Simultaneous determinations of τ polarisation and ν_τ helicity using the decay $\tau \to a_1 \nu_\tau$

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Abstract

A method is presented to extract tau polarisation P_τ and tau neutrino helicity γ_{VA} simultaneously using τ leptons produced at the Z^0 which decay into a_1 and ν_τ. It provides better sensitivity than independent fits to P_τ or γ_{VA} separately and in fact can be shown to be optimal. Estimates of sensitivity from Monte Carlo studies and comments regarding the use of the technique in experiments are also presented. We also discuss how results of tau polarisation determinations using other decay modes may be included. © 1997 Elsevier Science B.V.

1. Introduction

The production and decay of tau pairs from the Z^0 depend on the structure of both the weak neutral and charged current interactions of the tau. The neutral current couplings contain unequal right and left handed contributions resulting in a net tau polarisation averaged over all production angels of

$$P_\tau = -\frac{2v_\tau a_\tau}{v_\tau^2 + a_\tau^2},\qquad(1)$$

where v_τ and a_τ are the usual vector and axial vector coupling constants of the τ to the Z^0.

One can define a quantity analogous to the polarization, which characterizes the handedness of the charged leptonic current. This quantity is called the chirality parameter and is given by:

$$\gamma_{VA} = \frac{2g_V g_A}{g_V^2 + g_A^2},\qquad(2)$$

where g_V and g_A are the vector and axial vector coupling of the τ to the W. It measures the average helicity of the ν_τ and in the Standard Model with purely left-handed neutrinos, $\gamma_{VA} = -1$.

The decay $\tau \to a_1 \nu_\tau$ is of special interest, since the presence of two interfering amplitudes in the a_1 decay allows the effects due to charged and neutral currents to be separated, while in other hadronic decays such as $\tau \to \pi \nu_\tau$ and $\tau \to \rho \nu_\tau$, which have the greatest sensitivity to the neutral current couplings, only the product $\gamma_{VA} P$ can be measured. The structure of the charged current is extremely important as it is the assumed lefthandedness of the ν_τ that allows the tau polarisation, and thus neutral current couplings of the tau to be determined from a knowledge of $\gamma_{VA} P$.

Measurements by the ARGUS Collaboration [1] at energies near the production threshold of $\tau^+ \tau^-$ support the hypothesis of left-handed tau neutrinos using

unpolarised taus. Measurements have also been made at LEP, both of the absolute value of γ_{VA} [2] (suggesting either purely right-handed or purely left-handed neutrinos but being unable to distinguish between without information from additional experiments) using correlated tau decays, and of its signed value [3] using the decay $\tau \to a_1 \nu_\tau$. For measurements of the tau polarization [4], the value quoted as tau polarisation is in fact often $-\tau_{VA} P_\tau$, with the value $\tau_{VA} = -1$ assumed.

Efremov et al. [5] have previously examined the possibility of studying the tau charged current using handedness in the decay $\tau^- \to a_1^- \nu_\tau$ and suggested that the tau polarisation and chirality parameter might both be determined from this decay. We have previously presented a method to determine the chirality parameter based on the construction of an observable sensitive to γ_{VA}, but independent of the τ polarisation [6].

Here we extend that work to present a technique by which both the tau polarisation and chirality parameter may be determined simultaneously. It provides greater sensitivity to both these variables than analyses which attempt to extract either one alone and is based on a generalization of the "optimal variable" technique introduced in [7] and used to extract P_τ from τ decays.

2. Description of $\tau^- \to a_1^- \nu_\tau$ decays

The a_1 (1260) is a pseudovector resonance decaying into three pions. The decay process (we take the negatively charged tau lepton in the following, with the understanding that the corresponding statements for the charge conjugate processes are always implied) is as follows:

$$\tau^- \to a_1^- \nu_\tau, \quad a_1 \to \rho^0 \pi^-, \quad \rho^0 \to \pi^+ \pi^- .$$

There are two ways in which a ρ^0 can be formed from the three charged pions in the final state and as these alternative are indistinguishable, the corresponding amplitudes must both be included in the theoretical treatment.

