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Measurement of the average lifetime of b-hadrons in Z decays

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Abstract

We present a measurement of the average b-hadron lifetime τ_b at the e^+e^- collider LEP. Using hadronic Z decays collected in the period from 1991 to 1994, two independent analyses have been performed. In the first one, the b-decay

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position is reconstructed as a secondary vertex of hadronic b-decay particles. The second analysis is an updated measurement of $\tau_{\rm b}$ using the impact parameter of leptons with high momentum and high transverse momentum. The combined result is

 $\tau_{\rm b} = \left[1549 \pm 9\,(\text{stat}) \pm 15\,(\text{syst})\right] \text{fs}\,.$

In addition, we measure the average charged b-decay multiplicity $\langle n_b \rangle$ and the normalized average b-energy $\langle x_E \rangle_b$ at LEP to be

 $\langle n_{\rm b} \rangle = 4.90 \pm 0.04 \, (\text{stat}) \pm 0.11 \, (\text{syst}), \quad \langle x_{\rm F} \rangle_{\rm b} = 0.709 \pm 0.004 \, (\text{stat} + \text{syst}).$

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1. Introduction

Lifetimes of b-hadrons are basic ingredients for the determination of the Cabibbo-Kobayashi-Maskawa matrix (CKM) element $|V_{\rm eb}|$. The measurement of the b-hadron lifetime in 1983 at SLAC [1] provided the first evidence for the hierarchy of the CKM matrix elements. Today precision measurements of the b-lifetime are made at LEP, SLC and at the Tevatron [2,3].

All b-flavoured hadrons are expected to have a similar lifetime due to the high value of the b-quark mass. An inclusive analysis at the Z resonance results in an average of the individual lifetimes of B^+ , B^0_d , B^0_s and Λ_b hadrons and their antiparticles, weighted with their production cross section. In this letter we present a measurement of τ_b using two complementary and largely uncorrelated techniques.

In the first analysis, called the secondary vertex method, the hadronic decay topology of $b\bar{b}$ events is reconstructed. The track measurements are used to determine the primary e^+e^- interaction point and the decay positions of b-hadrons. The b-lifetime can be derived from the impact parameters of the b-decay products or from the distance of secondary vertices with respect to the primary event vertex.

The second analysis, the lepton impact parameter method, improves the statistical and systematic accuracy compared to our previous measurement [2]. It uses the same technique and is based on the impact parameter distribution of leptons with high momentum p and high transverse momentum p_{\perp} with respect to the nearest jet.

2. Hadronic Event Selection

The L3 detector and its modeling have been described elsewhere [4-6]. The selection of hadronic events is identical to the one used for the measurement of the total hadronic cross section [7]. For the lifetime analyses only events with more than four tracks measured in the Time Expansion Chamber (TEC) are kept to ensure a good vertex reconstruction. The lepton impact parameter method has been applied to a sample of 2.9 million hadronic events collected on and near the Z resonance in the years 1991 to 1994. The secondary vertex analysis has been performed on the data sample of the year 1994 consisting of 1.5 million hadronic events with a center-of-mass energy of 91.2 GeV.

In the L3 coordinate system the direction of the e^- beam defines the *z* direction. The *xy*, or $r\phi$ plane, is the bending plane of the magnetic field, with the *x* direction pointing to the center of the LEP ring. The coordinates ϕ and θ denote the azimuthal and polar angles of tracks or jets.

3. Secondary vertex method

The pair production of b-quarks is characterized by two decay positions (secondary vertices) separated on average by $\approx 3 \text{ mm}$ from the e⁺e⁻ interaction point (primary vertex). The daughter particles of b-hadrons arise at the secondary vertices and additional particles produced in the fragmentation process originate at the primary vertex. Therefore, in contrast to Z decays into other quark species, bb events consist of three distinct vertices: one primary and two secondary. The decay points of the b-hadron and of a secondary c hadron decay can not be separated and are merged into one decay position.

