Optimization of Scheduling for Hydro-thermal Power Generation and Transmission Including Pumped Storage Systems

Gonzalo E. Alvarez, Marian G. Marcovecchio, and Pío A. Aguirre

Abstract— This work presents a novel approach for solving the short-term scheduling of hydro-thermal power generation, including pumped storage systems and transmission constraints. The problem addressed is known as Security Constrained Unit Commitment (SCUC). Pumped Storage Units (PSUs) are important in electric systems during the off-peak and the peak demand periods, providing economic and technical benefits. Linear aproximations are applied to nonlinear equations of this kind of mathematical problems which are: fuel cost functions, generation-discharge curves of PSUs and transmission constraints modeled with Alternating Current power flow model. Thus, MILP models are presented for the problem addessed. To prove the efficiency of the proposed models, two systems with PSUs will be tested: a modified 6-bus and the IEEE 31-bus power system. Results show that the proposed MILP models allow: modeling the SCUC problem more realistically, obtaining feasible solutions within efficient computational times, and reaching production cost savings up to almost 15% compared to power systems that lack capacities to pumping water. Several indicators obtained from results are presented through graphs, as a tool for improving operation and maintenance of power systems. The analysis of these indicators and the graphic interpretation allow to identify and classify critical parts of systems as well as to make recommendations about future system improvements.

Index Terms— Electric power systems, Mixed integer linear programming, Pumped hydro storage, Security Constrained Unit Commitment, Transmission constraints

1 Introduction

Optimization of the electricity generation and transmission has become a crucial task in the power systems operations. Global demand of electricity has been increased over 100% in the last four decades [1]. Fossil fuels are the predominant energy source, but this source presents two important disadvantages: the pollution ([2], [3]) and the eventual depletion. Oil reserves have grown over recent times due to new extraction techniques as fracking, but this technique is also questioned due to the environmental impact [4]. Two possible solutions are considered at global level to mitigate the situation: increasing the participation of renewable sources in the energy matrix and enhancing the efficiency of power systems.

Hydroelectric generation also plays an important role and represents 26.6% of the energy matrix in Argentina [5]. Pumped Storage Units (PSUs) ([6], [7]) are growing significantly in the field of hydraulic energy ([6], [8], [9]). These units allow store energy in the form of water, with its consequent advantages. **Figure 1** shows the basic diagram of a hydraulic system with a PSU; it consists of two interconnected reservoirs, and a unit that works both as a turbine and as a pump. PSUs pump water to an upper reservoir during off-peak periods, increasing reservoir volume, and then, the water is turbined during peak periods, generating power as a conventional hydroelectric plant. PSUs can take advantage of fluctuations in the electricity demand, and also they can provide electric grid ancillary services.

Unlike older PSUs that have a single speed, the speed of modern PSUs can be adjusted. Adjustable speed increases efficiency during generation mode. Another advantage is a decrease on the amount of startups and shutdowns of units in the pumping mode. Furthermore, PSU with adjustable speed reduces the number of fluctuations on power grids and enhances the efficiency of power systems.

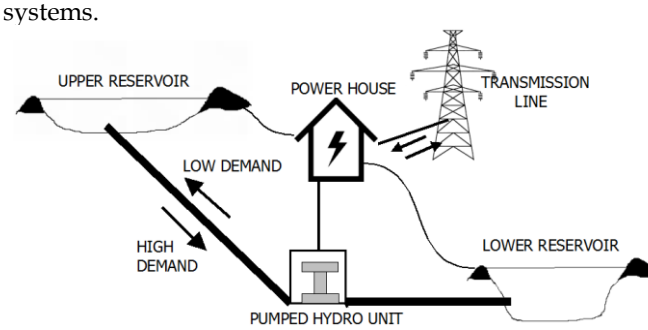

Fig 1. Diagram of a basic pumped storage system.

The problem which decides the best combination of startups and shutdowns from a number of generating units to meet a forecast demand is called Unit Commitment Problem (UC) [10]. This problem usually refers to the scheduling in thermoelectric power plants. When the

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problem also includes hydroelectric generation, it is known as Hydro Unit Commitment Problem (HUC) [11]. Moreover, if transmission constraints are considered, the problem is known as Security Constrained Unit Commitment Problem (SCUC) ([12]–[15]). Two models are mainly implemented to model transmission constraints: AC and DC power flow models. AC model is known by its high level of precision [16], but its main disadvantage is the high demand of computational requirement. DC power flow model, which is obtained from the linearization of AC model ([17], [18]), has a considerable computational saving with acceptable accuracy.

In order to model the short term hydrothermal generation scheduling, the following constraints must be represented mathematically. For thermal units, it must be taken into account: power output limits, minimum shutdown/start-up hours, ramp limits, hot/cold cost for unit startups/shutdowns [19]. Hydraulic power plants have nonlinear generation-discharge curves in addition to forbidden working zones. HUC problems also include constraints for modeling the levels of reservoirs [20]. The modeling for the PSU operation must consider both: the generation mode and the pumping mode [21]. Cycling conditions for the reservoirs must be considered, relating the first hour of the current programming horizon with the last hour of the previous one. Finally, there are initial values for the generating units and reservoirs.

Mathematical programming is a helpful tool for solving these kinds of problems. Particularly, Mixed Integer Linear Programming models (MILP) have advantages as the flexibility to add constraints, and global optimality [22], [23].

Several approaches of linear approximations have been developed in the literature to represent nonlinear equations of SCUC problems such as the PSU curves of generation-discharge ([20]), transmission constraints of AC model ([26]), and fuel cost functions ([24], [25]).

This paper presents a novel MILP optimization approach to solve SCUC problems including hydrothermal generation and PSUs. The approach considers generation-discharge curves obtained from a real case, and the modeling of reservoirs status. Security constraints included in this paper are formulated using DC power flow model which ensures the feasibility of the obtained solutions. The novel contributions of the present paper are: a more complete thermal unit constraint formulation, the calculation of angle bus variables for power flow transmission constraints without matrices, and more rigorous modeling of PSU constraints. An accurate representation of the aforementioned constraints is crucial in order to obtain realistic results. To prove the effectiveness of proposed models, two hydrothermal systems are tested, coupled to a 6-bus and the IEEE 31-bus power system. Both systems incorporate two PSUs. Models are implemented in GAMS, with solvers CLPEX and Gurobi. Status of transmission lines, generators, and reservoirs are analyzed. Insightful graphics are presented to better illustrate the results.

