

## Supply Chain Planning Considering the Production of Defective Products

Ferrara Miguel, Corsano Gabriela, Montagna Marcelo

INGAR – Instituto de Desarrollo y Diseño CONICET-UTN

Avellaneda 3657, Santa Fe, Argentina

E-mail: {miguel ferrara, g corsano, m montagna}@santafe-conicet.gov.ar

**Abstract.** When the markets become more competitive, the customer satisfaction level starts to be a more important factor. One of the principal components to achieve the customer satisfaction is to minimize the amount of defective or non-standard products. Therefore, it is convenient to consider decisions about product quality when the supply chain is planned. In this work, a mixed integer linear programming mathematical (MILP) model is developed for the supply chain planning, which determines impact of different decisions over the expected number of defective products. These decisions are: supplier selection, inbound control methods selection, and production process selection. The objective function is the cost minimization associated to the supply chain planning.

**Keywords:** Supply Chain; Planning; Customer Satisfaction; Defective Products

### 1 Introduction

Many companies when planning their supply chain (SC) face the challenge of taking the right decisions between two conflictive objectives: Cost versus Customer service level. Without a satisfied customer, the supply chain strategy cannot be deemed effective. Van Hoek et al. [1] emphasized that to assess supply chain performance, supply chain metrics must center on customer satisfaction. One of the principal ways to achieve it is to deliver the least amount of defective products. In order to do that, a model for the planning of the SC that considers the amount of defective products delivered to each client is proposed in this work to assess the Customer service level. There are few works in the literature considering the quality of the final products delivered to the client for the design or planning of the SC [2, 3]. Some exceptions are, for example, Franca et al. [4], who develop a multi-objective stochastic model for

adfa, p. 1, 2011.

© Springer-Verlag Berlin Heidelberg 2011

this problem. The quality of the total production is an objective function, and it is measured considering the quality of raw materials used for producing each product. Chung-Chi et al. [5] consider one supplier and one manufacturer both with imperfect production and inspection systems. The model establishes the amount of investment to increase the quality of the production and inspection system in order to maximize the expected profit per unit. Brojeswar et al. [6], propose a continuous-time production and inventory model considering defective raw materials and rework of defective products. The manufacturer engages in 100% perfect inspection and discards the defective raw materials, so they do not enter in the production process. Duffuaa et al. [7] develop a multi-objective optimization model for a process targeting problem. The quality of the product is a random variable with known variance, and the objective is to find the optimal value for its mean in order to maximize the expected profit per product, along with other objectives. A product is considered non-conforming if its quality is lower than a given value, and, in this case, it is sold at lower price to the client.

## 2 Problem Description

A MILP model is presented to solve a SC planning problem of one period. The SC considered in this work involves three echelons: suppliers, production plants, and clients. The product demands of each client must be fulfilled respecting the maximum amount of defective products that each client tolerates. The impact of 3 types of decisions on the product quality is considered: suppliers selection, selection of in-bound control methods, and selection of production processes.

The model determines the following planning decisions: how much raw materials must be purchased at each supplier and which inbound control systems are used. Also, how many products are produced by each production process (technologies) at each plant, how many defected products are discarded, and how many defective and non-defective products are sent to each client.

It is assumed that a type of raw material to produce a given product, at a given plant, must be purchased at a single supplier, and controlled by a single inbound control method. It is also considered that all the units of raw material of a certain type, delivered from a certain supplier, have the same probability to be defective. This probability is usually related with the price, in the sense that it will be higher if the price is cheaper. The raw materials that arrive to the plants are controlled by inbound control methods that are imperfect; this means that there is a probability of not detecting defective raw materials. This implies that, defective units of raw material may enter into the production system, causing a probability of producing defective products. The units of raw material detected as defective are automatically discarded and they do not enter in the production process. The best control method will be more expensive. The raw materials not discarded are used in any of the available production process in the plant. At each plant the model decides how many units are produced by each production process (technologies). A unit of product produced by a given technology has a known probability to be defective.

Therefore, according to the supplied raw material, inbound control method and process technology, a unit of final product will have a probability to be defective that depends on which combination of these decisions has been taken to produce it. Then, the expected number of produced defective products can be computed. Also, there is a penalty for each unit of defective final product delivered to a client. This penalty depends on the client. At each plant, 100% perfect outbound inspection over its final products is performed. This means that the total amount of defective products will be detected. After this inspection the model decides how many defective products are discarded and how many are sent to each client. This decision is made based on the tradeoff between the cost to produce a unit of product versus the penalty cost of deliver a defective unit of product. The objective function considers the sum of the following costs: raw materials, control, production, transport, and penalty. Finally, the model determines the SC planning with minimum cost that satisfies the demands of the clients and their tolerance for defective products.

