alpha = \frac{1}{2}$, $\frac{\psi}{k} = 0.05$, $\pi = 0.4$, $z_L = 3$, $z_H = 7$.

This implies the following result:

Proposition 14 *If $\widehat{D} = \emptyset$, then $D_N = \emptyset$.*

The corollary of this result is that no default under the T - good denominated contract implies no default under the perfectly indexed nominal contract. Under the assumption above, the ex-post utility for the borrower is given by

$$\begin{aligned}
& \alpha^{\alpha} (1-\alpha)^{1-\alpha} \bar{k} \frac{\left[z \left(\frac{1-\alpha}{\alpha}\right) \left(1 - \frac{A^{\alpha}}{(1-\alpha) A \left[\pi z_L^{\alpha} + (1-\pi) z_H^{\alpha}\right]}\right)\right]}{\left(\frac{z}{A} \frac{1-\alpha}{\alpha}\right)^{1-\alpha}} \\
= \;& \alpha A^{1-\alpha} \bar{k} \left[z^{\alpha} \frac{(1-\alpha)}{\alpha} - \frac{z^{\alpha} \frac{(1-\alpha)}{\alpha}}{(1-\alpha) A^{1-\alpha} \left[\pi z_L^{\alpha} + (1-\pi) z_H^{\alpha}\right]} \right] \\
= \;& A^{1-\alpha} z^{\alpha} \bar{k} (1-\alpha) - \frac{z^{\alpha}}{\left[\pi z_L^{\alpha} + (1-\pi) z_H^{\alpha}\right]} \bar{k}
\end{aligned}$$

So in ex-ante terms:

$$V_{ind}^{B} = A^{1-\alpha} (1-\alpha) \bar{k} \left[\pi z_L^{\alpha} + (1-\pi) z_H^{\alpha}\right] - \bar{k}$$

which is equal to the utility reached by the borrower with the other two contracts when no default occurs under both of them. This has the implication that the perfectly indexed contract can never be dominated by the T - good debt contract.

What happens if the borrower has to choose among the three contracts? We first have that

$$\begin{aligned}
V_{ind}^{B} - U_T^{B} &= A^{1-\alpha} (1-\alpha) \bar{k} \left[\pi z_L^{\alpha} + (1-\pi) z_H^{\alpha}\right] - \bar{k} \\
&\quad - \left[(1-\alpha) A^{1-\alpha} \bar{k} \sum_{(\lambda,z)} q(\lambda, z) z^{\alpha} - \psi (1-\alpha) A^{1-\alpha} \bar{k} \sum_{(\lambda,z) \in \widehat{D}} \frac{q(z,\lambda)}{z^{1-\alpha}} \right] \\
&= \psi (1-\alpha) A^{1-\alpha} \bar{k} \sum_{(\lambda,z) \in \widehat{D}} \frac{q(z,\lambda)}{z^{1-\alpha}}
\end{aligned}$$

and

$$
\begin{aligned}
V_{ind}^{B}-U_{nom}^{B} &= A^{1-\alpha}(1-\alpha)\bar{k}\left[\pi z_L^{\alpha}+(1-\pi)z_H^{\alpha}\right]-\bar{k} \\
&\quad -\left[(1-\alpha)A^{1-\alpha}\bar{k}\sum_{(\lambda,z)}q(\lambda,z)z^{\alpha}-\bar{k}-(1-\alpha)A^{1-\alpha}\bar{k}\psi\sum_{(\lambda,z)\in D}\frac{q(\lambda,z)}{z^{1-\alpha}}\right] \\
&= (1-\alpha)A^{1-\alpha}\bar{k}\psi\sum_{(\lambda,z)\in D}\frac{q(\lambda,z)}{z^{1-\alpha}}
\end{aligned}
$$

So clearly, as long as no default occurs with indexed contracts, these are preferred than non-indexed contracts (either nominal or in T-goods) except when there is no default in these other cases.

7 Indexed contract with noisy signals in general equilibrium (case of positively-correlated real and nominal exchange rates).

As in the partial equilibrium case, we analyze now the choice of contract's unit of account when there is a nominal indexed contract but the lender can only observe a noisy signal of the realization of λz. Again, assume that the lender observes $\lambda p_N(z)+x$, where $p_N(z)=\frac{z}{A}\frac{1-\alpha}{\alpha}$ and where x can take values on the set $\{-\varepsilon,0,\varepsilon\}$. Assume that x is *i.i.d.* and the probability for each possible realization is 1/3. We still assume also that $q_{LL}=q_{HH}=0$ and that $\pi=q_{HL}$. If there is no default under this contract, the borrower must pay

$$\lambda A p_N(z)\bar{k}-r\left[\lambda p_N(z)+x\right]\bar{k}=\lambda p_N(z)\left[A-r\right]\bar{k}-rx\bar{k}$$

where r is to be determined.

From the lender's side, we can derive his nominal income for this contract

$$I^{L}=\begin{cases}\lambda\left[z\bar{k}+rp_N(z)k+\frac{rx}{\lambda}k\right] & \text{if no default}\\ \lambda\bar{k}z+\lambda A p_N(z)k-\lambda\psi & \text{if default}\end{cases}$$

The ex-post utility for the lender is given by

$$U_{\exp}^{L}(\lambda,z,x)=\begin{cases}\alpha^{\alpha}(1-\alpha)^{1-\alpha}\left[\frac{z\bar{k}+rp_N(z)k+\frac{rx}{\lambda}k}{p_N(z)^{1-\alpha}}\right] & \text{if no default}\\ \alpha^{\alpha}(1-\alpha)^{1-\alpha}\left[\frac{z\bar{k}+Ap_N(z)k-\psi}{p_N(z)^{1-\alpha}}\right] & \text{if default}\end{cases}$$

The lender's problem is, as before:

$$\max_{k\in\left[0,\bar{k}\right]} \quad \bar{k}-k+E\left[U^{L}_{\text{exp}}\left(\lambda,z,x\right)\right]$$

subject to

$$\bar{k}-k+E\left[U^{L}_{\text{exp}}\left(\lambda,z,x\right)\right]\geq\bar{k}+E\left[\frac{z\alpha^{\alpha}\left(1-\alpha\right)^{1-\alpha}\bar{k}}{p_{N}\left(z\right)^{1-\alpha}}\right]$$