2 SCUC PROBLEM FOR HYDROTHERMAL GENERATION

The aim of the SCUC problem is to schedule the generating units in order to meet the energy demand, while the production cost is minimized. The system is composed of I thermal units, X PSUs, BU buses, L lines, and C loads. T is the length of the programming horizon.

The objective function to be minimized is the operating cost F (1), where variable $p_{i,t}$ is the power output, variable $cu_{i,t}$ is the startup cost and variable $cd_{i,t}$ is the shutdown costs $cd_{i,t}$ for each thermal unit and period of time along the programming horizon. $u_{i,t}$ is a binary variable which indicates the on/off status of unit i for each period of time. The function in (1) has a nonlinear term. In order to obtain an MILP model, it will be linearized.

$$min\ F = \sum_{t=1}^{T}\sum_{i=1}^{I}\left[a_i u_{i,t} + b_i p_{i,t} + c_i {p_{i,t}}^2 + cu_{i,t} + cd_{i,t}\right] \quad (1)$$

Demand system balance is calculated as follows:

$$\sum_{c=1}^{C} dl_{c,t} \le \sum_{i=1}^{I} p_{i,t} + \sum_{x=1}^{X} e_{x,t}\,, \quad t = 1, \dots, T \quad (2)$$

where $dl_{c,t}$ is a set for the power load of buses and $e_{x,t}$ is the power output of PSUs, for each period of time.

Spinning Reserve: R_t is the online power generation capacity available but unloaded which could respond quickly to compensate outages. This requirement is represented by equation (3):

$$\sum_{c=1}^{C} dl_{c,t} + R_t \le \sum_{i=1}^{I} p_i^{UP} * u_{i,t} + \sum_{x=1}^{X} e_{x,t}^{UP} * z_{x,t}^{gen} \quad t = 1, \dots, T \quad (3)$$

where p_i^{UP} and $e_{x,t}^{UP}$ are the power output upper bounds for units i and x, respectively. And $z_{x,t}^{gen}$ is a binary variable which indicates the generation mode of a PSU.

2.1 Thermal units constraints

The mathematical model implemented in this work for the operation of thermal units is detailed in [24].

Thermal units have a minimum and maximum power output:

$$u_{i,t}\, p_i^{LO} \le p_{i,t} \le u_{i,t}\, p_i^{UP}, \quad t = 1, \dots, T; i = 1, \dots, I \quad (4)$$

The initial status of each unit is modeled in eq. (5) and (6) as follows:

$$u_{i,t} = 0 \quad \forall i : T_i^{ini} < 0; \quad t = 1, \dots . (TD_i + T^{ini}) \quad (5)$$
$$u_{i,t} = 1 \quad \forall i : T_i^{ini} > 0; \quad t = 1, \dots . (TU_i - T^{ini}) \quad (6)$$

The status of unit i at the beginning of the programming horizon depends on the value of the parameter T_i^{ini}. This parameter indicates the number of hours that the unit has been online (if the value is positive) or offline (if the values are negative) at the beginning of the first time period. TD_i/TU_i are parameters which refer to the minimum down/up times of unit i.

The minimum up time of each unit is modeled as follows:

$$u_{i,t} - u_{i,t-1} \le u_{i,t+j} \quad i = 1, \dots \dots I;\ t = 2, \dots \dots T;\ j = 1, \dots \dots (TU_i - 1) \quad (7)$$

$$u_{i,1} \le u_{i,1+j} \qquad \forall i : T_i^{ini} < 0; \quad j = 1, \dots . . (TU_i - 1) \quad (8)$$

The minimum down time of unit i is modeled as follows:

$$u_{i,t+j} \le u_{i,t} - u_{i,t-1} + 1 \quad i = 1, \dots \dots I;\ t = 2, \dots \dots T;\ j = 1, \dots \dots (TD_i - 1) \quad (9)$$

$$u_{i,1+j} \le u_{i,1}\,, \qquad \forall i : T_i^{ini} > 0; \quad j = 1, \dots \dots (TD_i - 1) \quad (10)$$

Ramp rates limit the increase or decrease of unit power between two consecutive time periods and are modeled by equations (11-12). DR_i/UR_i are the ramp limits and SD_i/SU_i are the maximum shutdown/startup rates for unit i.

$$p_{i,t-1} - DR_i u_{i,t} - SD_i\left(1 - u_{i,t}\right) \le p_{i,t}, \quad i = 1, \dots \dots I;\ t = 2, \dots \dots T \quad (11)$$

$$p_{i,t} \le p_{i,t-1} + UR_i u_{i,t-1} - SU_i(1 - u_{i,t-1}) \quad i = 1, \dots \dots I;\ t = 2, \dots \dots T \quad (12)$$

The hot start-up cost of thermal unit i is modeled by equations (13) and (14). Hsc_i is a parameter which indicates the value of start-up cost when the off-time of the unit is lower than T_i^{cold}.

$$(u_{i,t} - u_{i,t-1})\, Hsc_i \le cu_{i,t}, \qquad i = 1, \dots, I;\ t = 2, \dots, T \quad (13)$$

$$u_{i,1} * Hsc_i \le cu_{i,t} \qquad \forall i : T_i^{ini} < 0 \quad (14)$$

The cold start-up cost is modeled as follows:

$$\left(u_{i,t} - \sum_{j<TD_i+T_i^{cold}+1} u_{i,t-j}\right) Csc_i \le cu_{i,t} \quad i = 1, \dots \dots I;\ TD_i + T_i^{cold} < t \le T \quad (15)$$

$$\left(u_{i,t} - \sum_{j<t} u_{i,t-j}\right) Csc_i \le cu_{i,t} \quad \forall i : T_i^{ini} < 0; \quad i = 1, \dots, I; (TD_i + T_i^{cold} + 1) < t \le (TD_i + T_i^{cold}) \quad (16)$$

where T_i^{cold} is the minimum of downtime for considering the cold cost and Csc_i is the value of the cold start-up cost.

