Terceras Jornadas de Astrofísica Estelar
AAA Workshop Series, Vol. 9, 2016
L. P. Bassino, Z. L. López García, L. S. Cidale & M. Lares, Eds.

INFORME INVITADO - INVITED REVIEW

Pulsating white dwarf stars and asteroseismology

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Resumen. Actualmente, un gran número de estrellas enanas blancas variables pulsantes está siendo decubierto ya sea a partir de relevamientos basados en observaciones desde Tierra, tales como el Sloan Digital Sky Survey (SDSS), u observaciones desde el espacio (por ejemplo, la Misión Kepler). Las técnicas astrosismólogicas permiten inferir detalles de la estratificación química interna, la masa total, e incluso el perfil de rotación estelar. En este trabajo, describimos en primer lugar las propiedades básicas de las estrellas enanas blancas y sus pulsaciones, así como también los diferentes sub-tipos de estas variables conocidos hasta el momento. Posteriormente, describimos algunos hallazgos recientes en relación a enanas blancas pulsantes de baja masa.

Abstract. At present, a large number of pulsating white dwarf (WD) stars is being discovered either from Earth-based surveys such as the Sloan Digital Sky Survey, or through observations from space (e.g., the Kepler mission). The asteroseismological techniques allow us to infer details of internal chemical stratification, the total mass, and even the stellar rotation profile. In this paper, we first describe the basic properties of WD stars and their pulsations, as well as the different sub-types of these variables known so far. Subsequently, we describe some recent findings about pulsating low-mass WDs.

1. WD stars in a nutshell

WD stars constitute the ultimate fate of most the stars that populate the Universe, including our Sun. Indeed, single stars that born with masses below $\sim 10.6 M_{\odot}$ (Woosley & Heger 2015) will end their lives as WDs. Typical ages of WDs are in the range $1-10$ Gyr (1 Gyr $\equiv 10^9$ yr). Details about the formation and evolution of WDs can be found in the review papers of Winget & Kepler (2008), Fontaine & Brassard (2008), and Althaus et al. (2010). Here, we only give a brief summary of the main characteristics of these stars. WDs are characterized by stellar masses comparable to the mass of the Sun ($M_{\star} \sim 0.15-1.2 M_{\odot}$) but with sizes of the order of the size of planets ($R_{\star} \sim 0.01 R_{\odot}$), which implies very high mean densities ($\overline{\rho} \sim 10^6$ gr/cm^3) and strong degeneration of matter. In particular, the equation of state describing the mechanical properties of WDs is that of a Fermi gas of degenerate electrons (Chandrasekhar 1939), which provides most of the pressure that counteracts the force of gravity, thus preventing the collapse. Immersed in this sea of electrons —and decoupled thereof— coexists a non-degenerate gas of ions that provide the heat reservoir of the star, resulting from the previous evolutionary history.

WDs cover a wide range of effective temperatures ($4000 \lesssim T_{\rm eff} \lesssim 200\,000$ K) and hence a large interval of luminosities ($0.0001 \lesssim L_\star/L_\odot \lesssim 1000$). Average mass WDs ($M_\star \sim 0.6 M_\odot$) harbor cores likely made of ^{12}C and ^{16}O, although massive WDs could have cores made of ^{16}O, ^{20}Ne y ^{24}Mg, and the lowest-mass WDs (extremely low mass WDs, abbreviated as ELM WDs) could contain cores of ^{4}He. Roughly speaking, WD evolution is nothing but a slow cooling (Mestel 1952), in which the star gets rid of its thermal energy content into space, being the nuclear energy sources (nuclear burning) almost extinct. The rate at which this heat is removed (i.e., the cooling rate of the WD) is controlled by the outer layers of the star. According to the chemical composition of the outer layers, WDs come in two main flavors: DAs (80%, almost pure-H atmospheres) and DBs (15%, almost pure-He atmospheres), although there are also some WDs which show atmospheres of He, C, and O (PG1159 stars) or C and He (DQ WDs). We have to add to this list the DZ WDs, which show only metal lines, without H or He present. Finally, there is the exotic object called SDSSJ1240+6710, that has an almost pure O atmosphere, diluted only by traces of Ne, Mg, and Si (Kepler et al. 2016a). In the case of DA and DB WDs, the purity in the surface chemical composition is the result of the extremely high surface gravity ($\log g \sim 6 - 9$ [cm/s^2]), which results in the settling of the heavier elements, leaving the lightest nuclear species to float at the stellar surface.