### 3 Mathematical Model

In this section, the mathematical formulation is presented. The model parameters and variables are defined in the Nomenclature section.

#### Selection of Suppliers and Inbound Control Systems for Raw Materials.

The Equations 1 and 2 determine which plants are going to be used and which products they are going to produce.

$$xc_{il} \leq xcd_l \quad \forall i \in I, l \in L \quad (1)$$

$$xcd_l \leq \sum_i xc_{il} \quad \forall l \in L \quad (2)$$

Let  $R_i = \{r_{1i}, r_{2i}, \dots, r_{m_i}\}$  be the set of raw materials necessary for producing product  $i$ , and  $R = \bigcup_i R_i$ , the set of all raw materials. Let denote an element of  $R$  simply by  $r$ . It is assumed that only one supplier can supply raw material  $r$  for the production of  $i$  in plant  $l$  if that product is produced in  $l$ :

$$\sum_{s \in S} x_{sr_{ji}il} = xc_{il} \quad \forall i \in I, l \in L, r_{ji} \in R_i \quad (3)$$

For a plant  $l$ , an inbound control method  $u$  with 100% inspection is used to control the raw material  $r$  to produce  $i$  only if the product  $i$  is produced at plant  $l$ :

$$\sum_{u \in U_{r_{ji}}} x_{1r_{ji}l u} = xc_{il} \quad \forall i \in I, l \in L, r_{ji} \in R_i \quad (4)$$

To make  $Q_{1sr_{ji}ilt}$  zero if  $x_{sr_{ji}il}$  is zero, a constraint of big-M type is stated:

$$Q_{1sr_{ji}ilt} \leq M_0 x_{sr_{ji}il} \quad \forall s \in S, i \in I, l \in L, t \in T_{il}, r_{ji} \in R_i \quad (5)$$

### Defective Raw Materials Detected by the Control System

According to the concepts of random variable and Binomial distribution  $X \sim \text{Bin}(n, p)$  denotes that a random variable  $X$  has Binomial distribution with parameters  $n$  (the number of trails) and  $p$  (the probability of success). In order to denote the expected value of a random variable  $X$  the notation  $E X$  is used.

It is assumed that suppliers can deliver defective raw material. Supposing that a supplier delivers  $N$  units of raw material  $r$ , let  $X_{sr}$  be a random variable that measures the quantity of defective units of raw material  $r$  delivered by  $s$ . We suppose that  $X_{sr} \sim \text{Bin}(N, p_{sr})$  where  $p_{sr}$  is a parameter that represents the probability for a unit of raw material  $r$  supplied by supplier  $s$  to be defective

On the other hand, each inbound control system  $u$  is imperfect. This means that a defective raw material  $r$  can be undetected by  $u$ , but a non-defective raw material  $r$  will never be detected as defective. The units of raw material are controlled one at a time by the control system. Given a unit of defective raw material  $r$ , the probability that the inbound control system  $u$  detects it as defective is the parameter  $c_{ur}$ . Also, when a unit of raw material  $r$  is detected as defective then it is automatically discarded. Then

$$Q1d_{srilt} = p_{sr}c_{ur} Q1_{srilt}$$

As this equation must be satisfied only when raw material  $r$  comes from  $s$  for producing  $i$  in  $l$  and it is controlled by  $u$ , we can state this using constraints of Big-M type

$$Q1d_{srjilt} - p_{srji}c_{urji}Q1_{srjilt} \geq -M_1(2 - x_{srjil} - x1_{rjiilu}) \quad (6)$$

$$Q1d_{srjilt} - p_{srji}c_{urji}Q1_{srjilt} \leq M_1(2 - x_{srjil} - x1_{rjiilu}) \quad (7)$$

$$\forall s \in S, i \in I, l \in L, r_{ji} \in R_i, t \in T_{il}, u \in U_r$$

Now, the raw material that is not discarded is used to produce. This is stated in the following two equations:

$$Q1nd_{srjilt} = Q1_{srjilt} - Q1d_{srjilt} \quad \forall s \in S, i \in I, l \in L, t \in T_{il}, r_{ji} \in R_i \quad (8)$$

$$Q2_{ilt} f_{irji} = \sum_{s \in S} Q1nd_{srjilt} \quad \forall i \in I, l \in L, t \in T_{il}, r_{ji} \in R_i \quad (9)$$

