Monopoly Intermediary and Information Transmission

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Seminario de Economía
19 de julio de 2002
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April 2002

Abstract

In this paper we extend Lizzeri’s simple model of information transmission through certification intermediaries. A seller with no means to signal his quality has the possibility to be certified by an institution that owns a technology to discover the true quality and can credibly commit to a disclosure rule. We study the incentives of this institution to disclose information to the buyers. When buyers are risk neutral, the intermediary cannot help to increase the total surplus and, therefore, there is no disclosure of information at equilibrium. Moreover, there always exists an equilibrium with no revelation of information. However, with an unrestricted space of contracts, self selection of sellers indirectly transmits some information. On the other hand, when buyers are risk averse, the intermediary can increase total surplus by inducing better risk sharing. We show that the equilibrium is to offer a menu of contracts where information will be fully disclosed for all types above a certain threshold and no announcement is made for the others.

Keywords: Intermediary, Certification, Information Transmission, Quality.

JEL Codes: D42, D82, L15

*We are indebted to Jacques Crémer, Jean-Jacques Laffont, Alessandro Lizzeri and Estelle Malavolti-Grimal for helpful comments. All remaining errors are ours.

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1 Introduction

Certification intermediaries are agents with the ability to reduce informational asymmetries between sellers and buyers. In this paper, we analyze to which extent a monopoly certification intermediary actually reveals some new information to uninformed (or less informed) parties. Without restrictions on the set of contracts, transmission of information can take two forms: an indirect revelation pattern through the self-selection of the informed parties or a direct revelation pattern through the direct disclosure of information by the intermediary. We show that if revealing information cannot increase total surplus, there will never be direct revelation of information by the intermediary. However, at equilibrium there may be indirect transmission of information. On the other hand, if the information transmitted increases social surplus, there will always be some direct but suboptimal revelation of information.

Examples of such intermediaries abound. When a firm decides to issue securities to the public, it almost always hires an intermediary, typically an investment banking firm. The bank audits and discloses information about the economic situation and perspectives of the firm to all potential buyers of the stocks. Marketing information services firms constitute another relevant example. They sell studies to manufacturers and provide consumers with quality assessments, market research, forecasting and consulting about different products they offer.

In Lizzeri (1999) a privately informed seller owns a good but has no means to signal the quality he offers. On the other hand, an intermediary owns a technology that enables him to test the good and discover the true quality. The intermediary moves first by choosing a unique price to charge to the seller and a disclosure rule that specifies how information will be revealed to the buyer. Then, the seller learns his quality and decides whether to pay or not the price and go through the intermediary. If the seller pays, the intermediary discovers the type and implements the disclosure rule. Finally two buyers bid for the good, taking into account the information revealed, if any, by the intermediary.

When trade is efficient in all states of nature, Lizzeri shows that there always exists an equilibrium in which all types of seller ask for certification and no information is revealed in equilibrium. Moreover, under certain reasonable conditions on the distribution of types, this equilibrium is unique. The intermediary captures all the extractable surplus, without revealing any valuable information, and the seller chooses to be certified only to avoid being taken as a low quality seller.

When trade is not always efficient the absence of any intermediary would
involve either too little or too much trade in equilibrium. The intermediary can increase social welfare by certifying only types for which trade is efficient. The level of trade becomes efficient in equilibrium and the good is sold if and only if it is valuable for the consumer. Still, the equilibrium does not (in general) involve full disclosure, but only reveals whether trade is efficient or not.

We depart from Lizzeri’s model in three respects: imperfect information on the seller’s side about the quality of the good, the possibility for the intermediary to indirectly transmit information through a menu of contracts and finally the introduction of risk averse buyers.

We assume that ex ante nobody knows exactly the quality of the good. However, the seller receives a private signal that is correlated with the true quality. Introducing signals allows for a very general analysis that seems economically reasonable and includes the case where the seller is perfectly informed about the quality he offers.

In the first part of the paper, we assume that all agents are risk neutral. We show that with more than two signals and a unique contract two types of equilibria always exist: in one equilibrium the seller chooses to test the good whatever his signal and no information is disclosed; in the second, all types of seller but the lowest ask for certification and no information is disclosed. In the latter equilibrium, there is always a minimum amount of information transmitted at equilibrium, namely that the seller is not of the lowest type. We show that a necessary and sufficient condition to have more revelation of information at equilibrium is that the intermediary offer a menu of \( N \) different contracts with no direct disclosure.

In the second part of the paper we introduce risk averse buyers. In this case, revelation of information improves welfare since it enables better risk sharing between the buyers and the intermediary who can extract a higher rent. We show that there is always direct revelation of information in equilibrium. More precisely, the intermediary offers a menu of contracts where the information will be fully disclosed for all types above a certain threshold and no disclosure is made for the others.

One illustration of such information transmission policy is given by JD Power in the US.\(^1\) This company characterized as a “marketing information firm” offers its expertise in certification to manufacturers. For the automobile industry, JD Power releases ranking for vehicles performing above industry average and simply lists below-average performers in alphabetical order. Thus, the information is revealed for all qualities above a given threshold and nothing is disclosed for lower quality cars.

\(^1\)We thank Alessandro Lizzeri for pointing out this example to us.
The paper is organized as follows. In Section 2, we sketch the main structural features of the model. In Section 3, we develop the model with \( N \) different signals, we state the necessary correspondence between the number of signals and the number of alternatives of the seller, and we characterize the equilibria of the game. In Section 4, we introduce risk averse buyers and show that there is always some direct revelation of information at equilibrium. Finally, some concluding remarks are given in Section 5.

2 The model

Consider an economy with one seller, an intermediary and \( n \) buyers. The seller produces a good of quality \( \tau \), which is valued \( \tau \) by the buyers but for which neither the seller nor the intermediary have any value.