Thermal units can incur in a cost when they are shutdown. Consequently, the shut down cost is represented by equations (17) and (18).

$$(u_{i,t-1} - u_{i,t})DC_i \le cd_{i,t}, \qquad i = 1, \dots \dots I;\ t = 2, \dots \dots T \quad (17)$$

$$\left(1 - u_{i,1}\right)DC_i \le cd_{i,t}\,, \qquad \forall i : T_i^{ini} > 0 \quad (18)$$

Equations (19) and (20) avoid startup/shutdown cost variables taking negative values, if the unit i is not startup/shutdown at period of time t.

$$0 \le cu_{i,t} \qquad i = 1, \dots \dots I;\ t = 1, \dots \dots T \quad (19)$$

$$0 \le cd_{i,t} \qquad i = 1, \dots \dots I;\ t = 1, \dots \dots T \quad (20)$$

2.2 Hydraulic system constraints

PSU power variable $e_{x,t}$ will assume the positive values generated if the unit is in generating mode, negative values if the PSU is in pumping mode since it is consuming electricity, and 0 if the unit is offline.

$$e_{x,t} = pg_{x,t} - pp_{x,t}, \qquad x = 1, \dots \dots X;\ t = 1, \dots \dots T \quad (21)$$

$pg_{x,t}$ is the variable of power generation and $pp_{x,t}$ is the power consumption of the PSU.

The calculation of power output of each PSU is detailed in [28] and represented by equation (22). Variable $pg_{x,t}$ is the power output of each PSU, while η_x^{gen} represents the turbine efficiency, which is a function of the head height and the flow conditions. This equation depends on water flow discharge $dg_{x,t}$ and the hydraulic head $h_{x,t}$.

$$pg_{x,t} = 9800 dg_{x,t} h_{x,t} \eta_x^{gen} / (1 * 10^6) \quad x = 1, \dots \dots X;\ t = 1, \dots \dots T \quad (22)$$

Equation (23) defines the power consumption of each PSU in pumping mode, where η_x^{pump} is the pumping efficiency. The equation depends on the pumped water flow $dp_{x,t}$ and hydraulic head. Equations (22-23) are nonlinear and they will be linearized to obtain a MILP model.

$$pp_{x,t} = 9800 dp_{x,t} h_{x,t} / (\eta_x^{pump} * 1 * 10^6) \quad x = 1, \dots \dots X;\ t = 1, \dots \dots T \quad (23)$$

If a PSU is pumping water, the rest of PSUs must not be generating power. Equation (24) and (25) avoid the PSU modes overlapping. α_t is a binary variable, that is equal to 1 when a PSU is in generating mode and 0 otherwise. Binary variables $z_{x,t}^{gen}$ and $z_{x,t}^{pump}$ correspond to generating and pumping mode, respectively.

$$\sum_{x=1}^{X} z_{x,t}^{gen} \le X\, \alpha_t\,, \qquad t = 1, \dots T \quad (24)$$

$$\sum_{x=1}^{X} z_{x,t}^{pump} \le X\left(1 - \alpha_t\right) \qquad t = 1, \dots, T \quad (25)$$

Constraints (26-27) define values for the water level of each reservoir, where h_t^{up}/h_t^{lo} are the variable for upper/lower reservoir level. Variables v_t^{up} and v_t^{lo} are the upper/lower reservoir volumes. Constants c_slop^{up}/c_slop^{lo} and ${c_{ord}}^{up}/{c_{ord}}^{lo}$ depend on geometrical characteristics of reservoirs.

$$h_t^{up} = c\_slop^{up} v_t^{up} + {c_{ord}}^{up}, \qquad t = 1, \dots \dots T \quad (26)$$

$$h_{x,t}^{lo} = c\_slop^{lo} v_t^{lo} + {c_{ord}}^{lo}, \qquad t = 1, \dots \dots T \quad (27)$$

Maximum and minimum bounds of reservoir volumes are imposed by equations (28-29). $\overline{v^{up}}/\underline{v^{up}}/\overline{v^{lo}}/\underline{v_x^{lo}}$ are the maximum/minimum limits for the volumes of upper/lower reservoirs.

$$\underline{v^{up}} \le v_t^{up} \le \overline{v^{up}}, \qquad t = 1, \dots T \quad (28)$$

$$\underline{v^{lo}} \le v_t^{lo} \le \overline{v^{lo}}, \qquad t = 1, \dots T \quad (29)$$

Constraints (30-31) calculate the level of the upper and lower reservoir volumes. These values depend on the water flows which are discharged/pumped by PSUs (variables qg_t/qp_t) and the water upper/lower reservoir inflow/outflows from rivers (parameters $\mu_{in}^{up}/\mu_{in}^{lo}/\mu_{out}^{up}/\mu_{out}^{lo}$), which are considered as constant in this paper.

$$v_t^{up} = v_{t-1}^{up} + \mu_{in}^{up} - \mu_{out}^{up} - qg_t + qp_t, \qquad t = 1, \dots \dots T \quad (30)$$

$$v_t^{lo} = v_{t-1}^{lo} + \mu_{in}^{lo} - \mu_{out}^{lo} + qg_t - qp_t\,, \qquad t = 1, \dots \dots T \quad (31)$$

Constraint (32) states the cyclic condition for upper reservoir and ensures enough reserve of potential energy for the following programming horizon.

$$v_{t=T}^{up} \geq v_{t=1}^{up} \quad (32)$$

2.3 Transmission constraints – AC model

AC power flow model adopted in this work for representing transmission lines is described in [25]. Four variables are associated to each bus for AC model: active power, reactive power, bus voltage, and bus voltage angle.

The goal of the model is to obtain the values of voltage angle and magnitude information for each bus in a power system. This model involves nonlinear constraints, thus the resolution of SCUC problems could require a high computational effort, especially for large scale power systems ([26], [27]).