### 3.3 Probability for Non-discarded Unit of Raw Material to Be Defective

In order to calculate the probability for a unit to be defective given that it has not been discarded, i.e. defective raw material not detected by the control system, two well known results of probability theory has been used: the Bayes theorem and the law of total probability.

$$P \text{ } r \text{ from } s \text{ is defective } | r \text{ from } s \text{ was not detected by } u = \frac{p_{sr} - c_{ur} p_{sr}}{1 - c_{ur} p_{sr}} = \theta_{sru}$$

In other words, the probability that a unit of raw material  $r$  from  $s$  controlled by  $u$  remains defective after the discarding is equal to the parameter  $\theta_{sru}$ . Let  $\psi_{sriu}$  be random variable that measures the quantity of raw material  $r$  from  $s$  **that is defective but not detected** by  $u$  and it is used to produce a unit of product  $i$  in  $l$ . Then, because of previous suppositions;  $\psi_{sriu} \sim Bin(f_{ri}, \theta_{sru})$ . Where  $f_{ri}$  is a parameter that denotes the amount of raw material  $r$  necessary to produce a unit of product  $i$ . This parameter must be a natural number.

### 3.4 Probability to Produce a Defective Unit of Product.

#### Defective Products Due to Defective Raw Materials

In the model we consider that one of the reasons for a product to be defective is to produce it with an amount of defective raw material that exceeds a threshold.

The parameter  $cm_{r_{ji}}$ , with  $r_{ji} \in R_i$ , is defined as follows: if a unit of product  $i$  is produced with more than  $cm_{r_{ji}}$  units of defective raw material  $r_{ji}$ , then the unit of product  $i$  is considered defective, otherwise it is considered non-defective. If at least one unit of defective raw material  $r_{ji}$  will produce a defective unit of product  $i$  then  $cm_{r_{ji}}$  is set to zero. If the condition of raw material  $r_{ji}$  does not affect the quality of the product  $i$  then  $cm_{r_{ji}} = f_{r_{ji}i}$ , this means that every unit of  $r_{ji}$  used to produce  $i$  can be defective.

The probability for a unit of product  $i$  produced in plant  $l$  to be defective due to defects in the raw materials must be determined. Then, the selected suppliers for each raw material to produce product  $i$  in plant  $l$  must be known, as well as, i.e. the procurement policy-, the chosen control system to check each raw material, i.e which was the inbound raw material control policy.

Let  $\alpha_{il}$  be a list of elements of  $S$ ,  $\alpha_{il} = s_1, s_2, \dots, s_{r_{m_i}}$  with  $s_j \in S$ , where  $s_j$  is the supplier of raw material  $r_{ji} \in R_i$  to produce  $i$  in  $l$ ,  $\alpha_{il}$  is a raw material procurement policy for product  $i$  in plant  $l$ . Let  $\rho_{il}$  be the set of all raw material procurement policies for product  $i$  in plant  $l$ . Let  $\beta_{il}$  be a list of elements of  $U$   $\beta_{il} = u_1, u_2, \dots, u_{r_{m_i}}$  with  $u_j \in U_{r_{ji}}$ , where  $u_j$  is the inbound control method applied to raw material  $r_{ji} \in R_i$  to produce product  $i$  in plant  $l$ ,  $\beta_{il}$  is an inbound raw material control policy for product  $i$  in plant  $l$ . Let  $\mu_{il}$  be the set of all inbound raw material control policies for product  $i$  in plant  $l$ .

Now we compute the probability for a unit of product  $i$  produced in plant  $l$  to be defective due to defects in the raw materials.

Let  $A_{\alpha_{il}\beta_{il}}$  be the event "a unit of product  $i$  produced in plant  $l$  is defective due to the use of defective raw materials from the set  $R_i$ , where  $\alpha_{il}$  is the raw material procurement policy and  $\beta_{il}$  is the inbound raw material control method policy". In order to calculate  $P A_{\alpha_{il}\beta_{il}}$  the random variable  $\psi_{sriu}$ , defined in section 3.2, is used:

$$P A_{\alpha_{il}\beta_{il}} = 1 - \prod_{j=1}^{m_i} P \psi_{s_j r_{ji} u_j} \leq cm_{r_{ji}}$$

$$P \psi_{s_j r_{ji} u_j} \leq c m_{r_j} = \sum_{k=0}^{c m_{r_j}} C(f_{r_{ji}}, k) (\theta_{s_j r_{ji} u_j})^k (1 - \theta_{s_j r_{ji} u_j})^{(f_{r_{ji}} - k)}$$