In Figure 1 we plot the internal chemical structure of a typical DA WD model with $M_\star = 0.56 M_\odot$ and $T_{\rm eff} \sim 12\,000$ K. For instructional purposes, we have included in the figure a brief explanation indicating the origin of each feature in the chemical structure of the star. Note that the adopted x-coordinate, $-\log(1 - M_r/M_\star)$, strongly amplifies the outer part of the star. As it can be seen in the figure, in a typical WD star the $\sim 99\%$ of the mass is made of a mixture of ^{12}C and ^{16}O in uncertain proportions, being the ^{4}He content of $M_{\rm He} \sim 0.01 M_\star$ at most, and the ^{1}H content at the envelope of $M_{\rm H} \lesssim 0.0001 M_\star$.

The number of known WD stars is increasing fast thanks to the Sloan Digital Sky Survey (SDSS; York et al. 2000). In fact, SDSS increased the number of spectroscopically-confirmed WD stars more than an order of magnitude prior to the SDSS (see Kepler et al. 2016b). At present, there is a total of around 37 000 WDs identified, $\sim 30\,000$ of which are from DR7 (Kleinman et al. 2013), DR10 (Kepler et al. 2015) and DR12 (Kepler et al 2016b) of SDSS, and $\sim$ 5000 correspond to the McCook & Sion (1999) catalog (S. O. Kepler, private communication).

2. Why we care about WDs?

WDs are very old objects, so they harbor valuable information about the history of our Galaxy. As such, these exotic objects have application to various fields of modern astrophysics:

- Cosmochronology: the derivation of the ages of stellar populations using the WD luminosity function. WDs provide independent chronometers for the age and star formation history of the Galactic disk (e.g., Harris et al.

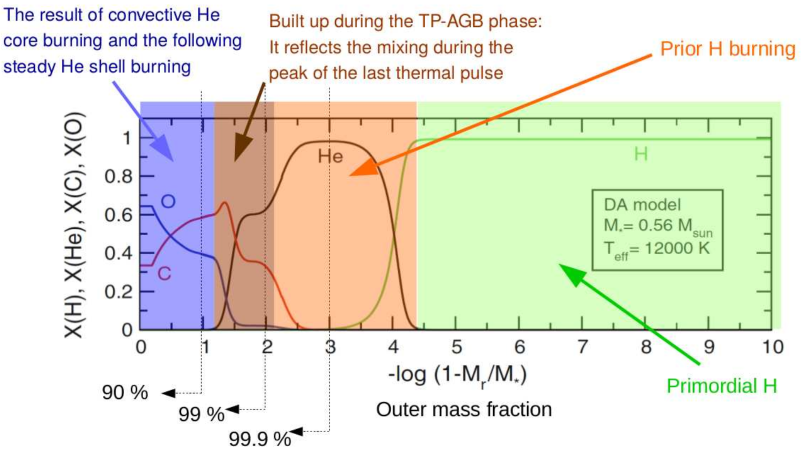

Figure 1. The internal chemical structure of a DA WD model with $M_\star = 0.56 M_\odot$ and $T_{\rm eff} \sim 12\,000$ K. Plotted is the mass fraction (X) of ^{16}O, ^{12}C, ^{4}He, and ^{1}H in terms of the outer mass coordinate. We include in the plot a brief explanation about the origin of each part of the internal chemical structure. Also, we indicate a few values of the percentage of the total mass of the star associated to the outer mass coordinate.

2006), and halo (Isern et al. 1998) through cosmochronology. Furthermore, WD cosmochronology can be applied to Galactic globular clusters (Hansen et al. 2013) and open clusters (e.g., García-Berro et al. 2010) as well.

- Initial to Final Mass Relation (IFMR; Catalán et al. 2008): more than 95 % of stars will end their lives as WDs, so that they can be considered as a boundary condition for stellar evolution. The difference between the initial Main Sequence masses and the final WD masses highlights the role of mass-loss during the AGB phase.
- Progenitors of SNIa, cataclysmic variables (novae): energetic events (10^{44}–10^{51} erg) of mass transfer on the WD by its companion.
- Cosmic laboratories: the interiors of WDs are composed of matter under extreme conditions (that is, extremely high density and pressure). This allows the study of equation of state (EoS), crystallization, particle physics, fundamental constant variations.