The active and reactive power flows $p\_l_{bui\text{-}buo,t}/q\_l_{bui\text{-}buo,t}$ from the input bus bu_i to the output bus bu_o are modeled by the nonlinear equations (33) and (34) as follows:

$$p\_l_{bu_i-bu_o,t} = V_{bu_i,t}V_{bu_o,t}\left(g_{bu_i-bu_o,t}Cos\left(\theta_{bu_i,t} - \theta_{bu_o,t}\right) + b_{bu_i-bu_o}Sin\left(\theta_{bu_{i,t}} - \theta_{bu_o,t}\right)\right)$$
$$bu_i = 1, \dots, BU; bu_o = 1, \dots, BU\,; t = 1, \dots, T \quad (33)$$

$$q\_l_{bu_i-bu_o,t} = V_{bu_i,t}V_{bu_o,t}\left(g_{bu_i-bu_o,t}Sin\left(\theta_{bu_i,t} - \theta_{bu_o,t}\right) - b_{bu_i-bu_o}Cos\left(\theta_{bu_{i,t}} - \theta_{bu_o,t}\right)\right)$$
$$bu_i = 1, \dots, BU; bu_o = 1, \dots, BU\,; t = 1, \dots, T \quad (34)$$

where $V_{bui,t}/V_{buo,t}$ are bus voltage variables, $\Theta_{bui,t}/\Theta_{buo,t}$ are bus voltage angle variables, $g_{bui,t}/g_{buo,t}$ are parameters of line susceptance, and $b_{bui,t}/b_{buo,t}$ are parameters of line conductance.

The active and reactive power balances for the electrical system are defined by equations (35) and (36). According to the adopted convention, the power flow entering to the bus is considered as positive, and otherwise, as negative.

$$\sum_{bu=1}^{BU}\sum_{i=1}^{I} p_{i,bu,t} + \sum_{bu=1}^{BU}\sum_{x=1}^{X} e_{x,bu,t} + \sum_{bu_i=1}^{BU}\sum_{bu_o=1}^{BU} p\_l_{bu_i,bu_o,t} - \sum_{bu_o=1}^{BU}\sum_{bu_i=1}^{BU} p\_l_{bu_i,bu_o,t} = \sum_{bu=1}^{BU}\sum_{c=1}^{C} dkt_{bu,c,t},$$
$$t = 1, \dots, T \quad (35)$$

$$\sum_{bu=1}^{BU}\sum_{i=1}^{I} pr_{i,bu,t} + \sum_{bu=1}^{BU}\sum_{x=1}^{X} er_{x,bu,t} + \sum_{bu_i=1}^{BU}\sum_{bu_o=1}^{BU} q\_l_{bu_i,bu_o,t} - \sum_{bu_i=1}^{BU}\sum_{bu_o=1}^{BU} q\_l_{bu_i,bu_o,t} = \sum_{bu=1}^{BU}\sum_{c=1}^{C} bktr_{bu,c,t}$$
$$t = 1, \dots, T \quad (36)$$

3 LINEAR APPROXIMATIONS FOR SCUC MODELS

3.1 Approximations for Pumped Hydro units

Characteristic curves of PSUs describing relationships between power generation and the corresponding water discharge are nonlinears. The goal is to obtain accurate linear approximations for these curves [28]. **Figure 2** represents an example of a generation-discharge curve. Efficiency of PSUs in pumping mode is 75-82% [29].

Minimum/maximum power output limits are imposed for ensuring the operation of the units in a safe zone [30]. These limits avoid problems as cavitation [31], mechanical vibrations, and low unit efficiency.

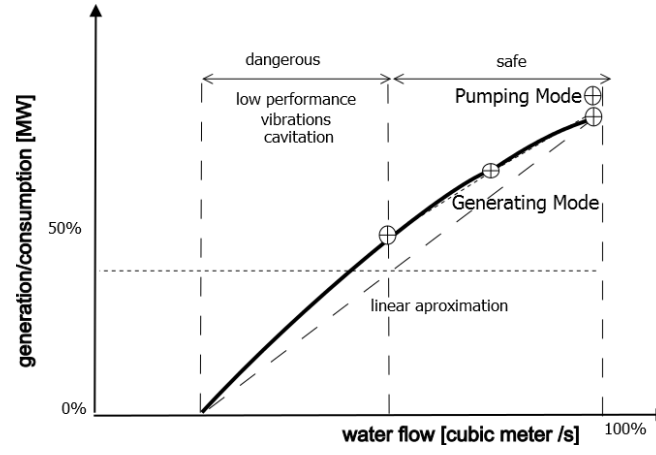

Fig 2. Illustration of a generation-discharge PSU curve.

Equation (37) ensures that if the PSU is working in generating mode, binary variable $z_{x,t}^{gen}$ is equal to 1. pg_x^{up} is a parameter that indicates the upper limit of power output of PSU x.

$$pg_{x,t} \leq pg_x^{up} z_{x,t}^{gen}, \qquad x = 1, \dots, X;\ t = 1, \dots, T \quad (37)$$

Variable of the power output for each PSU at time t ($pg_{x,t}$) is defined by (38). $pg_x^{lo}$is a parameter of lower bound of PSU output.

$$pg_x^{lo} \leq pg_{x,t} \leq pg_x^{up}, \quad x = 1, \dots, X;\ t = 1, \dots, T \quad (38)$$

Variable for the water discharge of each PSU x at the period of time t ($qg_{x,t}$) is computed by equation (39). c_g_x and ord_g_x are constants of slope and ordenate of PSUs.

$$qg_{x,t} = c\_g_x pg_{x,t} + ord\_g_x, x = 1, \dots, X;\ t = 1, \dots, T \quad (39)$$

For the pumping mode, equations (40) and (41) describe the power consumption and the pumped water by each PSUs respectively. Pp_x and Qp_x are parameters defining the values of power consumption and pumped water flow for the PSU in pumping mode, respectively.

$$pp_{x,t} = z_{x,t}^{pump} * Pp_x\,, \qquad x = 1,\dots,X;\ t = 1,\dots,T \qquad (40)$$

$$qp_{x,t} = z_{x,t}^{pump} * Qp_x, \qquad x = 1,\dots,X;\ t = 1,\dots,T \qquad (41)$$

3.2 Transmission constraints – DC model

DC power flow model, which is a linearization of the AC model, is generally preferred, due to the computational complexity of AC power flow model in UC problems for real systems. As a consequence, DC power flow model is adopted in the present work. This model requires less computational effort for its resolution while achieves an acceptable level of accuracy [17]. It is based on three assumptions:

1. Transmission losses are not considered, since the line resistances r_l are assumed as negligible compared with the line reactances x_l.

