# Digital speckle pattern interferometry applied to a surface roughness study

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**Abstract.** Surface roughness determination is of great interest for many applications. Several methods can be found in the literature, but most of them rely on indirect evaluation of the information, such as photographic techniques. We propose a method to measure surface roughness that takes advantage of digital speckle pattern interferometry for obtaining the data, and of digital image processing for evaluating it. After defining the problem, a theoretical description is presented, and finally it is compared with experimental results, showing good agreement.

Subject terms: *digital speckle pattern interferometry; speckle statistics; roughness measurement; interferometry.*

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## 1 Introduction

Since the fundamental work of Beckmann et al.[1] dealing with the scattering of electromagnetic waves from rough surfaces, their results have been extensively used to obtain information about surface roughness from the scattered light in the far-field region.

The correlation fringes of two speckle patterns, obtained from a rough surface under different angles of illumination, bear information about the roughness. Several optical roughness measurements techniques involve the use of a double-exposure specklegram method. The basic approach is to produce a shift in the incidence angle of the illumination beam between exposures, in order to produce, after a Fourier transformation, a set of interference fringes whose visibility depends on the rms roughness of the scattering surface.[2,3] This technique is known as angular speckle correlation (ASC).

Tribillon[4] showed that the degree of correlation between speckle patterns generated by two different wavelengths depends strongly on the surface roughness. The method based on this effect is known as spectral speckle correlation (SSC).[5]

All of the above-mentioned techniques suffer from the disadvantages of a photographic process, besides introducing a second step in the use of an optical Fourier transform. There also exist statistical methods that derive roughness information from the degree of decorrelation of imaged speckle patterns as a function of the wavelength,[6] where electronic detection and processing replaces the conventional photographic record. Electronic correlation of speckle patterns is currently accomplished by a technique called digital speckle

pattern interferometry (DSPI), normally used in various metrological applications.[7-9]

In DSPI a fringe pattern is obtained using video processing by storing an initial reference image of the surface in a frame grabber and then subtracting it from the one being acquired, displaying the modulus of the result on a TV monitor. Fringes, similar to those obtained in holographic interferometry, then appear when a phase change is introduced between the reference image and the subsequent ones. As noted by Wykes,[10] the rms difference of the speckle intensities from different wavelengths is directly related to the degree of speckle correlation.

In this work we apply the DSPI technique on a single-wavelength basis, combined with the ASC method, to show that the visibility of the correlation fringes in this case also depends on the roughness of the object surface. A theoretical approach to the proposed technique is also given.

## 2 Theoretical Description

Let us suppose the situation sketched in Fig. 1. A plane wave $\Sigma$ illuminates a rough surface S at an angle of incidence $\theta$ with respect to the normal to the surface. In our case, the surface is considered to be formed from a set of $n$ identical scattering elements of random height following a normal distribution of standard deviation $\sigma$ and zero mean. A CCD camera images the surface on the detector array. A separate reference beam is added to the first one via the beamsplitter BS. Since the DSPI detects a quantity proportional to $|\langle (I_1 - I_2)^2 \rangle^{1/2}|$, we examine how this quantity varies with a tilt of the beam illuminating the surface.

To do that we assume

$$A_1 = A_0 \sum_{j=1}^{n} \exp\left\{ \frac{2\pi i}{\lambda} [Z_j (1 + \cos\theta) + \phi_0] \right\} + A_R \exp(i\phi_R)$$
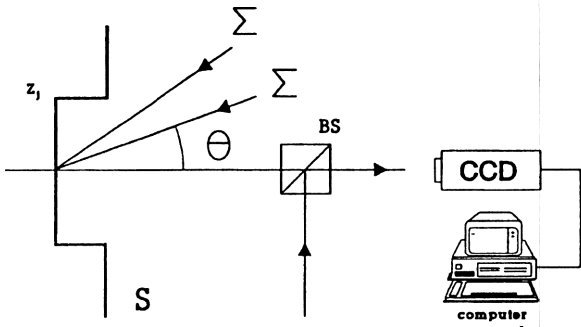
(1)

**Fig. 1** Schematic representation of two plane waves, $\Sigma$ and $\Sigma'$, illuminating a rough surface at two different angles, $\theta$ and $\theta + \Delta\theta$.

to be the field amplitude detected by the CCD array when the surface is illuminated at an angle $\theta$. In this equation the first term denotes the speckle pattern amplitude field, where $A_0$ is the light amplitude scattered by one element, $Z_j$ is the height of the $j$th scattering element, and $\phi_0$ allows for the constant phase of the incident light. In the second term, $A_R$ is the amplitude of the reference beam and $\phi_R$ its phase. In the following, all calculations are assumed to be made at the same point $(x,y)$.

