# Optimal supply chain design: a comparison of individual factories and industrial clusters allocation 

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#### Abstract

The aim of this work is to present a mathematical model for the optimal design and planning of a supply chain, where several plants that produce different products can be installed forming clusters. In order to analyze facilities integration, discounts in installation and production costs are assumed. In addition, the tradeoffs between clusters and individual facility configurations are assessed. This work also focuses on the effect of the production scale in the overall SC, and the costs reduction when plants share resources and services.


## 1 Introduction

The supply chain (SC) is composed by agents committed to meet the needs of customers. It is integrated by production units with their sources of supplies and their customers, and it coordinates all input and output flows (materials, information and finances) so that products are produced and distributed in the right amounts, in appropriate locations and at the right time. Success in operations depends on the coordination of its members, since they are composed of complex activities having different commitments among actors, and which require a significant effort to carry them out.

An industrial cluster is formed by companies located in a given area. They collaborate to gain advantages in cost and time through beneficial and effective exchange of resources and services, achieving improved competitiveness and market opportunities [1, 2].

There are several types of industrial clusters, and many factors may explain their formation. Porter (1998) [3] developed the theory of industrial clusters and postulated that the benefits of industrial clusters are based on economies of scale, technology transfer and the availability of human capital. He states that four major sources of productivity and cost benefits can be linked to industrial clusters: (1) access to inputs and infrastructure (2) labor and human resource pooling (3) access to information and performance measures and (4) complementary products.

There are some published works about the industrial clusters formation for improving the SC performance [4], for economic and industrial development of a region [5],
for knowledge transfer [6], as well as the risks of industry knowledge exchanges [7]. In some cases, an alliance of companies arises, with the leading enterprise as alliance leader, and the rest of the chain is organized around its needs [8, 9]. Also, there are some works considering customer clustering according to demand patterns and on this basis, the production, inventory and distribution problems are solved satisfying demands in each period of time considering limited production and transport capacity [10]. Ng and Lam (2014) [11] propose the formation of industrial clusters based on a specific function for each group, increasing its economic potential. They consider that the relationship between industries, their material sources and production sink depend on each cluster.

In this work, the clusters installation in a SC is analyzed. A mixed integer linear programming (MILP) model for the optimal SC design and planning is presented. The different tradeoffs between clusters and individual facilities allocation are evaluated. The costs reduction when plants share resources and services, and the effect of the production scale in the overall SC is highlighted through the proposed formulation. Also, transportation costs are analyzed, since it is expected that a greater effort to move materials in these big complexes is carried out. The approach addressed in this work represents a useful tool for analyzing production, resources and services integration in a SC involving different facilities in the diversification of an industry.

## 2 Problem statement

A SC with three echelons is considered in this work: regions of raw material $(s)$, production facilities sites ( $l$ ) where different plants $(p)$ can be installed, and consumer areas ( $k$ ).

Each raw material site has certain types of materials $(z)$ with different qualities $(r)$ to be used by production plants. Moreover, at each raw material site, a quantity of residue is generated to be consumed by some plants $p, p \in R P$. It is a priori known the possible raw material type and quality to be used in each plant $p \in Z P_{z p}$.

Each plant $p$ can produce a set of products $i \in I P_{i p}$. Also, some plants can produce byproducts $b, b \in B P_{b p}$, to be used as raw material for other products in the same plant or in a different plant, $p \in P B_{p b}$ for producing the products at plant $p$. The size of each plant, is a model decision and must be selected from a discrete set of alternatives, $t \in T_{p}$.

Various connections between facilities, such as flows of different types of products, byproducts, and residues, for diverse uses within the plant (raw material, energy source) arise. The final products are distributed from production facilities to the consumer regions, which have a maximum demand for the different products, $D_{i k}^{\max }$, to be fulfilled.

Plants have several alternatives for its location: close to the supply areas, near the consumption regions or at intermediate points. It is possible to install more than one plant of different type at the same site, but not two plants of the same type. The adequate allocation of these plants will influence economic performance of the global SC. Installing plants near raw material sites reduces transportation costs of raw materials, usually using smaller plants. If the factories are located near consumption areas, the cost of raw material transportation increases, large-scale production is also affected depending on the specific demands of each zone, but the product transportation is cheaper. Middle points are intermediate solutions where several factors must be considered: products to be elaborated, distances from raw material sources and customer areas, etc. The objective is to design and plan the SC with the maximum net profit given by the incomes from sales minus the raw material, investment, operation and transportation costs.