$$g_{bu_i bu_o} = \frac{r_{bu_i bu_o}}{r_{bu_i bu_o}^2 + x_{bu_i bu_o}^2} = \frac{r_l}{r_l^2 + x_l^2} \approx 0 \qquad (42)$$

$$b_{bu_i bu_o} = \frac{x_{bt_e bt_o}}{r_{bu_i bu_o}^2 + x_{bu_i bu_o}^2} = \frac{x_l}{r_l^2 + x_l^2} \approx -\frac{x_l}{x_l^2} \approx -\frac{1}{x_l} \qquad (43)$$

2. The voltage difference between buses is minimal. Then, all bus voltages are assumed as equal to 1 [p.u.].
3. The voltage angle differences between neighboring buses are minimal. As a consequence, sine and cosine terms of AC power flow model equations are linearized as follows:

$$Sen\left(\theta_{bu_i} - \theta_{bu_o}\right) \cong \theta_{bu_i} - \theta_{bu_o} \qquad (44)$$

$$Cos\left(\theta_{bu_i} - \theta_{bu_o}\right) \cong 1 \qquad (45)$$

Replacing (42-45) in the equation (33), and considering that the line l connects the bus bu_i with the bus bu_o, the power flow between theses buses is calculated by equation (46).

$$p\_l_{bu_i - bu_o,t} = -b_{bu_i - btu_o}\theta_{bu_i - bu_o,t} = \frac{\theta_{bu_i,t} - \theta_{bu_o,t}}{x_{bu_i - bu_o}} = \frac{\theta_{bu_i,t} - \theta_{bu_o,t}}{x_l}$$

$$bu_i = 1,\dots,BU; bu_o = 1,\dots,BU\,; t = 1,\dots,T \qquad (46)$$

If the equation (35) is reformed taking into consideration the equation (46), the active power balance for the DC model is obtained, as follows.

$$\sum_{bu=1}^{BU}\sum_{i=1}^{I} p_{i,bu,t} + \sum_{bu=1}^{BU}\sum_{x=1}^{X} e_{x,bu,t} + \sum_{bu_i=1}^{BU}\sum_{bu_o=1}^{BU}\left(\frac{\theta_{bu_i,t} - \theta_{bu_o,t}}{x_l}\right) - \sum_{bu_o=1}^{BU}\sum_{bu_i=1}^{BU}\left(\frac{\theta_{bu_i,t} - \theta_{bu_o,t}}{x_l}\right) = \sum_{bu=1}^{BU}\sum_{c=1}^{C} dkt_{c,bu,t}$$

$$t = 1,\dots,T \qquad (47)$$

4 NUMERICAL TESTS

The effectiveness of the proposed model is illustrated by solving a modified 6-bus hydrothermal system, based in [32], and the IEEE 31-bus hydrothermal power system. Mixed integer linear programming models are implemented in GAMS using the linear solvers CPLEX and Gurobi on a computer with an Intel i5 750 (2.67 GHz) processor and 3 GB of RAM.

Difference between the objective values of the best feasible solution found and the best known bound for the value of the objective function is known as absolute gap, while the relative gap is the absolute gap divided by the best bound of the objective value. The relative gap is set to zero for all the cases tested in this work. Time horizon is one day divided into periods of time of 1 hour and the spinning reserve is set as 10% of total load demand.

Six cases will be tested for both systems: First, the power system including only the thermal generation will be solved. Then, the power system with hydrothermal generation, but without the pumped storage capacity will be considered. Finally, the system with hydrothermal generation and pumped storage capacity will be addressed.

4.1. 6-bus power system – thermal generation

In this first case, SCUC problem is solved by considering only the thermal generation. The one-line diagram is presented in **Figure 3.** The power system is composed of three thermal units, eleven transmission lines, three power loads, and two PSUs.

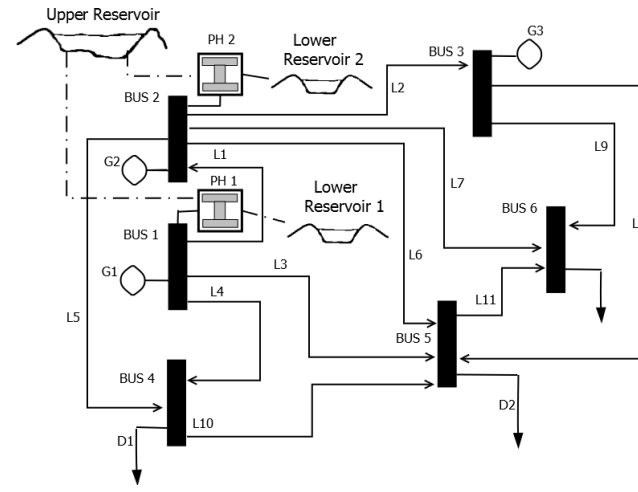

Fig 3. One-line diagram for the 6-bus system including 3 thermal units and 2 PSUs.

Data for line reactances are listed in **Table 1.** Transmission limits are set at 160 [MW] for all lines. Hourly load data for the system is reported in **Table 2**; load participation percentages of buses 4, 5, 6 are 60%, 20%, and 20%, respectively. Operation parameters for thermal units are shown in **Table 3**.

The solutions obtained with CPLEX and Gurobi have a total production cost of $222,334.15. The problem was solved in 0.094 and 0.035 sec. with CPLEX and Gurobi, respectively. The model has 1,604 equations, 697 continuous variables, and 72 binary variables.

TABLE 1. LINE REACTANCES. 6-BUS SYSTEM.

Line	1	2	3	4	5	6
Reactance [p.u]	0.2	0.2	0.3	0.25	0.1	0.3
Line	7	8	9	10	11	
Reactance [p.u]	0.2	0.26	0.1	0.4	0.3	

TABLE 2. HOURLY LOAD [MW]. 6-BUS SYSTEM.

1	2	3	4	5	6	7	8
121.1	104	092.7	087	081.4	084.2	187	198.4
9	10	11	12	13	14	15	16
226.7	249.4	255.1	257.9	255.1	249.4	246.6	246.6
17	18	19	20	21	22	23	24
226.7	249.4	255.1	257.9	255.1	249.4	246.6	246.6

TABLE 3. THERMAL GENERATION DATA. 6-BUS SYSTEM.