Recall that $\frac{\alpha^{\alpha}(1-\alpha)^{1-\alpha}}{p_N(z)^{1-\alpha}}=\alpha\frac{A^{1-\alpha}}{z^{1-\alpha}}$. Hence assuming that the constraint holds with equality at the solution then

$$\alpha A^{1-\alpha}\bar{k}\left[r\sum_{i,j,x\in\bar{\Delta}}\frac{q_{ij}}{3}\left[z_j^{\alpha}\left(\frac{1-\alpha}{\alpha A}\right)+\frac{x}{\lambda_i z_j^{1-\alpha}}\right]+\sum_{i,j,x\in\bar{D}}\frac{q_{ij}}{3}\left(z_j^{\alpha}\left(\frac{1-\alpha}{\alpha}\right)-\frac{\psi}{z_j^{1-\alpha}}\right)\right]=\bar{k}$$

so we can derive r from here:

$$r=\frac{\frac{1}{\alpha A^{1-\alpha}}-\sum_{i,j,x\in\bar{D}}\frac{q_{ij}}{3}\left(z_j^{\alpha}\left(\frac{1-\alpha}{\alpha}\right)-\frac{\psi}{\bar{k}}\right)}{\sum_{i,j,x\in\bar{\Delta}}\frac{q_{ij}}{3}\left[z_j^{\alpha}\left(\frac{1-\alpha}{\alpha A}\right)+\frac{x}{\lambda_i}\right]}=\frac{1-\alpha A^{1-\alpha}\sum_{i,j,x\in\bar{D}}\frac{q_{ij}}{3}\left(z_j^{\alpha}\left(\frac{1-\alpha}{\alpha}\right)-\frac{\psi}{\bar{k}z_j^{1-\alpha}}\right)}{\alpha A^{1-\alpha}\sum_{i,j,x\in\bar{\Delta}}\frac{q_{ij}}{3}\left[z_j^{\alpha}\left(\frac{1-\alpha}{\alpha A}\right)+\frac{x}{\lambda_i}\right]}$$

where $q_{ij}=\Pr\left[\lambda=\lambda_i,z=z_j\right]$, $\bar{\Delta}$ is the set of states where the borrower does not default and $\bar{D}$ is its complement. With this expression at hand, the ex-post utility for the borrower is:

$$\alpha^{\alpha}\left(1-\alpha\right)^{1-\alpha}\bar{k}\frac{\left[z\left(\frac{1-\alpha}{\alpha}\right)-\left(\frac{1-\alpha A^{1-\alpha}\sum_{i,j,x\in\bar{D}}\frac{q_{ij}}{3}\left(z_j^{\alpha}\left(\frac{1-\alpha}{\alpha}\right)-\frac{\psi}{\bar{k}z_j^{1-\alpha}}\right)}{\alpha A^{1-\alpha}\sum_{i,j,x\in\bar{\Delta}}\frac{q_{ij}}{3}\left[z_j^{\alpha}\left(\frac{1-\alpha}{\alpha A}\right)+\frac{x}{\lambda_i}\right]}\right)\left[\frac{z}{A}\frac{1-\alpha}{\alpha}+\frac{x}{\lambda}\right]\right]}{\left(\frac{z}{A}\frac{1-\alpha}{\alpha}\right)^{1-\alpha}}$$

$$=\alpha\bar{k}A^{1-\alpha}\left[z^{\alpha}\left(\frac{1-\alpha}{\alpha}\right)-\frac{\left[\frac{z}{A}\left(\frac{1-\alpha}{\alpha}\right)+\frac{x}{\lambda}\right]}{z^{1-\alpha}}\left(\frac{1-\alpha A^{1-\alpha}\sum_{i,j,x\in\bar{D}}\frac{q_{ij}}{3}\left(z_j^{\alpha}\left(\frac{1-\alpha}{\alpha}\right)-\frac{\psi}{\bar{k}z_j^{1-\alpha}}\right)}{\alpha A^{1-\alpha}\sum_{i,j,x\in\bar{\Delta}}\frac{q_{ij}}{3}\left[z_j^{\alpha}\left(\frac{1-\alpha}{\alpha A}\right)+\frac{x}{\lambda_i}\right]}\right)\right]$$

so in ex-ante terms

$$\begin{aligned} V^B_{indns} &= \alpha \bar{k} \left[A^{1-\alpha} \left(\frac{1-\alpha}{\alpha} \right) \sum_{i,j \in \bar{\Delta}} q_{ij} z_j^{\alpha} \right. \\ &- \left(\frac{1 - \alpha A^{1-\alpha} \sum_{i,j,x \in \bar{D}} \frac{q_{ij}}{3} \left(z_j^{\alpha} \left(\frac{1-\alpha}{\alpha} \right) - \frac{\psi}{\bar{k} z_j^{1-\alpha}} \right)}{\alpha \sum_{i,j,x \in \bar{\Delta}} \frac{q_{ij}}{3} \left[z_j^{\alpha} \left(\frac{1-\alpha}{\alpha A} \right) + \frac{x}{\lambda_i} \right]} \right) \left[\sum_{i,j,x \in \bar{\Delta}} \frac{q_{ij}}{3} \left[z_j^{\alpha} \left(\frac{1-\alpha}{\alpha A} \right) + \frac{x}{z_j^{1-\alpha} \lambda_i} \right. \right. \end{aligned}$$

We now consider the corresponding cases analyzed under partial equilibrium.

7.1 No default under indexed-nominal contract with noise.

Suppose first that $\bar{D}$ is empty. Therefore, for any λ, z it must happen that

$$\lambda z \left(\frac{1-\alpha}{\alpha} \right) \geq r \left[\lambda \frac{z}{A} \frac{(1-\alpha)}{\alpha} + \varepsilon \right]$$

Note that if this holds, the same inequality holds when $x = 0$ or $x = -\varepsilon$. Replacing by the expression of r :

$$\lambda z \left(\frac{1-\alpha}{\alpha} \right) \geq \left[\frac{\lambda \frac{z}{A} \frac{(1-\alpha)}{\alpha} + \varepsilon}{\alpha A^{1-\alpha} \sum_{i,j,x} \frac{q_{ij}}{3} \left[z_j^{\alpha} \left(\frac{1-\alpha}{\alpha A} \right) + \frac{x}{\lambda_i} \right]} \right]$$

We can say then that this is true when ε is bounded above by

$$\left[\lambda z \left(\frac{1-\alpha}{\alpha} \right) \right] \left[\left[\alpha A^{1-\alpha} \sum_{i,j,x} \frac{q_{ij}}{3} \left[z_j^{\alpha} \left(\frac{1-\alpha}{\alpha A} \right) + \frac{x}{\lambda_i} \right] \right] - \frac{1}{A} \right]$$

for any λ, z. Therefore a sufficient condition for this to hold is

$$\varepsilon \leq \max \left\{ \lambda_H z_L ; \lambda_L z_H \right\} \left(\frac{1-\alpha}{\alpha} \right) \left[\alpha A^{1-\alpha} \sum_{i,j,x} \frac{q_{ij}}{3} \left[z_j^{\alpha} \left(\frac{1-\alpha}{\alpha A} \right) + \frac{x}{\lambda_i} \right] - \frac{1}{A} \right]$$