Until recent years, the only available tools for studying WDs were spectroscopy, that provides $T_{\rm eff}$, $\log g$, the surface chemical composition, the magnitude of magnetic fields, and photometry, that allows to infer the stellar mass. These techniques provide global information of the WDs. But, in what way could we dig beneath the surface layers of these stars? WD asteroseismology, that is, the comparison of the observed pulsational spectrum of variable WDs with that emerging from theoretical models of evolution and pulsations, comes to help (Winget & Kepler 2008; Fontaine & Brassard 2008; Althaus et al. 2010). Asteroseismology allows to “see” inside these stars, otherwise inaccessible by other means (Catelan & Smith 2015). WD asteroseismology allows to infer the chemical stratification, the core chemical composition, and the stellar masses, to measure rotation and magnetic fields, to detect planets around WDs, and to probe for exotic particles that are strong candidates for dark matter, like axions.

3. A little bit about non-radial WD pulsations

Within the framework of the linear theory of stellar pulsations, in which the perturbations of the stellar fluid are assumed to be small, spheroidal nonradial stellar pulsations are described by the Lagrangian displacement vector (Unno et al. 1989):

$$\vec{\xi}_{k\ell m} = \left[\xi_r^{k\ell m}(r), \xi_h^{k\ell m}(r)\frac{\partial}{\partial\theta}, \xi_h^{k\ell m}(r)\frac{1}{\sin\theta}\frac{\partial}{\partial\phi}\right] Y_\ell^m e^{i\sigma_{k\ell m}t}, \tag{1}$$

where $Y_\ell^m(\theta,\phi)$ are the spherical harmonic functions, $\sigma_{k\ell m}$ is the pulsation eigenfrequency, and $\xi_r^{k\ell m}(r)$ y $\xi_h^{k\ell m}(r)$ are the radial and horizontal eigenfunctions, respectively, of the eigenmode characterized by the set of quantum numbers k, ℓ, m. Therefore, in the linear theory of pulsations, the eigenmodes are described by a sinusoidal temporal dependence given by the factor $e^{i\sigma_{k\ell m}t}$, an angular dependence defined by the spherical harmonics $Y_\ell^m(\theta,\phi)$, and a radial dependence through the eigenfunctions $\xi_r^{k\ell m}(r)$ y $\xi_h^{k\ell m}(r)$, which have to be determined by numerically solving the set of differential equations[1] that describe the nonradial pulsations (Unno et al. 1989). This set is a fourth-order system of equations in real variables in the adiabatic approximation, and a sixth-order system of equations in complex variables for the full problem of nonadiabatic pulsations. The quantum numbers are defined as: (1) harmonic degree, $\ell = 0, 1, 2, 3, \cdots, \infty$, that represents $(\ell - m)$ nodal lines (parallel circles) on the stellar surface; (2) azimuthal order, $m = -\ell, \cdots, -2, -1, 0, +1, +2, \cdots, +\ell$, that represents nodal lines (meridian circles) on the stellar surface, and (3) radial order, $k = 0, 1, 2, 3, \cdots, \infty$, that represent nodal concentric spherical surfaces on which the displacement is null. We note that in the absence of any physical agent able to remove spherical symmetry, such as magnetic fields or rotation, the eigenfrequencies are $(2\ell + 1)$-fold degenerate in m. In Figure 2 we show an illustration of single spherical harmonics.

There exist three families of spheroidal pulsation modes: (1) p modes: characterized by large pressure variations and displacements mostly in the radial direction, the dominant restoring force being the compressibility. p modes have high frequencies (short periods); (2) g modes: characterized by small pressure variations and displacements almost tangential, being the dominant restoring force the buoyancy. g modes have low frequency (long periods); (3) f modes: have intermediate characteristics between p and g modes, and exist only for $\ell > 1$. Generally, these modes do not have nodes in the radial direction ($k = 0$), except for stellar models with a high central density.

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[1] An exception is the case of a self-gravitating homogeneous (ρ= constant) sphere, that admits an analytic solution (Pekeris 1938), although such a configuration is not of much astronomical relevance.