We consider the realistic situation of $e^+ e^- \to Z^0 \to \tau^+ \tau^-$ decays into tau pairs, in which the tau energy is known, and only the energies and momenta of the three charged pions in the final state are
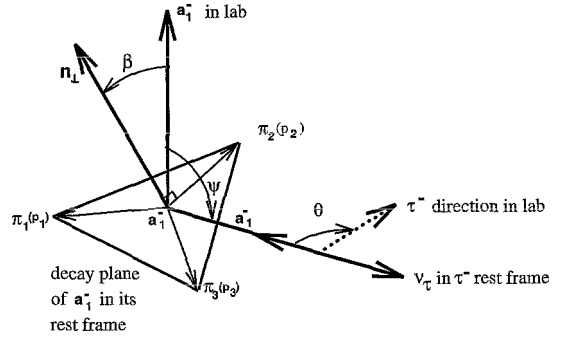

Fig. 1. Kinematics for the a_1 decay.

measurable. The neutrino escapes detection, but the kinematics of the system is well enough constrained to allow a partial reconstruction of the events, despite the fact that one cannot reconstruct the τ-rest frame. The appropriate frame for the analysis here is the a_1-rest frame where the pions are coplanar as illustrated in Fig. 1.

The angular distribution for the decay can be written following Ref. [8], where the unknown tau flight direction has been integrated out:

$$dГ = N \Big[h_1^+ W_A + (h_1^- \cos\theta - h_2 \sin\theta) W_A \gamma_{VA} P_\tau$$
$$+ h_3 \cos\beta \, W_E P_\tau + 3Q^2 \cos\psi \cos\beta \, W_E \gamma_{VA} \Big]$$
$$\times \frac{(m_\tau^2 - Q^2)^2}{Q^2} \frac{dQ^2}{Q^2} ds_1 ds_2 \frac{d\cos\theta}{2} \frac{d\cos\beta}{2} ,$$

$$(3)$$

with $Q = q_1 + q_2 + q_3$ where q_1, q_2 and q_3 are the final pion 4-momenta in the laboratory frame.

The τ decay angle θ measured with in its rest frame with respect to its momentum, and the angle ψ between the direction of the τ and the laboratory as seen from the a_1 rest frame, can be reconstructed from the energy of the hadronic system:

$$\cos\theta = \frac{2 x m_\tau^2 - m_\tau^2 - Q^2}{(m_\tau^2 - O^2)\sqrt{1 - m_\tau^2/E_{beam}^2}} ; \quad \theta \in [Q, \pi],$$

$$(4)$$

$$\cos\psi = \frac{x(m_\tau^2 + Q^2) - 2Q^2}{(m_\tau^2 Q^2)\sqrt{x^2 - Q^2/E_{beam}^2}} ,$$

$$(5)$$

with

$$x = \frac{E_1 + E_2 + E_3}{E_{\text{beam}}}, \qquad (6)$$

and E_{beam} is the beam energy. β denotes the angle between the normal $n_{\perp}$ to the plane spanned by the three pions in the a_1 rest frame and the momentum of the three-pion system in the laboratory frame. $\cos\beta$ is obtained from the measured pion momenta using the analytic approximation of Ref. [9]:

$\cos\beta$

$$= \frac{8Q^2 p_1 \cdot (p_2 \times p_3) / |p_1 + p_2 + p_3|}{[-\lambda(\lambda(Q^2, s_1, m_\pi^2), \lambda(Q^2, s_2, m_\pi^2), \lambda(Q^2, s_3, m_\pi^2))]^{1/2}}, \qquad (7)$$

where $\lambda(x, y, z) = x^2 + y^2 + z^2 - 2xy - 2yz - 2zx$.