From these equations, it can be noted that  $P A_{\alpha_{il}\beta_{il}}$  is a function of  $i, l, \alpha_{il}$  and  $\beta_{il}$ . Let define the parameter  $\delta_{\alpha_{il}\beta_{il}} = P A_{\alpha_{il}\beta_{il}}$

### Defective Products Due to Defects of the Technology

A unit of product  $i$  can be defective not only because of defective raw materials but also because of defects of the productive process or technology  $t$ . Let  $B_{it}$  be the event "a unit of product  $i$  produced in plant  $l$  by technology  $t$  is defective due to defects in the technology  $t$ ". Suppose that  $P B_{it} = d r r_{it}$  where  $d r r_{it}$  is a parameter. Let  $V_{it}$  be a random variable that measures the quantity of defective products  $i$  produced by technology  $t$  due to defects of the technology  $t$ , given that  $N$  units of product  $i$  are produced by technology  $t$ . We suppose that  $V_{it} \sim \text{Bin}(N, d r r_{it})$

### Defective Products Due to Defective Raw Materials or Defects of the Technology

Let  $C_{t\alpha_{il}\beta_{il}}$  be the event "one unit of product  $i$  produced in plant  $l$ , is defective due to any or both of the next reasons: "

- The use of defective raw materials from the set  $R_i$  where  $\alpha_{il}$  is the raw material procurement policy and  $\beta_{il}$  is the inbound raw material control method policy
- Defects in the technology  $t$ .

$$\text{Then } P C_{t\alpha_{il}\beta_{il}} = P A_{\alpha_{il}\beta_{il}} B_{it} = P A_{\alpha_{il}\beta_{il}} + P B_{it} - P A_{\alpha_{il}\beta_{il}} B_{it}$$

But  $P A_{\alpha_{il}\beta_{il}} B_{it} = P B_{it} A_{\alpha_{il}\beta_{il}} P(A_{\alpha_{il}\beta_{il}})$ . Let define the parameter  $\varphi_{it}$  with  $\varphi_{it} \geq 1$ . It measures how much  $P B_{it}$  increases when defective raw materials are used to produce product  $i$  using technology  $t$ . Then for simplicity we assumed that  $P B_{it} A_{\alpha_{il}\beta_{il}} = P B_{it} \varphi_{it}$  and  $P B_{it} A_{\alpha_{il}\beta_{il}} \approx P B_{it}$  so the parameter  $\varphi_{it} \approx 1$ . Let's define the parameter  $\delta_{t\alpha_{il}\beta_{il}} = P C_{t\alpha_{il}\beta_{il}}$

### 3.5 Calculation of the Expected Amount of Defective Units of Product

Let  $Z_{t\alpha_{il}\beta_{il}}$  be a random variable that measure the quantity of defective product  $i$  due to the event  $C_{t\alpha_{il}\beta_{il}}$  when  $N$  units of product  $i$  are produced by  $t$  in  $l$  with purchase and control policies  $\alpha_{il}$  and  $\beta_{il}$ , respectively. It is assumed that  $Z_{t\alpha_{il}\beta_{il}} \sim \text{Bin}(N, \delta_{t\alpha_{il}\beta_{il}})$ . Let  $Q2d_{ilt}$  be the expected number of defective products  $i$  produced in plant  $l$  using technology  $t$  due to the event  $C_{t\alpha_{il}\beta_{il}}$ , given that the purchase and control policy were  $\alpha_{il}$  and  $\beta_{il}$  respectively. So  $Q2d_{ilt} = \delta_{t\alpha_{il}\beta_{il}} Q2_{ilt}$

Suppose that  $x^*$  is a feasible solution of the problem. In order to compute  $Q2d_{ilt}$  for this feasible solution, suppose that  $x^*$  has a purchase policy  $\alpha_{il}^* = s_1^*, s_2^*, \dots, s_{m_i}^*$  and an inspection policy  $\beta_{il}^* = u_1^*, u_2^*, \dots, u_{m_i}^*$ . For  $x^*$ :  $Q2d_{ilt} = \delta_{t\alpha_{il}^* \beta_{il}^*} Q2_{ilt}$  if  $(x_{s_1^* r_{1i}} = 1 \text{ and } \dots \text{ and } x_{s_{m_i}^* r_{m_i}} = 1 \text{ and } \dots)$