3.1. Reconstruction of Z decays into $b\bar{b}$

Hadronic jets are reconstructed from energy clusters in the calorimeter using the JADE algorithm [8] with a cut on the scaled invariant mass squared y_{cut} of 0.04. This results mostly in two-jet events with a jet axis resolution of 40 mrad.

The tracks in the event are associated to the closest jet. The two most energetic jets in the event have been considered to contain the original b-quarks. Both of these jets must point into a fiducial region delimited by the barrel part of the detector, $|\cos \theta| < 0.74$.

The three-vertex hypothesis is tested with tracks that fulfill the following quality cuts:

- The track should contain a minimum of 25 TEC hits and at least one hit in the Silicon Microvertex Detector (SMD) from the inner $r\phi$ ring to ensure a good extrapolation to the interaction point.
- The transverse momentum with respect to the beam axis should be greater than 300 MeV in order to restrict the uncertainty due to multiple scattering.
- Tracks from identified decays of K^0 and Λ hadrons are rejected to reduce the number of downstream secondary vertices.

Tracks which satisfy the above criteria are used for the calculation of the xy position of the track vertices. If the track is also well measured in the zprojection it has been used in the calculation of the 3D vertex position.

In order to assign tracks to one of the three vertices we consider all associations and select the most likely combination by a χ^2 fit, which included lifetime and kinematical information:

$$\chi_{\text{vertex}}^{2} = \sum_{i=1}^{n} \left[\left(\frac{\delta_{i}^{r\phi}(\mathbf{x}_{j})}{\sigma_{\delta_{i}^{r\phi}}} \right)^{2} + \left(\frac{\delta_{i}^{z}(\mathbf{x}_{j})}{\sigma_{\delta_{i}^{z}}} \right)^{2} \right] + \sum_{k=1}^{3} \left(\frac{(\mathbf{x}_{1} - \mathbf{x}_{\text{BS}})\mathbf{e}_{k}}{\sigma_{\text{BS}}^{k}} \right)^{2} - 2\sum_{i=1}^{n} \ln P_{l}(\eta_{i}),$$

- $i = 1, \cdots, n$ tracks, j = 1, 2, 3 vertices,
- k = x, y, z projections,
- l = 1,2 b hadron/fragmentation.

The first two terms contain the sum of quadratic deviations of all track positions to the respective vertices in $r\phi$ and z. The parameter $\delta_i^{r\phi}$ denotes the impact parameter of the track in the bending plane and δ_i^z is the z difference, both with respect to the vertices x_i .

The associated errors σ_{δ_i} are defined as the sum of uncertainties from the track fit and multiple scattering, added in quadrature. The impact parameter resolution for high-energy tracks is measured to 32 μ m through the distribution of the distance at which tracks of dilepton events miss each other at the interaction point. The additional multiple scattering error for tracks with an inner SMD hit is $110/(p_{\perp}/\text{GeV}\sqrt{\sin\theta}) \mu \text{m}$. If a track does not belong to the z selected sample, the error $\sigma_{\delta_i^z}$ is set to infinity.

The primary vertex x_1 is constrained to be identical to the beam spot position $x_{\rm BS}$, within the horizontal beam spot size of $\sigma_{\rm BS}^x = 135 \,\mu$ m, the vertical size of $\sigma_{\rm BS}^y = 25 \,\mu$ m and the longitudinal size of $\sigma_{\rm BS}^z = 1 \,\mathrm{cm}$. The average beam interaction point $x_{\rm BS}$ is reconstructed using tracks collected from 200 hadronic events accumulated every few minutes.

The last term in the expression for χ^2 exploits the high mass and high-energy of b-quarks to distinguish b-decay from fragmentation tracks. A suitable variable for b-decay tracks is the rapidity $\eta = 1/2 \ln(E + p_{\parallel})/(E - p_{\parallel})$, where *E* stands for the energy of particles (assuming pions) and p_{\parallel} for the momentum parallel to the jet axis [9]. $P_l(\eta_i)$ is the probability density for the rapidity of either b-decay or fragmentation tracks.

