Sensitivities of one-prong tau branching fractions to tau neutrino mass, mixing, and anomalous charged current couplings

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Maria Teresa Dova\textsuperscript{a}, John Swain\textsuperscript{b} and Lucas Taylor\textsuperscript{b}

\textsuperscript{a}Universidad Nacional de La Plata, La Plata, Argentina
\textsuperscript{b}Department of Physics, Northeastern University, Boston, MA02115, USA

We analyse the sensitivities of exclusive one-prong tau branching fractions to: the tau neutrino mass; its mixing with a fourth generation neutrino; the weak charged current magnetic and electric dipole moments of the tau; and the Michel parameter \( \eta \). Quantitative constraints are derived from current experimental data and the future constraints derivable from tau-charm factory measurements are estimated. The anomalous coupling constraints are used to constrain the tau compositeness scale and the allowed parameter space for Higgs doublet models.

1. INTRODUCTION

We analyse the sensitivity to new physics of the \( \tau \) partial widths for the following decays\textsuperscript{1}: \( \tau^- \to e^- \bar{\nu}_e \nu_\tau \), \( \tau^- \to \mu^- \bar{\nu}_\mu \nu_\tau \), \( \tau^- \to \pi^- \nu_\tau \), and \( \tau^- \to K^- \nu_\tau \). We determine constraints on the mass \( m_{\nu_3} \) of the third generation neutrino \( \nu_3 \), its mixing with a fourth generation neutrino \( \nu_4 \) of mass \( > M_Z/2 \), anomalous weak charged current magnetic and electric dipole couplings \[ 1, 2 \], and the Michel parameter \( \eta \) \[ 3 \]. In each case, we present quantitative results using current experimental data (which update our previous analyses \[ 1, 2, 3 \]) and estimate the future constraints which would be achievable using the expected precision of measurements at a tau-charm factory. The results for the \( \eta \) parameter are used to constrain extensions of the Standard Model which contain more than one Higgs doublet and hence charged Higgs bosons.

2. THEORETICAL PREDICTIONS

2.1. Tau neutrino mass and mixing

The theoretical predictions for the branching fractions \( B_\ell \) allowing for the \( \nu_\tau \) mass and mixing with a fourth lepton generation are given by \[ 4 \]:

\[
B_\ell^{\text{th.}} = \frac{G_F^2 m_\tau^5 \tau_\ell}{192 \pi^3} \left( 1 - 8x - 12x^2 \ln x + 8x^3 - x^4 \right) \\
\times \left[ \left( 1 - \frac{\alpha(\tau_\ell)}{2\pi} \left( \frac{x^2}{4} - \frac{25}{4} \right) \right) \left( 1 + \frac{3m_\tau^2}{5m_W^2} \right) \right] \\
\times [1 - \sin^2 \theta \left[ 1 - 8y(1 - x)^3 + \cdots \right] \right] \tag{1}
\]

where \( x = m_\ell^2/m_\tau^2 \), \( y = m_\nu_3^2/m_\tau^2 \), \( G_F = (1.16639 \pm 0.00002) \times 10^{-5} \text{GeV}^{-2} \) is the Fermi constant \[ 4 \], and \( \tau_\ell \) is the tau lifetime. The tau mass, \( m_\tau \), is taken only from production measurements at tau-pair threshold since values derived from kinematic reconstruction of tau decays depend on tau neutrino mass. The first term in square brackets allows for radiative corrections\[ 5, 6, 7, 8 \], where \( \alpha(\tau_\ell) \simeq 1/133.3 \) is the QED coupling constant \[ 7 \] and \( m_W = 80.41 \pm 0.10 \text{GeV} \) is the W mass \[ 9 \].

