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Optical velocimetry based on the spatial correlation of off-axis image speckle patterns

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Abstract

A method is proposed for measuring the linear velocity of a moving rough surface. It is based on maximizing the spatial correlation between two laterally shifted image speckle patterns. Besides, an expression of a spatially variant point-spread function is given, which modifies the spatial distributions of the off-axis image speckle patterns.

Keywords: Speckle; Velocimetry; Image formation

1. Introduction

The finite 'mean life' of the speckle pattern is a limitation to be considered for studying moving rough surfaces. For the case of a plane object describing a linear motion, the speckle pattern originating from the surface under coherent illumination encodes information about velocity, roughness, texture, etc. In order to obtain a real-time measurement of such surface parameters, several methods were developed where the speckle intensity distribution is detected and processed following a spatial or a temporal approach [1–13]. In most cases, the speckle pattern is analogically or digitally integrated for performing a correlation operation between successive signals from which the magnitude under study can be derived. The condition to be fulfilled to achieve a reliable measurement becomes: $a/v \gg \tau$, being a the spot diameter of the illuminating beam, v the surface velocity, and τ the detection time, which depends on the detector response, the laser intensity and the optical configuration. However, if the surface velocity is large enough so that the above condition does not fully accomplish, each speckle pattern to be correlated becomes 'blurred', so resulting in a loss of measuring accuracy. Therefore, it is worthwhile for measurement purposes to investigate optical arrangements that allow to record and process high contrast speckle patterns originated from such moving surfaces.

In this paper, we analyze a method where a laser beam is linearly scanned in the same direction of the moving surface. The light scattered from the surface at the different locations of each scan gives rise to laterally shifted speckle patterns, which are uncorrelated unless two conditions are fulfilled: (i) the scanning rate of the laser beam matches the surface velocity, and (ii) the angle of incidence of the laser beam remains constant over the whole scanning region. Therefore, the proposed optical arrangement is designed to satisfy these two conditions in order to achieve a measurement of a spatial correlation pa-

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rameter between two speckle patterns, from which the surface velocity can be derived. As the employed optical system images on a photodetector the laterally shifted speckle patterns to be correlated, an additional effect should be considered; namely, a spatially variant point-spread function that modifies the measured correlation parameter.

In Section 2, we describe the principle of the method. Then, in Section 3, we derive a general expression for the off-axis spatially variant point-spread function and a particular case is analyzed: a system affected by distortion, which is one of the dominant off-axis aberrations. Finally, in Section 4, the experimental conditions and the range of operation of the method are discussed and some experimental results are presented.

2. Description of the method

By referring to Fig. 1, a scanning laser beam illuminates the moving surface S under study along the x-axis with constant velocity and angle of incidence. This illumination condition is provided by a K–theta lens system [14,15], which consists of a two-element lens that originates an emerging beam parallel to its optical axis, at a distance proportional to the product of the focal length and the angle made by the incident beam. This beam emanates from the front focal point of the lens where a rotating polygon mirror is placed.

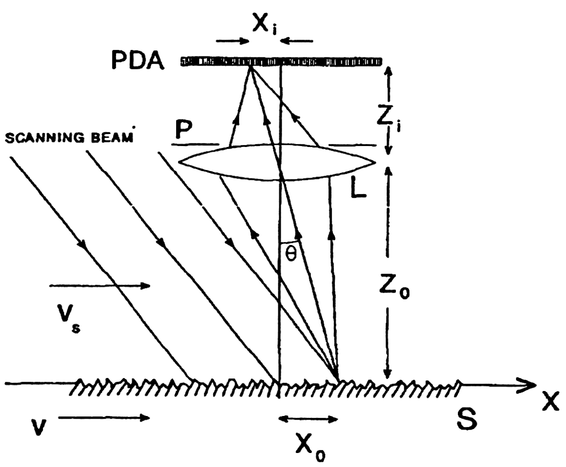

Fig. 1. Experimental setup.

For a certain beam location x_0, the light scattered from S is collected by the optical system L to form an image speckle pattern, centered at $x = x_i = mx_0$, being m the lateral magnification. The intensity distribution recorded by the linear photodiode array PDA is given by the imaging relationship

$$I(x;\, x_i) = \int_{-\infty}^{\infty} p_\theta(x;\, x')\, p_\theta^*(x;\, x'') \times R_s(x';\, x'')\, \mathrm{d}x'\, \mathrm{d}x'', \qquad (1)$$

where $R_s(x';\, x'')$ is a function which depends on the statistical properties of S, and $p_\theta(x;\, x')$ is the point-spread function corresponding to the coordinate $x_i = mz_0 \operatorname{tg}\theta$, being z_0 the distance between L and S.