But

$$\sum_{i,j,x} \frac{q_{ij}}{3} \frac{x}{\lambda_i} = \sum_{i,j} \frac{q_{ij}}{\lambda_i} \sum_{x} \frac{x}{3} = \sum_{i,j} \frac{q_{ij}}{\lambda_i} 0 = 0$$

so

$$\varepsilon \leq \max\{\lambda_H z_L; \lambda_L z_H\} \left(\frac{1-\alpha}{\alpha}\right) \left[(1-\alpha) \frac{A^{1-\alpha}}{A} \sum_{i,j,x} \frac{q_{ij}}{3} \left[z_j^\alpha\right] - \frac{1}{A}\right]$$

In this case then the ex-post utility for the borrower is

$$\begin{aligned} & \alpha \bar{k} A^{1-\alpha} \left[z^\alpha \left(\frac{1-\alpha}{\alpha}\right) - \frac{\left[z\left(\frac{1-\alpha}{\alpha}\right) + \frac{x}{\lambda}\right]}{z^{1-\alpha}} \left(\frac{1}{\alpha A^{1-\alpha} \sum_{i,j,x} \frac{q_{ij}}{3} \left[z_j^\alpha \left(\frac{1-\alpha}{\alpha}\right) + \frac{x}{\lambda_i} \right]} \right) \right] \\ = \; & \alpha \bar{k} A^{1-\alpha} \left[z^\alpha \left(\frac{1-\alpha}{\alpha}\right) - \frac{\left[\frac{z}{A}\left(\frac{1-\alpha}{\alpha}\right) + \frac{x}{\lambda}\right]}{z^{1-\alpha}} \left(\frac{1}{\alpha A^{1-\alpha} \sum_{i,j} \frac{q_{ij}}{3} \left[z_j^\alpha \left(\frac{1-\alpha}{\alpha A}\right) \right]} \right) \right] \end{aligned}$$

and the ex-ante expected utility is equal to:

$$\begin{aligned} V^B_{indns} &= \alpha \bar{k} \left\{ A^{1-\alpha} \left(\frac{1-\alpha}{\alpha}\right) \sum_{i,j} q_{ij} z_j^\alpha - \left(\frac{\sum_{i,j} \frac{q_{ij}}{3} \left[\frac{z_j^\alpha}{A} \left(\frac{1-\alpha}{\alpha}\right) \right]}{\alpha \sum_{i,j} \frac{q_{ij}}{3} \left[\frac{z_j^\alpha}{A} \left(\frac{1-\alpha}{\alpha}\right) \right]} \right) \right\} \\ &= \bar{k} \left[A^{1-\alpha} (1-\alpha) \sum_{i,j} q_{ij} z_j^\alpha - 1 \right] = \bar{k} \left[A^{1-\alpha} (1-\alpha) \left[\pi z_L^\alpha + (1-\alpha) z_H^\alpha\right] - 1 \right] \end{aligned}$$

This is clearly equal to the ex-ante utility obtained by borrowers when indexed contract without noise are used. Hence all the welfare results related to that contract applies here.

7.2 Default with sufficiently large noise

Suppose now that ε is large enough such that, for any pair (λ, z) borrowers default. Hence $\bar{D} = \{\Lambda \times Z\} \times \{\varepsilon\} = \{(\lambda_L, z_H, \varepsilon); (\lambda_H, z_L, \varepsilon); (\lambda_L, z_L, \varepsilon); (\lambda_H, z_H, \varepsilon)\}$. Note that the last two elements are 0-probability events. For this to be a possible equilibrium we must observe that:

$$\lambda_i z_j \left(\frac{1-\alpha}{\alpha}\right) < r_{def} \left[\lambda_i \frac{z_j}{A} \left(\frac{1-\alpha}{\alpha}\right) + \varepsilon\right]$$

where

$$r_{def}=\frac{1-\frac{\alpha A^{1-\alpha}}{3}\left(\pi\left(z_L^{\alpha}\left(\frac{1-\alpha}{\alpha}\right)-\frac{\psi}{\bar{k}z_L^{1-\alpha}}\right)+(1-\pi)\left(\left(\frac{1-\alpha}{\alpha}\right)z_H^{\alpha}-\frac{\psi}{\bar{k}z_H^{1-\alpha}}\right)\right)}{\alpha A^{1-\alpha}\left[\frac{2}{3}\left(\frac{1-\alpha}{\alpha A}\right)\left[\pi z_L^{\alpha}+(1-\pi)z_H^{\alpha}\right]-\frac{\varepsilon}{3}\left[\frac{\pi}{\lambda_H z_L^{1-\alpha}}+\frac{1-\pi}{\lambda_L z_H^{1-\alpha}}\right]\right]}$$

Note that a sufficient condition for the inequality above to hold is:

$$\begin{aligned}\varepsilon \;\; > \;\; & \left(\frac{1-\alpha}{\alpha}\right)\max\left\{\lambda_H z_L;\lambda_L z_H\right\} \\ & \left\{\frac{\alpha A^{1-\alpha}\left[\frac{2}{3}\left(\frac{1-\alpha}{\alpha A}\right)\left[\pi z_L^{\alpha}+(1-\pi)z_H^{\alpha}\right]-\frac{\varepsilon}{3}\left[\frac{\pi}{\lambda_H z_L^{1-\alpha}}+\frac{1-\pi}{\lambda_L z_H^{1-\alpha}}\right]\right]}{1-\frac{\alpha A^{1-\alpha}}{3}\left(\pi\left(z_L^{\alpha}\left(\frac{1-\alpha}{\alpha}\right)-\frac{\psi}{\bar{k}z_L^{1-\alpha}}\right)+(1-\pi)\left(\left(\frac{1-\alpha}{\alpha}\right)z_H^{\alpha}-\frac{\psi}{\bar{k}z_H^{1-\alpha}}\right)\right)}-\frac{1}{A}\right\}\end{aligned}$$