The tau neutrino weak eigenstate is given by the superposition of two mass eigenstates \( |\nu_\ell\rangle = \cos \theta |\nu_3\rangle + \sin \theta |\nu_4\rangle \), such that the mixing is parametrised by the Cabibbo-like mixing angle \( \theta \). The second term in square brackets describes mixing with a fourth generation neutrino which, being kinematically forbidden, causes a suppression of the decay rate. The third term in brackets parametrises the suppression due to a non-zero mass of \( \nu_3 \), where the ellipsis denotes negligible higher order terms \[ 4 \].

\textsuperscript{1}Throughout this paper the charge-conjugate decays are also implied. We denote the branching ratios for these processes as \( B_\ell, B_\mu, B_\pi, B_K \) respectively; \( B_\text{th} \) denotes either \( B_\ell, B_\mu, B_\pi, B_K \).
The branching fractions for the decays $\tau^- \rightarrow h^- \nu_\tau$, with $h = \pi/K$, are given by \[ B^h_{\ell} = \left( \frac{G_F^2 m_\ell^3}{16 \pi^2} \right) \tau f_\ell^2 |V_{\alpha \beta}|^2 (1 - x)^2 \times \left[ 1 + \frac{2\alpha}{\pi} \ln \left( \frac{m_Z}{m_\tau} \right) + \cdots \right] [1 - \sin^2 \theta] \times \left[ 1 - y \left( \frac{2 + x - y}{1 - x} \right) \left( 1 - y \left( 2 + 2x - y \right) \left( 1 - x \right)^2 \right)^{\frac{3}{2}} \right] (2) \]

where $x = m_\nu^2 / m_\tau^2$, $m_\ell$ is the hadron mass, $f_\ell$ are the hadronic form factors, and $V_{\alpha \beta}$ are the CKM matrix elements. $V_{ud}$ and $V_{us}$, for $\pi^- \rightarrow K^-$ respectively. The quantities $f_\pi, f_K$ are obtained from analyses of $\pi^- \rightarrow \mu^- \bar{\nu}_\mu$ and $K^- \rightarrow \mu^- \bar{\nu}_\mu$ decays \[ [23 \text{ and references therein}]. \]

The first term in square brackets describes mixing with a fourth generation neutrino while the second parametrizes the effects of a non-zero $m_{\nu_4}$.

The fourth generation neutrino mixing affects all the tau branching fractions with a common factor whereas a non-zero tau neutrino mass affects all channels with different kinematic factors. Therefore, given sufficient experimental precision, these two effects could in principle be separated.

Analyses which determine the tau mass from a kinematic reconstruction of the tau decay products are also sensitive to tau neutrino mass. For example, from an analysis of $\tau^+ \tau^- \rightarrow (\pi^+ n \pi^0 \nu_\tau)$ ($\pi^- m \pi^0 \nu_\tau$) events (with $n \leq 2, m \leq 2.1 \leq n + m \leq 3$), CLEO determined the mass to be $m_\tau = (1777.8 \pm 0.7 \pm 1.7) + |m_{\nu_4}(\text{MeV})|^2/1400$ MeV \[ [4 \text{]. Such measurements may be used to further constrain } m_{\nu_4} \].

### 2.2. Anomalous couplings

The theoretical predictions for the branching fractions $B_\ell$ for the decay $\tau^- \rightarrow \ell^- \bar{\nu}_\ell \nu_\tau (X_{EM})$, with $\ell^- = e^- , \mu$, and $X_{EM} = \gamma, \gamma \gamma, e^+ e^- , \ldots$, are given by:

\[ B^h_{\ell} = \left( \frac{G_F^2 m_\ell^5 \tau^3}{192 \pi^3} \right) (1 - 8x - 12x^2 \ln x + 8x^3 - x^4) \times \left( 1 - \frac{\alpha(m_\tau)}{2\pi} \left( \frac{\pi^2 - 25}{4} \right) \right) \left( 1 + \frac{3}{5} \frac{m_\ell^2}{m_W} \right) \times \left[ 1 + \Delta_\ell \right] (3) \]

The term in square brackets describes the effects of new physics where the various $\Delta_\ell$ we consider are defined below.