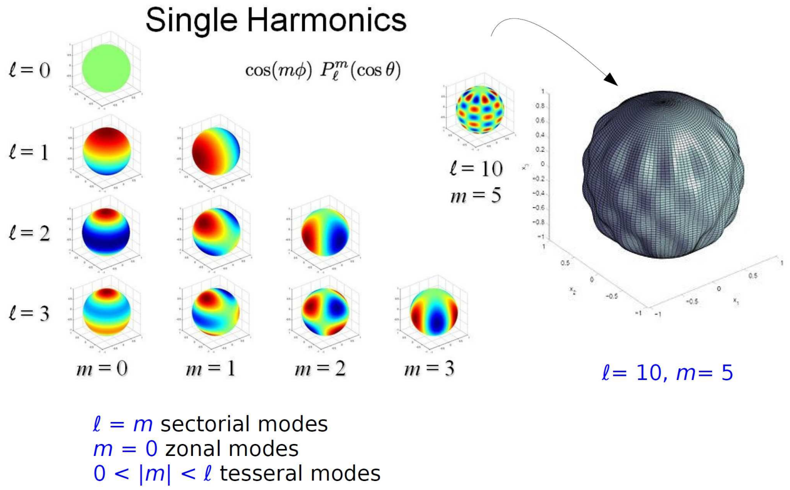

Figure 2. *Left:* some examples of single spherical harmonics. *Right:* The specific case of $\ell = 10, m = 5$ and its three-dimensional representation. Figures adapted from `www.atmos.albany.edu`.

The first pulsating WD star, HL Tau 76, was discovered by Landolt (1968). Pulsating WDs exhibit g-mode pulsations with $\ell = 1$ and 2[2]. There is only one reported case of a pulsating WD (specifically, a low-mass WD) that could be pulsating with periods at ~ 100 s associated to p modes or even radial ($\ell = 0$) modes and low radial order k (Hermes et al. 2013a). The amplitude of the variations of WD pulsations are typically between 0.005 y 0.4 mag. A plethora of light curves, some sinusoidal and with small amplitudes, other nonlinear and with large amplitudes, are observed. Generally, pulsating WDs are multimode pulsators, that is, they exhibit more than just a single period. In some cases, the pulsation spectrum contains linear frequency combinations of genuine pulsation eigenmodes, likely produced by the outer convection zone of the star.

4. The zoo of pulsating WDs

An increasing number of kinds of WD pulsators has been discovered in the last years. At present, there are seven classes of pulsating WDs. To begin with, the variables ZZ Ceti or DAVs (pulsating WDs with almost pure H atmospheres) are the most numerous ones. The other classes comprise the DQVs (atmospheres rich in He and C), the variables V777 Her or DBVs (atmospheres almost pure in He), the Hot DAVs (H-rich atmospheres), and the variables GW Vir or pulsating PG1159 pre-WD stars (atmospheres dominated by C, O, and He) that include the DOVs and PNNVs objects. To these families of pulsating WD stars, we have to add the ELMVs (extremely low-mass WDs variable) and the pre-ELMVs (the

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[2] Modes with higher values of ℓ could be excited in WDs, but their detection should be hampered by geometric effects of cancellation (Dziembowski 1977).