## 3 Mathematical model

In this section, the mass balances among the different nodes of the SC , the design equations and the objective function are presented. Fig. 1 shows the links with the different optimization variables used in the model in a generic case.


Fig. 1. Flows among SC nodes

### 3.1 Raw material sites

Eq. (1) states that each raw material area has a maximum capacity for each commodity $z$ of quality $r\left(\right.$ Maxrm $\left._{z r s}\right)$. Therefore, the total amount used by different plants $p\left(Q h_{z x p l}\right)$ should not exceed this capacity.

$$
\begin{equation*}
\sum_{p \in Z P_{z r p}} \sum_{l} Q h_{z s p l} \leq \text { Maxrm }_{z z s} \quad \forall s, z, r \tag{1}
\end{equation*}
$$

At each raw material site, the produced residue is determined through the parameter $f h r$. Therefore, the amount of residue transported from raw material sites to all the production plants that consume residue ( $Q h r_{\text {spl }}$ ) is given by:

$$
\begin{equation*}
\sum_{p \in R P} \sum_{l} Q h r_{s p l} \leq\left(\sum_{p} \sum_{l} \sum_{z} Q h_{z s p l}\right) f h r \quad \forall s \tag{2}
\end{equation*}
$$

### 3.2 Production facilities

The total input material for producing products at each plant of type $p$ in site $l$ $\left(R M_{p l}\right)$ is equal to the amount of raw material ( $f l_{z} Q h_{z x p l}$ ) and residue ( $f r Q h r_{s p l}$ ) coming from raw material sites $s$, plus byproduct ( $f e_{b} Q r_{b p^{\prime} p l^{\prime} l}$ ) from production plants:

$$
\begin{equation*}
R M_{p l}=\sum_{\substack{r, z \\ p \in P_{z r p}^{\prime}}} \sum_{s} f l_{z} Q h_{z s p l}+\sum_{\substack{p^{\prime} l^{\prime} \\ b \in B P_{b p p^{\prime}} \\ p \in P P_{p b}}} \sum_{b} e_{b} Q r_{b p^{\prime} p^{\prime} l^{\prime} l}+\sum_{\substack{s l \\ p \in R P}} f r Q h r_{s p l} \quad \forall p, l \tag{3}
\end{equation*}
$$

$R M_{p l}$ is converted to final products according to a conversion factor $f_{i p}$ which depends on the plant type and product to be produced. The variable $P m_{i p t l}$ represents the amount of product $i$ produced at plant $p$ with size $t$ in location $l$

$$
\begin{equation*}
\sum_{\substack{t, i l \\ i \in I T_{i p}}} P m_{i p t l}=f_{i p} R M_{p l} \quad \forall p, l \tag{4}
\end{equation*}
$$

The following equation provides the amount of each type of byproducts $b$ generated in each industry, depending on the amount of material input.

$$
\begin{equation*}
Q r t_{b p l}=f c_{b} R M_{p l} \quad \forall p, l, b \in B P_{b p} \tag{5}
\end{equation*}
$$

$f c_{b}$ is a conversion factor for byproduct type $b$.
The generated byproducts may be used as raw material for other industries $\left(Q r_{b p p^{\prime} l l^{\prime}}\right)$, selling them to third parts $\left(Q s_{b p l}\right)$ or use them for energy $\left(Q b_{b p l}\right)$ in the same plant, if necessary:

$$
\begin{equation*}
Q r t_{b p l}=\sum_{\substack{p^{\prime}, l^{\prime}, l \\ p^{\prime} \in P B_{p^{\prime} b}^{\prime}}} Q r_{b p p^{\prime} l l^{\prime}}+Q b_{b p l}+Q s_{b p l} \quad \forall b \in B P_{b p}, p, l \tag{6}
\end{equation*}
$$

For each installed plant a size $t$ from a set $T_{p}$ must be selected related to the size of the equipment used in it. Thus the effect of the production scale is represented.

