1. INTRODUCTION

The tau lepton in the Standard Model is an exact duplicate of the electron and muon, apart from its greater mass and separately conserved quantum number. Its charged current interactions are expected to be mediated by the $W$ boson with pure $V-A$ coupling. In this paper we present constraints on anomalous charged current couplings of the $\tau$ [\cite{ref1, ref2}]. These are derived from an analysis of the branching fractions for $\tau^- \rightarrow e^- \bar{\nu}_e \nu_\tau$ and $\tau^- \rightarrow \mu^- \bar{\nu}_\mu \nu_\tau$, where charge-conjugate decays are implied. In particular, we consider derivative terms in the Hamiltonian which describe anomalous weak charged current magnetic and electric dipole moments of the tau and the Michel parameter $\eta$ with unprecedented precision. These results are then used to constrain the tau compositeness scale and the allowed parameter space for Higgs doublet models. We also present new constraints on the mass of the tau neutrino and its mixing with a fourth generation neutrino.

2. THEORETICAL PREDICTIONS

2.1. Anomalous couplings

The theoretical predictions for the branching fractions $B_\ell$ of the decay $\tau^- \rightarrow \ell^- \bar{\nu}_\ell \nu_\tau(X_{EM})$, with $\ell^- = e^-, \mu^-$ and $X_{EM} = \gamma, \gamma\gamma, e^+e^-$, are given by:

$$B_\ell^{th} = \frac{G_F^2 m_\tau^5}{192\pi^3} \left(1 - 8x - 12x^2 \ln x + 8x^3 - x^4\right) \times \left[\left(1 - \frac{\alpha(m_\tau)}{2\pi} \left(\frac{\pi^2}{4} - 25\right)\right) \left(1 + \frac{1}{5} \frac{m_\tau^2}{m_W^2}\right)\right] \times [1 + \Delta_\ell],$$

where $G_F = (1.16639 \pm 0.00002) \times 10^{-5}$GeV$^{-2}$ is the Fermi constant [\cite{ref3}]; $\tau^- = (200.55 \pm 1.06)\text{fs}$ is the tau lifetime [\cite{ref4}]; $m_\tau = (1776.96^{+0.18}_{-0.21} + 0.25)\text{MeV}$ [\cite{ref5}]; $\alpha(m_\tau)$ is the fine-structure constant; $X_{EM}$ are the electromagnetic processes; and $x = m_\tau^2/m_W^2$. The first term in square brackets allows for radiative corrections [\cite{ref6, ref7, ref8}],

\footnote{Throughout this paper the charge-conjugate decays are also implied. We denote the branching ratios for these processes as $B_e$, $B_\mu$, $B_\tau$, $B_K$ respectively; $B_e$ denotes either $B_\bar{e}$ or $B_\bar{\mu}$ while $B_\tau$ denotes either $B_\bar{\tau}$ or $B_K$.}
where \( \alpha(m_\tau) \simeq 1/133.3 \) is the QED coupling constant \([\text{?}]\) and \( m_W = 80.400 \pm 0.075 \text{ GeV} \) is the \( W \) mass \([\text{?}]\). The second term in brackets describes the effects of new physics where the various \( \Delta_\ell \) we consider are defined below.

The effects of anomalous weak charged current dipole moment couplings at the \( \tau \nu_\tau W \) vertex are described by the effective Lagrangian

\[
\mathcal{L} = \frac{g}{\sqrt{2}} \left[ \gamma_\mu + i \frac{\sigma_{\mu\nu}}{2m_\tau} (\kappa_\tau \gamma_\nu - i \kappa \gamma_\nu) \right] P_L \nu_\tau W^\mu + (\text{Hermitian conjugate}),
\]

where \( P_L \) is the left-handed projection operator and the parameters \( \kappa \) and \( \bar{\kappa} \) are the (CP-conserving) magnetic and (CP-violating) electric dipole form factors respectively \([\text{?}]\). They are the charged current analogues of the weak neutral current dipole moments, measured using \( Z \to \tau^+ \tau^- \) events \([\text{?}]\), and the electromagnetic dipole moments \([\text{?}, \text{?}, \text{?}]\) recently measured by L3 and OPAL using \( Z \to \tau^+ \tau^- \gamma \) events \([\text{?}, \text{?}, \text{?}]\). In conjunction with Eq. 1, the effects of non-zero values of \( \kappa \) and \( \bar{\kappa} \) on the tau leptonic branching fractions may be described by \([\text{?}]\)

\[
\Delta_\kappa = \kappa/2 + \bar{\kappa}^2/10; \quad \Delta_{\bar{\kappa}} = \bar{\kappa}^2/10. \tag{3}
\]