The effects of anomalous weak charged current dipole moment couplings at the $\tau \nu_\tau W^\pm$ vertex are described by the effective Lagrangian \[ \mathcal{L} = \frac{g}{\sqrt{2}} \left[ \gamma_\mu + \frac{i\sigma_{\mu\nu}q^\nu}{2m_\tau}(\kappa_\tau - i\bar{\kappa}_\gamma) \right] P_L \nu_\tau W_\mu \]

\[ + \text{(Hermitian conjugate)} \]

where $P_L$ is the left-handed projection operator and the parameters $\kappa$ and $\bar{\kappa}$ are the (CP-conserving) magnetic and (CP-violating) electric dipole form factors respectively \[ [1 \text{]. They are the charged current analogues of the weak neutral current dipole moments, measured using $Z \rightarrow \tau^+ \tau^-$ events \[ [4 \text{}, and the electromagnetic dipole moments \[ [4 23 \text{ recently measured by L3 and OPAL using } Z \rightarrow \tau^+ \tau^- \gamma \text{ events \[ [18 19 23 \text{. In conjunction with Eq. } 3 \text{ the effects of non-zero values of } \kappa \text{ and } \bar{\kappa} \text{ on the tau leptonic branching fractions may be described by } \Delta_{\ell_\tau} \].}

\[ \Delta_{\ell_\gamma} = \frac{\kappa}{2} + \frac{\bar{\kappa}^2}{10}; \]

\[ \Delta_{\ell_\gamma} = \frac{\bar{\kappa}^2}{10}. \]

The dependence of the tau leptonic branching ratios on $\eta$ is given, in conjunction with Eq. 3 by \[ [24 \text{]}

\[ \Delta_{\ell_\eta} = 4\eta_{\ell_\tau} \sqrt{\eta}. \]

where the subscripts on $\eta$ denote the initial and final state charged leptons. Both leptonic tau decay modes probe the charged current couplings of the transverse $W$, and are sensitive to $\kappa$ and $\bar{\kappa}$. In contrast, only the $\tau^- \rightarrow \mu^- \bar{\nu}_\mu \nu_\tau$ channel is sensitive to $\eta$, due to a relative suppression factor of $m_e / m_\mu$ for the $\tau^- \rightarrow e^- \bar{\nu}_e \nu_\tau$ channel. Semi-leptonic tau branching fractions are not considered since they are insensitive to $\kappa$, $\bar{\kappa}$, and $\eta$.

### 3. RESULTS

Three sets of fits are performed, as follows.

- **Case 1**
  - We use current world averages of the experimental measurements.
• **Case 2**  
  We use estimated errors on measurements which would be possible with a tau-charm factory assuming that there is no improvement in the tau lifetime compared to current measurements.

• **Case 3**  
  This is identical to Case 2 except that, in order to assess the limiting factors of our method, we assume somewhat arbitrarily that CLEO and the b-factories succeed in reducing the tau-lifetime error by a factor of two.

For Cases 2 and 3 the central values are clearly unknown, therefore in making our predictions we adjust the branching fractions to their standard model values, such that our predictions is not arbitrarily biased by the current experimental central values. The input parameters for the three cases are summarised in Tab. 1.

Table 1  
Input parameters used in the determinations of \(m_{\nu_\tau}, \sin^2 \theta, \kappa, \bar{\kappa}, \text{and } \eta_{\tau\mu}\).