Since ε also appears on the left hand side, we solve for ε so that

$$\begin{aligned}&\varepsilon\left[1+\frac{\left[\frac{\pi}{\lambda_H z_L^{1-\alpha}}+\frac{1-\pi}{\lambda_L z_H^{1-\alpha}}\right]\left(\frac{1-\alpha}{\alpha}\right)\max\left\{\lambda_H z_L;\lambda_L z_H\right\}}{3-\alpha A^{1-\alpha}\left(\pi\left(z_L^{\alpha}\left(\frac{1-\alpha}{\alpha}\right)-\frac{\psi}{\bar{k}z_L^{1-\alpha}}\right)+(1-\pi)\left(\left(\frac{1-\alpha}{\alpha}\right)z_H^{\alpha}-\frac{\psi}{\bar{k}z_H^{1-\alpha}}\right)\right)}\right] \\ &> \left(\frac{1-\alpha}{\alpha}\right)\max\left\{\lambda_H z_L;\lambda_L z_H\right\} \\ &\left\{\frac{\alpha A^{1-\alpha}2\left(\frac{1-\alpha}{\alpha A}\right)\left[\pi z_L^{\alpha}+(1-\pi)z_H^{\alpha}\right]}{3-\alpha A^{1-\alpha}\left(\pi\left(z_L^{\alpha}\left(\frac{1-\alpha}{\alpha}\right)-\frac{\psi}{\bar{k}z_L^{1-\alpha}}\right)+(1-\pi)\left(\left(\frac{1-\alpha}{\alpha}\right)z_H^{\alpha}-\frac{\psi}{\bar{k}z_H^{1-\alpha}}\right)\right)}-\frac{1}{A}\right\}\end{aligned}$$

or

$$\varepsilon>\frac{\left(\frac{1-\alpha}{\alpha}\right)\max\left\{\lambda_H z_L;\lambda_L z_H\right\}\left\{\frac{\alpha A^{1-\alpha}2\left(\frac{1-\alpha}{\alpha A}\right)\left[\pi z_L^{\alpha}+(1-\pi)z_H^{\alpha}\right]}{3-\alpha A^{1-\alpha}\left(\pi\left(z_L^{\alpha}\left(\frac{1-\alpha}{\alpha}\right)-\frac{\psi}{\bar{k}z_L^{1-\alpha}}\right)+(1-\pi)\left(\left(\frac{1-\alpha}{\alpha}\right)z_H^{\alpha}-\frac{\psi}{\bar{k}z_H^{1-\alpha}}\right)\right)}-\frac{1}{A}\right\}}{\left[1+\frac{\left[\frac{\pi}{\lambda_H z_L^{1-\alpha}}+\frac{1-\pi}{\lambda_L z_H^{1-\alpha}}\right]\left(\frac{1-\alpha}{\alpha}\right)\max\{\lambda_H z_L;\lambda_L z_H\}}{3-\alpha A^{1-\alpha}\left(\pi\left(z_L^{\alpha}\left(\frac{1-\alpha}{\alpha}\right)-\frac{\psi}{\bar{k}z_L^{1-\alpha}}\right)+(1-\pi)\left(\left(\frac{1-\alpha}{\alpha}\right)z_H^{\alpha}-\frac{\psi}{\bar{k}z_H^{1-\alpha}}\right)\right)}\right]}$$

If this holds, then the ex-post utility for the borrower in state (λ_i, z_j, x) is given by:

$$U_{ex-p-def}^{B,indns}=A^{1-\alpha}z_j^{\alpha}(1-\alpha)-r_{def}\alpha A^{1-\alpha}\left[\frac{\left(\lambda_i\left(\frac{z_j}{A}\frac{1-\alpha}{\alpha}\right)+x\right)}{\lambda_i\left(z_j\right)^{1-\alpha}}\right]$$

(given that $x \neq \varepsilon$. If $x = \varepsilon$ the borrower gets just 0). Therefore the ex-ante utility for the borrower is (see appendiz for details).

$$V_{indns}^{B,def} = A^{1-\alpha}(1-\alpha)\,\bar{k}\left[\pi z_L^{\alpha} + (1-\pi)\, z_H^{\alpha}\right] - \bar{k} - \psi A^{1-\alpha}\frac{\alpha}{3}\left[\frac{\pi}{z_L^{1-\alpha}} + \frac{1-\pi}{z_H^{1-\alpha}}\right]$$

Suppose now that the parameters are such that, if the borrower must choose between a nominal-indexed contract (with noise) and a T-good contract that induces default at (λ_H, z_L) then the difference in ex-ante utilities is equal to:

$$\begin{aligned} U_T^B - V_{indns}^{B,def} &= -\frac{A^{1-\alpha}\alpha\psi}{3}\left[\frac{3\pi}{z_L^{1-\alpha}} - \left[\frac{\pi}{z_L^{1-\alpha}} + \frac{1-\pi}{z_H^{1-\alpha}}\right]\right] \\ &= -\frac{A^{1-\alpha}\alpha\psi}{3}\left[\frac{2\pi}{z_L^{1-\alpha}} - \frac{1-\pi}{z_H^{1-\alpha}}\right] \end{aligned}$$

Therefore the borrower chooses a default-inducing $T - good$ contract rather than a default-inducing indexed-nominal contract if $\frac{2\pi}{z_L^{1-\alpha}} - \frac{1-\pi}{z_H^{1-\alpha}} < 0$ or

$$\frac{2\pi}{z_L^{1-\alpha}} < \frac{1-\pi}{z_H^{1-\alpha}}$$

which is equivalent to

$$\frac{1-\pi}{\pi} > 2\left(\frac{z_H}{z_L}\right)^{1-\alpha}$$

Note that since the right hand side is strictly greater than 2 then this implies that $\pi < \frac{1}{3}$. In other words, for a borrower to make such a choice the probability of state (λ_H, z_L) must be smaller than the probability that $x = \varepsilon$.

8 Concluding Remarks

The preliminary exploration in this paper tends to confirm in general some simple intuitions. Unsteady monetary policies, which make uncertain the real outcome of nominal contracts and which, when very erratic, also make nominal contracts less attractive (due to reporting lags) tend to induce dollarization of liabilities as long as the volatility of the real exchange rate is not too large. However, the vulnerability of those contracts to real exchange rate shocks (which may result in some instances in widespread defaults) is one of the large costs associated with the failure to provide a workable "domestic" unit of account for transactions among residents.

The model above assumes that the real shocks are completely driven by the relative price of non - tradeables (which is exogenously random). This assumption provides sufficient intuition to start analyzing the role of nominal and real shocks in the choice of the currency denomination of debt contracts. However, the model above clearly ignores general equilibrium effects. Also, the model above is not completely suitable to be interpreted as a domestic lender - domestic borrower relationship, since it is not natural to think that domestic lenders do not care about non - traded goods.