The seller does not know the quality of the good he offers but receives a partially informative signal \( \sigma \in S \). Since the true quality offered by the seller is unknown, all the agents will have a prior on the value of \( \tau \) represented by the cumulative distribution \( F(\cdot) \) on the closed support \( [\underline{\tau}, \overline{\tau}] \), where \( \overline{\tau} > \underline{\tau} \geq 0 \). Once he has observed the signal, the seller updates his beliefs about the quality of his good. Let \( G_i(\cdot) \) be the assessment about this quality if signal \( \sigma_i \) is received.

The signal can take \( N + 1 \) values, \( S = \{\sigma_0, \ldots, \sigma_N\} \), and the ex post probability distributions

\[
G_i(t) = \Pr(\tau < t / \sigma = \sigma_i)
\]

satisfy

\[
G_i(t) \geq G_{i+1}(t) \quad \forall t, \forall i < N,
\]

that is, \( G_{i+1} \) first order stochastically dominates \( G_i \). In other words, receiving a high signal is a good news in Milgrom (1981) sense.

Define \( \rho_i \) as the unconditional probability of receiving \( \sigma_i \). Note that

\[
\sum_i \rho_i G_i(t) = F(t) \quad \forall t.
\]  

We will call \( E_i[\tau] \), \( i = 0, \ldots, N \), the expected quality when signal \( \sigma_i \) is received and \( E_F[\tau] \) the ex ante expected quality.

\[2\]This assumption allows us to conduct a broader analysis that includes the case where the signal is perfectly informative. Our results still hold if the seller is perfectly informed about the quality of his good.
The intermediary has a technology to test the quality of the good provided by the seller at a fixed cost normalized to 0. If the good is tested, the intermediary discovers $\tau$ which becomes hard information.

**Timing of the game**

- At date 0, the intermediary commits to a menu of (at most) $N + 1$ fees $P$ and disclosure rules $D$.
- At date 1, the seller receives the signal $\sigma$ on the quality of the good.
- At date 2, the seller decides not to have his product tested ($x = N$) or, he chooses contract $j$ among the menu ($x = Y_j$) for some $j \in \{0, ..., N\}$.
- At date 3, the intermediary observes the type if $x = Y$. He discloses the information he has committed to disclose. We call $R(\tau, D)$ the information revealed given the type observed and the disclosure rule.
- At date 4, the buyers observe $(P, D, x)$ and the information disclosed by the intermediary, $R(\tau, D)$.
- At date 5, buyers bid independently and simultaneously for the good.

**Strategies and equilibria**

Because of their tractability and economic appeal, we will limit our analysis to pure-strategy sequential equilibria.

A strategy for the intermediary is a choice of $(P, D)$ where $P \in \mathbb{R}^{N+1}_+$ and $D \in \Psi^{N+1}$, the set of all possible disclosure rules. Since $\tau$ is hard information once the good is certified, we can restrict our attention to disclosure rules of the form “the information is perfectly revealed ($R(\tau, D) = \tau$) with probability $\alpha$ and nothing is revealed ($R(\tau, D) = \emptyset$) with probability $1 - \alpha$”. Without loss of generality, we can set $\Psi = [0, 1]$ and the parameter $\alpha$, the probability of fully revealing the true quality, completely identifies the disclosure rule.

A strategy for the seller is a function $x(P, D, \sigma) : \mathbb{R}_+^{N+1} \times [0, 1]^{N+1} \times S \rightarrow \{Y_1, ..., Y_{N+1}, N\}$. When a menu of contracts is offered, the seller has a richer set of strategies that includes the choice of the particular contract.

Buyers will bid the expected value of the good conditional on the information disclosed by the intermediary, the price of certification, the decision of the seller to be tested or not, and the offer made by the intermediary. Hence, a strategy for the buyer is a function $\beta(\cdot) : \mathbb{R}_+^{N+1} \times [0, 1]^{N+1} \times \{Y_1, ..., Y_{N+1}, N\} \times \emptyset \cup \mathbb{R}_+ \rightarrow \mathbb{R}_+$.

\[\text{This result would not hold anymore if there were some uncertainty about what the intermediary knows (Shin (1994))}.\]
3 Direct versus indirect information transmission

3.1 Equilibrium with a unique contract

We first consider the case where the intermediary is constrained to offer a unique contract \((P, \alpha)\) to all types of seller. We show that there is never direct disclosure of information. Indeed, only a minimum amount of information can be indirectly transmitted in equilibrium.

The following proposition describes the equilibria of the game.

**Proposition 1** \(In the case of a unique contract, there only exist two equilibria: a fully pooling equilibrium and a semi-separating equilibrium where only the seller with the lowest signal does not ask for certification. Both of them involve no disclosure of information.\)

**Proof.** We start by stating three lemmas that we prove in Appendix B.

**Lemma 1** There is no equilibrium in which no seller asks for certification.

**Lemma 2** If a pooling equilibrium exists, it never involves full disclosure of information.

**Lemma 3** There is no equilibrium in which more than one type of seller do not ask for certification.

Using lemmas 1 and 2, we rule out two pooling equilibrium candidates, one in which no one asks for certification and the other where all types ask for certification and the information is fully disclosed by the intermediary. Lemma 3 shows that it is never optimal to leave more than the lowest type without certification.

There are two remaining candidates: a fully pooling equilibrium with no disclosure and a semi-separating equilibrium in which all types but the lowest ask for certification and with no disclosure.

We show now that the intermediary is indifferent between those two candidates. Suppose there is a fully pooling continuation equilibrium and that the out-of-equilibrium beliefs are such that a deviator is assumed to have received the lowest signal: \(\sigma_0\). The price that can be charged by the intermediary is then

\[ P_{FP} = E_F [\tau] - E_0 [\tau], \]
and the profit of the intermediary is 

$$\pi_{FP} = P_{FP} = E_F[\tau] - E_0[\tau].$$

One can then easily check that all the incentive compatibility constraints of the seller are weakly satisfied.