In a previous work [9], a temporal correlation parameter was defined,

$$\Gamma = 1 - K \sum_{j=1}^{N} |I(x_j;\, t) - I(x_j;\, t+\tau)|, \qquad (2)$$

for the case in which a stationary laser beam illuminates, on-axis, the moving surface. In Eq. (2), $I(x_j;\, t)$ and $I(x_j;\, t+\tau)$ denote the successive (separated by τ seconds) detected intensities by a linear N-photodiode array, and K is a normalization constant. The illuminated zone of the surface changes between two data samples and, consequently, Γ drops as the surface velocity increases. Therefore, a transference curve $\Gamma = \Gamma(v)$ is obtained from which the velocity v can be derived. However, the measuring range using this approach is limited to velocities up to 1 m/s.

For the optical arrangement shown in Fig. 1, we rewrite the parameter Γ given by Eq. (2) in the form of a spatial correlation

$$\Gamma_i = 1 - K \sum_{j=1}^{N} |I(x_j;\, 0) - I(x_j - x_i;\, x_i)|, \qquad (3)$$

where $I(x_j;\, 0)$ and $I(x_j - x_i;\, x_i)$ are the speckle intensities corresponding to the on-axis beam location ($x_i = x_0 = 0$), and to an off-axis situation ($x_i \neq 0$), respectively. The value of x_i should be selected large enough to avoid overlapping between both speckle intensity distributions to be correlated. Taking into account Eq. (3), the relationship $\Gamma_i =$

$\Gamma(x_i)$ defines a curve where Γ diminishes as the value of the coordinate x_i increases. The slope of this curve depends on the difference between the laser scanning velocity v_s and the surface velocity v, reaching a minimum value when the condition $v_s = v$ is met. Under this situation, the illuminated area S remains unchanged over the whole scan, so originating maximum correlation between the on-axis and any of the off-axis image speckle patterns. Thus, for performing a measurement of the surface velocity, the scanning rate of the laser beam (i.e. the scanning velocity v_s) should be varied to obtain the above mentioned condition.

The measuring procedure can be summarized in the following steps: (i) for a given value of v_s, the on-axis ($x_i = 0$) image speckle pattern is recorded and stored; (ii) an off-axis (centered at $x_i \neq 0$) image speckle pattern, corresponding to the same scan, is also recorded and stored. For obtaining maximum measurement sensitivity, x_i is chosen at the extreme of PDA: $x_i = x_{i,\max}$. (iii) The correlation parameter $\Gamma(x_{i,\max})$ is calculated by means of Eq. (3). (iv) The scanning velocity v_s is varied and the steps (i) to (iii) repeated until $\Gamma(x_{i,\max})$ is maximized, and so, $v_s = v$.

In the above description, the maximum attainable value of $\Gamma(x_i)$ never reaches the ideal value of unity due to the variations of the point-spread function of the imaging lens L as the illuminated region of S moves away from the optical axis. In order to give an explanation of this effect, we derive in the following section an expression for this spatially variant point-spread function.

3. Off-axis point-spread function

For simplicity, we perform a one-dimensional treatment. If we consider the line joining the object source point x_0 with the corresponding image point x_i as a new optical axis z', the point-spread function $p(x;\ \theta)$ can be thought as resulting from the Fraunhofer diffraction pattern of a 'tilted' aperture given by the pupil function $P(\xi)$ of the imaging lens L (see Fig. 1). For the particular on-axis case ($x_0 = 0$, and so $z' = z$), the well-known Fourier transform relationship,

$$p(x;\ \theta=0) = \exp\left(\frac{i\pi x^2}{\lambda z_i}\right)\int_{-\infty}^{\infty} P(\xi) \times \exp\left(-\frac{2\pi i x\xi}{\lambda z_i}\right) d\xi, \tag{4}$$

is obtained where $z_i = m z_0$ is the image distance. For the general off-axis case ($x_0 \neq 0$), the amplitude distribution $p(x';\ \theta)$, along a tilted axis x', orthogonal to z', centered at the image point x_i, is still given by the Fourier transform of $P(\xi)$ but in a new set of spatial frequencies which depends on the value of θ [16]. Therefore, all the properties of the Fourier transform can be straightforwardly reformulated for the off-axis case, but calculated in a modified coordinate system. By using the result found in Ref. [16] for the Fraunhofer pattern diffracted by a tilted aperture, we obtain the following expression for the off-axis point-spread function:

$$p(x';\ \theta) = \int_{-\infty}^{\infty} P(\xi)\ \exp[-2\pi i \alpha(x')\xi]\ d\xi, \tag{5}$$

where the spatial frequency $\alpha(x')$ becomes

$$\alpha(x') = \frac{\cos\theta\ x'}{\lambda r_i} - \frac{\sin\theta\ x'^2}{2\lambda r_i^2}, \tag{6}$$