In the fit at least one track from each jet is required to originate at the primary vertex and at least three tracks at the secondary vertex to reduce fake vertices of random track crossing. Therefore the minimum number of selected tracks per jet is four. Due to the higher multiplicity of bb events, the b quark content is slightly increased from 22 % to 27 %. In light-flavour events the algorithm produces artificial secondary vertices with small decay lengths compatible with zero. No attempt is made to remove this background and to enrich the fraction of b events in the hadronic sample.

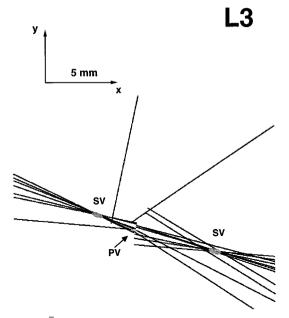


Fig. 1. A $b\bar{b}$ candidate event in 1994 data shown in the $r\phi$ plane of the L3 detector. The secondary vertices (SV) in both jets of the event are separated from the primary vertex (PV) by more than five standard deviations. The shaded ellipses represent the positions of the calculated vertices within one standard deviation.

The result of the χ^2 fit are the three-dimensional positions x_j of the three vertices j = 1,2,3. The b flight direction is fixed to the jet axis, which resolves it better than the line connecting primary and secondary vertices. The assumption of a fixed direction allows a three-dimensional interpretation of χ^2 even if no track belongs to the z selected sample. An event selected in 1994 data with a high probability to be a bb event is shown in Fig. 1.

3.2. Lifetime fit

Lifetime measurements with hadronic b events at LEP already reach a statistical precision that is small compared to the systematic uncertainties in modeling heavy quark decays [2,3] Our main purpose is thus to perform a fit where the values of poorly known model parameters are determined from the data itself. The most relevant parameters are the average b-hadron energy, the average b-decay multiplicity and the background of long-lived mesons.

Two variables are sensitive to the b-lifetime. One is the impact parameter, here defined as the impact

parameter of those tracks attached to the secondary vertices in the minimum χ^2 configuration. The other is the decay length, which measures the distance between primary and secondary vertex. The advantage of the impact parameter is that for relativistic b-hadrons its mean value is proportional to $c\tau$, whereas the decay length is proportional to $(E/m)c\tau$. The average impact parameter of tracks from b-decays at LEP is therefore less sensitive to the b-hadron momentum and to a precise knowledge of the b-quark fragmentation parameters. We have performed two lifetime fits, one to the impact parameter distribution and the other to the decay length distribution. The comparison of both results is used to measure the value of the average b-hadron energy *E*.

The τ_b value is obtained from a binned χ^2 fit to the respective distributions, with the lifetime dependent expected distribution predicted by a Monte Carlo simulation [10]. The lifetime dependence is introduced by reweighting the simulated events according to

$$\left(\frac{\tau_{\rm MC}}{\tau_{\rm b}}\right)^2 \frac{e^{-t_1/\tau_{\rm b}}}{e^{-t_1/\tau_{\rm MC}}} \; \frac{e^{-t_2/\tau_{\rm b}}}{e^{-t_2/\tau_{\rm MC}}},$$

where t_1 and t_2 are the proper times of the weakly decaying b-hadrons in jet 1 and 2 and $\tau_{\rm MC}$ is the lifetime value in the Monte Carlo, 1.55 ps. The one-parameter lifetime fit is based on the assumption that all b-hadrons have the same lifetime.

In order to account for small discrepancies in the resolution function between data and Monte Carlo additional resolution parameters are introduced. The Monte Carlo distribution is convoluted with two Gaussian functions with a standard deviation $\sigma_{1,2}$ applied to a small fraction $f_{1,2}$ of Monte Carlo events. The resolution correction parameters have been measured by the shape of the respective distributions on the negative side.

Another important source of uncertainty concerns the background of fragmentation tracks being attached to the secondary vertices. Monte Carlo studies show that this background fraction depends on the average b-decay charged multiplicity $\langle n_b \rangle^8$.