Let us now show that the previously described semi-separating equilibrium is payoff equivalent for the intermediary. The maximum price that can be charged by the intermediary is:

$$P_{SS1} = \sum_{i=1}^{N} \frac{\rho_i}{1 - \rho_0} E_i[\tau] - E_0[\tau],$$

and the expected profit of the intermediary is

$$E(\pi_{SS1}) = (1 - \rho_0)P_{SS1} = \sum_{i=1}^{N} \rho_i E_i[\tau] - (1 - \rho_0)E_0[\tau].$$

Using (1),

$$\sum_{i=1}^{N} \rho_i E_i[\tau] = E_F[\tau] - \rho_0 E_0[\tau].$$

Plugging this equality in the expression of $E(\pi_{SS1})$, we obtain

$$E(\pi_{SS1}) = E_F[\tau] - E_0[\tau] = \pi_{FP}.$$ 

This shows that the two continuation strategies constitute an equilibrium of the subgame starting after the offer of the intermediary. Then we just need to show that the intermediary has no incentives to deviate. But this is trivially true, since the intermediary captures all the possible rent, $E_F[\tau] - E_0[\tau]$. No deviation can then give a higher payoff to the intermediary. ■

The intuition for this result is as follows. First of all, it is always optimal to attract at least one type. Indeed, the intermediary has the ability to modify any proposal that excludes all types in order to attract, at least, one of them and get a positive profit. This eliminates any equilibrium where no type chooses not to be certified.

Furthermore, it is never optimal to offer a pooling contract with full disclosure. By offering a contract that reveals all the information for a given type $i$, all types above $i$ will choose this contract and get a positive rent. This strategy is strictly dominated by one in which only the highest types ask for
certification. Similarly, leaving more than one type without certification would increase the outside opportunity of each type of seller and decrease the rent captured by the intermediary. Thus, the intermediary never wants to offer a contract in which more than the lowest type stays out of the certification market.

Finally, a pooling equilibrium and a semi-separating equilibrium excluding the lowest type both with no disclosure are payoff equivalent for the intermediary. On the one hand, the intermediary loses by letting the lowest type out of the certification market since with positive probability the seller will choose not to be certified. On the other hand, if the lowest type is excluded, the expected quality of a seller asking for certification is higher and, therefore, the buyers’ willingness to pay increases. The intermediary can then extract this additional rent by increasing the certification price. It turns out that this gain completely compensates the loss.

Notice that under both equilibria the intermediary is able to capture all the rent \((E_F[\tau] - E_0[\tau])\). Indeed, the seller can always guarantee himself \(E_0[\tau]\), even without asking for certification. Then, in any equilibrium the intermediary must leave him at least this payoff to satisfy the participation constraint. However, although there is no disclosure and the intermediary captures all the extractable surplus in the two equilibria, their information content is different.

Let us define the minimum information transmission concept as follows.

**Definition 1** *The Minimum Information Transmission is the one that only reveals that the seller is not of the lowest type.*

The existence of the semi-separating equilibrium implies that a minimum amount of information can always be transmitted. The information indirectly revealed in such an equilibrium is that whenever the seller chooses to be certified, the buyer is sure that he did not receive the lowest signal, whereas when the seller does not ask for certification, the buyer knows that he is the lowest type.

### 3.2 Design of the optimal contract with information transmission

In the previous section we restricted the strategy set of the intermediary to unique contracts. We now ask whether setting a unique price for certification is the only optimal strategy of the intermediary. Indeed, we will show that he can do as well, but no better, by offering a menu of prices and let the seller self-select.
Assume now that the intermediary can offer a menu of contracts \( \{(P_i, \alpha_i)\}_{i=1}^N \). We are looking for a separating equilibrium in which all sellers except the seller with the lowest possible signal\(^4\) ask for certification and where a seller chooses contract \((P_i, \alpha_i)\) after receiving signal \(\sigma_i\).

**Proposition 2** There exist a continuum of sequential equilibria in which the intermediary offers a menu of contracts \( \{(P_i, \alpha_i)\}_{i=1}^N \) such that each type of seller self-selects and type 0 does not ask for certification. In all this equilibria \( P_i = E_i[\tau] - E_0[\tau] \) and no information is disclosed for all \( i < N \). Moreover, any \( \alpha_N \in [0,1] \) is equally optimal.

**Proof.** We first state the following lemma, proved in Appendix B:

**Lemma 4** Suppose that at least two types choose different contracts. Then in any optimal menu the intermediary offers \( N \) different contracts.

Using Lemma 4, we know that the intermediary will offer \( N \) different contracts.

The following incentive compatibility conditions must be verified:

\[
\begin{align*}
E_0[\tau] &\geq \alpha_i E_0[\tau] + (1 - \alpha_i) E_i[\tau] - P_i \quad \forall i > 0, \\
E_i[\tau] - P_i &\geq \max \{ E_0[\tau], \alpha_j E_i[\tau] + (1 - \alpha_j) E_j[\tau] - P_j \} \quad \forall i, j.
\end{align*}
\]

Let us assume that only the downward adjacent incentive compatibility constraints are binding. That is

\[
P_i = P_{i-1} + (1 - \alpha_{i-1}) (E_i[\tau] - E_{i-1}[\tau]), \\
P_1 = E_1[\tau] - E_0[\tau].
\]

By induction,

\[
P_i = (1 - \alpha_{i-1}) E_i[\tau] + \sum_{j=2}^{i-1} (\alpha_j - \alpha_{j-1}) E_j[\tau] - E_0[\tau],
\]

and therefore the expected profit of the intermediary is

\[
E(\pi) = \sum_{i=1}^{N} \rho_i \left[ (1 - \alpha_{i-1}) E_i[\tau] + \sum_{j=2}^{i-1} (\alpha_j - \alpha_{j-1}) E_j[\tau] - E_0[\tau] \right].
\]

\(^4\)The intermediary will never extract any rent by offering a contract to the lowest type since the maximum price he can charge would be 0.
This expression is independent of the value of $\alpha_N$, so any $\alpha_N \in [0, 1]$ is equally optimal. Moreover, it is straightforward to see that $\alpha_{N-1} = 0$. This implies that $\alpha_{N-2}$ is also equal to 0 and, by induction, that all $\alpha_i$, $i < N$, are equal to 0 at the optimum. Thus,

$$P_i = E_i[\tau] - E_0[\tau],$$

and the missing incentive compatibility constraints are verified.