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Future Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>(m_\tau) (MeV)</td>
<td>1776.96^{+0.31}_{-0.27}</td>
<td>0.1 [23] 0.1 [24]</td>
</tr>
<tr>
<td>(\tau_\tau) (fs)</td>
<td>290.5 ± 1.0</td>
<td>1.0 [23] 0.5 [24]</td>
</tr>
<tr>
<td>(B_{\tau} (%))</td>
<td>17.81 ± 0.06</td>
<td>0.018 [23] 0.018 [24]</td>
</tr>
<tr>
<td>(B_{\mu} (%))</td>
<td>17.36 ± 0.06</td>
<td>0.017 [23] 0.017 [24]</td>
</tr>
<tr>
<td>(B_{e} (%))</td>
<td>11.08 ± 0.13</td>
<td>0.011 [23] 0.011 [24]</td>
</tr>
<tr>
<td>(B_{K} (%))</td>
<td>0.695 ± 0.026</td>
<td>0.003 [23] 0.003 [24]</td>
</tr>
</tbody>
</table>

We derive constraints on \(m_{\nu_\tau}\) and \(\sin^2 \theta\) from combined likelihood fits to the four tau decay channels, using equations 1 and 2. The likelihood for the CLEO and BES measurements of \(m_\tau\) to agree, as a function of \(m_{\nu_\tau}\), is included in the global likelihood. We derive constraints on \(\kappa, \bar{\kappa}, \text{and } \eta_{\tau\mu}\) using the two leptonic tau decay channels and Eq. 3. Each of the five parameters is analysed separately, conservatively assuming in each case that the other four parameters are zero.

In the fit, the uncertainties on all the quantities in Eqs. 1, 2 and 3 are taken into account. The likelihood is constructed numerically following the procedure of Ref. 27 by randomly sampling all the quantities used according to their errors.

Tab. 2 summarises the results obtained. For Cases 2 and 3 the limiting error is that on the tau lifetime; arbitrarily setting all other errors to zero yields negligible improvement in the fit results.

4. DISCUSSION

4.1. Tau neutrino mass

The limit on \(m_{\nu_\tau}\) can be reasonably interpreted as a limit on \(m_{\nu_\tau}\), since \(\sin^2 \theta\) is small as well as the mixing of \(m_{\nu_\tau}\) with lighter neutrinos. The best direct experimental constraint on the tau neutrino mass is \(m_{\nu_\tau} < 18.2\,\text{MeV}\) at the 95% confidence level, which was obtained using many-body hadronic decays of the \(\tau\). While our constraint is less stringent, it is statistically independent. Moreover, it is insensitive to fortuitous or pathological events close to the kinematic limits, the absolute energy scale of the detectors, and the details of the resonant structure of multi-hadron \(\tau\) decays.

Although the constraint on \(m_{\nu_\tau}\), which we estimate does improve with the tau-charm input, this method would not be competitive with direct reconstruction analyses which are predicted to be sensitive at the \(O(2\,\text{MeV})\) level.

4.2. Fourth generation mixing

Our upper limit on \(\sin^2 \theta\) is already the most stringent experimental constraint on mixing of the third and fourth neutrino generations. This constraint will improve by a factor of up to two using future tau-charm factory data, depending on the improvement in the error on \(\tau_\tau\). We anticipate that this technique will continue to provide the most stringent constraints in the foreseeable future.

4.3. Anomalous couplings and tau compositeness

Our results for \(\kappa\) and \(\bar{\kappa}\) are currently the most precise. The less stringent constraint on \(\bar{\kappa}\) compared to that on \(\kappa\) is due to the lack of linear
Table 2
Constraints on $m_{\nu_3}$, $\sin^2 \theta$, $\kappa$, $\tilde{\kappa}$, and $\eta_{\tau \mu}$ at the 95% confidence level.

<table>
<thead>
<tr>
<th>Case 1</th>
<th>Case 2</th>
<th>Case 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>$m_{\nu_3} &lt; 36$ MeV</td>
<td>$m_{\nu_3} &lt; 34$ MeV</td>
<td>$m_{\nu_3} &lt; 28$ MeV</td>
</tr>
<tr>
<td>$\sin^2 \theta &lt; 0.0053$</td>
<td>$\sin^2 \theta &lt; 0.0039$</td>
<td>$\sin^2 \theta &lt; 0.0024$</td>
</tr>
<tr>
<td>$-0.011 &lt; \kappa &lt; 0.017$</td>
<td>$-0.011 &lt; \kappa &lt; 0.009$</td>
<td>$-0.006 &lt; \kappa &lt; 0.005$</td>
</tr>
<tr>
<td>$</td>
<td>\tilde{\kappa}</td>
<td>&lt; 0.26$</td>
</tr>
<tr>
<td>$-0.030 &lt; \eta_{\tau \mu} &lt; 0.052$</td>
<td>$-0.030 &lt; \eta_{\tau \mu} &lt; 0.029$</td>
<td>$-0.017 &lt; \eta_{\tau \mu} &lt; 0.016$</td>
</tr>
</tbody>
</table>