⁸ Our definition includes only the direct decay products pions, charged kaons, leptons and protons, not including decay products from K_s^0 and Λ .

Secondary vertices far from the primary event vertex tend to include one or two tracks from decays of long-lived strange hadrons, mainly K_{e}^{0} and Λ

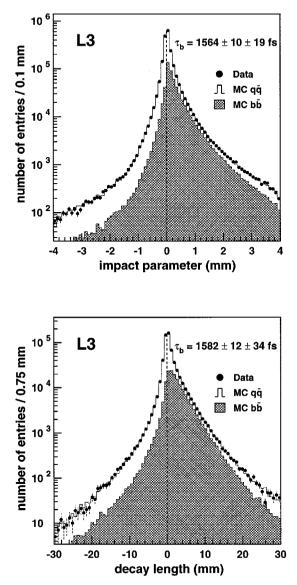


Fig. 2. a) Impact parameter distribution of tracks from secondary vertices. The points represent the 1994 data and the histogram shows the MC distribution for the fitted lifetime $\tau_b = 1564$ fs. The b component in the MC sample is indicated by the shaded histogram. b) Decay length distribution of reconstructed secondary vertices. The points represent the 1994 data and the histogram shows the MC distribution for the fit result $\tau_b = 1582$ fs. The b component in the MC sample is indicated by the shaded histogram.

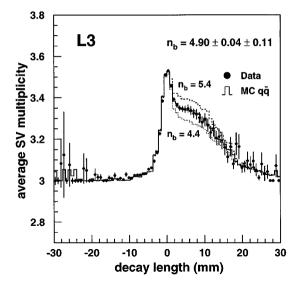


Fig. 3. Average multiplicity of tracks at the secondary vertices as a function of the decay length. The points represent the 1994 data and the histogram shows the MC distribution for the fitted average b-decay multiplicity $\langle n_b \rangle = 4.9$. The dotted (dashed) line indicates the multiplicity distribution for $\langle n_b \rangle = 4.4(5.4)$.

decays. In the following $\langle n_{\rm K} \rangle$ denotes the fraction of ${\rm K}^0_s$ and Λ hadrons produced in Z decays.

In order to make the measurement of the b-lifetime independent of the described parameters, the resolution correction parameters $f_{1,2}, \sigma_{1,2}$, the average b-decay multiplicity $\langle n_b \rangle$ and the fraction of long-lived hadrons $\langle n_K \rangle$ are fitted together with τ_b . We perform a binned χ^2 fit simultaneously to the multiplicity distribution and to either the impact parameter or decay length distribution.

The total number of secondary vertices used in the decay length fit is $583\,972$. The fit to the impact parameter distribution is performed with a total of $2\,007\,415$ tracks. The fit results are:

$$\tau_{\rm b} = (1564 \pm 10) \, {\rm fs}$$

 $\langle n_{\rm b} \rangle = 4.90 \pm 0.04, \quad \langle n_{\rm K} \rangle = 1.29 \pm 0.02,$

for the impact parameter distribution and

 $\tau_{\rm b} = (1582 \pm 12) \, {\rm fs}$,

 $\langle n_{\rm h} \rangle = 4.89 \pm 0.04$, $\langle n_{\rm K} \rangle = 1.33 \pm 0.05$

for the decay length distribution, where the errors are statistical only.

The impact parameter and decay length distribu-

tion for the fitted parameters are shown in Fig. 2. A good agreement between data and MC is observed and the positive tail due to b-decays is clearly visible.

Fig. 3 shows the average number of tracks at the secondary vertices as a function of the decay length. The peak at zero decay length is due to tracks from light quark fragmentation. The enhancement at positive decay lengths is due to b and c events and its magnitude depends on the number of charged particles from b-decays. At very small and very big decay lengths the secondary vertex multiplicity is dominated by the constraint of a minimum of three tracks.

The τ_b value is an average value according to the b-hadron composition in Z decays. It has been checked in a Monte Carlo study that the composition of the accepted events was consistent with the world average [11] within the statistical errors.