The expected profit of the intermediary is

$$\sum_{i=1}^{N} \rho_i (E_i[\tau] - E_0[\tau]) = E_F[\tau] - \rho_0 E_0[\tau] - (1 - \rho_0) E_0[\tau] = E_F[\tau] - E_0[\tau],$$

which is the maximum profit that he can extract in any equilibrium. Therefore, offering such a menu of contracts is an optimal strategy.

The optimal menu involves the same disclosure rule $\alpha_i = 0$ for all types $i < N$ together with a price schedule $P_i$ increasing in $i$. Thus, the intermediary is able to screen the different types of seller with a single instrument. This is possible because if the buyers observe a seller choosing a higher certification price, they expect a higher quality and, thus, are ready to pay more for the good. This is analogous to the burning money effect in standard signalling models. The difference is that, in our model, this “burned money” constitutes a transfer from the seller to the intermediary.

The intermediary extracts again the whole possible surplus. However, if $\alpha_i = 1$ for some $i$, then incentive compatibility implies that the price charged by the intermediary has to be the same for all types from $i$ to $N$. Therefore, all types above $i$ choose the same contract and the intermediary leaves them a rent. The intermediary can do better by hiding the information to the buyers and no disclosure will happen at equilibrium for types below $N$.

Let us now introduce the following definitions:

**Definition 2** An equilibrium has a Direct Revelation Pattern if the disclosure rule satisfies $\alpha_i \neq 0$ for some $i$, that is, the intermediary reveals the true quality with positive probability for some type.

**Definition 3** An equilibrium has an Indirect Revelation Pattern if the disclosure rule satisfies $\alpha_i = 0$ for all $i$ and different types of seller choose different contracts, that is, the only information transmitted is through the separation of types.
By allowing the intermediary to offer a menu of contracts, we can now state one of the main results of the paper:

**Proposition 3** An optimal contract generates more than the minimum information transmission if and only if the intermediary offers a menu of $N$ different contracts.

**Proof.** a) We show first that if there is more than the minimum information transmission, then the intermediary offers a menu of $N$ different contracts. By lemma 2, there is never revelation of information in a pooling equilibrium, therefore, the continuation must involve some separation of types. By lemma 3, the intermediary offers a contract that attracts all types except the lowest one. If only one contract is offered, it has to involve no disclosure of information, and only the minimum information is revealed. Thus, because more than the minimum is revealed, the intermediary offers at least two contracts. Then, by lemma 4, the intermediary offers $N$ different contracts.

b) Now, according to definition 3, if the intermediary offers $N$ different contracts such that the different types of seller self-select, then, at least the signal received by the seller is indirectly revealed. Moreover, by proposition 2 this is an equilibrium of the game. Hence, more than the minimum amount of information is transmitted.

So far we have shown that there are three possible classes of equilibrium in this certification game: a pooling equilibrium and a semi-separating equilibrium both with no disclosure and a unique certification price and also a menu of $N$ different contracts with no disclosure for all $i < N$. The information content of all these equilibria is very different. In the pooling equilibrium, no information is transmitted to the buyers, neither directly nor indirectly. The semi-separating equilibrium with no disclosure reveals whether the seller received the lowest signal or not. That is, just the minimum amount of information is transmitted. Finally, the equilibrium with a menu of contracts involves a richer information transmission, since the choice of the seller perfectly reveals the signal he received and, furthermore, the true quality of the highest type may be directly disclosed.

Moreover, the amount of information transmitted is an increasing function of the number of signals. As an illustration, consider the limit case where the seller receives a perfectly informative signal. Applying Theorem 3 to a continuum of types, it is optimal for the intermediary to offer a continuum of prices, $P(\tau)$, such that each type of seller has incentives to choose the contract designed for him. Extending Proposition 2, the optimal price function proposed by the intermediary is $P(\tau) = \tau - \tau$. The intermediary
extracts the maximum possible rent, $E_F[\tau] - \tau$, but here the signal received by the seller is perfectly revealed to the buyers through the self selection of the seller.

However, the expected profit of the intermediary is exactly the same as if a unique price was implemented. Therefore, our result is not robust to the introduction of a small cost of writing contracts. Indeed, in the presence of such a cost, the intermediary would always be strictly better off by offering a single contract with no revelation of information.

Moreover, ex ante, information has no particular value since none of the agents is ready to pay for it. Indeed, ex ante total surplus in this economy is always equal to $E_F[\tau]$ and is not affected by the action of the intermediary. Therefore, there will never be direct revelation of information when the intermediary cannot contribute to increase total surplus.

**Corollary 1** When total surplus is independent of the intermediary’s action, no direct revelation of information for $i \neq N$ can arise at equilibrium.

**Proof.** Because the total surplus is fixed, the intermediary can only affect the share he can extract. By revealing information for types below $N$, this share decreases.

In the following section we show that the intermediary might do strictly better with a menu of contracts rather than a unique price if buyers are risk averse. Intuitively, the introduction of risk aversion on the buyers’ side implies that the intermediary can increase social welfare and, in particular, his own payoff, by inducing a better risk-sharing through the revelation of information.