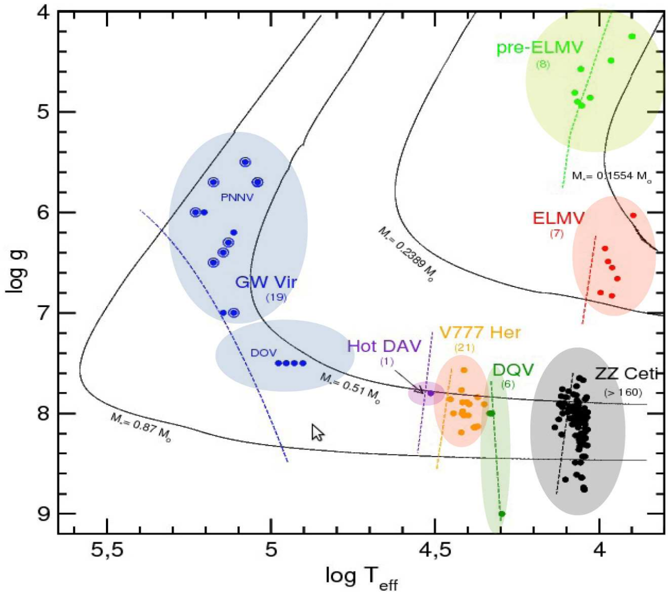

Figure 3. The location of the known classes of pulsating WD and pre-WD stars (dots of different colors) in the $\log T_{\rm eff} - \log g$ plane. We emphasize in particular the ELMV and pre-ELMV stars with big red and light green circles, respectively. In parenthesis we include the number of known members of each class. Two post-VLTP (Very Late Thermal Pulse) evolutionary tracks for H-deficient WDs and two evolutionary tracks for low-mass He-core WDs are plotted for reference. Also shown with dashed lines are the theoretical blue edges of the different instability domains.

probable precursors of ELMVs). In Figure 3 we depict the location of the several families of pulsating WDs known hitherto in the $\log T_{\rm eff} - \log g$ plane.

Regarding the driving mechanisms involved in the excitation of the pulsations in WDs, there is a strong consensus that they correspond to thermal processes that give place to self-excited pulsations. The more relevant mechanism is the $\kappa-\gamma$ mechanism acting in partial ionization regions of the dominant chemical element: $\rm H_{I-II}$ (DAVs, ELMVs), $\rm He_{I-II}$ (DBVs), $\rm He_{II-III}$ (pre-ELMVs), $\rm C_{V-VI}$ and and $\rm O_{VII-VIII}$ (pulsating PG119 pre-WD stars). When the outer convection zone deepens, WD pulsations are excited by the "convective driving" mechanism (DAVs, DBVs, ELMVs). Finally, the ε mechanism due to stable nuclear burning could be responsible for the excitation of short-period g modes in GW Vir stars (He burning; Córsico et al. 2009), ELMVs (H burning; Córsico et al. 2014a), and in average-mass DAV WDs evolved from low-metallicity progenitors (H burning; Camisasa et al. 2016).

In the next section, we concentrate in the last theoretical results about pre-ELMV and ELMV variable stars. A thorough description of the pulsation properties of these pulsating WD stars can be found in the papers by Steinfadt et al. (2010), Córsico et al. (2012), Jeffery & Saio (2013), Van Grootel et al. (2013), Córsico & Althaus et al. (2014ab, 2016), Córsico et al. (2016), and Gianninas et al. (2016).

5. Pulsating low-mass He-core WDs

Low-mass WDs ($M_\star \lesssim 0.45 M_\odot$), including ELM WDs ($M_\star \lesssim 0.18 - 0.20 M_\odot$, H-rich atmospheres), which are currently being detected through the ELM survey (see Brown et al. 2016 and references therein), likely harbor cores made of He. They are thought to be the outcome of strong mass-loss episodes at the red giant branch stage of low-mass stars in binary systems before the He flash onset that, in this way, is avoided (Althaus et al. 2013; Istrate et al. 2016). Some ELM WDs exhibit long-period ($\Pi \sim 1000 - 6300$ s) g-mode pulsations (Hermes et al. 2012, 2013ab; Kilic et al. 2015; Bell et al. 2015); they are called ELMV pulsating stars. The asteroseismological study of ELMVs ($7000 \lesssim T_{\rm eff} \lesssim 10\,000$ K and $6 \lesssim \log g \lesssim 7$; red circles in Figure 3) can helps to sound their interiors and ultimately to yield valuable clues about their formation scenarios. On the other hand, short-period ($\Pi \sim 300 - 800$ s) p- or even radial-mode ($\ell = 0$) pulsations in five objects that are probably precursors of ELM WDs have been recently discovered (Maxted et al. 2013, 2014; Gianninas et al. 2016). These stars have $8000 \lesssim T_{\rm eff} \lesssim 13\,000$ K and $4 \lesssim \log g \lesssim 5.5$ (green circles in Figure 3) and show a mixture of H and He on the surface. They are known as pre-ELMV stars and constitute the newest class of pulsating WD stars [3].

Below, we describe the predictions of current stability analysis on pre-ELMVs and ELMVs and how these compare with the observations.