The functions h_i are given by

$$h_1^{\pm} = m_\tau^2 \pm 2Q^2$$
$$- (m_\tau^2 \mp Q^2) \frac{3\cos^2\psi - 1}{2} \frac{3\cos^2\beta - 1}{2}, \qquad (8)$$

$$h_2 = 3m_\tau \sqrt{Q^2} \frac{\sin 2\psi}{2} \frac{\cos^2\beta - 1}{2}, \qquad (9)$$

$$h_3 = -3Q^2 \left(\cos\theta\cos\psi + \frac{m_\tau}{\sqrt{Q^2}} \sin\theta\sin\psi \right), \qquad (10)$$

$$N = \frac{G_F^2}{8m_\tau^3} (g_V^2 + g_A^2)\cos^2\theta_C \frac{1}{64(2\pi)^5}, \qquad (11)$$

with G_F the Fermi constant and W_A and W_E are given by

$$W_A = |F_1|^2 \left[(s_2 - 4m_\pi^2) + \frac{(s_3 - s_1)^2}{4Q^2} \right]$$
$$+ |F_2|^2 \left[(s_1 - 4m_\pi^2) + \frac{(s_3 - s_2)^2}{4Q^2} \right]$$
$$+ \left[(Q^2 - 2s_3 - m_\pi^2) + \frac{(s_3 - s_1)(s_3 - s_2)}{2Q^2} \right]$$
$$\times \text{Re}(F_1 F_2^*), \qquad (12)$$

$$W_E = -3 \left[\frac{s_1 s_2 s_3 - m_\pi^2 (Q^2 - m_\pi^2)^2}{Q^2} \right]^{\frac{1}{2}} \text{Im}(F_1 F_2^*). \qquad (13)$$

The Dalitz variables s_1 and s_2 are defined by

$$s_i = (q_j + q_+)^2; \quad i \neq j = 1, 2, \qquad (14)$$

where q_+ is the momentum of the positive pion and $q_{i,j}$ are momenta of negative pions. The functions W_A and W_E are model-dependent. We follow the work of Kuhn and Mirkes [8] which is based on a hadronic current of the form:

$$J^\mu = F_1(s_1, s_2, Q^2) V_1^\mu + F_2(s_1 s_2, Q^2) V_2^\mu, \qquad (15)$$

with

$$V_1^\mu = q_1^\mu - q_3^\mu - Q^\mu \frac{Q \cdot (q_1 - q_3)}{Q^2}, \qquad (16)$$

$$V_2^\mu = q_2^\mu - q_3^\mu - Q^\mu \frac{Q \cdot (q_2 - q_3)}{Q^2}, \qquad (17)$$

$$F_1 = F(s_1, s_2, Q_2); \quad F_2 = F(s_2, s_1, Q^2)$$

and a choice for F defined by

$$F(s_2, s_2, Q^2) = -\frac{2\sqrt{2}i}{3f_\pi} BW_a(Q^2) B_\rho(s_2),$$

where BW_a and BW_ρ denote Breit–Wigner resonances for the ρ and a_1 respectively. This model for the current is due to Kuhn and Wagner [10] and is implemented in the KORALZ event generator [11] widely used to simulate τ production and decays.

As noted earlier, the two negative pions are not distinguishable, and there are two possible ways to form the ρ-meson. The interference between them is contained in the function W_E through the imaginary part of the structure functions F_1 and F_2. Notice that the only term in the angular distribution that contains γ_{VA} without the presence of P_τ is proportional to W_E. The interference in this decay makes the $\tau \to a_1 \nu_\tau$ the unique hadronic channel in which we can disentangle the dependence on the chirality parameter from the polarisation.

While Eq. (2) shows the dependence of the tau decays on the polarisation and the chirality parameter, it is not directly useful, involving a six-dimensional phase space, which, for LEP statistics, would cause difficulties in statistical treatment of binned data. In the next section we address this problem.