being $r_i = z_i/\cos\theta$ the distance between the center of the exit pupil of L and the image point. It should be noted that a quadratic phase term has been neglected in Eq. (5). Hence, the image formation of an off-axis point source can be stated as follows. The Fourier transform which relates the impulse response with the pupil function remains valid but the coordinate axis (i.e. the spatial frequency) changes accordingly with Eq. (6). However, since we are interested to process image information in a fixed recording plane, parallel to the x-axis, Eq. (6) is rewritten as

$$\alpha(x) = \frac{\cos\theta\ x}{\lambda z_i} - \frac{\sin\theta\ x^2}{2\lambda z_i^2} = \alpha_0\left[\cos\theta - \frac{\sin\theta\ x}{2 z_i}\right], \tag{7}$$

where $\alpha_0 = x/\lambda z_i$ is the spatial frequency coordinate for the on-axis case. Thus, the point-spread function becomes

$$p(x;\theta) = \int_{-\infty}^{\infty} P(\xi)\exp[-2\pi i\alpha(x)\xi]\,d\xi. \tag{8}$$

Next, we analyze the effect of the impulse response, given by Eq. (8), on the image speckle pattern assuming a pupil function $P(\xi)$ of the form

$$P(\xi) = P_0(\xi)\exp\left[\frac{2\pi i}{\lambda}\frac{W_d}{a}\xi\right], \tag{9}$$

where $P_0(\xi)$ represents a uniform rectangular aperture of width $2a$, and W_d is the Seidel distortion coefficient measured in units of the wavelength λ. By employing Eqs. (7), (8) and (9), the intensity impulse response results in

$$|p(x;\theta)|^2 = (2a)^2\,\mathrm{sinc}^2\left[2a\left(\frac{\cos\theta}{\lambda z_i}x - \frac{W_d}{a\lambda} - \frac{\sin\theta}{2\lambda z_i^2}x^2\right)\right]. \tag{10}$$

As in the on-axis, free aberration case, where the mean width of the point-spread function is $\Delta x_0 = \lambda z_i/a$, in the off-axis situation the corresponding mean width can be derived from Eq. (10) to give

$$\Delta x \simeq \frac{\lambda z_i}{a\cos\theta}\left(1 + \frac{2W_d}{\lambda}\right). \tag{11}$$

Therefore, for increasing values of the angle θ, the extent where the impulse response is significantly different from zero increases proportional to both, $1/\cos\theta$ and the amount of distortion. The effect of other lens aberrations, such as spherical aberration or astigmatism, will further modify the shape of the point-spread function.

An alternative approach to space-variant image formation (by analyzing the effects of shift-variance on the decay of the modulation transfer function of coherent systems) was performed by Tichenor and Goodman [17], who concluded that the isoplanatic region is restricted to object points lying inside a circle of about 1/4 of the aperture radius.

4. Experimental results and discussion

In order to experimentally verify the method proposed in Section 2, we employed the optical setup shown in Fig. 1. A rotating polygon, illuminated by an unexpanded laser beam at the location of the front focal point of a K–theta lens, produces a uniform beam scanning along a direction that coincides with that of the moving surface S. Taking into account the finite length of the linear photodiode array PDA: $l^{(i)} = N\Delta_0$, N being the number of photodiodes and Δ_0 the separation between adjacent photodiodes, a linear illuminated region of S given by $l^{(0)} = l^{(i)}/m$ can be imaged on PDA. In our case, $N = 2048$, $\Delta_0 = 13$ μm, and L is a one-element positive lens of focal length $f = 60$ mm (for which $m = z_i/z_0 =$ 90 mm/180 mm = 0.5), so resulting in $l^{(0)} = 53$ mm and $\theta_{max} \simeq 16°$. On the other hand, at a given instant, the laser spot illuminates a region of S having a diameter $\delta x^{(0)} \simeq 2$ mm, producing an image speckle pattern at PDA with a spatial extent $\delta x^{(i)} = m\,\delta x^{(0)} \simeq 1$ mm. As it was explained in Section 2, the distance x_i between the on-axis and a certain off-axis image speckle pattern should be larger than the extent of each individual speckle distribution for avoiding overlapping when Eq. (3) is used. Thus, in our case, the minimum value of x_i results in $x_{i,min} \simeq \delta x^{(i)} \simeq 1$ mm, and of course the maximum attainable value becomes $x_{i,max} \simeq l^{(i)}/2 = 13$ mm. Besides, as the speckle pattern should be resolved by PDA, each individual speckle grain, having a mean size $\Delta x_i \simeq \lambda z_i/a$ is sampled by more than one photodiode (e.g. 4–5 photodiodes). As $\lambda = 0.633$ μm and $z_i = 90$ mm, this condition requires that the aperture radius becomes $a \simeq 2$ mm. If the detection time of PDA is $\tau = 1$ ms, then the time required to detect a given image speckle pattern becomes $\tau' = \tau/(l^{(i)}/\delta x^{(i)}) = 38$ μs. Thus, for recording a high contrast image speckle pattern, $\tau' < \delta x^{(0)}/v$ should be satisfied, from which the surface velocity to be measured is limited to the range $v < 50$ m/s.