$x_{1r_1 i l u_1^*} = 1$  and ... and  $x_{1r_{m_i} i l u_{m_i}^*} = 1$ )

So this condition is stated using a big-M restriction:

$$\delta_{t\alpha_{il}\beta_{il}} Q_{2ilt} - Q_{2dilt} \leq \sum_{1 \leq j \leq m_i} M_3 (1 - x_{s_j r_{j i l}}) + \sum_{1 \leq j \leq m_i} M_3 (1 - x_{1r_{j i l} u_j})$$

$$\forall i, l, t \in T_{il}, s_1 \in S, \dots, s_{m_i} \in S; u_1 \in U_{r_{1i}}, \dots, u_{m_i} \in U_{r_{m_i i}}$$

with  $\alpha_{il} = s_1, s_2, \dots, s_{m_i}$  and  $\beta_{il} = u_1, u_2, \dots, u_{m_i}$  (10)

$$\delta_{t\alpha_{il}\beta_{il}} Q_{2ilt} - Q_{2dilt} \geq \sum_{1 \leq j \leq m_i} M_3 (x_{s_j r_{j i l}} - 1) + \sum_{1 \leq j \leq m_i} M_3 (x_{1r_{j i l} u_j} - 1)$$

$$\forall i, l, t \in T_{il}, s_1 \in S, \dots, s_{m_i} \in S; u_1 \in U_{r_{1i}}, \dots, u_{m_i} \in U_{r_{m_i i}}$$

with  $\alpha_{il} = s_1, s_2, \dots, s_{m_i}$  and  $\beta_{il} = u_1, u_2, \dots, u_{m_i}$  (11)

Restriction (10) and (11) are active only when ( $\alpha_{il} = \alpha_{il}^*$  and  $\beta_{il} = \beta_{il}^*$ )

### 3.6 Fulfillment of the Demand

A 100% outbound inspection is assumed to be applied at each plant, i.e. with a control method that is perfect. So the expected number of defective products detected is equal to the expected number of defective products.

$${}_t Q_{2dilt} = {}_k Q_{3dilk} + Q_{2ddiscard_{il}} \quad \forall i, l \quad (12)$$

$$Q_{2nd_{ilt}} = Q_{2ilt} - Q_{2dilt} \quad \forall i, l, t \quad (13)$$

$${}_t Q_{2nd_{ilt}} = {}_k Q_{3nd_{ilk}} \quad \forall i, l \quad (14)$$

$${}_l Q_{3dilk} \leq \rho_{ik} D_{ik} \quad \forall i, k \quad (15)$$

$${}_l Q_{3nd_{ilk}} + Q_{3dilk} \geq D_{ik} \quad \forall i, k \quad (16)$$

Equation (12) is a mass balance between the defective products produced, the defective products sent to each client, and the defective products discarded. Equations (13) to (16) state the mass balance of non-defective products to clients, the compliance of the tolerance of defective products of each client, and the fulfillment of the demand.

### 3.7 Objective Function

The objective function is to minimize the total cost:

$$F_1 = \sum_{s,r,i,l,t} (co_{sr} + trm_{sl}) Q_{1srilt} + \sum_{i,l,t} cp_{it} Q_{2ilt} + \sum_{s,r,i,l,t} Cont_{srilt} + \sum_{i,l,k} pty_{ik} Q_{3dilk} + \sum_{i,l,k} trp_{lk} (Q_{3dilk} + Q_{3nd_{ilk}}) \quad (17)$$

$Cont_{srilt}$  is a positive variable, it represents the inbound control method costs. The constraints to settle its value are not shown for space reasons, but they can be derived from the model presented here.

#### 4. Examples

**Case Study 1.** Consider a SC with 3 suppliers ( $s1-s3$ ) which can provide 3 types of raw material ( $r1-r3$ ) to 2 production plants ( $l1$  and  $l2$ ) where 3 products ( $i1-i3$ ) can be produced to cover the demand of 2 clients ( $k1$  and  $k2$ ). Also 2 inbound control methods ( $u1$  and  $u2$ ) can be used at any plant to control any raw material, and 2 technologies ( $t1$  and  $t2$ ) can be used at any plant to produce any product. In Table 1, 2 and 3 the model parameters are shown. Parameters  $\delta_{t\alpha_{il}\beta_{il}}$  can be obtained from the values of the parameters in Table 1, 2 and 3. The rest of the parameters are not shown for space reasons, but they are available for the interested reader. The MILP model was implemented and solved in GAMS with an Intel (R) Core2Duo, 2.66 GHz using the solver CPLEX. It has 7920 equations, 1966 continuous variables, and 98 binary variables. A solution with a 1% optimality gap was found in 550 CPU s.