Let $w_{p t l}$ be a binary variable for selecting the discrete size $t$ when a plant $p$ is assigned in the location $l$
$w_{p t l}= \begin{cases}1 & \text { if a plant } p \text { in site } l \text { with capacity } t \text { is installed } \\ 0 & \text { otherwise }\end{cases}$
Then, $P c_{t p}^{\max }$ represent the different possible maximum capacities $t \in T$, for plant $p$ :

$$
\begin{equation*}
\sum_{i \in I P_{i p}} P m_{i p t l} \leq w_{p t l} P c_{t p}^{\max } \quad \forall p, l, t \tag{7}
\end{equation*}
$$

Moreover, if plant $p$ is installed at location $l$, only one size is selected for it:

$$
\begin{equation*}
\sum_{t} w_{p t l}=x_{p l} \quad \forall p, l \tag{8}
\end{equation*}
$$

where $\quad x_{p l}= \begin{cases}1 & \text { if a plant } p \text { in site } l \text { is installed } \\ 0 & \text { otherwise }\end{cases}$

### 3.3 Demand constraints

Let $k$ be the different regions of customers and $D_{i k}^{m a x}$ its maximum demand for each product type. The amount of each product provided from facilities to each region $k\left(Q p_{i p l k}\right)$ cannot exceed the demand in that region.

$$
\begin{equation*}
\sum_{p, l} Q p_{i p l k} \leq D_{i k}^{\max } \quad \forall p, i \in I P_{i p}, k \tag{9}
\end{equation*}
$$

Also, the product flow to each region should not exceed the production of each plant

$$
\begin{equation*}
\sum_{t} P m_{i p t l} \geq \sum_{k} Q p_{i p l k} \quad \forall p, i \in I P_{i p}, l \tag{10}
\end{equation*}
$$

### 3.4 Objective Function

The adopted objective function represents the profit maximization given by the difference between the incomes for sales of products and byproducts with the costs of raw materials, transportation, installation and production.

$$
\begin{equation*}
G=I-R M c-T c-I c-P c \tag{11}
\end{equation*}
$$

Income $(I)$ is represented as follows

$$
\begin{equation*}
I=\sum_{\substack{i, p, l, k / \\ i \in \in P_{i p}}} S_{i p} Q p_{i p l k}+\sum_{\substack{b, p, l \\ b \in B P_{p p}}} S_{b} Q r r_{b p l} \tag{12}
\end{equation*}
$$

Raw material costs $(R M c)$ are related to the acquisition of the raw materials and residues needed for production. Therefore, they are calculated from the unit cost of the materials and the quantities supplied to each plant.

$$
\begin{equation*}
R M c=C r m \sum_{\substack{z, r, r, p, l / \\ p \in Z \not P_{r p p}}} Q h_{z s p l}+C h r \sum_{\substack{s, p, l / l \\ p \in R P}} Q h r_{s p l} \tag{13}
\end{equation*}
$$

Transportation cost $(T c)$ includes transportation of raw materials and residues from the supply area to the production plants, transportation of byproducts among plants, and shipping products to customers regions. It is calculated by multiplying the amount of transported material by the distance between the involved nodes and their relative cost depending on the delivered material:

$$
\operatorname{Tc}=\mathrm{Ctrm} \sum_{\substack{z, r, s, p, l l^{\prime} \\ p \in P_{z p l}}} \operatorname{Dh} p_{\text {spl }} Q h_{z s p p l}+C t h r \sum_{\substack{s, p, l \\ p \in R P}} D h p_{\text {spl }} Q h r_{\text {spl }}+
$$

Installation cost (Ic) is determined for each type of production plant in every possible location taking into account its capacity:

$$
\begin{equation*}
I c=\text { fam } \sum_{p, t, l} \alpha_{p l}\left(P C_{t p}^{\max }\right)^{\beta_{l}} w_{p t l} \tag{15}
\end{equation*}
$$

where fam represents the capital charges factor, including the amortization and maintenance, and $\alpha$ and $\beta$ are cost coefficients defined for each type of installation.

Production cost $(P C)$ involves the costs of labor and materials needed for the various products. This cost depends on the production capacity of each facility and it is obtained by multiplying in each case the total production by its production cost

$$
\begin{equation*}
P c=\sum_{\substack{i, p, t . l \\ i \in I P_{p}}} C P_{i p t} P M_{i p t l} \tag{16}
\end{equation*}
$$

### 3.5 Formation of clusters

In order to encourage the formation of industrial clusters, discounts on installation and production costs are proposed. These benefits depend on the number of plants to be installed in a particular location and the relative size of production between plants. When several plants are jointly installed, large plants will have a lower discount than that obtained by a factory of smaller size, i.e. a better benefit is reached for smaller plants.