5.1. Pre-ELMVs

Here, we describe the main nonadiabatic pulsation results of the study by Córsico et al. (2016). The stability properties of He-core, low-mass pre-WD models extracted from the computations of Althaus et al. (2013), calculated assuming the ML2 prescription for the MLT theory of convection (Tassoul et al. 1990), covering a range of effective temperatures of $25\,000$ K $\gtrsim T_{\rm eff} \gtrsim 6000$ K and a range of stellar masses of $0.1554 \lesssim M_\star/M_\odot \lesssim 0.2724$, were analyzed. Gravitational settling was neglected. For each model, the pulsational stability of radial ($\ell = 0$), and nonradial ($\ell = 1, 2$) p and g modes with periods in the range 10 s $\lesssim \Pi \lesssim 20\,000$ s was assessed. The results are shown in the left panel of Figure 4, in which we depict a spectroscopic HR diagram (the $T_{\rm eff} - \log g$ plane) that includes low-mass He-core pre-WD evolutionary tracks (dotted curves) along with the location of the known pre-ELMV stars (green circles) and stars not observed to vary (black dots). The theoretical blue edge of the dipole ($\ell = 1$) pre-ELMV instability domain (gray area) due to the $\kappa - \gamma$ mechanism acting at the ${\rm He}_{\rm II-III}$ ($\log T \sim 4.7$) and ${\rm He}_{\rm I-II}/{\rm H}_{\rm I-II}$ ($\log T \sim 4.42/\log T \sim 4.15$) partial ionization regions is displayed with a dashed blue line. These results are in excellent agreement with the predictions of the stability analysis carried out by Jeffery & Saio (2013). The location of the theoretical blue edge is almost insensitive to the value of ℓ and the prescription for the MLT theory of convection (that is, the efficiency of convection) adopted in the construction of the stellar mod-

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[3] There are other three stars (Corti et al. 2016, Zhang et al. 2016) that exhibit long-period ($\Pi \sim 1600 - 4700$ s) pulsations, located at nearly the same region of the HR diagram, but their nature as low-mass proto-WDs cannot be confirmed at the moment; i.e., they could be, alternatively, δ Scuti- and/or SX Phe-like pulsators.

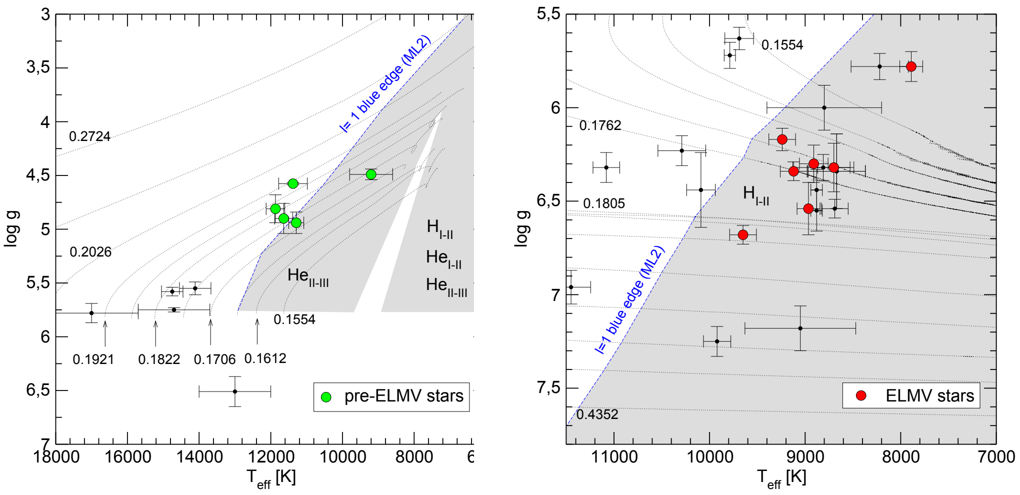

Figure 4. *Left:* The $T_{\rm eff} - \log g$ diagram showing low-mass He-core pre-WD evolutionary tracks (dotted curves) computed neglecting element diffusion. Numbers correspond to the stellar mass of some sequences. Green circles with error bars correspond to the confirmed pre-ELMV stars, and black dots depict the location of stars not observed to vary. The dashed blue line indicates the nonradial dipole ($\ell = 1$) blue edge of the pre-ELMV instability domain (emphasized as a gray area) of p and radial modes due to the $\kappa - \gamma$ mechanism acting at the He$_{\rm II-III}$ (at high $T_{\rm eff}$) and He$_{\rm I-II}$/H$_{\rm I-II}$ (at low $T_{\rm eff}$) partial ionization regions, as obtained by Córsico et al. (2016). *Right:* The same diagram but for low-mass He-core WD evolutionary tracks (final cooling branches). The locations of the seven known ELMVs are marked with red circles ($T_{\rm eff}$ and $\log g$ computed with 1D model atmospheres after 3D corrections). The gray region bounded by the dashed blue line corresponds to the instability domain of $\ell = 1$ g modes due to the $\kappa - \gamma$ mechanism acting at the H$_{\rm I-II}$ partial ionization region, according to Córsico & Althaus (2016).