3. New variables in phase space

Introducing four functions $H_i, i = 1, \ldots, 4$ we can rewrite Eq. (2) in the form

$$d\Gamma = \{ H_1 + H_2 P_\tau \gamma_{VA} + H_3 \gamma_{VA} + H_4 P_\tau \}$$
$$\times \frac{(m_\tau^2 - Q^2)}{Q^2} \frac{dQ^2}{Q^2} ds_1 ds_2 \frac{d\cos\theta}{2} \frac{d\cos\beta}{2} \frac{d\gamma}{2\pi}, \tag{18}$$

where the dependence of the expression on the three quantities γ_{VA}, P_τ, and $\gamma_{VA} P$ is clearly visible.

The functions H_i are defined by

$$H_1 = h_1^+ W_A, \tag{19}$$

$$H_2 = (h_1^- \cos\theta - h_2 \sin\theta) W_A, \tag{20}$$

$$H_3 = h_3 \cos\beta W_E, \tag{21}$$

$$H_4 = 3Q^2 \cos\psi \cos\beta W_E. \tag{22}$$

Now let us introduce a set of three variables

$$\omega = \left(\frac{H_2}{H_1}, \frac{H_3}{H_1}, \frac{H_4}{H_1} \right), \tag{23}$$

and the Jacobian for the change of variables $\{\gamma, \cos\theta, \cos\beta\} \rightarrow \{\omega_2, \omega_3, \omega_4\}$ given by $|J|$ with

$$J = \begin{vmatrix} \dfrac{\partial \gamma}{\partial \omega_2} & \dfrac{\partial \gamma}{\partial \omega_3} & \dfrac{\partial \gamma}{\partial \omega_4} \\[2mm] \dfrac{\partial \cos\theta}{\partial \omega_2} & \dfrac{\partial \cos\theta}{\partial \omega_3} & \dfrac{\partial \cos\theta}{\partial \omega_4} \\[2mm] \dfrac{\partial \cos\beta}{\partial \omega_2} & \dfrac{\partial \cos\beta}{\partial \omega_3} & \dfrac{\partial \cos\beta}{\partial \omega_4} \end{vmatrix}. \tag{24}$$

Making the change of variables in Eq. (3) one obtains

$$d\Gamma \propto H_1 \{ 1 + P_\tau \gamma_{VA} \omega_2 + \gamma_{VA} \omega_3 + P_\tau \omega_4 \}$$
$$\times |J| d\omega_2 d\omega_3 d\omega_4 ds_1 ds_2 dQ^2$$
$$= \tilde{H}(\omega) \{ 1 + P_\tau \gamma_{VA} \omega_2 + P_\tau \omega_4 \} d^3\omega, \tag{25}$$

where

$$\tilde{H} = \int ds_1 ds_2 dQ^2 H_1 |J|. \tag{26}$$

This expression generalizes the "optional variable" of Ref. [7] for P_τ alone. The motivation was to resolve difficulties of limited statistics fits in many dimensions by combining a large number of variables into only one so-called "optimal" variable,

but without loss of sensitivity. In a similar way, the use of the variables ω here is easily seen to incur no loss of sensitivity to P_τ or γ_{VA} by a simple generalization of the arguments of [7].

4. Fit technique and Monte Carlo studies

For application to an experimental situation, analytical approximations to the functions involved could be used with appropriate detector resolutions applied. In practice, the effects of cuts, radiative corrections etc. are more easily handled by Monte Carlo techniques, as is commonly done for the extraction of P_τ. The procedure then would be to generate four 3-dimensional distributions in $\omega_1, \omega_2, \omega_3$ (histograms) with Monte Carlo events generated according to $\gamma_{VA} = \pm 1$ and $P_\tau = \pm 1$ and passed through the appropriate detector simulation and reconstruction. Defining $MC_i(j,k)$ as the number of Monte Carlo events in bin i generated with $\gamma_{VA} = j$ and $P_\tau = k$ where j and k take the values ± 1, we write the likelihood