In order to tackle this problem, we propose to analyze a more general model, sketched in the appendix. This model assumes symmetric preferences between (domestic) lenders and domestic borrowers, both having Cobb - Douglas preferences defined over tradeables and non - tradeables. Domestic lenders receive a shock (in the future period) on their traded - good endowments, which substitutes for the shock on p_N in the sections above. We actually deduce the equilibrium value of p_N as a function of those endowment shocks. We also consider the possibility of strategic default by borrowers in the model of the appendix. The reader may verify that this second framework, although more complete, implies a much more cumbersome algebra to obtain the results. Further work is needed to make more trasparent the intuition behind some of those results.

Actually this second model also considers the case of indexed contracts and also it discusses why these contracts may not arise in equilibrium. The argument we use is the presence of costly state verification when considering indexed contracts. Indeed, in reality most of these contracts should be indexed to prices of various non - tradeables, most of which are services. Thus, it is unclear how a lender may monitor for free the ex-post state of the borrower which produces such services. On the other hand, publicly available price indices are only known with a lag with respect to time to which prices refer. The non - free - verification assumption is sufficient to exclude indexed contracts in some cases. Hence, there are insyances where the relevant comparison reduces again to that between nominal (non - indexed) and traded - good denominated debt contracts.

These results are related to the general discussion about indexed contracts as presented, for example, in Fischer (1983). This also tests the absence of indexed bonds in the U.S. market based on his theoretical earlier work (see Fischer, 1975). His main conclusion is that the factors that, according to his theory, should help to explain this absence are in fact false in practice. Even though the aim of this paper was clearly not to explain this *puzzle* posed by Fischer, our model stresses also an aspect ignored by these explanations, namely, the fact that *optimal* indexation may imply high verification costs.

We have explored the choice of contractual units in a very simple setting. A natural extension of the argument would be to incorporate hedging behavior on the part of agents, or different motives for borrowing (e.g. by producers of traded goods) which could result in a representation of states with multiple contractual forms. Another noticeable simplification of the argument was in the assumption that the "nominal shock" is independent of the denomination of assets. Modeling the policy-game that determines monetary policies in conjunction with the choice of denomination can be interesting to undertake. In this regard, to the extent that "asset dollarization" would discourage monetary variability, it need not be the case that large monetary shocks are actually observed in "dollarized economies". The relevant parameter to define the incentives for private agents to dollarize would be the variability under contracting in "domestic units".

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A Proofs of propositions (general equilibrium model)

Proof of proposition 5. The proof is a simple comparison of the ex-ante borrower's utility from either contract. The utility from the dollar contract is

$$V^{b,T}_{nodef} = \Lambda \left[A \left[\pi p^{\alpha}_{NL} + (1-\pi) p^{\alpha}_{NH} \right] - \left[\pi p^{\alpha-1}_{NL} + (1-\pi) p^{\alpha-1}_{NH} \right] \right]$$

Hence the comparison is just between

$$-\frac{\left[\pi p^{\alpha}_{NL} + (1-\pi) p^{\alpha}_{NH} \right]}{\pi p_{NL} + (1-\pi) p_{NH}}$$

and

$$-\left[\pi p^{\alpha-1}_{NL} + (1-\pi) p^{\alpha-1}_{NH} \right]$$

To prove this, suppose that the $T-good$ contract gives more utility. Hence $-\left[\pi p_{NL}^{\alpha-1}+(1-\pi)\,p_{NH}^{\alpha-1}\right]\geq-\frac{\left[\pi p_{NL}^{\alpha}+(1-\pi)p_{NH}^{\alpha}\right]}{\pi p_{NL}+(1-\pi)p_{NH}}$. Or

$$\left[\pi p_{NL}^{\alpha-1}+(1-\pi)\,p_{NH}^{\alpha-1}\right]\leq\frac{\left[\pi p_{NL}^{\alpha}+(1-\pi)\,p_{NH}^{\alpha}\right]}{\pi p_{NL}+(1-\pi)\,p_{NH}}$$

equivalent to

$$\left[\pi p_{NL}^{\alpha-1}+(1-\pi)\,p_{NH}^{\alpha-1}\right]\left[\pi p_{NL}+(1-\pi)\,p_{NH}\right]\leq\left[\pi p_{NL}^{\alpha}+(1-\pi)\,p_{NH}^{\alpha}\right]$$

$\Leftrightarrow$

$$\pi^{2}p_{NL}^{\alpha}+(1-\pi)^{2}\,p_{NH}^{\alpha}+\pi\,(1-\pi)\left[p_{NL}^{\alpha}\left(\frac{p_{NH}}{p_{NL}}\right)+p_{NH}^{\alpha}\left(\frac{p_{NL}}{p_{NH}}\right)\right]\leq\left[\pi p_{NL}^{\alpha}+(1-\pi)\,p_{NH}^{\alpha}\right]$$

$\Leftrightarrow$

$$\pi\,(1-\pi)\left[p_{NL}^{\alpha}\left(\frac{p_{NH}}{p_{NL}}\right)+p_{NH}^{\alpha}\left(\frac{p_{NL}}{p_{NH}}\right)\right]\leq\pi\,(1-\pi)\,p_{NL}^{\alpha}+(1-\pi)\,\pi p_{NH}^{\alpha}$$

$\Leftrightarrow$

$$p_{NL}^{\alpha}\left(\frac{p_{NH}}{p_{NL}}\right)+p_{NH}^{\alpha}\left(\frac{p_{NL}}{p_{NH}}\right)\leq p_{NL}^{\alpha}+p_{NH}^{\alpha}$$

$\Leftrightarrow$

$$p_{NL}^{\alpha}\left(\frac{p_{NH}}{p_{NL}}-1\right)\leq p_{NH}^{\alpha}\left(1-\frac{p_{NL}}{p_{NH}}\right)\Leftrightarrow p_{NL}^{\alpha}\left(\frac{p_{NH}-p_{NL}}{p_{NL}}\right)\leq p_{NH}^{\alpha}\left(\frac{p_{NH}-p_{NL}}{p_{NH}}\right)$$

Since $p_{NH}\geq p_{NL}$ then this is equivalent to $p_{NL}^{\alpha-1}\leq p_{NH}^{\alpha-1}$, or

$$p_{NH}^{1-\alpha}\leq p_{NL}^{1-\alpha}$$

and since $1-\alpha>0$ then $p_{NH}\leq p_{NL}$, a contradiction. Then we must have that $V_{nodef}^{b,T}<V_{nodef}^{b,ind}$. ■