els. The blue edge of radial modes is ~ 1000 K cooler than for nonradial modes. Our computations roughly account for the location of the known pre-ELMVs, although the theoretical blue edge should be ~ 900 K hotter in order to achieve a better agreement. This can be accomplished with higher He abundances at the driving region. This issue should be investigated in more detail.

A crucial aspect of these computations is that they neglect the effect of gravitational settling. When we take into account gravitational settling, the driving regions quickly becomes depleted in He, and the instability region in the $T_{\rm eff} - \log g$ diagram shrinks, leading us to the inability to explain the existence of *any* of these pulsating stars. This indicates that *gravitational settling could not be operative, or could be slowed, in the pre-WD stage.*

5.2. ELMVs

For the study of ELMVs, stellar models of He-core, low-mass WDs extracted from the computations of Althaus et al. (2013), having H-pure atmospheres and taking into account three different prescriptions for the MLT theory of convection (ML1, ML2, ML3; see Tassoul et al. 1990 for their definitions), covering a range of effective temperatures of $13\,000$ K $\lesssim T_{\rm eff} \lesssim 6\,000$ K and a range of stellar masses of $0.1554 \lesssim M_{\star}/M_{\odot} \lesssim 0.4352$, were considered. For each model, the pulsation stability of radial ($\ell = 0$) and nonradial ($\ell = 1, 2$) p and g modes with periods from a range 10 s $\lesssim \Pi \lesssim 18\,000$ s for the sequence with $M_{\star} = 0.1554 M_{\odot}$, up to a range of periods of 0.3 s $\lesssim \Pi \lesssim 5\,000$ s for the sequence of with $M_{\star} = 0.4352 M_{\odot}$ was assessed. Complete details of these calculations are given in Córsico & Althaus (2016). The results are shown in the right panel of Figure 4, in which we depict the spectroscopic HR diagram including the low-mass He-core WD evolutionary tracks (final cooling branches), along with the location of the seven known ELMVs (red circles), where $T_{\rm eff}$ and $\log g$ have been computed with 1D model atmospheres after 3D corrections. The instability domain of $\ell = 1$ g modes due to the $\kappa - \gamma$ mechanism acting at the $\rm H_{I-II}$ ($\log T \sim 4.15$) partial ionization region is emphasized with a gray region bounded by a dashed blue line corresponding to the blue edge of the instability domain. Our results are in good agreement with those of Córsico et al. (2012) and Van Grootel et al. (2013). Some short-period g modes are destabilized mainly by the ε mechanism due to stable nuclear burning at the basis of the H envelope, particularly for model sequences with $M_{\star} \lesssim 0.18 M_{\odot}$ (see Córsico & Althaus 2014a for details). The blue edge of the instability domain is hotter for higher stellar mass and larger convective efficiency. The ML2 and ML3 versions of the MLT theory of convection are the only ones that correctly account for the location of the seven known ELMV stars. We found a weak sensitivity of the blue edge of g modes with ℓ, and the blue edges corresponding to radial and nonradial p modes are somewhat (~ 200 K) hotter than the blue edges of g modes.

6. Conclusions

WD asteroseismology is consolidating as one of the most attractive avenues to explore the properties of WDs. It can provide crucial information about the internal structure of these stars, which leads to a detailed knowledge of evolutionary processes experienced by their parent stars. It is expected that in the coming years, unparalleled progress in the asteroseismic study of the origin and evolution of WDs will be reached with the help of space missions such as TESS (Transiting Exoplanet Survey Satellite, `https://tess.gsfc.nasa.gov/`).

Agradecimientos. I want to warmly thank the members of the LOC and SOC of the "Terceras Jornadas de Astrofísica Estelar" for the valuable work that led to the realization of this excellent scientific meeting.

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