3.3. Systematic uncertainties

The contributions to the systematic error in the b-lifetime measurement and $\langle n_{\rm b} \rangle$ due to detector

resolution and the incomplete knowledge of heavyflavour hadronization and decay processes are summarized in Table 1.

The agreement between data and Monte Carlo in the average number of quality tracks per event is better than 0.1 tracks, resulting in the uncertainties given in Table 1. The uncertainty in the resolution function has been estimated by varying the resolution correction parameters within errors. The uncertainty in the knowledge of the beam spot size and position is determined by varying the size of the beam spot within the accuracy of the 200-eventvertex (15 μ m in x and 10 μ m in y) and allowing for a bias of 10 μ m in x and 5 μ m in y for the central beam position. The definition of the sign of the decay length depends on the jet direction. The resolution on the jet direction is estimated from comparisons between data and Monte Carlo on a two-jet event sample. The maximum discrepancy found is 2 mrad.

The precision of the lifetime measurement is further limited due to heavy-quark physics modeling.

Table 1

Systematic errors in the measurement of τ_b with a) secondary vertices and b) with leptons. $\Delta \tau_b^{\text{IP}}$ is the error for the impact parameter and $\Delta \tau_b^{\text{DL}}$ for the decay length method. The corresponding errors in the b-decay multiplicity Δn_b are also given. $\Delta \tau_b^{\text{e}}$ and $\Delta \tau_b^{\mu}$ show the errors from the lepton measurement for electrons and muons, respectively

	a) Secondary vertex method			b) Lepton method	
Error source	$\Delta { au}_{ m b}^{ m IP}$ (fs)	$\varDelta au_{ m b}^{ m DL}$ (fs)	$\Delta n_{\rm b}$	$\Delta \tau^{\rm e}_{\rm b}~{ m (fs)}$	$\Delta au_{ m b}^{\mu}~({ m fs})$
Reconstructed track multiplicity	5	2	0.04	-	-
Resolution function	5	5	0.07	3	2
TEC calibration and SMD alignment	3	3	< 0.01	15	10
Beam spot size and position	6	7	< 0.01	6	6
Jet direction resolution	5	5	< 0.01	4	5
Lepton Selection	-	-	-	15	15
Sum of detector uncertainties	11	11	0.08	22	19
b fragmentation	9	30	0.02	5	3
c fragmentation	< 1	2	< 0.01	1	1
b-decay multiplicity distribution	4	4	0.06	-	-
D fraction in $c\overline{c}$ events	3	2	< 0.01	< 1	< 1
D content in b-decays	11	10	0.04	4	4
D lifetimes	3	2	< 0.01	< 1	< 1
R _b	4	4	0.02	< 1	< 1
R _c	1	1	0.01	1	1
Semileptonic branching fractions	-	-	-	1	1
Inclusive lepton spectra	-	-	-	3	5
Fit Method	2	2	< 0.01	16	14
Heavy quark physics modeling	16	32	0.08	18	16

The dominant contributions arise from the uncertainty in the inclusive description of b-decays. Theoretical uncertainties have been estimated by a variation of the model parameters as recommended by the LEP Heavy Flavour Electroweak Group in [12]. Systematic errors due to b and c fragmentation are determined by varying $\epsilon_{\rm b}$ and $\epsilon_{\rm c}$ in the Peterson fragmentation function [13] according to $\langle x_E \rangle_{\rm b} =$ 0.702 \pm 0.008 for b fragmentation and $\langle x_{\rm H} \rangle_{\rm c} =$ 0.484 ± 0.008 for c fragmentation [12]. x_F is the fraction of the beam energy carried by the b or c hadron. The uncertainty in $\tau_{\rm b}$ from b fragmentation is estimated to 9 fs and 30 fs for the impact parameter and decay length measurement, respectively. The b fragmentation error has also a contribution from the uncertainty in the shape of the fragmentation function. Several models [14] have been tuned on the mean value $\langle x_F \rangle_b = 0.702$ and the maximum shift in the lifetime found is 2 fs. The error from the uncertainty in c fragmentation has been estimated to be negligible.