**Table 1.** Parameters for Case Study 1

Raw materials	$s1$	$co_{sr}$		$s1$	$p_{sr}$		$cto_{ur}$		$c_{ur}$	
		$s2$	$s3$		$s2$	$s3$	$u1$	$u2$	$u1$	$u2$
$r1$	0.165	0.15	0.108	0.1	0.13	0.15	0.001	0.0015	0.9	0.95
$r2$	0.16	0.2	0.18	0.12	0.08	0.09	0.001	0.0013	0.87	0.93
$r3$	0.15	0.13	0.2	0.12	0.15	0.1	0.001	0.0013	0.9	0.96

**Table 2.** Parameters for Case Study 1

Products	$f_{ri}$			$cm_{ri}$			$cp_{it}$		$drr_{it}$		$\rho_{ik}$		$pty_{ik}$	
	$r1$	$r2$	$r3$	$r1$	$r2$	$r3$	$t1$	$t2$	$t1$	$t2$	$k1$	$k2$	$k1$	$k2$
$i1$	3	4	6	0	0	0	0.4	0.6	0.07	0.03	0.12	0.12	3.5	7.5
$i2$	4	5	5	0	2	0	0.5	0.8	0.06	0.05	0.12	0.12	3	5
$i3$	2	3	3	0	0	0	3	3.2	0.05	0.03	0.12	0.12	3.5	4

**Table 3.** Parameters for Case Study 1

Pro-ducts	$\varphi_{it}$		$D_{ik}$	
	$t1$	$t2$	$k1$	$k2$
$i1$	1.11	1.09	1500	1300
$i2$	1.08	1.08	1000	1200
$i3$	1.04	1.07	900	1100

Figure 1 shows SC flows for the solution. The total cost was equal to \$ 27,369. Table 4 shows that the solution includes different decisions for each product regarding how much percentage of defective products are delivered. Once a product has a given procurement policy, control policy, and it is known how many units are going to be produce with each technology, the model determines the percentage of defective products delivered to each client. The main factor to make this decision is the relation between the parameter  $pty_{ik}$  and the sum of all the necessary costs to produce a unit of that product. For example, for product  $i3$  the percentage of defective products delivered is at its maximum for each client, because the sum of all the necessary costs to produce a unit of product  $i3$  is \$4.26 which is higher than the penalty costs for both clients  $pty_{31} = 3.5$ ;  $pty_{32} = 4$ . So, in this case, it is more convenient to deliver a defective

unit of product and to pay the penalty. Conversely, for product  $i1$ , there are 2915 units produced by  $t1$  and 725 produced by  $t2$ . The cost to produce a unit with  $t1$  is \$2.4, and \$2.6 with  $t2$ . Because these costs are lower than the penalty cost for both clients  $pty_{11} = 3.5$ ;  $pty_{12} = 7.5$ , it is more convenient to produce again. Figure 1 shows that the decisions about procurement, control policy, and the technology used to produce are almost the same for all the products. It can be noted that  $u2$  is always chosen except to control  $r2$  for product  $i2$ . This is because of  $cm_{r2 i2} = 2$  so is less likely that defective raw material  $r2$  can provoke a defective unit of  $i2$ . Therefore, it is not important to use a more accurate control system. The procurement policy is the same for all products which consist of the cheapest and worst suppliers in terms of quality. This is based on the fact that control systems are so accurate and cheap in relation with the purchase cost of the other suppliers, that is better to buy and discard more raw materials, than to pay a higher price for raw materials and decrease the discarded materials. Also the defected materials are detected with a very high probability so their impact on the production is not important. In Figure 1 the notation  $s.r.i.u=x$  represents that supplier  $s$  supplies  $x$  units of raw material  $r$  to produce  $i$  and are controlled by  $u$ . The notation  $i.nd=x$  represents that  $x$  non-defective units of product  $i$  are sent to a client and  $i.d=x$  represents that  $x$  defective units of product  $i$  are sent to a client.