terms in Eq. 4.

Derivative couplings necessarily involve the introduction of a length or mass scale. Anomalous magnetic moments due to compositeness are expected to be of order $m_\tau / \Lambda$ where $\Lambda$ is the compositeness scale [13]. We can then interpret the 95% confidence level on $\kappa$, the quantity for which we have a more stringent bound, as a statement that the $\tau$ appears to be a point-like Dirac particle up to an energy scale of $\Lambda \approx m_\tau / 0.017 = 105$ GeV. These results are comparable to those obtained from anomalous weak neutral current couplings [32] and more stringent than those obtained for anomalous electromagnetic couplings [13, 19, 21]. While the decay $W \to \tau \nu$ which is measured at LEP II is also sensitive to charged current dipole terms, given that the energy scale is $m_W$, the interpretation in terms of the static properties $\kappa$ and $\tilde{\kappa}$ is less clear.

The results for $\kappa$ and $\tilde{\kappa}$ will improve by using tau-charm data, and will probe the point-like nature of the tau up to a scale of $\Lambda = O(180$ GeV) (assuming no improvement in $\tau$) or $\Lambda = O(300$ GeV) (assuming a factor of two improvement in the error on $\tau$).

4.4. $\eta_{\tau \mu}$ and extended Higgs sector models

Our value for $\eta_{\tau \mu}$ is currently the most precise. The uncertainty is significantly smaller than determinations using the shape of momentum spectra of muons from $\tau$ decays, $(\eta_{\tau \mu} = -0.04 \pm 0.20)$ [13].

Many extensions of the Standard Model, such as supersymmetry (SUSY), involve an extended Higgs sector with more than one Higgs doublet. Such models contain charged Higgs bosons which contribute to the weak charged current with couplings which depend on the fermion masses. Of all the Michel parameters, $\eta_{\tau \mu}$ is especially sensitive to the exchange of a charged Higgs. Following Stahl [21], $\eta_{\tau \mu}$ can be written as

$$\eta_{\tau \mu} = -\left(\frac{m_\tau m_\mu}{2 m_H}\right)^2 \left(\frac{\tan \beta}{m_H}\right)^2$$

where $\tan \beta$ is the ratio of vacuum expectation values of the two Higgs fields, and $m_H$ is the mass of the charged Higgs. This expression applies to type II extended Higgs sector models in which the up-type quarks get their masses from one doublet and the down-type quarks get their masses from the other. From current data we determine the one-sided constraint $\eta_{\tau \mu} > -0.0232$ at the 95% C.L.

An almost identical constraint on the high $\tan \beta$ region of type II models may be obtained from the process $B \to \tau \nu$ [34]. The most stringent constraint, from the L3 experiment, rules out the region $m_H < (2.09 \tan \beta)$ GeV at the 95% C.L. [34]. Within the specific framework of the minimal supersymmetric standard model, the process $B \to \tau \nu X$ rules out the region $m_H < (2.33 \tan \beta)$ GeV at the 95% C.L. [54].