Similarly, if the illumination direction is changed by an amount $\Delta\theta$, the new field amplitude can be described by

$$A_2 = A_0 \sum_{j=1}^{n} \exp\left(\frac{2\pi i}{\lambda}\{Z_j[1+\cos(\theta+\Delta\theta)]+\phi_0\}\right)$$

$$+ A_R \exp(i\phi_R) \ . \tag{2}$$

Equations (1) and (2) can be rewritten as the following more compact expressions:

$$A_1 = A_0 e^{i\phi} \sum_{j=1}^{n} e^{i\alpha Z_j} + A_R e^{i\beta} \tag{3}$$

and

$$A_2 = A_0 e^{i\phi} \sum_{j=1}^{n} e^{i\gamma Z_j} + A_R e^{i\beta} \ , \tag{4}$$

where $e^{i\phi}$ is the common phase term $\exp[i2\pi\phi_0/\lambda]$, $A_R$ is the amplitude of the reference beam, and $\alpha$ and $\gamma$ are abbreviations for $(2\pi/\lambda)(1+\cos\theta)$ and $(2\pi/\lambda)[1+\cos(\theta+\Delta\theta)]$, respectively.

To obtain $\langle(I_1-I_2)^2\rangle$ we need to calculate $\langle I_1^2\rangle$, $\langle I_2^2\rangle$ and $\langle I_1 I_2\rangle$, since

$$\langle(I_1-I_2)^2\rangle = \langle I_1^2\rangle + \langle I_2^2\rangle - 2\langle I_1 I_2\rangle \ . \tag{5}$$

Using Eqs. (3) and (4) we obtain

$$I_1 = A_1 A_1^*$$

$$= A_0^2 \sum_{j=1}^{n} e^{i\alpha Z_j} \sum_{k=1}^{n} e^{-i\alpha Z_k} + A_R^2 + A_0 B e^{i(\phi-\beta)} \sum_{j=1}^{n} e^{i\alpha Z_j}$$

$$+ A_0 A_R e^{-i(\phi-\beta)} \sum_{j=1}^{n} e^{-i\alpha Z_j} \tag{6}$$

and

$$I_2 = A_2 A_2^*$$

$$= A_0^2 \sum_{j=1}^{n} e^{i\gamma Z_j} \sum_{k=1}^{n} e^{-i\gamma Z_k} + A_R^2 + A_0 A_R e^{i(\phi-\beta)} \sum_{j=1}^{n} e^{i\gamma Z_j}$$

$$+ A_0 A_R e^{-i(\phi-\beta)} \sum_{j=1}^{n} e^{-i\gamma Z_j} \ . \tag{7}$$

Equations (6) and (7) can be reduced to the following expressions:

$$I_1 = A_0^2 \left(n + 2\sum_{\substack{j=1 \\ j<k}}^{n-1} \cos\alpha(Z_j - Z_k)\right) + A_R^2$$

$$+ 2A_0 A_R \sum_{j=1}^{n} \cos(\alpha Z_j + \phi - \beta) \ , \tag{8}$$

$$I_2 = A_0^2 \left(n + 2\sum_{\substack{j=1 \\ j<k}}^{n-1} \cos\gamma(Z_j - Z_k)\right) + A_R^2$$

$$+ 2A_0 A_R \sum_{j=1}^{n} \cos(\gamma Z_j + \phi - \beta) \ . \tag{9}$$

Squaring Eq. (8) yields

$$I_1^2 = A_0^4 n^2 + 4nA_0^4 \sum_{j<k} \cos\alpha(Z_j - Z_k)$$

$$+ 4A_0^4 \sum_{j<k} \cos^2\alpha(Z_j - Z_k) + A_R^4$$

$$+ 8A_0^4 \sum_{\substack{j<k \\ l<m \\ j\neq l, \ k\neq m}} \cos\alpha(Z_j - Z_k) \cos\alpha(Z_l - Z_m)$$

