

**A NOTE ON A SIMPLE INTERPRETATION OF THE GINI
COEFFICIENT ¹**

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Even though the Gini coefficient is likely the most frequently used measure of income inequality, policy makers and communicators rarely appreciate its conceptual and statistical advantages (see Cowell (2011) for an extensive discussion), and only exploit the fact that the higher its value the worse income inequality is in a certain region or period. Most intuitive presentations of the Gini coefficient are based on areas of the Lorenz curve, a rather abstract concept for many practitioners. This lack of obvious interpretability has led both experts and policy makers to cast doubts on the relevance of the Gini coefficient in practice. To the point, the widely cited book by Thomas Piketty (2014) claims that ‘Statistical indices such as the Gini coefficient give an abstract and sterile view of inequality...’. This short note provides an intuitive interpretation of the Gini coefficient based on simple averages of income differences, without resorting to the Lorenz curve explicitly, and hence being more amenable to those with minimal technical background.

Assume a society with n individuals with incomes y_1, y_2, \dots, y_n . Let \bar{y} be the average income and T the total income ($T = \sum_{i=1}^n y_i = n \bar{y}$). Even though there are several equivalent ways to define the Gini index for this population (see Yitzhaki and Schechtman (2012) for a comprehensive review), a convenient one is the following (see Gasparini, Cicowiez and Sosa Escudero (2013) for a detailed discussion and empirical applications to the case of Latin America):

$$G = \frac{\sum_i \sum_j |y_i - y_j|}{n^2 2 \bar{y}}.$$

Let PD be the average of all differences among all income pairs, in absolute value:

$$PD = \frac{\sum_i \sum_j |y_i - y_j|}{n^2}.$$

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A convenient way to handle PD is through an inequality matrix M , with $d_{ij} \equiv |y_i - y_j|$:

$$M = \begin{vmatrix} d_{11} & d_{12} & \cdots & d_{1n} \\ d_{21} & d_{22} & & \\ \vdots & & \ddots & \vdots \\ d_{n1} & \cdots & & d_{nn} \end{vmatrix}.$$

Clearly, PD is the average of the n^2 elements of M . Intuitively, the Gini coefficient measures how ‘large’ is this matrix of income disparities, in a relative sense. In a completely egalitarian society, all individuals have the same income ($y_i = y_j = \bar{y}$), hence all elements of M are zero. In the case of extreme inequality one individual has the whole income and the rest nothing. Assuming (without loss of generality) that the only individual that has everything is the last one. The M matrix for this case (M_T) is:

$$M_T = \begin{vmatrix} 0 & 0 & \cdots & 0 & T \\ 0 & 0 & & 0 & T \\ \vdots & & \ddots & & \vdots \\ 0 & 0 & \cdots & 0 & T \\ T & T & \cdots & T & 0 \end{vmatrix}.$$

Naturally, PD is zero for the completely egalitarian case. It is easy to see that PD for the other extreme case (PD_T) is:

$$PD_T = \frac{2(n-1)T}{n^2} = 2 \frac{n-1}{n} \frac{T}{n} \sim 2\bar{y},$$

where the last expression is valid for large n . Then,

$$G = \frac{PD}{PD_T}.$$

For example, if in a society of three individuals incomes in dollars are 1,2 and 9, $T = 12$ and $\bar{y} = 4$. The average of all income discrepancies (PD) is 3.55. If all income is concentrated in only one person PD_T is 5.33 (using the exact formula, since in this case $n = 3$ is very small). Then, the Gini coefficient for this example is $G = 3.55/5.33 = 0.66$.

The previous expression shows that $G = 0$ if the distribution of incomes is completely egalitarian ($PD = 0$) and $G = 1$ in the other extreme case ($PD = PD_T$). Obviously, $G \geq 0$ since $PD \geq 0$, by construction. If all incomes are non-negative, based on the triangle inequality it is easy to show (see the Appendix) that $PD \leq PD_T$, hence $0 \leq G \leq 1$.

To summarize, the Gini coefficient measures how far are the aggregate income discrepancies of a society with respect to two extreme societies: one completely egalitarian ($G = 0$) and another one with maximal inequality ($G = 1$). A final remark is that computing the Gini coefficient using average discrepancies has no obvious empirical advantages with respect to alternative formulae based the Lorenz curve, since all of them led to consistent estimators of the population Gini index.

References

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Appendix: Proof that $PD \leq PD_T$.

By the triangle inequality

$$|y_i - y_j| \leq |y_i| + |-y_j|$$

$$y_i + y_j,$$

since all incomes are non-negative. Then

$$\frac{1}{n^2} \sum_i \sum_j |y_i - y_j| \leq \frac{1}{n^2} \sum_i \sum_j (y_i + y_j) = 2\bar{y}$$

hence $PD \leq PD_T$.