Unit	a_i	b_i	c_i	p_i^{LO}	p_i^{LO}	Hsc_i
1	142.73	10.69	0.00463	20	260	200
2	2183.4	181	0.0612	20	220	100
3	1188.2	378.9	0.1433	5	80	70
Unit	Csc_i	TU_i	TD_i	T_i^{cold}	T_i^{Ini}	RD/RU_i
1	200	4	3	5	4	40
2	100	2	2	5	2	60
3	70	1	1	4	2	30

Table 4 shows the daily schedule of thermal units for case 4.1. Unit 1 is committed along the programming horizon due to its low cost of production. In fact, unit 1 produces almost 88% of the total power generated. Units 2 and 3 are committed 6 and 7 hours, respectively. Their participation in the total power generated is low: 11.3% (649.7 [MW]) for unit 2 and 0.7% for unit 3 (40[MW]).

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TABLE 4. Daily Schedule for Thermal Units. Case 4.1.

G1	1	2	3	4	5	6	7	8	9	10	11	12
	1	1	1	1	1	1	1	1	1	1	1	1
	13	14	15	16	17	18	19	20	21	22	23	24
	1	1	1	1	1	1	1	1	1	1	1	1
G2	1	2	3	4	5	6	7	8	9	10	11	12
	0	0	0	0	0	0	0	0	0	0	0	0
	13	14	15	16	17	18	19	20	21	22	23	24
	0	0	0	0	1	1	1	1	1	1	0	0
G3	1	2	3	4	5	6	7	8	9	10	11	12
	0	0	0	0	0	0	0	0	0	1	1	1
	13	14	15	16	17	18	19	20	21	22	23	24
	1	1	1	1	0	0	0	0	0	0	1	0

4. 2. 6-bus system - hydrothermal generation

Hydrothermal generation is considered for the second case, but the pumped storage capacity is disabled, thus $z_{x,t}^{pump}$ is set to zero.

The hydraulic system is based on a real case presented in [33]. The system is composed of two PSUs, with a common upper reservoir and two lower reservoirs. Maximum capacities of reservoirs are 150, 8, and 12 [Mm³], respectively. Upper reservoir must maintain its volume at a level higher than the 80% of its maximum capacity in order to guarantee a level of energy storage.

Table 5 shows the data for power generation and water discharge of PSUs as was illustrated in the previous section (see Figure 1). The characteristics of the reservoirs are shown in **Table 6.**

TABLE 5. GENERATION [MW] - DISCHARGE DATA [M^3/S].

Generation Limits		Discharge Limits	
min	max	min	max
135.24	259.63	42.3	79.1

TABLE 6. DATA FOR RESERVOIRS.

Data [unit] / Reservoir	Upper	Lower 1	Lower 2
Min. volume [Mm³]	135	0	0
Max. volume [Mm³]	150	8	12
Initial volume [Mm³]	145	4	5
μ_{in} [m^3/s]	0.83	1.94	1.94
μ_{out} [m^3/s]	0.83	1.39	1.39
c_slop	0.13	2.5	2.5
c_{ord} [m]	728	355	3.55

TABLE 7. Schedule of Hydrothermal Units. Case 4.2.

G1	1	2	3	4	5	6	7	8	9	10	11	12
	1	1	1	1	1	1	1	1	1	1	1	1
	13	14	15	16	17	18	19	20	21	22	23	24
	1	1	1	1	1	1	1	1	1	1	1	1
G2	1	2	3	4	5	6	7	8	9	10	11	12
	0	0	0	0	0	0	0	0	0	0	0	0
	13	14	15	16	17	18	19	20	21	22	23	24
	0	0	0	0	1	1	1	1	1	1	0	0
G3	1	2	3	4	5	6	7	8	9	10	11	12
	0	0	0	0	0	0	0	0	0	1	1	1
	13	14	15	16	17	18	19	20	21	22	23	24
	1	1	1	1	0	0	0	0	0	0	1	0
PSU1	1	2	3	4	5	6	7	8	9	10	11	12
	1	0	0	0	0	0	0	0	0	0	0	0
	13	14	15	16	17	18	19	20	21	22	23	24
	0	0	0	0	0	0	0	0	0	0	0	0
PSU2	1	2	3	4	5	6	7	8	9	10	11	12
	0	0	0	0	0	0	0	0	0	0	0	0

	13	14	15	16	17	18	19	20	21	22	23	24
	0	0	0	0	0	0	0	0	0	0	0	0

The operating cost of the optimal solution obtained with both solvers is $221,674.76; the computational time required was of 0.156 sec. for CPLEX and 0.079 sec. for Gurobi. The operating cost is $659.39 lower than the one obtained for case 4.1. The mathematical model implemented for this case is composed of 2,469 equations, 1,441 continuous variables, and 192 binary variables.

Table 7 presents the daily hydrothermal unit schedule of case 4.2.

Thermal unit 1 is committed along the programming horizon and its production represents almost 84% of the total power generated. Thermal units 2 and 3 are committed the same number of hours as in the previous case. The thermal generators, G2 and G3, produce 649.7 and 40 [MW], respectively along the programming horizon.

At the first hour, PSU 1 generates 259.63 [MW] while PSU 2 is offline. Neither of the two PSUs generates power during the rest of the programming horizon, since the water inflow from the rivers is relatively low. In these conditions the volume of the upper reservoir is not enough to overcome the minimum volume required for operation. In this case, the hydraulic generation represents only 4.58% of the total power generated.

4. 3. 6-bus system – hydrothermal generation including pumped storage units

For the third case, SCUC problem for hydrothermal generation is considered and the pumping mode is enabled. The pumping efficiency rate of each PSU is 82.4%.

The mathematical model was solved with CPLEX and Gurobi, and the same optimal solution was obtained.

TABLE 8. Schedule of Hydrothermal Units. Case 4.3.