$$+ 4A_0^2 A_R^2 \sum_{j=1}^{n} \cos^2(\alpha Z_j + \phi - \beta)$$

$$+ 8A_0^2 A_R^2 \sum_{j<k} \cos(\alpha Z_j + \phi - \beta) \cos(\alpha Z_k + \phi - \beta)$$

$$+ 2A_0^2 A_R^2 \left(n + 2\sum_{j<k} \cos\alpha(Z_j - Z_k)\right)$$

$$+ 4A_0^3 A_R n \sum_{j=1}^{n} \cos(\alpha Z_j + \phi - \beta)$$

$$+ 8A_0^3 A_R \sum_{j<k} \cos\alpha(Z_j - Z_k) \cos(\alpha Z_l + \phi - \beta)$$

$$+ 4A_0 A_R^3 \sum_{j=1}^{n} \cos(\alpha Z_j + \phi - \beta) \ , \tag{10}$$

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from which we obtain, taking into account[11] that the mean value of the cosine function evaluates to zero when the phase is uniformly distributed in the interval $[-\pi, \pi]$,

$$\langle I_1^2 \rangle = A_0^4 n^2 + 4A_0^4 \frac{n^2 - n}{4} + A_R^4 + 4A_0^2 A_R^2 \frac{n}{2} + 2A_0^2 A_R^2 n .$$

(11)

Clearly, since this last expression for $\langle I_1^2 \rangle$ does not depend on either $\alpha$ or on $\gamma$, the same result holds for $\langle I_2^2 \rangle$.

To obtain the third term in Eq. (5) we need now to calculate the mean value of the product $I_1 I_2$:

$$
\begin{aligned}
I_1 I_2 = A_0^4 &\Bigg[ n^2 + 2n \Bigg( \sum_{j<k} \cos\alpha(Z_j - Z_k) + \sum_{j<k} \cos\gamma(Z_j - Z_k) \Bigg) \\
&+ 4\sum_{j<k} \cos\alpha(Z_j - Z_k) \sum_{j<k} \cos\gamma(Z_j - Z_k) \Bigg] \\
&+ A_R^4 + 4A_0^2 A_R^2 \Bigg( \sum_{j=1}^{n} \cos(\alpha Z_j + \phi - \beta) \sum_{j=1}^{n} \\
&\times \cos(\gamma Z_j + \phi - \beta) \Bigg) \Bigg] + A_0^2 A_R^2 \Bigg[ 2n + 2\Bigg( \sum_{j<k} \\
&\times \cos\alpha(Z_j - Z_k) + \sum_{j<k} \cos\gamma(Z_j - Z_k) \Bigg) \Bigg] + 2A_0^3 A_R \\
&\times \Bigg[ \Bigg( n + 2\sum_{j<k} \cos\alpha(Z_j - Z_k) \Bigg) \sum_{j=1}^{n} \cos(\gamma Z_j + \phi - \beta) \\
&+ \Bigg( n + 2\sum_{j<k} \cos\gamma(Z_j - Z_k) \Bigg) \sum_{j=1}^{n} \cos(\alpha Z_j + \phi - \beta) \Bigg] \\
&+ 2A_0 A_R^3 \Bigg( \sum_{j=1}^{n} \cos(\alpha Z_j + \phi - \beta) + \sum_{j=1}^{n} \\
&\times \cos(\gamma Z_j + \phi - \beta) \Bigg) .
\end{aligned}
$$

(12)

Taking into account that

$$\cos\alpha(Z_j - Z_k) \cos\gamma(Z_j - Z_k) = \frac{1}{2}\bigg\{ \cos\bigg[ (\alpha + \gamma)(Z_j - Z_k) \bigg] + \cos\bigg[ (\alpha - \gamma)(Z_j - Z_k) \bigg] \bigg\} ,$$

the mean value $\langle \cos\alpha(Z_j - Z_k) \cos\gamma(Z_j - Z_k) \rangle$ evaluates to

$$\frac{1}{2} \langle \cos(\alpha - \gamma)(Z_j - Z_k) \rangle .$$

Taking into account that we assumed a normal distribution with standard deviation $\sigma$ and zero mean, this last equation can be expressed as

$$\frac{1}{2} \langle \exp[i(\alpha - \gamma)(Z_j - Z_k)] \rangle .$$

(13)