Let be $n$ the number of installed industries at a given site. This value is determined by the binary variable $y_{l n}$ defined by the following expression

$$
\begin{array}{ll}
\sum_{n=1}^{N_{p l a n t}} n y_{l n}=\sum_{p, t} w_{p t l} & \forall l \\
\sum_{n=1}^{N_{p l a n t}} y_{l n} \leq 1 & \forall l \tag{18}
\end{array}
$$

where $y_{l n}= \begin{cases}1 & \text { if } n \text { plants are installed on thesite } l \\ 0 & \text { otherwise }\end{cases}$
In order to establish the relation among the sizes of the installed factories at site $l$, binary variables $a u x 1_{p t l}, a u x 2_{p p^{\prime} t t^{\prime} l}, a u x 3_{p p^{\prime} p^{\prime \prime} t t^{\prime} t^{\prime \prime} l}$, and so on, are defined for stating the number of plants installed in a site is one, two or three, etc., respectively. These variables must be defined according to the maximum number of plants that can be considered in a given location. In this work, in order to give a brief description about the definition of these variables, it is assumed an upper bound of three plants to be installed for each site $l$, but this can be easily extended for a higher number of plants.

$$
a u x 1_{p t l}= \begin{cases}1 & \text { if a only plant } p \text { of size } t \text { is installed at site } l \\ 0 & \text { otherwise }\end{cases}
$$

aux $2_{p p^{\prime} t t^{\prime} l}= \begin{cases}1 & \text { if plants } p \text { and } p^{\prime} \text { with sizet and } t^{\prime} \text { respective ly are installed at sitel } \\ 0 & \text { otherwise }\end{cases}$ $a u x 3_{p p^{\prime} p^{\prime \prime} t t^{\prime} t^{\prime \prime} l}= \begin{cases}1 & \text { if plants } p, p^{\prime} \text { and } p^{\prime \prime} \text { are installed of size } t, t^{\prime} \text { and } t^{\prime \prime} \text { respectively at site } l \\ 0 & \text { otherwise }\end{cases}$

To determine its value, the following restrictions are used:

$$
\begin{array}{cc}
a u x 1_{p t l} \geq w_{p t l}+y_{l n 1}-1 & \forall p, l, t \\
a u x 2_{p p^{\prime} t t^{\prime} l} \geq w_{p t l}+w_{p^{\prime} t^{\prime} l}+y_{l n 2}-2 & \forall p, p^{\prime}, l, t, t^{\prime}, p \neq p^{\prime} \\
a u x 3_{p_{p^{\prime} p^{\prime \prime} t t^{\prime} t^{\prime \prime} l} \geq w_{p t l}+w_{p^{\prime} t^{\prime} l}+w_{p^{\prime \prime \prime} t^{\prime \prime} l}+y_{l n 3}-3} & \forall p, p^{\prime}, p^{\prime \prime}, l, t, t^{\prime}, t^{\prime \prime}, p \neq p^{\prime} \neq p^{\prime \prime} \\
a u x 1_{p t l} \leq y_{l n 1} & \forall p, l, t \\
a u x 2_{p p^{\prime} t t^{\prime} l} \leq y_{l n 2} & \forall p, p^{\prime}, l, t, t^{\prime}, p \neq p^{\prime} \\
a u x 3_{p p^{\prime} p^{\prime \prime} t t^{\prime} t^{\prime \prime} l} \leq y_{l n 3} & \forall p, p^{\prime}, p^{\prime \prime}, l, t, t^{\prime}, t^{\prime \prime}, p \neq p^{\prime} \neq p^{\prime \prime} \tag{24}
\end{array}
$$

where the subscript $n 1$ represents that only one plant is installed at $l$ if $y_{l n 1}=1$, and so on for $n 2$ and $n 3$.

Thus, if for example two plants, $p$ and $p^{\prime}$, with capacity $t$ and $t^{\prime}$ are installed at site $l$, then $y_{l n 2}=1, w_{p t l}=1$ and $w_{p^{\prime} t^{\prime} l}=1$, and therefore, by Eqs. (20) and (23), $a u x 2_{p p^{\prime} t t^{\prime} l}=1$, while $a u x 1_{p t l}=0$ and $a u x 3_{p p^{\prime} p^{\prime \prime t} t^{\prime} t^{\prime \prime} l}=0$ by Eqs. (18), (22) and (24).

Using these binary variables, the installation and production costs are calculated.