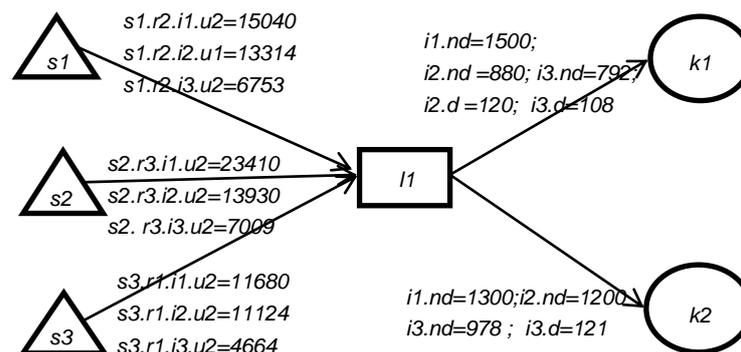


Fig. 1. SC flows for Case Study 1 solution

Table 4. Information of the solution of Case Study 1

Products	Percentage of defective products delivered (%)		Amount produced	Amount discarded	Penalty cost paid
	$k1$	$k2$			
$i1$	0	0	3340	540	0
$i2$	12	0	2348	184	360
$i3$	12	12	2000	0	863

**Case Study 2.** In this example a variation on the values of the penalties is performed in order to analyze the changes that it causes on the solution of the study case 1. Table 5 shows the new penalties in the last column.

**Table 5.** Penalties and information of the solution of Case Study 2

Products	Percentage of defective products delivered (%)		Amount produced	Amount discarded	Penalty cost paid	$pty_{ik}$	
	$k1$	$k2$				$k1$	$k2$
$i1$	12	12	2921	121	695	2	2.15
$i2$	12	0	2372	172	360	3	5
$i3$	0	0	2250	250	0	6	7

A solution with 1% optimality gap for this new case was found in 1900 CPU s. The total cost was equal to \$26,301. In this solution the procurement and control policies for raw materials are the same as in study case 1 and also only the plant  $11$  is used to produce. The most significant changes are in the percentage of defective products delivered to each client, and in the amount of produced and discarded products. Table 5 shows these values that confirm some of the conclusions for study case 1.

## 5. Conclusion

In this work a model is presented to integrate decisions that impact on the quality of the produced product with decisions regarding the planning of the SC. This approach provides a useful tool for the evaluation of the quality impact on different scenarios of provider selection, inbound control methods, production processes, and percentage of defective products delivered and produced, and also to assess the trade-offs between all of them. In order to add uncertainty to the model, some decisions are represented using a probability distribution. The problem was formulated as a MILP model, where several variables represent expected quantities. This model is a tool to support decision making in order to face negotiation processes with suppliers and clients. Also, the knowledge of the impact of the quality of the raw materials over the quality of the production is useful to establish more fruitful and cooperative relation with them.

It is worth highlighting the importance of combining the decisions about SC planning with the ones that affect the product quality in the competitive context of modern world. Numerical examples were presented and solved with the proposed approach. Case Study 2 shows how the changes in the penalties from the clients affect planning decisions regarding production.

## Nomenclature

### Indices:

$i$  for products;  $j$  is an auxiliary index used to enumerate generic elements of specific sets;  $k$  for clients;  $l$  for plants;  $r$  for raw materials;  $r_{ji}$  for raw materials necessary to produce product  $i$ ;  $s$  for Suppliers;  $t$  for technologies;  $u$  for control methods;

### Sets:

$S$  set of Suppliers;  $I$  set of products;  $L$  set of plants;  $R$  set of all raw materials

$R_i$  set of raw materials necessary to produce product  $i$ ;  $T_{il}$  set of technologies available in plant  $l$  to produce product  $i$ ;  $U$  set of all control methods;  $U_r$  set of available control methods to control raw material  $r$ ;  $K$  set of clients

**Binary Variables:**

$x_{cd_l} = 1$  if plant  $l$  is used to produce products and 0 otherwise

$x_{c_{il}} = 1$  if product  $i$  is produced in plant  $l$  and 0 otherwise

$x_{sril} = 1$  if raw material  $r$  is supplied by supplier  $s$  to produce product  $i$  in plant  $l$  and 0 otherwise

$x_{1_{rilu}} = 1$  if raw material  $r$  to produce product  $i$  in plant  $l$  is controlled by control method  $u$  and 0 otherwise

**Positive Variables:**

$Q1_{srilt} =$  amount of raw material  $r$  supplied by supplier  $s$  to produce product  $i$  in plant  $l$  using technology  $t$

$Q1d_{srilt} =$  expected amount of raw material  $r$  supplied by supplier  $s$  to produce product  $i$  in plant  $l$  using technology  $t$  that is discarded