Proof of proposition 7. It is true that for z_L large enough such that $\pi A^{1-\alpha}z_L^{\alpha}>1$, then

$$\frac{z_L^{1-\alpha}}{A^{1-\alpha}}<\pi\,(1-\alpha)\,z_L$$

which implies that there is no default under z_L, and since z_H and $1-\pi$ are both strictly positive

$$\frac{\frac{\pi}{z_L^{1-\alpha}}}{\left(\frac{\pi}{z_L^{1-\alpha}}+\frac{1-\pi}{z_H^{1-\alpha}}\right)}<1$$

which implies that there is no default under state z_H. ■

Proof of proposition 8. We basically need first that

$$\left(\frac{\frac{1}{A^{1-\alpha}}+\frac{\alpha\chi}{k}\frac{\pi}{z_L^{1-\alpha}}-(1-\alpha)\pi z_L^{\alpha}}{\alpha(1-\pi)}\right)z_H^{1-\alpha}\leq z_H\left(\frac{1-\alpha}{\alpha}\right)$$

or equivalently:

$$\left(\frac{\frac{1}{A^{1-\alpha}}+\frac{\alpha\chi}{k}\frac{\pi}{z_L^{1-\alpha}}-(1-\alpha)\pi z_L^{\alpha}}{(1-\alpha)(1-\pi)}\right)\leq z_H^{\alpha}$$

Clearly, for given z_L, there exists values for z_H such that this holds. In fact there exists a unique htreshold value $z_H(z_L)$ such that, for any $z_H>z_H(z_L)$ the inequality holds strict. On the other hand we also need that z_L be sufficiently low so that the borrower does not repay his debt under the non-default interest rate, that is $\frac{1}{A^{1-\alpha}\alpha\left(\frac{\pi}{z_L^{1-\alpha}}+\frac{1-\pi}{z_H^{1-\alpha}}\right)}>z_L\left(\frac{1-\alpha}{\alpha}\right)$, or equivalently,

$$\frac{1}{A^{1-\alpha}(1-\alpha)\left(\frac{\pi}{z_L^{1-\alpha}}+\frac{1-\pi}{z_H^{1-\alpha}}\right)}>z_L$$

It can be shown that $\frac{1}{A^{1-\alpha}(1-\alpha)\left(\frac{\pi}{z_L^{1-\alpha}}+\frac{1-\pi}{z_H^{1-\alpha}}\right)}$ is less than $\left(\frac{\frac{1}{A^{1-\alpha}}+\frac{\alpha\chi}{k}\frac{\pi}{z_L^{1-\alpha}}-(1-\alpha)\pi z_L^{\alpha}}{(1-\alpha)(1-\pi)}\right)$ [4],

[4] This is because $\frac{1}{A^{1-\alpha}(1-\alpha)\left(\frac{\pi}{z_L^{1-\alpha}}+\frac{1-\pi}{z_H^{1-\alpha}}\right)}<\left(\frac{\frac{1}{A^{1-\alpha}}+\frac{\alpha\chi}{k}\frac{\pi}{z_L^{1-\alpha}}-(1-\alpha)\pi z_L^{\alpha}}{(1-\alpha)(1-\pi)}\right)z_H^{1-\alpha}$ is equivalent to $\frac{\frac{1-\pi}{z_H^{1-\alpha}}}{\left(\frac{\pi}{z_L^{1-\alpha}}+\frac{1-\pi}{z_H^{1-\alpha}}\right)}<1+A^{1-\alpha}\frac{\pi}{z_L^{1-\alpha}}\left[\phi\alpha-(1-\alpha)z_L\right]$. However, we know that $z_L<\frac{1}{A^{1-\alpha}(1-\alpha)\left(\frac{\pi}{z_L^{1-\alpha}}+\frac{1-\pi}{z_H^{1-\alpha}}\right)}$ so $1-z_L(1-\alpha)A^{1-\alpha}>1-\frac{1}{\left(\frac{\pi}{z_L^{1-\alpha}}+\frac{1-\pi}{z_H^{1-\alpha}}\right)}=\frac{\frac{1-\pi}{z_H^{1-\alpha}}}{\left(\frac{\pi}{z_L^{1-\alpha}}+\frac{1-\pi}{z_H^{1-\alpha}}\right)}$. Given that $\phi>0$ we obtained the desired result.

so that $z_L < \left(\frac{\frac{1}{A^{1-\alpha}} + \frac{\alpha\chi}{k}\frac{\pi}{z_L^{1-\alpha}} - (1-\alpha)\pi z_L^{\alpha}}{(1-\alpha)(1-\pi)} \right) z_H^{1-\alpha}$, so default occurs effectively at z_L. This shows the result. ■

Proof of Proposition 9. Note that since $\frac{\pi}{z_H^{1-\alpha}} > 0$ and $\frac{\lambda_H}{\lambda_L} > 1$ then $\frac{\lambda_H}{\lambda_L}\frac{1-\pi}{z_H^{1-\alpha}} + \frac{\pi}{z_L^{1-\alpha}} > \frac{1-\pi}{z_H^{1-\alpha}} + \frac{\pi}{z_L^{1-\alpha}} > \frac{\pi}{z_L^{1-\alpha}}$ for any $\lambda_j's$ and $z_k's$. So

$$\frac{1}{A^{1-\alpha}\alpha\left(\frac{\lambda_H}{\lambda_L}\frac{1-\pi}{z_H^{1-\alpha}} + \frac{\pi}{z_L^{1-\alpha}}\right)} < \frac{1}{A^{1-\alpha}\alpha\frac{\pi}{z_L^{1-\alpha}}} = \frac{z_L^{1-\alpha}}{\alpha A^{1-\alpha}\pi}$$

Hence as z_L is large enough $\frac{z_L^{1-\alpha}}{\alpha A^{1-\alpha}\pi}$ must be less than z_L since the first is a strictly concave function. Therefore z_L is greater than $\frac{1}{A^{1-\alpha}\alpha\left(\frac{\lambda_H}{\lambda_L}\frac{1-\pi}{z_H^{1-\alpha}} + \frac{\pi}{z_L^{1-\alpha}}\right)}$. As $z_H > z_L$, so happens with z_H. On the other hand, since $\frac{1-\pi}{z_H^{1-\alpha}} + \frac{\lambda_L}{\lambda_H}\frac{\pi}{z_L^{1-\alpha}} > \frac{1-\pi}{z_H^{1-\alpha}}$ then

$$\frac{1}{A^{1-\alpha}\alpha\left(\frac{1-\pi}{z_H^{1-\alpha}} + \frac{\lambda_L}{\lambda_H}\frac{\pi}{z_L^{1-\alpha}}\right)} < \frac{1}{A^{1-\alpha}\alpha\left(\frac{1-\pi}{z_H^{1-\alpha}}\right)} = \frac{z_H^{1-\alpha}}{A^{1-\alpha}\alpha(1-\pi)}$$