## 4 Equilibrium with risk averse buyers

Consider the model of Section 3 and assume now that the utility function of a buyer is $v(\tau - \beta)$ if he buys a good of quality $\tau$ and pays a price equal to $\beta$. We assume that $v$ is increasing and strictly concave, and we normalize $v(0) = 0$. If the intermediary commits to fully disclose the information, the buyer will be willing to pay the true quality and will get an utility of 0 whatever the quality of the good. On the other hand, if he commits not to reveal anything, the buyer will be willing to pay a price such that his expected utility is 0.

Let $\beta_i$ be the price that a buyer is willing to pay for the good when he knows the seller has received signal $\sigma_i$. Formally,
\[ \beta_i : E_i [v (\tau - \beta_i)] = v (0) = 0, \]

and \( \beta_i < E_i [\tau] \) by concavity of \( v \).

Let \( \beta_{ij} \) \((i > j)\) be the price that buyers are willing to pay when they believe that the seller has received some signal between \( \sigma_j \) and \( \sigma_i \). Formally,

\[ \beta_{ij} : \sum_{k=j}^{i} \frac{\rho_k}{\sum_{t=j}^{i} \rho_t} E_k v (\tau - \beta_{ij}) = 0. \]

Since \( v \) is concave,

\[ \beta_{ij} \leq \sum_{k=j}^{i} \frac{\rho_k}{\sum_{t=j}^{i} \rho_t} \beta_k \quad \forall i, j, \quad i > j. \quad (2) \]

We first show that the pooling equilibrium with no disclosure is no longer an equilibrium when buyers are risk averse. The intermediary can always do better by revealing some information.

**Proposition 4** For any \( j < N \), any contract \((\overline{P}, 0)\) offered to all types from \( j \) to \( N \) is strictly dominated by a menu of two contracts \((\overline{P}, 0)\) for all types from \( j \) to \( N - 1 \) and \((P_N, \alpha_N = 1)\) for type \( N \).

**Proof.** Assume that there is an equilibrium in which the intermediary offers contract \( (\overline{P}, 0) \) and all types of seller from \( j \) to \( N \) ask for certification, while all other types do not.

If the seller does not ask for certification, he reveals that his signal is lower than \( j \) and, thus, buyers will be ready to pay \( \beta_{(j-1)0} \). If the seller asks for certification, buyers will infer that he is of a type between \( j \) and \( N \) and will be ready to pay \( \beta_{Nj} \).

The equilibrium price, \( \overline{P} \), is then the maximum price the intermediary can charge given the continuation strategies. It is defined as follows:

\[ \overline{P} = \beta_{Nj} - \beta_{(j-1)0}. \]

The expected profit of the intermediary is:

\[ E [\pi_1] = \sum_{i=j}^{N} \rho_i (\beta_{Nj} - \beta_{(j-1)0}). \quad (3) \]
Suppose now that the intermediary offers two contracts \( \{ \left( \bar{P}, 0 \right), (P_N, \alpha_N = 1) \} \) defined as follows:

\[
\bar{P} = \beta_{(N-1)j} - \beta_{(j-1)0} \\
P_N = E_N [\tau] - \beta_{(j-1)0}.
\]

It is straightforward to show that there is a continuation equilibrium in which a seller of type \( N \) will choose the contract \((P_N, \alpha_N = 1)\), all types from \( j \) to \( N-1 \) choose the contract \((\bar{P}, 0)\) and all remaining types do not ask for certification. The expected profit of the intermediary is then:

\[
E[\pi_2] = \sum_{i=j}^{N-1} \rho_i \left( \beta_{(N-1)j} - \beta_{(j-1)0} \right) + \rho_N \left[ E_N [\tau] - \beta_{(j-1)0} \right]. \tag{4}
\]

Using equation (2) we get:

\[
\sum_{i=j}^{N} \rho_i \beta_{Nj} \leq \rho_N \beta_N + \sum_{i=j}^{N-1} \rho_i \beta_{(N-1)j},
\]

and

\[
\sum_{i=j}^{N} \rho_i \left[ \beta_{Nj} - \beta_{(j-1)0} \right] \leq \rho_N \left[ \beta_N - \beta_{(j-1)0} \right] + \sum_{i=j}^{N-1} \rho_i \left[ \beta_{(N-1)j} - \beta_{(j-1)0} \right] \leq \sum_{i=j}^{N-1} \rho_i \left( \beta_{(N-1)j} - \beta_{(j-1)0} \right) + \rho_N \left[ E_N [\tau] - \beta_{(j-1)0} \right].
\]

So (4) is higher than (3) for all \( j \) and therefore offering \((\bar{P}, 0)\) was not an equilibrium.

The result stems from the fact that the menu of contracts (even with \( \alpha_N = 0 \)) creates a better risk-sharing between the intermediary and the buyers and, hence, the intermediary is able to extract a higher rent. On the other hand, by directly revealing the information of type \( N \), buyers bear even less risk and, both the total rent and the share extracted by the intermediary are larger. Finally, there is no cost for the intermediary in disclosing the information of the highest type, because there are no types above \( N \) to whom a rent should be given. In the following proposition we show that, in general, more information will be directly disclosed at equilibrium.
Proposition 5 When there is risk aversion on the buyers’ side, there is always direct revelation of information. In equilibrium, there exists a type $k \in \{0, \ldots, N\}$ such that for all types $i < k$ there is no disclosure and for all types $i \geq k$ there is full disclosure.

Proof. Let us state the following lemma, proved in Appendix B:

Lemma 5 In any optimal contract at most the lowest type of seller does not ask for certification.