This ensemble average evaluates to

$$\frac{1}{2} \exp\bigg( -\frac{1}{2}(\alpha - \gamma)^2 \langle (Z_j - Z_k)^2 \rangle \bigg)$$

(14)

and $\langle (Z_j - Z_k)^2 \rangle = 2\sigma^2(1 - r_{jk})$, where $r_{jk}$ is the normalized surface correlation. Since from Eq. (12) we have $j \neq k$, and if the individual steps $Z_j$ are uncorrelated with each other, then Eq. (13) reduces to

$$\frac{1}{2} \exp[-(\alpha - \gamma)^2 \sigma^2] .$$

(15)

With this result, the final expression for $\langle I_1 I_2 \rangle$ becomes

$$
\begin{aligned}
\langle I_1 I_2 \rangle = A_0^4 &\Bigg( n^2 + 2\frac{n^2 - n}{2} \exp(-\xi^2 \sigma^2) \Bigg) \\
&+ A_R^4 + 2nA_0^2 A_R^2 \exp\bigg( -\frac{\xi^2 \sigma^2}{2} \bigg) + 2nA_0^2 A_R^2 ,
\end{aligned}
$$

(16)

where $\xi = \alpha - \gamma = (2\pi/\lambda)[\cos\theta - \cos(\theta + \Delta\theta)] \approx (2\pi/\lambda) \sin\theta \, \Delta\theta$. From Eqs. (11) and (16) the following expression for $\langle (I_1 - I_2)^2 \rangle$ can be obtained:

$$
\begin{aligned}
\langle (I_1 - I_2)^2 \rangle = 2nA_0^2 &\bigg\{ (n-1)A_0^2[1 - \exp(-\xi^2 \sigma^2)] \\
&+ 2A_R^2 \bigg[ 1 - \exp\bigg( -\frac{\xi^2 \sigma^2}{2} \bigg) \bigg] \bigg\} .
\end{aligned}
$$

(17)

The number of scatterers producing the speckle field, $n$, is very large, and thus $n - 1 \approx n$. In addition, we suppose that $nA_0^2 = A_S^2$ represents the total light intensity emanating from the surface and $A_R$ can be chosen to be equal to $A_S$. With these assumptions, the final expression for $\langle (I_1 - I_2)^2 \rangle$ results:

$$\langle (I_1 - I_2)^2 \rangle = 2A_S^4 \bigg[ 3 - \exp(-\xi^2 \sigma^2) - 2\exp\bigg( -\frac{\xi^2 \sigma^2}{2} \bigg) \bigg] .$$

(18)

We are primarily concerned with the determination of the local visibility $V_L$ and its variation with respect to $\Delta\theta$. By using the substitution $\langle (I_1 - I_2)^2 \rangle = \langle I^2 \rangle$, the local visibility is defined as

$$V_L = \frac{\langle I^2 \rangle_{max}^{1/2} - \langle I^2 \rangle_{min}^{1/2}}{\langle I^2 \rangle_{max}^{1/2} + \langle I^2 \rangle_{min}^{1/2}} .$$

(19)

In practice, it is equivalent to measure the global visibility through the fringe pattern that arises because of the tilt $\Delta\theta$ of the illuminating beam, defined by the equation

$$V = \frac{I_{max} - I_{min}}{I_{max} + I_{min}} ,$$

(20)

where $I_{max}$ and $I_{min}$ are the maximum and minimum intensity values present in the fringe pattern, respectively.

For the case of $\Delta\theta = 0$, we have $\langle (I_1 - I_2)^2 \rangle^{1/2} = 0$ (no fringes are present), and Eq. (19) becomes undefined. The corresponding visibility can be taken as one, as for all other cases, having $\Delta\theta \neq 0$, lower values are obtained.