An uncertainty in the shape of the b-decay multiplicity distribution is taken into account by replacing the Monte Carlo distribution with a binomial distri-

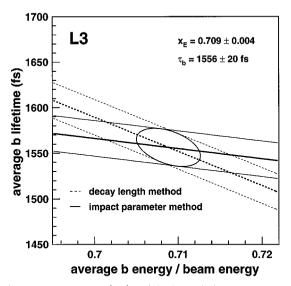


Fig. 4. Dependence on $\langle x_E \rangle_b$ of the hadronic impact parameter (full line) and the decay length (dashed line) measurement. The central values are shown together with the upper and lower one sigma bounds. Both measurements are compatible with each other at an average b-energy of $\langle x_E \rangle_b = 0.709 \pm 0.004$ and a b-lifetime of $\tau_b = 1556 \pm 20$ fs.

bution of the same mean and variance as in the Monte Carlo.

Sensitivity to the modeling of c quarks, occurring as primary $c\bar{c}$ events and as decay products of bhadrons, is estimated by varying the lifetimes of c hadrons and the fraction of c hadrons in $c\bar{c}$ events and b-decays. The production of the different D mesons and Λ_c baryons in $c\bar{c}$ events is varied according to the LEP averages [12]. The branching fractions $B_d^0 \rightarrow D^+, D_d^0$ and $B_s^0 \rightarrow D_s^0$ as well as the lifetimes of c hadrons are varied following the world averages [11].

3.3.1. Combination of impact parameter and decay length results

The impact parameter and decay length measurement are within their different sensitivity to the b momentum consistent with each other. We have combined the two results and simultaneously determined $\langle x_E \rangle_b$, taking into account the correlation between the two τ_b measurements and assuming linear dependences on $\langle x_E \rangle_b$ with coefficients determined from Monte Carlo, as illustrated in Fig. 4. The result is

$$\tau_{\rm b} = (1556 \pm 10 \pm 17) \, {\rm fs}, \quad \langle x_{\rm F} \rangle_{\rm b} = 0.709 \pm 0.004 \, {\rm s}$$

The average b-decay multiplicity is

$$\langle n_{\rm h} \rangle = 4.90 \pm 0.12$$
.

The contributions to the total systematic error of the b-lifetime are similar to those in Table 1, except the uncertainty due to the b fragmentation, which is reduced to 5 fs. The errors on $\langle x_E \rangle_b$ and $\langle n_b \rangle$ contain both statistical and systematic uncertainties.

4. Lepton impact parameter method

4.1. Event selection

Electron candidates have been identified as electromagnetic showers in the BGO calorimeter associated to a track in the TEC. The fiducial volume covered the barrel part, $|\cos \theta| < 0.69$. Hadronic background has been rejected by requiring an energetic and angular matching between TEC track and BGO cluster.

Table 2 Composition of the 1994 sample of selected lepton candidates, as estimated from the Monte Carlo simulation

Composition (1994)	Electrons	Muons
$b \rightarrow 1X$ (Prompt)	83.1%	76.2%
$b \rightarrow c \rightarrow 1X$ (Cascade)	9.6%	9.1%
$c \rightarrow 1X$ (Charm)	4.6%	7.8%
Background	2.7%	6.9%

Muon candidates are identified as charged tracks in the muon spectrometer, satisfying $|\cos \theta| < 0.72$. Only muons with an associated TEC track and momentum and polar angle measurements are accepted for the analysis. Punch-through tracks from longlived hadrons are rejected by requiring a good consistency between the track extrapolation to the center of the beam collision point.

The track quality cuts for leptons are chosen to be more stringent than in the hadron track selection because a single lepton track is sufficient to perform a lifetime fit, whereas several tracks are needed for the reconstruction of a secondary vertex. In the 1994 sample, the presence of a hit in the inner $r\phi$ layer of the SMD is required, which improves the impact parameter resolution from 112 µm to 32 µm.