Suppose the intermediary offers a menu of contracts $(P_i, \alpha_i)_{i=0}^N$. From lemma 5 the intermediary will certify, at least, the $N$ highest types. Incentive compatibility requires that for all $i$ and $j$

$$\alpha_i E_i [\tau] + (1 - \alpha_i) \beta_i - P_i \geq \alpha_j E_j [\tau] + (1 - \alpha_j) \beta_j - P_j,$$

and

$$\alpha_j E_j [\tau] + (1 - \alpha_j) \beta_j - P_j \geq \alpha_i E_i [\tau] + (1 - \alpha_i) \beta_i - P_i,$$

which implies,

$$(\alpha_i - \alpha_j) (E_i [\tau] - E_j [\tau]) \geq 0.$$

In particular, if $\alpha_i = 0$ then $\alpha_j = 0$ for all $j < i$ and if $\alpha_i = 1$ then $\alpha_j = 1$ for all $j > i$.

Now, since the intermediary’s profit function is linear in $\alpha_i$, for all $i$, the optimal disclosure rule for type $i$ is either equal to 0 or 1.

Finally, the highest price, $P_N$ that can be set is

$$P_N = P_{N-1} + \alpha_N E_N [\tau] + (1 - \alpha_N) \beta_N - \alpha_{N-1} E_N [\tau] - (1 - \alpha_{N-1}) \beta_{N-1},$$

and the profit of the intermediary $E[\pi] = \rho_N P_N + \sum_{i=1}^{N-1} \rho_i P_i$ is an increasing function of $\alpha_N$. The intermediary will therefore set $\alpha_N = 1$.

Furthermore, there exists a threshold $k < N$ such that there is full disclosure for all types from $k$ to $N$ and no disclosure for the remaining ones. So, the optimal menu $\left\{ (P_i, 0)_{i<k}, (\overline{P}, 1)_{i\geq k} \right\}$ is characterized by

$$P_i = \beta_i - \beta_0,$$

$$\overline{P} = E_k [\tau] - \beta_0.$$
The expected profit of the intermediary for a given \( k \) is

\[
E[\pi(k)] = \begin{cases}
  E_k[\tau] \sum_{i=k}^{N} \rho_i + \sum_{i=1}^{k-1} \rho_i \beta_i - (1 - \rho_0) \beta_0 & k > 1 \\
  (1 - \rho_0) (E_1[\tau] - \beta_0) & k = 1 \\
  E_0[\tau] - \beta_0 & k = 0,
\end{cases}
\]

and, the intermediary will choose \( k \) in order to maximize his expected profit.

We have shown that it is always optimal for the intermediary to fully disclose the information of the highest type, because by doing so he increases the total rent without any cost. However, it is not always optimal to fully disclose all the information whatever the type of the seller. Indeed, even if the total rent is increased when all the information is revealed to the buyers, the intermediary is not able to extract all this extra rent because of the incentive compatibility conditions.

The trade-off behind this result can be highlighted in the following way. In the case of a pooling contract with full disclosure of information, the total rent is maximized since it is equal to \( E_F[\tau] \).\(^5\) But the intermediary has to leave in expectation \( E_F[\tau] - E_0[\tau] + \beta_0 \) to the seller.

On the other hand, with the optimal menu that involves full revelation from type \( k \) on, the total rent is \( E_F[\tau] - \sum_{i=0}^{k-1} \rho_i (E_i[\tau] - \beta_i) < E_F[\tau] \) but the intermediary has to leave a smaller rent \( \sum_{i=k}^{N} \rho_i \beta_i \) to the seller.

Following this intuition, the assumption that the intermediary can perfectly discover the true quality is not essential. All our results would hold as long as the information acquired by the intermediary is slightly better than the precision of the signal. Indeed, in the limit case in which the intermediary can just observe the same signal as the seller, the equilibrium would be to offer a menu of \( N \) contracts with no disclosure for \( i < N \).

Finally, the intermediary’s contribution to total surplus is, in general, sub-optimal. Indeed, it would be socially efficient to reveal all the information to the buyers but, as we have shown, this strategy is not always in the interest of the intermediary.

In Appendix A, we construct an example with three different signals in which, for different values of the parameters, the equilibrium involves either a unique contract for all types with full disclosure or a menu of contracts leaving the lowest type out of the certification market.

\(^5\)With a unique contract with no disclosure, the total surplus is \( \beta_F < E_F[\tau] \).
5 Summary and Conclusions

In this paper we developed a simple model of information transmission through certification intermediaries. A seller with no means to signal his quality has the possibility to be certified by an institution that owns a technology to discover the true quality and can credibly commit to a disclosure rule. The focus of the paper is the extent to which a monopoly intermediary has incentives to disclose some information to the buyers.

We distinguish two types of revelation of information. The buyers can learn some new information directly through the announcement made by the intermediary or indirectly through the separation of types induced by the mechanism offered to the seller. That is, even without any announcement, the intermediary can actually transmit some information if he offers a menu of incentive compatible contracts to the seller.

We have shown that Lizzeri’s result relies on a limitation of the space of contracts and the inability of the intermediary to increase total surplus, since information has no ex ante value for the buyers. When information becomes valuable, the intermediary acts as a traditional monopolist. At equilibrium, he always discloses some information but less than what would be socially optimal. Introducing competing intermediaries should then favor more revelation.
A Numerical example

We consider the following example with three signals, $\sigma_0$, $\sigma_1$ and $\sigma_2$.

- $v(\tau - \beta) = 1 - e^{-\gamma(\tau - \beta)}$
- $\tau \in [1, 4]$
- $F(\tau) = \frac{\tau - 1}{3}$
- $G_0(\tau) = \begin{cases} \frac{\tau - 1}{1.5} & \text{if } \tau \leq 2.5 \\ 1 & \text{otherwise} \end{cases}$
- $G_1(\tau) = \begin{cases} 0 & \text{if } \tau \leq 2 \\ \frac{\tau - 2}{1.5} & \text{if } \tau \leq 3.5 \\ 1 & \text{otherwise} \end{cases}$
- $G_2(\tau) = \begin{cases} 0 & \text{if } \tau \leq 3 \\ \tau - 3 & \text{otherwise} \end{cases}$

We can easily determine $\beta_i$ such that $E_i [v(\tau - \beta_i)] = 0$ for $i = 0, 1, 2$.