## Installation cost

The installation cost for clusters involving one, two or three plants $\left(I C C_{l n}\right)$ is presented. For each site $l$, at most one of these expressions is positive:

$$
\begin{array}{cc}
I C C_{l n 1}=\sum_{p, t} I C P a u x 1_{p t l} \quad \forall l \\
I C C_{l n 2}=\sum_{\substack{p, p^{\prime}, t, t^{\prime} \\
p \neq p^{\prime}}} \operatorname{parP}_{t t t^{\prime}} I C P a u x 2_{p p p^{\prime} t t^{\prime} l} & \forall l \\
I C C_{l n 3}=\sum_{\substack{p, p^{\prime}, p^{\prime \prime}, t t^{\prime}, t^{\prime \prime} \\
p \neq p^{\prime} \neq p^{\prime}}} \operatorname{par}_{t t^{\prime} t^{\prime}} I C P a u x 3_{p p^{\prime} p^{\prime \prime} t t^{\prime \prime} l} & \forall l \tag{27}
\end{array}
$$

$\operatorname{par} P 2_{t t^{\prime}}$ and $\operatorname{par} P 3_{t t^{\prime} t^{\prime \prime}}$ are the discount factors for each industry relating the sizes of the factories. If only one plant is installed, no discount is applied.

ICPaux $1_{p t l}$, ICPaux $2_{p p^{\prime} t t^{\prime} l}$ and ICPaux $3_{p p^{\prime} p^{\prime \prime} t t^{\prime} t^{\prime \prime \prime} l}$ are the installation costs of each plant in a cluster of one, two or three plants respectively. The subscript represents the type and size of the involved plants. This allows applying a correct discount factor for each facility according to the number and size of the plants installed in the same cluster, and they are calculated as:

$$
\begin{gather*}
I C P a u x 1_{p t l} \leq I C P_{p t l}  \tag{28}\\
I C P a u x 1_{p t l} \leq I C P^{U P} \text { aux } 1_{p t l} \quad \forall p, l, t  \tag{29}\\
I C P a u x 1_{p t l} \geq I C P_{p l t}-I C P^{U P}\left(1-a u x 1_{p t l}\right) \tag{30}
\end{gather*} \quad \forall p, l, t
$$

where $I C P^{U P}$ is an upper bound for the investment cost, and $I C P_{p l t}$ is the installation costs of each plant given by the terms of Eq. (15), i.e.:

$$
\begin{equation*}
I C P_{p l t}=\operatorname{fam} \alpha_{p l}\left(P C_{p t}^{\max }\right)^{\beta_{l}} w_{p l t} \quad \forall p, l, t \tag{31}
\end{equation*}
$$

Analogously, for ICPaux $2_{p p^{\prime} t t^{\prime} l}$ and ICPaux $3_{p p^{\prime} p^{\prime \prime \prime} t t^{\prime} t^{\prime \prime} l}$, Eqs. (28)-(30) are formulated replacing $a u x 1_{p t l}$ by $a u x 2_{p p^{\prime} t t^{\prime} l}$ and $a u x 3_{p p^{\prime} p^{\prime} \prime t t^{\prime} t^{\prime \prime}}$, respectively.

Therefore, total installation cost is calculated as:

$$
\begin{equation*}
I c=\sum_{l, n} I C C_{l n} \tag{32}
\end{equation*}
$$

## Production cost

Discounts applied to production costs have a similar formulation to that made with installation costs. Production cost of each cluster $\left(P C C_{l n}\right)$ is obtained as follows:

$$
\begin{array}{cl}
P C C_{l n 1}=\sum_{p, t} P C P a u x 1_{p t l} & \forall l \\
P C C_{l n 2}=\sum_{\substack{p, p^{\prime}, t t^{\prime} \\
p \neq p^{\prime}}} p_{t} P 2_{t t^{\prime}} P C P a u x 2_{p p^{\prime} t t^{\prime} l} & \forall l \tag{34}
\end{array}
$$

$$
\begin{equation*}
P C C_{l n 3}=\sum_{\substack{p, p^{\prime}, p^{\prime \prime}, t, t t^{\prime}, t^{\prime \prime} \\ p \neq p^{\neq p^{\prime}}}} \operatorname{parP} 3_{t t^{\prime} t^{\prime \prime}} \text { PCPaux } 3_{p p^{\prime} p^{\prime \prime} t t^{\prime} t^{\prime \prime \prime l}} \quad \forall l \tag{35}
\end{equation*}
$$

PCPaux $1_{p l t}$, PCPaux $2_{p p^{\prime} l t t^{\prime}}$ and PCPaux $3_{p p^{\prime} p^{\prime \prime} l t t^{\prime} t^{\prime \prime \prime}}$ are the production costs of each plant in a cluster conformed by one, two or three plants respectively.