The fraction of $b\bar{b}$ events in the sample is enhanced by selecting leptons with high momentum and transverse momentum with respect to the closest jet. The jet parameters are redefined subtracting the lepton four-momentum. For electrons the cluster energy has to be larger than 3 GeV and the transverse momentum greater than 1 GeV. For muons the cuts are 4 GeV and 1.5 GeV, respectively.

After all cuts have been applied, a sample of 17647 electron candidates and 23062 muon candidates is obtained. The expected sample composition is shown in Table 2 for the 1994 detector setup.

4.2. Lifetime fit

The inclusive b-lifetime has been determined from a maximum-likelihood fit to the distribution of the impact parameter δ in the $r\phi$ plane of the lepton candidates. The impact parameter distribution for the lepton candidates has been expressed as a convolution of two functions: the true impact parameter distribution, which depends on the lifetime of b or c hadrons, and the resolution function, which describes the smearing of δ due to finite resolution. The following likelihood function

$$\begin{split} \mathscr{L}(\boldsymbol{\tau}_{\mathrm{b}}) &= \prod_{i}^{\mathrm{tracks}} \left(f_{\mathrm{b}} \mathscr{P}_{\mathrm{b}}(\boldsymbol{\tau}_{\mathrm{b}}; \boldsymbol{\delta}_{i}) + f_{\mathrm{bc}} \mathscr{P}_{\mathrm{bc}}(\boldsymbol{\tau}_{\mathrm{b}}; \boldsymbol{\delta}_{i}) \right. \\ &+ f_{\mathrm{c}} \mathscr{P}_{\mathrm{c}}(\boldsymbol{\delta}_{i}) + f_{\mathrm{bkg}} \mathscr{B}(\boldsymbol{\delta}_{i}) \Big) \\ & \otimes \mathscr{R}(\boldsymbol{\delta}_{i}, \boldsymbol{\phi}_{i}, \boldsymbol{p}_{\perp}, \boldsymbol{\sigma}_{\mathrm{FIT}, i}) \end{split}$$

is maximized. $\mathscr{P}_{\rm b}$, $\mathscr{P}_{\rm bc}$, and $\mathscr{P}_{\rm c}$ are the impact parameter distributions for the prompt, cascade and charm components. They included the effects of lifetimes and the occasional mis-signing of δ , due to the approximation of the b-hadron direction by the jet axis. The underlying distributions $\mathscr{P}_{\rm b}$, $\mathscr{P}_{\rm bc}$, and $\mathscr{P}_{\rm c}$ and the one for background tracks, \mathscr{R} , are extracted from Monte Carlo simulated events. \mathscr{R} , the track-by-track impact parameter resolution function, is parametrized as a superposition of three Gaussian functions whose standard deviations depend on the track azimuthal angle ϕ_i , the momentum p_{\perp} transverse to the beam, and the impact parameter error from the track fit $\sigma_{\rm EUT}$, i:

$$\mathcal{R}(\delta_i, \phi_i, p_{\perp}, \sigma_{\text{FIT}, i}) = \sum_{j=1}^3 \frac{f_j}{\sqrt{2\pi} \sigma_{ij}} \exp\left(-\frac{\delta_i^2}{2\sigma_{ij}^2}\right)$$

with $\sum_{j=1}^3 f_j = 1$,

as described in our publication [2]. σ_{ij} denotes the impact parameter resolution for the *i*th track.

In order to reduce systematic uncertainties which would otherwise limit the increase in statistical accuracy compared to our previous publication additional free parameters have been introduced in the maximum-likelihood fit. The relevant changes are:

- The parameters f_j have been left free. These fractions are not supposed to be similar for primary and secondary tracks, because they follow different distributions in phase space. Monte Carlo studies show that $f_1 \approx 1$, $f_2 \approx 0$ for secondaries while $f_1 \approx 0.8$, $f_2 \approx 0.2$ for the sample enriched in primary tracks.
- The background fraction, f_{bkg} , is also treated as a free parameter. The background contribution \mathscr{B} is almost invariant under the change $\delta \leftrightarrow -\delta$, which can be disentangled from the asymmetric shape of \mathscr{P}_{b} . This approach reduces the systematic error, in particular those caused by uncertain-

ties in the parameters R_b and $Br(b \rightarrow \ell \nu X)$, by a sizeable amount.