$Q1nd_{srilt} =$  expected amount of raw material  $r$  supplied by supplier  $s$  to produce product  $i$  in plant  $l$  using technology  $t$  that is not discarded

$Q2_{ilt} =$  amount of product  $i$  produced in plant  $l$  using technology  $t$

$Q2d_{ilt} =$  expected amount of product  $i$  produced in plant  $l$  using technology  $t$  that is defective

$Q2nd_{ilt} =$  expected amount of product  $i$  produced in plant  $l$  using technology  $t$  that is not defective

$Q2ddiscard_{il} =$  amount of product  $i$  in  $l$  that is discarded

$Q3d_{ilk} =$  amount of defective products  $i$  produced in  $l$  and send to client  $k$

$Q3nd_{ilk} =$  amount of non-defective products  $i$  produced in  $l$  and send to client  $k$

$Cont_{srilt} =$  cost of control all the units of raw material  $r$  from  $s$  to produce  $i$  in  $l$  using  $t$

**Parameters:**

$D_{ik} =$  demand of product  $i$  from client  $k$

$\rho_{ik} =$  maximum percentage of the demand from client  $k$  of product  $i$  that is tolerated as defective

$p_{sr} =$  probability for a unit of raw material  $r$  supplied by supplier  $s$  to be defective

$c_{ur} =$  probability for a defective unit of raw material  $r$  to be detected by control  $u$

$\theta_{sru} =$  probability that a unit of raw material  $r$  supplied by supplier  $s$  and controlled by control  $u$  remains defective after the discarding

$f_{ri} =$  raw material conversion factor from raw material  $r$  to product  $i$

$cm_{ri} =$  if a unit of product  $i$  is produced with more than  $cm_{ri}$  units of defective raw material  $r$  then the unit of product  $i$  is considered defective

$\delta_{\alpha_{il}\beta_{il}} =$  probability for a unit of product  $i$  produced in plant  $l$  to be defective due to defective raw materials given that the purchase and control policy were  $\alpha_{il}$  and  $\beta_{il}$  respectively

$\delta_{t\alpha_{il}\beta_{il}}$  = probability for a unit of product  $i$  produced in plant  $l$  to be defective due to defective raw materials or defects on the technology given that the purchase and control policy were  $\alpha_{il}$  and  $\beta_{il}$  respectively, and it was produced by technology  $t$   
 $drr_{it}$  = probability for a unit of product  $i$  to be defective given that it was produced by technology  $t$   
 $pty_{ik}$  = penalty for a defective unit of product  $i$  delivered to client  $k$   
 $cto_{ur}$  = unitary control cost of raw material  $r$  controlled by  $u$   
 $cp_{it}$  = unitary production cost of product  $i$  produced by technology  $t$   
 $co_{sr}$  = unitary procurement cost of raw material  $r$  from supplier  $s$   
 $\varphi_{it}$  = interaction level of technology  $t$  for product  $i$   
 $trm_{sl}$  = unitary transport cost of raw materials from  $s$  to  $l$   
 $trp_{lk}$  = unitary transport cost of products from  $l$  to  $k$   
 $m_i$  = cardinal of the set  $R_i$

## References

1. van Hoek, R.I. "The contribution of performance measurement to the expansion of third party logistics alliances in the supply chain". *International Journal of Operations & Production Management* 21, 15–29, (2001)
2. Farahani R.Z., Rezapour S., Drezner T., Fallah S. "Competitive supply chain network design: An overview of classifications, models, solution techniques and applications", *Omega*, in press, accepted in (2013).
3. Mula J., Peidro D., Díaz-Madroñero M., Vicens E. "Mathematical programming models for supply chain production and transport planning", *European Journal of Operational Research*, 204, 377–390, (2010)
4. Franca R. B., Jones E.C., Richards C.N., Carlson J.P. "Multi-objective stochastic supply chain modeling to evaluate tradeoffs", *Int. J. Production Economics*, 127, 292–299, (2010)
5. Hsieh C.C., Liu Y.T., "Quality investment and inspection policy in a supplier–manufacturer supply chain", *Euro. J. of Oper. Research*, 202, 717–729, (2010)
6. Pal B., Sana S.S, Chaudhuri K. "Three-layer supply chain – A production-inventory model for reworkable items", *Applied Mathematics and Computation*, 219, 530–543, (2012)
7. Duffuaa SO., El-Ga'aly A., "A multi-objective optimization model for process targeting using sampling plans", *Computers & Industrial Engineering*, 64, 309–317, (2013)