In order to determine the value of these costs, the following constraints are used for PCPaux1 $1_{p t l}$ (they are similarly formulated for PCPaux $2_{p p^{\prime} t t^{\prime} l}$ and $P C P a u x 3_{p p^{\prime} p^{\prime} t t^{\prime} t^{\prime} l}{ }^{\prime}$ ):

$$
\begin{array}{cc}
P C P a u x 1_{p t l} \leq P C P_{p t l} & \forall p, p^{\prime}, l, t, t^{\prime}, p \neq p^{\prime} \\
P C P a u x 1_{p t l} \leq P C P^{U P} \text { aux }_{p t l} & \forall p, p^{\prime}, l, t, t^{\prime}, p \neq p^{\prime} \\
P C P a u x 1_{p t l} \geq P C P_{p t l}-P C P^{U P}\left(1-a u x 1_{p t l}\right) & \forall p, p^{\prime}, l, t, t^{\prime}, p \neq p^{\prime} \tag{38}
\end{array}
$$

Where $P C P^{U P}$ is the upper bound of the cost, and $P C P_{p l t}$ is the production cost of each plant, calculated as follows:

$$
\begin{equation*}
P C P_{p t l}=\sum_{i \in I P_{p}} C p_{p t l} P m_{i p l l} \quad \forall p, l, t \tag{39}
\end{equation*}
$$

Therefore, the total production cost is given by:

$$
\begin{equation*}
P c=\sum_{l n} P C C_{l n} \tag{40}
\end{equation*}
$$

## 4 Case study

In order to apply the general proposed formulation, a forestry SC is used. The possible facilities to be installed in each location are sawmills, woodboards and pellets factories. In sawmills, the raw material (logs), becomes lumber through various stages. At the same time it produces a large amount of byproducts. These byproducts are classified into: wood chips, firewood chips, bark and sawdust. Two types of lumber are sold to consumers, while the byproducts can be used as fuel for boilers of sawmills and woodboard plants, raw material for the production of woodboards or pellets, or be sold to third parties.

The woodboard factories use sawdust, wood chips and firewood from sawmills and logs from harvest areas as raw materials. Two types of boards and bark as byproduct can be produced.

The wood pellet is a solid biofuel produced from wood residues. The raw material for pellet production can proceed from harvest areas (harvest residues) and byproducts from sawmills and woodboard plants (wood chips, firewood chips, bark and sawdust). The pellets can be used as a source of energy in sawmills or woodboards factories, or sold to consumer regions.

Eight raw material sites with two types of raw materials and two types of qualities
each are considered. A lot of waste is produced due to the forest pruning. A portion of these residues can be used as raw material for pellets production, achieving a more efficient SC.

A total of ten possible locations for production facilities distributed in the supply areas, in the customer regions and intermediate sites is assumed. Each facility adopts a discrete capacity and installation cost varies according to its size.

There are four consumer regions with a maximum demand for each type of wood, woodboards and pellets. It is assumed that raw material cost does not vary with the geographical location. The distance between locations, costs and selling price, efficiency factors and other model parameters are not presented due to lack of space, but they are available for interested readers.

Two examples are shown: the first considers installation and production cost without discount, while in the second, different discounts according to number and size of plants jointly installed are applied in order to favor cluster formation. Both examples were implemented and solved in GAMS [12] using the CPLEX solver

## 5 Results

### 5.1 Individual factories

The optimal attained SC configuration consists in 4 sawmills, 4 woodboards facilities, and 7 pellets facilities. Production plants are located as follows: seven in harvested areas, five in an intermediate point and three near to consumer region. There are 2 clusters of three plants, 4 clusters of two plants and 1 individual facility. The total benefit is $\$ 247.67 \mathrm{MM}$, and the detailed list of incomes for sales and costs is shown in Table 1. The raw material is completely utilized, and $86.6 \%$ of the total generated residues is used. The installed capacity for each facility and the production of the different products varies in each site, producing a total of $247078 \mathrm{~m}^{3}$ of lumber, $1200000 \mathrm{~m}^{3}$ of woodboards and 150831 t of pellets. The installed capacity is totally utilized in the case of woodboards, sawmills are utilized in a $98.1 \%$ and pellets facilities in a 94.3\%.