Given that this is a strictly concave function, there exists z_L is large enough so that

$$z_H > z_L > \frac{z_H^{1-\alpha}}{A^{1-\alpha}\alpha(1-\pi)}$$

This shows the result. ■

Proof of Proposition 10. Start by assuming that

$$A^{1-\alpha}z_L < \frac{1}{(1-\alpha)\left[\frac{\pi}{z_L^{1-\alpha}} + \frac{1-\pi}{z_H^{1-\alpha}}\right]}$$

so that it is not true that no default occurs with dollar contracts. However, no default occurs under nominal contracts with z_L if

$$A^{1-\alpha}z_L \geq \frac{1}{(1-\alpha)\left[\frac{\pi}{z_L^{1-\alpha}} + \left(\frac{\lambda_H}{\lambda_L}\right)\frac{1-\pi}{z_H^{1-\alpha}}\right]}$$

Solving for $\left(\frac{\lambda_H}{\lambda_L}\right)$ this inequality is equivalent to:

$$\frac{\lambda_H}{\lambda_L} \geq \left(\frac{z_L}{z_H}\right)\left(\frac{z_H^{1-\alpha}}{1-\pi}\right)\left[\frac{1}{(1-\alpha)\,A^{1-\alpha}} - \pi z_L^{\alpha}\right]$$

There is no default if z_H under nominal contracts if and only if

$$A^{1-\alpha} z_H \geq \frac{1}{(1-\alpha)\left[\left(\frac{\lambda_L}{\lambda_H}\right)\frac{\pi}{z_L^{1-\alpha}} + \frac{1-\pi}{z_H^{1-\alpha}}\right]}$$

solving for $\left(\frac{\lambda_L}{\lambda_H}\right)$ yields:

$$\frac{\lambda_L}{\lambda_H} \geq \left(\frac{z_L}{z_H}\right)\left(\frac{z_L^{1-\alpha}}{\pi}\right)\left(\frac{1}{A^{1-\alpha}(1-\alpha)} - (1-\pi)\,z_H^{\alpha}\right)$$

or

$$\frac{\lambda_H}{\lambda_L} \leq \left(\frac{z_H}{z_L}\right)\left(\frac{\pi}{z_L^{1-\alpha}}\right)\left(\frac{1}{\frac{1}{A^{1-\alpha}(1-\alpha)} - (1-\pi)\,z_H^{\alpha}}\right)$$

The combination of both inequalities give the desired result. ■

Proof of Proposition 11. The first inequality is the same as

$$z_L\left(\frac{1-\alpha}{\alpha}\right)\lambda_H < \frac{\lambda_L\lambda_H}{\left[\frac{\pi}{z_L^{1-\alpha}}\lambda_L + \frac{1-\pi}{z_H^{1-\alpha}}\lambda_H\right]\alpha A^{1-\alpha}} \Leftrightarrow z_L < \frac{\lambda_L}{A^{1-\alpha}(1-\alpha)\left[\frac{\pi}{z_L^{1-\alpha}}\lambda_L + \frac{1-\pi}{z_H^{1-\alpha}}\lambda_H\right]}$$

which proves the first part. The second inequality is

$$\left(\frac{\frac{1}{A^{1-\alpha}} + \frac{\alpha\psi}{\bar{k}}\frac{\pi}{z_L^{1-\alpha}} - (1-\alpha)\,\pi z_L^{\alpha}}{\alpha}\right)\left(\frac{z_H^{1-\alpha}}{1-\pi}\right)\lambda_L \leq \lambda_L z_H\left(\frac{1-\alpha}{\alpha}\right)$$

so

$$\left(\frac{\frac{1}{A^{1-\alpha}} + \frac{\alpha\psi}{\bar{k}}\frac{\pi}{z_L^{1-\alpha}} - (1-\alpha)\,\pi z_L^{\alpha}}{(1-\alpha)(1-\pi)}\right) \leq z_H^{\alpha}$$

This holds for sufficiently large z_H. ■

Proof of Proposition 12. We have default at (λ_L, z_H) when $\frac{\lambda_H}{A^{1-\alpha}\alpha\left(\frac{\pi}{z_L^{1-\alpha}}\lambda_L+\frac{1-\pi}{z_H^{1-\alpha}}\lambda_H\right)}$ $= \frac{1}{A^{1-\alpha}\alpha\left(\frac{\pi}{z_L^{1-\alpha}}\frac{\lambda_L}{\lambda_H}+\frac{1-\pi}{z_H^{1-\alpha}}\right)} > z_H\left(\frac{1-\alpha}{\alpha}\right)$. Note that when $\left(\frac{\lambda_H}{\lambda_L}\right)$ is large enough, the expression $\frac{1}{A^{1-\alpha}(1-\alpha)\left(\frac{\pi}{z_L^{1-\alpha}}\frac{\lambda_L}{\lambda_H}+\frac{1-\pi}{z_H^{1-\alpha}}\right)}$ is close enough to $\frac{1}{A^{1-\alpha}(1-\alpha)\left(\frac{1-\pi}{z_H^{1-\alpha}}\right)}$ $=\frac{z_H^{1-\alpha}}{A^{1-\alpha}(1-\alpha)(1-\pi)}$. When z_H is small enough then this is strictly larger than z_H. In particular, this is true whenever $z_H < \left[\frac{1}{A^{1-\alpha}(1-\alpha)(1-\pi)}\right]^{\frac{1}{1-\alpha}}$. Hence under this conditoin and the fact that $\left(\frac{\lambda_H}{\lambda_L}\right)$ is large enough guarantees that there is default at the state (λ_L, z_H). On the other hand, there is no default in state (λ_H, z_L) if

$$\left(\frac{\frac{1}{A^{1-\alpha}}+\frac{\alpha\psi}{k}\left(\frac{1-\pi}{z_H^{1-\alpha}}\right)-(1-\alpha)(1-\pi)z_H^{\alpha}}{\alpha\pi}\right)\lambda_H z_L^{1-\alpha} \leq \lambda_H z_L\left(\frac{1-\alpha}{\alpha}\right)$$