Depending on the values of the parameters, there are three possible equilibria.

1. A unique contract with full revelation of information chosen by all types. The price offered to the seller is then:

\[ \hat{P} = \frac{7}{4} - \frac{1}{\gamma} \ln \left( \frac{1.5\gamma}{e^{-\gamma} - e^{-2.5\gamma}} \right), \]

and the expected profit of the intermediary is:

\[ \pi(\hat{P}) = \hat{P}. \]

2. A contract with full revelation of information that excludes the lowest type. The price offered to the seller is then:

\[ \tilde{P} = \frac{11}{4} - \frac{1}{\gamma} \ln \left( \frac{1.5\gamma}{e^{-\gamma} - e^{-2.5\gamma}} \right), \]

and the expected profit of the intermediary is:

\[ \pi(\tilde{P}) = (1 - \rho_0) \tilde{P}. \]
3. A menu of contracts with full revelation of information for the highest type and no revelation for type 1. The prices offered to the seller are then:

\[ P_1 = \frac{1}{\gamma} \ln \left( \frac{e^{-\gamma} - e^{-2.5\gamma}}{e^{-2\gamma} - e^{-3.5\gamma}} \right), \]

and

\[ P_2 = \frac{7}{2} - \frac{1}{\gamma} \ln \left( \frac{1.5\gamma}{e^{-\gamma} - e^{-2.5\gamma}} \right), \]

and the expected profit of the intermediary is:

\[ \pi(P_1, P_2) = \rho_1 P_1 + \rho_2 P_2. \]

Consider the case where \( \rho_0 = \frac{8}{9}, \rho_1 = \rho_2 = \frac{1}{18} \) shown in Figure 1.

![Figure 1: Profit of the intermediary as a function of the degree of risk aversion in the case where \( \rho_0 = \frac{8}{9}, \rho_1 = \rho_2 = \frac{1}{18} \).](image)

When \( \gamma \) is close to 0, buyers are nearly risk neutral and therefore, the equilibrium is to offer a menu of contracts that excludes the lowest type and do not disclose information for type 1.

However, as buyers become more risk-averse, the positive effect of increasing total surplus through revelation of information regarding all types is larger than the negative effect of leaving some rents to types 1 and 2. This stems from the fact that \( \rho_0 \) is sufficiently large and therefore the probability of actually having to leave a rent is low.
Figure 2: Profit of the intermediary as a function of the degree of risk aversion in the case where $\rho_0 = 0.1$, $\rho_1 = 0.6$, $\rho_2 = 0.3$.

Consider finally the case where $\rho_0 = 0.1$, $\rho_1 = 0.6$, $\rho_2 = 0.3$, in Figure 2. When $\rho_0$ is small enough, offering a unique contract as above can never be an equilibrium. Again, when $\gamma$ is low, the “leaving the rent” effect dominates and the intermediary prefers not to reveal the information for type 1. However, this willingness to pay for information increases with $\gamma$ while the “leaving the rent effect” is constant. Therefore, from a certain threshold of $\gamma$, the intermediary prefers to offer a unique contract that excludes the lowest type and fully reveals the information. $\rho_0$ is low, so leaving type 0 out of the certification market is not very costly.

B Proof of lemmas

B.1 Proof of Lemma 1

Proof. At the first stage, the intermediary offers a price $P$ and a disclosure rule $\alpha \in [0, 1]$.

Suppose there is a continuation pooling equilibrium in which no seller goes through the intermediary. Because buyers do not learn anything new, they will be willing to pay the expected value, $E_F[\tau]$ for the good. If the buyers observe a seller going through the intermediary and no information is revealed, they will think that he is a $\sigma_i$ seller with probability $\nu_i$.

So, a seller of type $i$ does not go through the intermediary if

$$E_F[\tau] \geq (1 - \alpha) \sum_j \nu_j E_j[\tau] + \alpha E_i(t) - P.$$  (5)
The payoff of the intermediary when offering \((P, D)\) is equal to 0. Now, this is an equilibrium if there are no alternative price and disclosure rule that could increase the payoff of the intermediary. Consider the following deviation:

\[
P' = E_N[\tau] - E_F[\tau] - \epsilon
\]

\[
\alpha' = 1,
\]

with \(\epsilon\) small.

Then, a seller of type \(N\) will go to the intermediary since

\[
E_F[\tau] < (1 - \alpha') \sum_j \nu_j E_j[\tau] + \alpha' E_N[\tau] - P'
\]

\[
< E_F[\tau] + \epsilon.
\]

Indeed, either no type asks for certification and the price paid by the buyer will be \(E_F[\tau]\) or only the highest types ask for certification and the outside option is smaller than \(E_F[\tau]\).

But then, the profit of the intermediary is at least equal to

\[
P' \rho_N > 0,
\]

and therefore, this is a profitable deviation for the intermediary. This is true whatever the out-of-equilibrium beliefs. \(\blacksquare\)

### B.2 Proof of Lemma 2

**Proof.** Suppose it exists a pooling equilibrium with full disclosure, \(\alpha = 1\) and a price \(P\).

Since the true type of the seller will be fully revealed, then the buyers will pay a price equal to \(\tau\). So, ex ante, an \(i\) signal seller’s expected payoff when playing the equilibrium strategy is

\[
E_i[\tau] - P \quad i = 0, \ldots, N.
\]

If a seller plays an out-of-equilibrium strategy (not go to the intermediary), the buyers will be ready to pay \(E_i[\tau]\) if they expect the seller to have received signal \(i\). Suppose that the out-of-equilibrium beliefs of the buyers are that the seller has received signal with probability \(\nu_i \in [0, 1]\), then the seller’s expected payoff is

\[
\sum_j \nu_j E_j[\tau] \quad \forall i.
\]
So, type $i$ seller plays the equilibrium strategy if

$$E_i[\tau] - P \geq \sum_j \nu_j E_j[\tau] \quad i = 0, ..., N.$$ 

For the continuation strategies to be a pooling equilibrium, the price paid to the intermediary must satisfy for any $i$:

$$P \leq E_i[\tau] - \sum_j \nu_j E_j[\tau] \leq 0. \quad (6)$$

Since all types of seller have their product tested, the expected profit of the intermediary is $P \leq 0$.