G1	1	2	3	4	5	6	7	8	9	10	11	12
	1	1	1	1	1	1	1	1	1	1	1	1
	13	14	15	16	17	18	19	20	21	22	23	24
	1	1	1	1	1	1	1	1	1	1	1	1
G2	1	2	3	4	5	6	7	8	9	10	11	12
	0	0	1	1	1	1	0	0	0	0	0	0
	13	14	15	16	17	18	19	20	21	22	23	24
	0	0	0	0	1	1	1	1	1	1	0	0
G3	1	2	3	4	5	6	7	8	9	10	11	12
	0	0	0	0	0	0	0	0	0	1	1	1
	13	14	15	16	17	18	19	20	21	22	23	24
	1	1	1	1	0	0	0	0	0	0	1	0
PSU1	1	2	3	4	5	6	7	8	9	10	11	12
	0	0	0	0	0	0	0	0	0	0	0	0
	13	14	15	16	17	18	19	20	21	22	23	24
	0	0	0	0	1	1	1	0	0	1	0	0
PSU2	1	2	3	4	5	6	7	8	9	10	11	12
	0	0	0	0	0	0	0	0	0	0	0	0
	13	14	15	16	17	18	19	20	21	22	23	24
	0	0	0	0	0	0	0	1	1	0	0	0

The optimal cost production attains savings of $20,403.11 (9.20%) and $21,062.50 (9.47%), compared with cases 4.2 and 4.1, respectively. This cost reduction evidences the convenience of implementing pumping storage units. CPLEX required 4.34 sec. to solve the problem, meanwhile Gurobi reached optimality in only 0.91 sec. The model implemented for this case has 2,469 equations, 1,441 continuous variables, and 240 binary variables.

Table 8 shows the daily schedule of hydrothermal units for case 4.3. Similarly to the previous cases, thermal unit 1 is committed along the programming horizon, though its contribution has decreased to 80.6% of the total generation. Thermal unit 2 is committed 10 hours and generates a total amount of 463.82 [MW] during the time intervals of hours 3-6 and 17-22, while unit 3 is online 8 hours with identical power production than in the two previous cases. Performances of pumping storage units will be analyzed in the next section.

Only the schedule for line 2 will be presented in **Table 9**, since this lines presents the highest average of power flow through the programming horizon (95.6 [MW]).

TABLE 9. Schedule of line 2 [MW]. Case 4.3.

Transmission line 2					
1	2	3	4	5	6
50.38	79.80	9.49	90.67	8.35	90.39
7	8	9	10	11	12
77.80	82.54	94.32	102.29	104.66	105.83
13	14	15	16	17	18
104.66	102.29	101.12	101.12	142.61	153.22
19	20	21	22	23	24
148.51	101.68	101.39	143.77	101.12	95.53

4.4. IEEE 31-bus power system – thermal generation

This system is composed by 31 buses, 16 thermal units, 2 PSUs, 43 transmission lines, and 11 loads. The system is about four times larger than the previous one. Its one-line diagram is presented in **Figure 4,** and information regarding this power system can be found in [34].

In this case, SCUC problem is solved by considering only the thermal generation. The solutions obtained with CPLEX and Gurobi have a total production cost of $4,150,036. The CPU times are 38.08 and 27.99 sec. with CPLEX and Gurobi. The model has 8.699 equations, 3.313 continuous variables, and 384 binary variables.

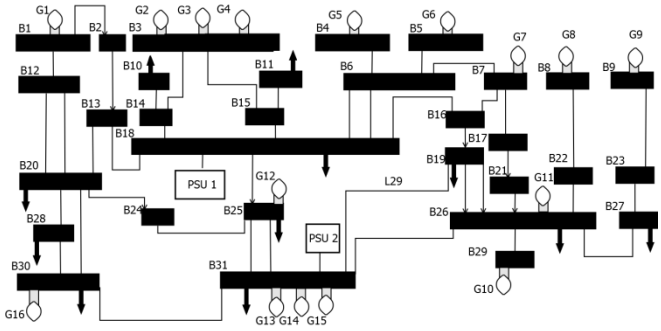

Fig 4. One-line diagram for the IEEE 31-bus system.

4. 5. IEEE 31-bus system - hydrothermal generation

Hydrothermal generation is considered here, and the pumped storage capacity is disabled ($z_{x,t}^{pump}$ is set to zero).

The hydraulic system is the same of cases 4.2 and 4.3: two PSUs, with a common upper reservoir and two lower reservoirs. Reservoir capacities are 150, 8, and 12 [Mm³], respectively. Solution report by both solvers is $4,149,154. CPU times are 41.17 sec for CPLEX and 27.77 sec for Gurobi. The model has 9,660 equations, 4,081 continuous variables, and 528 binary variables.

It can be seen that there are a saving cost of only $882 compared to the fact that the previous case. The slight reduction in cost is due to the tributaries have a low water flow. As a consequence the water volume of upper reservoir does not increase enough to produce power. In this case, the two PSUs only produce 259.63 [MW] during the first hour.

4. 6. IEEE 31-bus system – hydrothermal generation including pumped storage units

In this case, the SCUC problem for hydrothermal generation is addressed and the pumping mode is included (pumping efficiency rate is 82.4%).

The total operating cost presents savings of $618.822 (14.91%) and $617.940 (14.89%) comparing with cases 4.4 and 4.5, respectively. The reduction costs of almost 15% are due to the enabling of pumping mode. A total amount of 3.98 [Mm³] of water are pumped to increase the upper reservoir volume. CPU times are 288.43 sec. for CPLEX and 930.41 sec. for Gurobi.

During the programming horizon, the two PSUs produce 3,847.79 [MW] and consume 4,413.71 [MW]. The model is composed by 9,660 equations, 4,081 continuous variables, and 576 binary variables.

5 ANALYSIS OF RESULTS

Figure 5 shows the generation profile of case 4.3. The power consumed by the PSUs at pumping mode must be added to load demands. Thermal generation constitutes 87.57% of the total produced power.

For the hydraulic power generation, the two PSUs generate 832.40 [MW] between hours 17 and 22. This time interval coincides with peak loads. The rest of hours of the programming horizon, neither of the two PSUs is generating power. Hydraulic generation increased 220% compared to case 4.2, due to the possibility of pumping water to the upper reservoir.

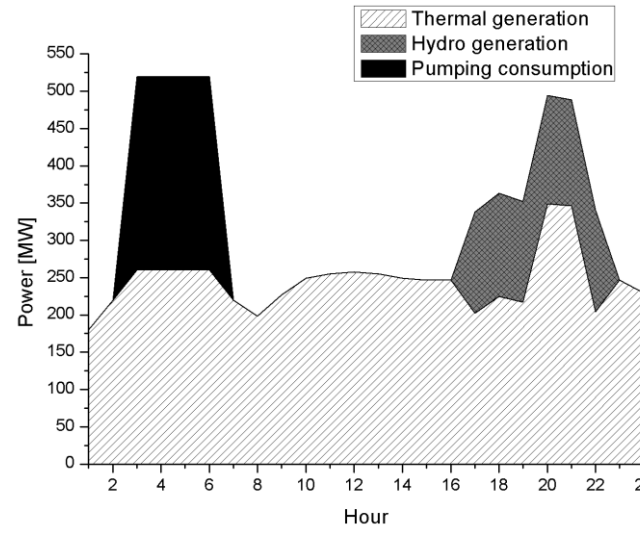

Fig 5. Generation profile. Case 4.3.