Rearranging gives directly:

$$z_L^{\alpha}(1-\alpha)\pi \geq \left[\frac{1}{A^{1-\alpha}}+\frac{\alpha\psi(1-\pi)}{z_H^{1-\alpha}}-(1-\alpha)(1-\pi)z_H^{\alpha}\right]$$

which is the second desired inequality. ■

Proof of Proposition 13. If the total absence of default were true then the first order conditions from the lender imply

$$r = \frac{A^{\alpha}}{(1-\alpha)\left[\pi z_L^{\alpha}+(1-\pi)z_H^{\alpha}\right]}$$

On the other hand, there is no default under any state if and only if

$$r \leq A$$

equivalent to

$$\frac{1}{(1-\alpha)\left[\pi z_L^{\alpha}+(1-\pi)z_H^{\alpha}\right]A^{1-\alpha}} \leq 1$$

which holds given the assumption in the statement. This ends the proof. ■

Proof of Proposition 14. If $\widehat{D} = \emptyset$ then

$$\frac{1}{A^{1-\alpha}\alpha\left(\frac{\pi}{z_L^{1-\alpha}} + \frac{1-\pi}{z_H^{1-\alpha}}\right)} \leqq \left(\frac{1-\alpha}{\alpha}\right)\theta z_L + (1-\chi)\left(\frac{\psi}{\bar{k}}\right)$$

which is the same as

$$\begin{aligned}
\frac{1}{(1-\alpha)A^{1-\alpha}} &\leqq \left(\frac{\pi}{z_L^{1-\alpha}} + \frac{1-\pi}{z_H^{1-\alpha}}\right) z_L\theta + (1-\chi)\left(\frac{\alpha}{1-\alpha}\right)\left(\frac{\psi}{\bar{k}}\right)\left(\frac{\pi}{z_L^{1-\alpha}} + \frac{1-\pi}{z_H^{1-\alpha}}\right) \\
&< \theta\left(\pi z_L^{\alpha} + (1-\pi) z_H^{\alpha}\right) + (1-\chi)\left(\frac{\alpha}{1-\alpha}\right)\left(\frac{\psi}{\bar{k}}\right)\left(\frac{\pi}{z_L^{1-\alpha}} + \frac{1-\pi}{z_H^{1-\alpha}}\right) \\
&= \pi\left[\theta z_L^{\alpha} + \frac{\eta}{z_L^{1-\alpha}}\right] + (1-\pi)\left[\theta z_H^{\alpha} + \frac{\eta}{z_H^{1-\alpha}}\right]
\end{aligned}$$

where the last equality comes from the definition of η. Hence the inequality to have $D_N = \emptyset$ also holds here. ■

Proof of the derivation of $V_{indns}^{B,def}$. According to the main text, ex-post we have (for the no-default states)

$$U_{ex-p-def}^{B,indns} = A^{1-\alpha} z_j^{\alpha}(1-\alpha)\bar{k} - r_{def}\alpha A^{1-\alpha}\left[\frac{\left(\lambda_i\left(\frac{z_j}{A}\frac{1-\alpha}{\alpha}\right) + x\right)}{\lambda_i (z_j)^{1-\alpha}}\right]\bar{k}$$

So in ex-ante terms we obtain

$$\begin{aligned}
V_{indns}^{B,def} &= A^{1-\alpha}(1-\alpha)\left[\pi z_L^{\alpha} + (1-\pi) z_H^{\alpha}\right]\left(\frac{2}{3}\right)\bar{k} \\
&\quad - r_{def}\alpha A^{1-\alpha}\left\{\left(\frac{2}{3A}\left(\frac{1-\alpha}{\alpha}\right)\right)\left[\pi z_L^{\alpha} + (1-\pi) z_H^{\alpha}\right] - \frac{\varepsilon}{3}\left[\frac{\pi}{\lambda_H z_L^{1-\alpha}} + \frac{1-\pi}{\lambda_L z_H^{1-\alpha}}\right]\right\}\bar{k}
\end{aligned}$$

Now, the lender's FOC in this case can be written as:

$$r_{def}\alpha A^{1-\alpha}\sum_{i,j,x\in\bar{\Delta}}\frac{q_{ij}}{3}\left[z_j^{\alpha}\left(\frac{1-\alpha}{\alpha A}\right) + \frac{x}{\lambda_i z_j^{1-\alpha}}\right] = 1 - \alpha A^{1-\alpha}\sum_{i,j,x\in\bar{D}}\frac{q_{ij}}{3}\left(z_j^{\alpha}\left(\frac{1-\alpha}{\alpha}\right) - \frac{\psi}{z_j^{1-\alpha}}\right)$$

$\Leftrightarrow$

$$\begin{aligned}
& r_{def}\alpha A^{1-\alpha}\left[\left(\frac{1-\alpha}{\alpha A}\right)\frac{2}{3}\left[\pi z_L^{\alpha} + (1-\pi) z_H^{\alpha}\right] - \frac{1}{3}\varepsilon\left[\frac{\pi}{\lambda_H z_L^{1-\alpha}} + \frac{1-\pi}{\lambda_L z_H^{1-\alpha}}\right]\right] \\
&= 1 - \frac{\alpha A^{1-\alpha}}{3}\left[\left(\frac{1-\alpha}{\alpha}\right)\left[\pi z_L^{\alpha} + (1-\pi) z_H^{\alpha}\right] - \psi\left[\frac{\pi}{z_L^{1-\alpha}} + \frac{1-\pi}{z_H^{1-\alpha}}\right]\right]
\end{aligned}$$

Hence replacing this expression in that of $V_{indns}^{B,def}$ we get

$$\begin{aligned}
V_{indns}^{B,def} &= A^{1-\alpha}(1-\alpha)\left[\pi z_L^{\alpha}+(1-\pi)z_H^{\alpha}\right]\left(\frac{2}{3}\right)\bar{k} \\
&\quad -\bar{k}+\frac{A^{1-\alpha}(1-\alpha)}{3}\left[\pi z_L^{\alpha}+(1-\pi)z_H^{\alpha}\right]\bar{k}-\psi\frac{\alpha A^{1-\alpha}}{3}\left[\frac{\pi}{z_L^{1-\alpha}}+\frac{1-\pi}{z_H^{1-\alpha}}\right] \\
&= A^{1-\alpha}(1-\alpha)\left[\pi z_L^{\alpha}+(1-\pi)z_H^{\alpha}\right]\bar{k}-\bar{k}-\psi\frac{\alpha A^{1-\alpha}}{3}\left[\frac{\pi}{z_L^{1-\alpha}}+\frac{1-\pi}{z_H^{1-\alpha}}\right]
\end{aligned}$$

which is what we wanted to show. ■