Now, consider the following deviation: $P' \neq P$, $P' = \varepsilon \geq 0$, $\alpha' = 1$. Suppose that the ex-post beliefs of the buyers if the seller does not go through the intermediary, $\nu'_i$, are such that $\exists i < N : \nu'_i \neq 0$ then, if $\varepsilon \leq E_N[\tau] - \sum_j \nu'_j E_j[\tau]$ the high signal seller prefers to be certified:

$$E_N[\tau] - \varepsilon > \sum_j \nu'_j E_j[\tau].$$

So, the equilibrium ex-post beliefs are given by $\nu'_N > 0$. But then $\varepsilon$ can be made strictly positive, and the intermediary’s payoff with this deviation is at least

$$\varepsilon \rho_N > 0,$$

and therefore, the intermediary is strictly better-off than with the previous strategy.

On the other hand, suppose $\nu'_i = 0$ for any $i < N$ and $\nu_0 \neq 1$. From (6), $P < 0$. Therefore, setting $\varepsilon = 0$ is a profitable deviation. Finally, if $\nu'_i = 0$ for any $i < N$ and $\nu_0 = 1$, $P = P' = 0$, a contradiction (the out of equilibrium beliefs cannot be different if all strategies are the same).

Thus, there is no $P$ such that $(P, \alpha = 1)$ could induce a pooling continuation equilibrium. ■

B.3 Proof of Lemma 3

Proof. Suppose that

$$x = Y_1 \quad \text{for all } \sigma_i \text{ with } i \geq j > 1$$

$$x = N \quad \text{for all } \sigma_i \text{ with } i < j.$$
The highest price that can be charged by the intermediary is then:

$$P_{SSJ} = \sum_{i=j}^{N} \frac{\rho_i}{\sum_{k=j}^{N} \rho_k} E_i \tau - \sum_{i=0}^{j-1} \frac{\rho_i}{\sum_{k=0}^{j-1} \rho_k} E_i \tau.$$  

The expected profit of the intermediary is

$$E(u_{SSJ}^I) = P_{SSJ} \sum_{k=j}^{N} \rho_k$$

$$= E_F \tau - \frac{1}{\sum_{i=0}^{j-1} \rho_k} \sum_{i=0}^{j-1} \rho_i E_i \tau$$

$$< E_F \tau - E_0 \tau = u_{FP}^I.$$

\[ \text{B.4 Proof of Lemma 4} \]

**Proof.** Consider 2 different types: \(m, k\) with \(m < k\). Suppose \(m\) and \(k\) choose the same contract: \((P_m, \alpha_m)\). On the other hand, all remaining types, \(i \neq m, k\) self-select by choosing contract \((P_i, \alpha_i)\). This implies

$$E_m \tau - P_m = \alpha_i E_m \tau + (1 - \alpha_i) E_i \tau,$$

$$\alpha_m E_k \tau + (1 - \alpha_m) E_m \tau - P_m = \alpha_i E_k \tau + (1 - \alpha_i) E_i \tau,$$

$$E_i \tau - P_i \geq \alpha_m E_i \tau + (1 - \alpha_m) E_m \tau \quad \forall i \neq m, k,$$

and the expected profit of the intermediary is

$$\sum_{i \neq m,k} \rho_i P_i + (\rho_m + \rho_k) P_m.$$  

(7)

Consider the introduction of a new contract, \((P_k, \alpha_k)\) with \(\alpha_k = \alpha_m\) such that

$$E_m \tau - P_m \geq \alpha_k E_m \tau + (1 - \alpha_k) E_k \tau$$

and

$$E_k \tau - P_k \geq \alpha_m E_k \tau + (1 - \alpha_m) E_m \tau.$$

6It is straightforward to show the same result by assuming that some of the remaining types may choose the same contract.
We have that

\[ P_k = P_m + (1 - \alpha_m) (E_k [\tau] - E_m [\tau]) > P_m. \]

No type \( i \notin \{m, k\} \) has incentives to choose \( (P_k, \alpha_k) \).

On the other hand, the expected profit of the intermediary is now

\[
\sum_{i \neq m, k} \rho_i P_i + \rho_m P_m + \rho_k P_k. \tag{8}
\]

So, noting that (8) is larger than (7), the intermediary always has incentives to offer a menu of contracts in which two different types choose two different contracts. \( \blacksquare \)

**B.5 Proof of Lemma 5**

**Proof.** Suppose the intermediary offers an incentive compatible menu of contracts \( (P_i, \alpha_i)_{i=j}^N \) such that all types below \( j \) prefer not to ask for certification. The expected profit of the intermediary is \( \sum_{i \geq j} \rho_i P_i \).

Consider now the introduction of a new contract for type \( j - 1 \) with \( \alpha_{j-1} = 0 \) and \( P_{j-1} > 0 \) that satisfies

\[
\alpha_j E_{j-1} + (1 - \alpha_j) \beta_j - \beta_{j-1} \leq P_j - P_{j-1} \leq \alpha_j E_j + (1 - \alpha_j) \beta_j - \beta_{j-1}.
\]

All contracts are still incentive compatible and the intermediary’s expected profit is \( \sum_{i \geq j} \rho_i P_i + \rho_{j-1} P_{j-1} > \sum_{i \geq j} \rho_i P_i. \) \( \blacksquare \)

**References**