Figure 2 shows the variation of the local visibility defined in Eq. (19) versus the rotation $\Delta\theta$ for $\sigma = 6.3$ μm.
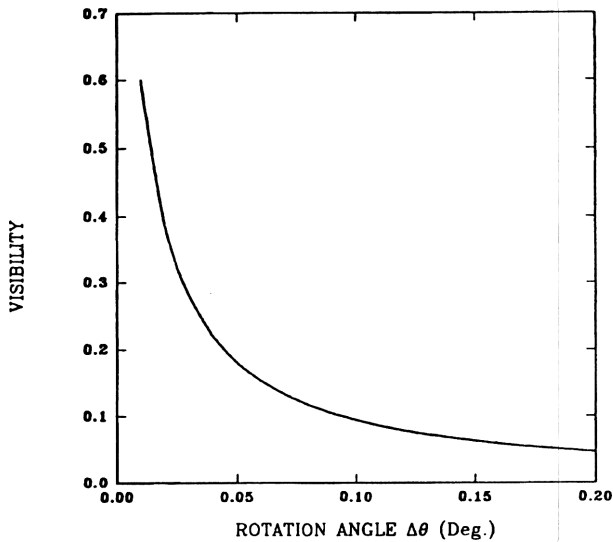
**Fig. 2** Theoretical variation of the visibility as a function of the change in the illumination direction, $\Delta\theta$.



**Fig. 4** Visibility of the fringe pattern as a function of $\Delta\theta$ for three different values of the surface roughness. Solid curves represent the best fit to the experimental data.

## 3 Experimental Results

The experimental arrangement of Fig. 3 was used to reproduce the conditions used in the theoretical approach. A 10-mW HeNe laser beam was divided by the beamsplitter BS1. One of the beams obtained in this way was expanded, collimated, and used to illuminate the rough surface under study at an angle $\theta$ with its normal. The second beam was overlapped with the image of the surface, which is a speckle pattern, at the detector array of the CCD camera by using the beamsplitter BS2. It is clear that the optical system does not resolve the individual scatterers. The video signal from the camera is sent to the frame grabber for further processing.

At the frame grabber the following operation, which can be accomplished at near-real-time rates, is performed: a first image of the surface is acquired and stored as a reference,
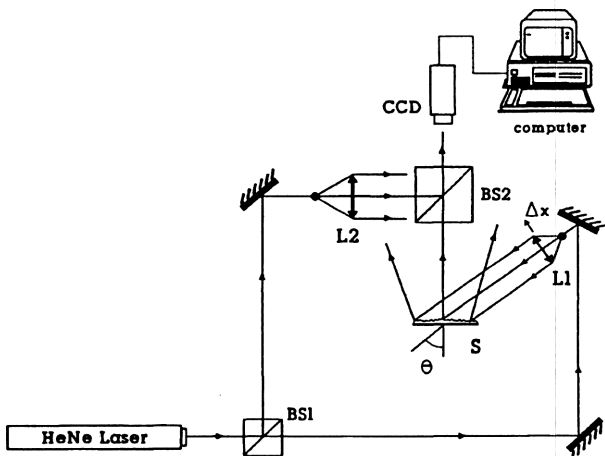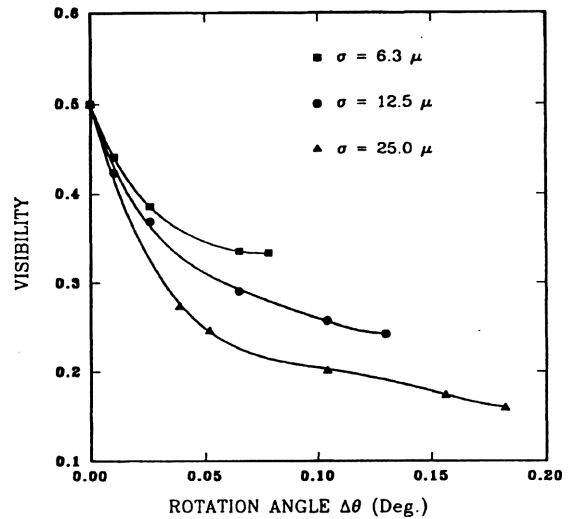
the subsequent images coming from the camera are subtracted from the first one, and the result is then displayed, in modulus, on a TV monitor. If an image is obtained under the same illumination conditions as the reference, then no differences are present between them, and the TV monitor appears dark. On the other hand, when the beam illuminating the surface is rotated by an angle $\Delta\theta$, a linear phase change occurs, altering the speckle pattern generated by the surface. At points where this phase change equals $2m\pi$, with $m$ an integer, the speckle pattern reproduces itself and the TV monitor shows a dark spot. On the contrary, for phase changes other than $2m\pi$, the subtraction gives a nonzero value and the monitor shows a spot whose gray level varies accordingly to the phase change. Thus the monitor shows a set of fringes immersed in speckle noise and following the zones of constant phase change. Both the spatial frequency and the visibility of these fringes depends on the rotation $\Delta\theta$ of the beam illuminating the surface, as demonstrated above.