The fits are performed separately for the four years of data taking, to account for changes in detector setup and performance. The results for 1993 and 1994 are shown in Fig. 5. Due to the SMD the resolution has improved significantly in 1994.

The different sources of systematic errors can be found in Table 1. Theoretical uncertainties have been assigned following the recommendations of the LEP Heavy Flavour Electroweak Group in [12]. We observed small dependences on the Monte Carlo parameters that govern the heavy-flavour production and decay fractions. The error due to uncertainties in the shape of the underlying distributions \mathscr{P} , mainly caused by Monte Carlo statistics, is combined under fit method error.

The fitted lifetimes obtained for the four data taking periods and two lepton channels agree with each other:

$$\tau_{\rm h}({\rm e}) = (1530 \pm 23 \pm 23) \,{\rm fs},$$

 $\tau_{\rm b}(\mu) = (1553 \pm 22 \pm 23)$ fs.

Taking into account the correlation between uncertainties in the muon and the electron channel, the combined result from the lepton impact parameter method is:

 $\tau_{\rm h} = (1544 \pm 16 \pm 21) \, {\rm fs}$.

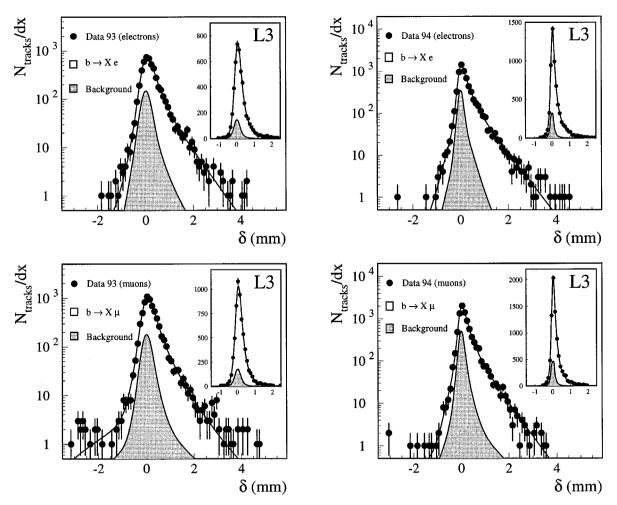


Fig. 5. Results of the maximum-likelihood fits for the lepton impact parameter distribution in 1993 and 1994. Both logarithmic and linear scales are shown.

5. Combined results

The results from the secondary vertex and lepton analysis is combined as weighted average of nine measurements. The measurements with electrons and muons in the years 1991 to 1994 are added to the result from the secondary vertex analysis in 1994 taking into account the correlations between the measurements. The statistical correlation between the lepton and secondary vertex measurement is negligible. The dominant systematic errors, the uncertainty in the underlying lepton distributions and the lepton track selection criteria on the one side and the charm hadron composition on the other side, are also independent in both measurements. The combined result for the average b-hadron lifetime at the Z resonance is

 $\tau_{\rm b} = (1549 \pm 9 \pm 15) \, {\rm fs}$,

where the first error is statistical and the second is systematic. The result is in good agreement with our previous measurements of $\tau_{\rm b}$ and with the world average of 1550 ± 20 fs [11].

The secondary vertex analysis yields in addition a measurement of the average b-decay multiplicity and the normalized average b-energy at LEP:

$$\langle n_{\rm b} \rangle = 4.90 \pm 0.12, \quad \langle x_{\rm F} \rangle_{\rm b} = 0.709 \pm 0.004.$$

These precise results are improvements over the other measurements [12,15]. The uncertainty in $\langle x_E \rangle_b$ reached by the novel method presented here is significantly lower than in previous analyses.

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