$$L = \prod_{\text{bins } i} e^{-\mu_i} \frac{\mu_i^{N_i}}{N_i!}, \tag{27}$$

where

$$\mu_i = \mathcal{N} \sum_{j,k = \pm 1} MC_i(j,k) \left(\frac{1 + j\gamma_{VA}}{2} \right) \left(\frac{1 + kP_\tau}{2} \right), \tag{28}$$

and $\mathcal{N}$ is the relative normalization factor between numbers of data and Monte Carlo events which can be fixed, or left free as a check of the fit. If the number of Monte Carlo events available is not much greater than the number of data events, the likelihood must be corrected to include this effect.

The likelihood L is then maximized with respect to γ_{VA} and P_τ in the usual way. This is a generalization of the technique commonly used to fit for P_τ.

We have performed a Monte Carlo study using the Koralz [11] program to generate samples of 60000 events with a_1 decays assuming pure V-A and pure V + A charged current couplings, divided into equal samples with $P_\tau = +1$ and $P_\tau = -1$, as well as 6000 events with nonstandard values of γ_{VA} to represent a hypothetical data sample. Note that a sample of τ^- decays with a V + A charged current

interaction can be obtained simply by using τ^+ decays with a V-A interaction and simply reversing the signs of the charges of all particles.

Six bins were used in each dimension, with the sizes of the bins chosen to make the number of events in each bin comparable. A maximum likelihood fit for the best linear combination of V-A and V + A samples with $P_\tau = +1$ and $P_\tau = -1$ to match the hypothetical data sample with $\gamma_{VA} = -0.768$ and $P_\tau = -.136$ gave statistical errors of 0.06 and 0.15 for P_τ and γ_{VA} respectively. Fixing the polarization gave an error on γ_{VA} of 0.13. Monte Carlo studies using samples of fully right or left-handedly polarized taus with $\gamma_{VA} = 1$ and $\gamma_{VA} = -1$ gave consistent answers. A perfect detector was assumed, so the errors are those expected from finite statistics. Detector effects and studies of systematic errors must be left to specific experiments.

This method can also be applied to the process in which $a_1^- \rightarrow \pi^0 \pi^0 \pi^-$. To this end one has only to change $q_+ \rightarrow q_-$ in Eq. (14) and $q_j; j = 1,2$ are now the neutral pion momenta. Clearly, in this case one is dealing with a negative ρ-meson rather than a neutral one.

To include other channels is straightforward. The likelihood considered above can be expressed in terms of $\gamma_{VA} P_\tau$ and γ_{VA} and then multiplied by the likelihood for $\gamma_{VA} P_\tau$ measured in other channels, or better, the likelihood for $\gamma_{VA} P_\tau$ and $|\gamma_{VA}|$ obtained from correlated tau decays. In the event that only $|\gamma_{VA}|$ is available, half the likelihood must be assigned to $\gamma_{VA} = 1$ and half to $\gamma_{VA} = -1$.

The resulting likelihood integrated over γ_{VA} is then the correct likelihood for P_τ, and vice versa. This constitutes the most complete determination of P_τ and γ_{VA} possible from LEP/SLCτ decay data without recourse to outside data to fix the sign of γ_{VA}.

5. Summary

In summary, we have found a method to determine γ_{VA} and P_τ simultaneously using the decay mode $\tau \rightarrow a_1 \nu_\tau$. The method is optimal, in the sense of using a reduced phase space to facilitate statistical treatment of data with no loss of sensitivity. We have described in detail how the technique can be applied

to tau pairs produced from the Z^0, checked it with simulated events generated by Monte Carlo and estimated the sensitivity. We hope that this technique will be useful in the final analyses of the LEP I and SLC tau pair data.

Acknowledgements

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