The maximum demand for woodboards is satisfied in a $96.8 \%$, while for lumber and pellets are $41.9 \%$ and $27.5 \%$ fulfilled respectively. It can be deduced that the woodboards demand is not totally satisfied because it is not profitable the installation of another facility for satisfying the remaining small amount of demand.

The byproducts generated in the sawmill are sold to third parties (80.9\%) and used as a source of energy ( $19.1 \%$ ), while bark produced in woodboard facilities is sent to the pellet plant $(94.1 \%)$ and the rest is used as fuel. A $20.4 \%$ of the total pellet production is used as a source of energy for sawmills and woodboard facilities and the rest is sold to consumers.

Some model statistics are shown in table 2 .

### 5.2 Industrial clusters

In this case, the formation of industrial clusters is promoted applying discounts on installation and production costs.

The optimal SC selects 5 clusters of three facilities and 1 cluster of two facilities. The attained SC configuration consists in 6 sawmills, 5 woodboards facilities, and 6 pellets facilities, with a total of 17 industries, two more than the previous case. The clusters are installed as follows: 3 are located near the harvest area, 2 near customer region, and 1 in an intermediate area. The total benefit is $\$ 290.44 \mathrm{MM}, 17.3 \%$ greater than the previous case. The raw material is completely utilized, while $96.1 \%$ of harvest residues are used. The woodboards and pellets production increase $3.1 \%$ and $15.6 \%$ respectively, while the lumber production is $6.6 \%$ decreased. The installed capacity is totally utilized in the case of woodboards, while sawmills and pellets are utilized in $84.1 \%$ and $99.1 \%$ respectively.

The maximum demand for woodboards is $99.8 \%$ satisfied, while lumber requirement is fulfilled in $39.1 \%$ and $28.8 \%$ of the total demand of pellets. The byproducts generated in sawmills are sold to third parties $(80.9 \%)$ and used as a source of energy ( $19.1 \%$ ). The byproducts produced in woodboard facilities are sent to the pellet plants $(94.1 \%)$ and the rest is used as fuel. Meanwhile, $18.2 \%$ of the total pellets production is used as a source of energy for sawmills and woodboard facilities, and the rest is sold to consumers. The increase of transportation costs is due to a greater delivery of residues, byproduct and product. It can be noted that, besides decreased total costs, the total income for sales is increased. That means that more profitable production/distribution scheme is attained.

Table 1. Economic report [M \$/year]

|  |  | Individual <br> factories | Industrial <br> clusters |
| :---: | :---: | :---: | :---: |
| Sawmills | Raw material costs | 19.52 | 79.23 |
|  | Installation cost | 0.81 | 17.95 |
|  | Production cost | 6.62 | 4.76 |
|  | Income | 385.83 | 396.86 |
|  | Raw material costs | 33.99 | 35.24 |
|  | Installation cost | 45.92 | 39.39 |
|  | Production cost | 95.96 | 71.33 |
| Pellet |  |  |  |
|  | Income | 30.15 | 35.00 |
|  | Raw material costs | 1.64 | 1.95 |
|  | Installation cost | 1.00 | 0.86 |
|  | Production cost | 6.08 | 5.28 |
|  | Transportation cost | 41.60 | 43.46 |
|  | Net benefit | $\mathbf{2 4 7 . 6 7}$ | $\mathbf{2 9 0 . 4 4}$ |

Table 2. Statistics model

|  | Equation | Continuous <br> variables | Discrete <br> variables | CPU time <br> (sec.) |
| :---: | :---: | :---: | :---: | :---: |
| Individual factories | 5996 | 7561 | 210 | 0.61 |
| Industrial clusters | 238117 | 62072 | 36360 | 58.55 |

## Conclusions

In this work, the cluster installation in a SC is analyzed. A more realistic approach is addressed considering cost discounts for facilities clusters. A MILP model for the optimal SC design and planning is presented. The different tradeoffs between clusters and individual facility configuration are evaluated. The costs reduction when plants share resources and services, and the effect of the production scale in the overall SC are highlighted through the proposed example. The approach addressed in this work represents a useful tool for analyzing production, resources and services integration in a SC involving different facilities in the diversification of an industry.

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