In order to evaluate the influence of both the space-variance and the distortion aberration effects on the image speckle patterns to be correlated, some computer simulations were carried out, which illustrate the results found in Eqs. (10) and (11). As can be derived from Fig. 2, the correlation degree be-

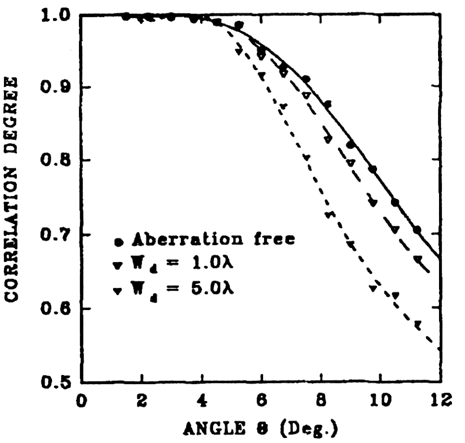

Fig. 2. Influence of space-variance and distortion aberration on the speckle correlation degree.

tween the on-axis ($x_i = 0$) and the successive off-axis ($x_i \neq 0$) image speckle patterns remains almost unchanged for values of $\theta \leq 4°$. For larger values of θ, the correlation degree drops, this effect being more noticeable for increasing W_d.

In Fig. 3, the parameter Γ is plotted for different values of x_i. Each measurement is obtained by performing the spatial correlation between the on-axis image speckle pattern and a given off-axis image speckle pattern, centered at x_i, as is defined in Eq. (3). These results were obtained while both the

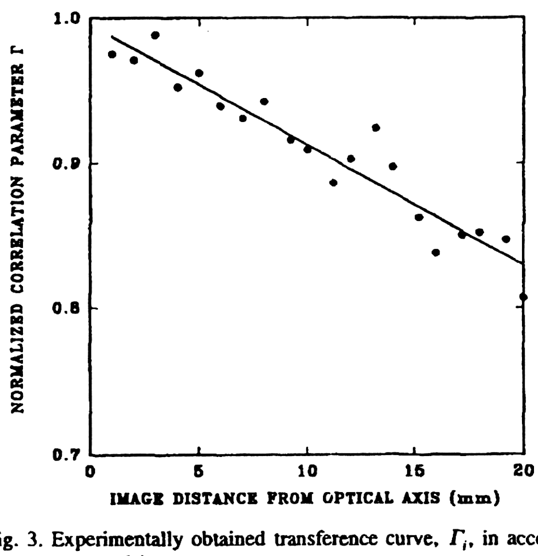

Fig. 3. Experimentally obtained transference curve, Γ_i, in accordance with Eq. (3).

illuminating beam and the portion of the surface S that originates each speckle pattern, remained unchanged. The correlation operation is properly normalized in order to remove the vignetting effect. As can be expected, the speckle correlation parameter diminishes when the illuminated surface is displaced out of the optical axis. However, as was explained in Section 2, in order to achieve the maximum measuring range, the value $\Gamma = \Gamma(x_{i,\max} \cong l^{(i)}/2)$ should be chosen as the reference value to be obtained when the laser scanning velocity v_s is varied to match the surface velocity v.

Next, we analyze the performance of the optical system when surface movements in directions other than the x-axis are considered. If the surface displaces along the z-axis, the condition to be fulfilled for obtaining a reliable velocity measurement along the x-axis is that the 'defocus' effect produced in the image speckle pattern be small enough so as not to modify the correlation parameter Γ. Since the average speckle size along the z-axis becomes $\Delta z_i \cong \lambda(z_i/a)^2 \cong 1.3$ mm, the tolerance for the maximum surface displacement is $\Delta z^{(0)}_{\max} = \Delta z_i/m^2 \cong 5.1$ mm. However, in the lateral direction (y-axis), each individual speckle has an average size $\Delta y_i \cong \lambda z_i/a \cong 60$ μm, so resulting in the maximum allowed displacement $\Delta y^{(0)}_{\max} = \Delta y_i/m \cong 0.12$ mm. Taking into account these restrictions, for practical applications such as in-line monitoring moving strip-type flat rolled products, the measuring system should be properly located close to the contact region cylinder/moving-surface, in order to minimize motions along the y- and z-directions.

Regarding the measurement time, by considering the aquisition time of the signal detected by PDA ($\tau = 1$ ms), the algorithm employed for obtaining the correlation parameter, and an averaging process, a measuring rate of 20 s^{-1} can be achieved.

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