Both PSUs are working in pumping mode during off-peak period, i.e. from hours 3 to 6. During this time interval, PSUs consume 1,038.52 [MW] to pump water.

The analysis of solutions for SCUC problems can provide helpful information not only for the daily schedule, but also for maintenance and future investment decisions in power systems.

In order to illustrate the analysis, the time period with higher hourly load demand is chosen, i.e. time period 18. **Figure 6** presents power output and limit for each generating unit for this time period. **Figure 6** also illustrates power flows transmitted by each line and their transmission limit capacities.

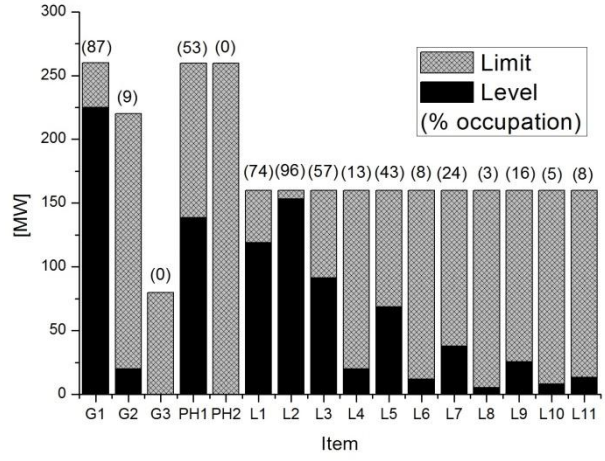

Fig 6. Power levels and limits for units and lines at time period 18. Case 4.3.

Thermal unit 1 produces 87% of their maximum power capacity. This occurs because this unit has lower cost per MWh generated in addition to high startup costs compared to the other thermal units. Thus, unit 1 is committed along the programming horizon.

PSU 1 is generating power in a point of operation near to the half of its limit, and it helps to mitigate the production of thermal generator 1 at this peak hour. Besides, PSU 2 is offline at this period of time.

None one of the transmission lines is occupied at their full capacity. With the aim of analyzing this aspect, the occupation line factor is defined as the power flow through a line divided by the maximum line capacity. There are three lines with occupation factor over 0.5: lines 1, 2, and 3. Eventhough the aforementioned lines are not at their full capacity, in case of an increase in the hourly demands, these lines can reach high values of occupation factor, leading to a critical scenario.

The occupation factor of the remainder lines are lower than 0.5. As a consequence, these lines have enough idle capacity to support potential increases of power flows without overloads. Lines 4-11 also help to mitigate the transmission of power flows in lines that are close to their limit or at full capacity.

Now, the obtained solutions for the hydroelectric system will be analyzed. **Figure 7** shows the level of the three reservoirs.

It can be seen that upper reservoir volume is inversely proportional to the sum of the two lower reservoir volumes. PSUs 1 and 2 pump 0.936 [Mm^3] of water to upper reservoir between period of times 3 and 6. Also, a total of 0.937 [Mm^3] of water is discharged by the two PSUs to generate power. Water inflow and outflow from rivers also influence on reservoir volumes. As a consequence, upper reservoir collects and leaks 0.072 [Mm^3] of water along the programming horizon. Besides, the total of water inflow for the two lower reservoirs is 0.336 [Mm^3], while the water outflow is 0.24 [Mm^3].

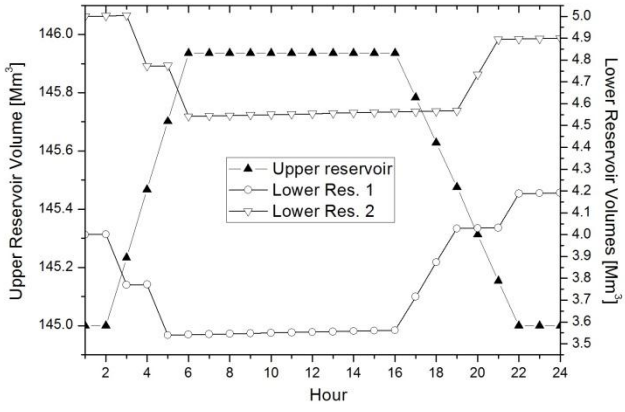

Fig 7. Reservoir volumes. Case 4.3.

6 Conclusion

This work presents a novel MILP formulation for the SCUC problem. The proposed model considers hydrothermal generation including pumped storage units. Classic formulations for modeling SCUC problems present nonlinear equations as fuel cost, transmission constraints for AC power flow model, and generation-discharge curves for hydraulic units. Consequently, in this paper, tight linear approximations of these nonlinear equations are implemented. Then, the MILP problems formulated allow to obtain accurate solutions for the short-term scheduling in hydrothermal power generation.

In order to prove the efficiency of the presented approach, a modified 6-bus system, with 3 thermal generators and two PSUs is solved using GAMS linear solvers CPLEX and Gurobi. The obtained results indicate that the incorporation of PSUs in thermal generation systems achieves cost savings of up to 9.47%, in comparison with systems without pumped hydraulic capacity. The computational time required for solving the full system is 0.91 sec. The results also show that Gurobi exhibits better performance than CPLEX for the cases solved of this system.

A modified IEEE 31-bus system, with 16 thermal generators and two PSUs is also solved. It is about four times larger than the 6-bus power system. Gurobi is faster than CPLEX in cases without hydro generation and without pumping mode. But, for the case with pumping capacity enabled, CPLEX is 3.23 times faster. The implementation of pumping mode achieves cost savings up to 15%, in comparison with systems of thermal generation and hydrothermal generation without pumping.

Results are also presented trough insightful graphs which constitute a powerful tool for planning the maintenance and operation in power systems. Reservoir volumes and power generated and consumed by the PSUs were analyzed for the hydraulic system.

Finally, the generation profiles were illustrated and analyzed, as well as the occupation status of generators and lines, determining the elements that can be critical for the whole system.

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