In our experiment, the rotation $\Delta\theta$ was obtained by a lateral shift $\Delta x$ of the collimating lens L1, which was mounted on a translator driven by a stepper motor capable of a minimum displacement of 2 $\mu$m. Lens L1 was chosen to have a focal length of 22 cm. Thus, the rotation $\Delta\theta$ can be obtained from the simple equation

$$\tan \Delta\theta = \frac{\Delta x}{f} \approx \Delta\theta \ ,$$

with $\Delta x$ varying from several microns to a few millimeters.

Although this produces not only a rotation but also a displacement of the object beam, using plane waves ensures that all phase changes are only a consequence of the rotation, provided that the whole field of view of the camera (which can always be chosen smaller than the illuminated area) remains unchanged.

The curves plotted in Fig. 4 (fringe visibility as a function of $\Delta\theta$) were obtained for three different values of the surface roughness, $\sigma = 6.3$, 12.5, and 25 $\mu$m, and for $\theta = 19$ deg,



**Fig. 3** Experimental DSPI setup for obtaining speckle correlation fringes and measuring the angular variation $\Delta\theta$.
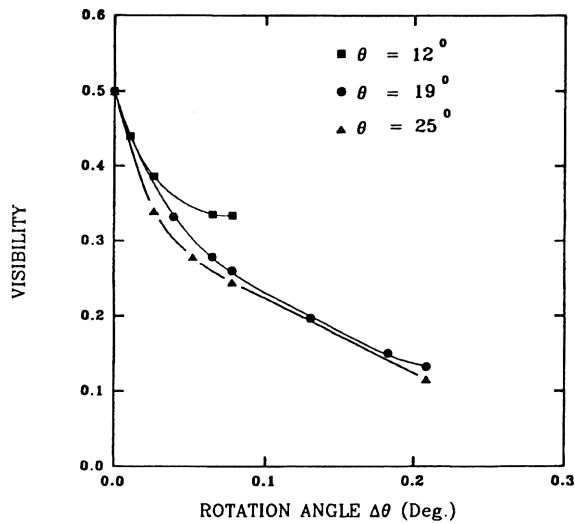
**Fig. 5** Best-fit curves for the visibility of the fringe pattern as a function of $\Delta\theta$ for three different values of the initial illumination angle $\theta$ and for a fixed value of the surface roughness, $\sigma = 6.3$ $\mu$m.

using standard surface roughness test plates. Clearly the visibility drops faster as the roughness increases, as can be expected from Eq. (18). Visibility measurements were carried out using an indirect digital method,[12] where a peak ratio at a Fourier plane is measured, instead of direct fringe visibility analysis. In this way, speckle noise is reduced and the error in the visibility remains around 1.4%.

For a given roughness, if the incidence angle $\theta$ of the beam illuminating the surface is changed, the maximum value of $\Delta\theta$ for which a variation in the fringes' visibility can be detected also changes, as illustrated in Fig. 5 for $\sigma = 6.3$ $\mu$m and $\theta = 12$, 19, and 25 deg. The sensitivity of the method (which is limited by the fringe density) can thus be controlled by setting the proper value of $\theta$.

For both, Figs. 4 and 5, solid curves represent the best fit to the experimental values.

## 4 Conclusions

A method for surface roughness determination using digital speckle pattern interferometry has been described theoretically and tested experimentally, showing good agreement. The experimental setup requires an image processor but not sophisticated optical components, making the method suitable, for example, for industrial applications. In addition, the whole process can be accomplished digitally, thus reducing the time needed for a measurement. The range of operation can be controlled by adjusting the parameters.

### Acknowledgments

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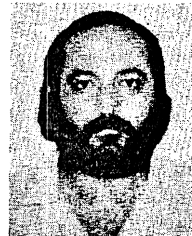
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