This limit, however, depends on the value of the Higgsino mixing parameter $\mu$ and can be evaded completely for $\mu > 0$. The non-observation of proton decay also tends to rule out the large $\tan \beta$ region but these constraints are particularly model-dependent. The very low $\tan \beta$ region is ruled out by measurements of the partial width $\Gamma(Z \to b\bar{b})$. For type II models the ap-
proximate region excluded is $\tan \beta < 0.7$ at the $2.5\sigma$ C.L. for any value of $M_H$ [35]. Complementary bounds for the full $\tan \beta$ region are derived from the CLEO measurement of $BR(b \to s\gamma) = (2.32 \pm 0.57 \pm 0.35) \times 10^{-4}$ which rules out, for type II models, the region $M_H < 244 + 63/\tan \beta^{1.3}$ [36]. This constraint can, however, be circumvented in SUSY models where other particles in the loops can cancel out the effect of the charged Higgs. Direct searches at LEP II exclude the region $m_H < 54.5$ GeV for all values of $\tan \beta$ [37]. The CDF search for charged Higgs bosons in the process $t \to bH^+$ rules out the region of low $m_H$ and high $\tan \beta$ [38]. The 95% C.L. constraints in the $m_H$ vs. $\tan \beta$ plane, from this and other analyses, are shown in Fig. 1.

![Graph](image)

Figure 1. Constraints on $m_H$ as a function of $\tan \beta$ at the 95% C.L., from this analysis of $\eta_{\tau\mu}$ and the other analyses described in the text.

We anticipate that the constraints from $Z \to bb$ and $b \to s\gamma$ will improve somewhat with new measurements from LEP, CLEO, and the $b$-factories and from refinements in the theoretical treatment [39]. CLEO and the $b$-factories may also improve the measurements of $B \to \tau\nu(X)$ which rule out a similarly-shaped region of the $m_H - \tan \beta$ plane as that of this analysis.

Some caution is advised in the interpretation of the large $\tan \beta$ regime which becomes non-perturbative for $\tan \beta > O(70)$. Future improved measurements of the tau branching fractions and lifetime will, however, extend the constraints on $\tan \beta$ towards lower values, where perturbative calculations are more applicable.

In particular, for the tau-charm factory we estimate the one-sided constraint $\eta_{\tau\mu} > -0.014$ at the 95% C.L. This rules out the region $m_H < (2.55 \tan \beta)$ GeV at the 95% C.L., as shown in Fig. 1, and corresponds to $\sim 25\%$ reduction in the maximum allowed value of $\tan \beta$ for a given value of $m_H$, compared to current constraints.

5. SUMMARY

From an analysis of tau leptonic and semileptonic branching fractions we determine constraints on $m_{\tau\nu}$, $\sin^2 \theta$, $\kappa$, $\bar{\kappa}$, and $\eta_{\tau\mu}$ using current experimental data. We then assess the future sensitivity to these parameters using predictions for the uncertainties on experimental quantities measured at a tau-charm factory. We find that in each case the future sensitivity is completely limited by the uncertainty on the tau lifetime.

The constraint on $m_{\tau\nu}$ using current data is complementary to, but less stringent than, that already obtained from multi-hadronic tau decays. Our technique will benefit slightly from improved tau-charm factory data but will be considerably less competitive than other techniques available at such a facility.

Using current experimental data we find that our technique yields the most stringent constraints to-date on $\sin^2 \theta$, $\kappa$, $\bar{\kappa}$, and $\eta_{\tau\mu}$. All these constraints are expected to improve by a factor of approximately two using future data from a tau-charm factory and, in the absence of novel competing techniques, will continue to yield the most precise determinations of these quantities.

The result for $\kappa$ indicates that the tau is point-like up to an energy scale of approximately 105 GeV (today) and $O(300 \text{ GeV})$ (using tau-
charm data and assuming a factor of two improvement in the tau-lifetime error).

The result for $\eta_{\tau\mu}$ constrains the charged Higgs of type II two-Higgs doublet models such that we can exclude, at the 95% C.L., the region $m_H < (2.01 \tan \beta)$ GeV (today) and $m_H < (2.55 \tan \beta)$ GeV (using tau-charm data and assuming a factor of two improvement in the tau-lifetime error).

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