The dependence of the tau leptonic branching ratios on \( \eta \) is given, in conjunction with Eq. 1, by \([\text{?}]\)

\[
\Delta_\eta = 4 \eta_\tau \sqrt{x}, \tag{5}
\]

where the subscripts on \( \eta \) denote the initial and final state charged leptons. Both leptonic tau decay modes probe the charged current couplings of the transverse \( W \), and are sensitive to \( \kappa \) and \( \bar{\kappa} \). In contrast, only the \( \tau^- \to \mu^- \bar{\nu}_\mu \nu_\tau \) channel is sensitive to \( \kappa \), due to a relative suppression factor of \( m_\tau/m_\mu \) for the \( \tau^- \to e^- \bar{\nu}_e \nu_\tau \) channel. Semileptonic tau branching fractions are not considered since they are insensitive to \( \kappa \), \( \bar{\kappa} \), and \( \eta \).

### 2.2. Tau neutrino mass and mixing

The theoretical predictions for the branching fractions \( B_\ell \) allowing for the \( \nu_\tau \) mass and mixing with a fourth generation generation are given by \([\text{?}]\):

\[
B_\ell^{\nu_\tau} = \frac{G_F^2 m_\tau^5}{192\pi^3} \left( 1 - 8x - 12x^2 \ln x + 8x^3 - x^4 \right) \times \left( 1 - \frac{\alpha(m_\tau)}{2\pi} \left( \frac{\pi^2 - \frac{25}{4}}{4} \right) \right) \left( 1 + \frac{3}{5} \frac{m_\nu^2}{m_W^2} \right) \times \left[ 1 - \sin^2 \theta \right] \left[ 1 - 8y(1-x)^3 + \cdots \right] \tag{6}
\]

where the tau mass used is determined by BES from the \( \tau^+ \tau^- \) production rate with no dependence on the tau neutrino mass and \( x = m_\nu^2/m_\tau^2 \). The first term in square brackets describes mixing with a fourth generation neutrino which, being kinematically forbidden, causes a suppression of the decay rate. The second term in brackets parametrises the suppression due to a non-zero mass of \( \nu_3 \), where \( y = m_\nu^2/m_\tau^2 \) and the ellipsis denotes negligible higher order terms \([\text{?}]\).

The branching fractions for the decays \( \tau^- \to h^- \nu_\tau \), with \( h = \pi/K \), are given by

\[
B_h^{\tau_h} = \left( \frac{G_F^2 m_H^3}{16\pi} \right) \tau_h f_h^2 |V_{\alpha\beta}|^2 (1-x)^2 \times \left( \frac{1+2\alpha_\tau}{\pi} \ln \left( \frac{m_H}{m_\tau} \right) + \cdots \right) \left[ 1 - \sin^2 \theta \right] \times \left[ 1 - y \left( \frac{2 + x - y}{1 - x} \right) \left[ 1 - y(2 + 2x - y) \right] \frac{1}{(1-x)^2} \right] \tag{7}
\]

where \( x = m_H^2/m_\tau^2 \), \( m_H \) is the hadron mass, \( f_h \) are the hadronic form factors, and \( V_{\alpha\beta} \) are the CKM matrix elements, \( V_{ud} \) and \( V_{us} \), for \( \pi^- \) and \( K^- \) respectively. From an analysis of \( \pi^- \to \mu^- \bar{\nu}_\mu \) and \( K^- \to \mu^- \bar{\nu}_\mu \) decays, one obtains \( f_\pi |V_{ud}| = (127.4 \pm 0.1) \text{ MeV} \) and \( f_K |V_{us}| = (35.18 \pm 0.05) \text{ MeV} \), and references therein. The ellipsis represents terms, estimated to be \( \mathcal{O}(0.01) \)\([\text{?}]\), which are neither explicitly treated nor implicitly absorbed into \( G_F, f_\tau |V_{ud}| \), or \( f_K |V_{us}| \). The first term in square brackets describes mixing with a fourth generation neutrino while the second parametrises the effects of a non-zero \( m_\nu \) \([\text{?}]\).

The fourth generation neutrino mixing affects all the tau branching fractions with a common factor whereas a non-zero tau neutrino mass affects all channels with different kinematic factors. Therefore, given sufficient experimental precision, these two effects could in principle be separated.

### 3. RESULTS
3.1. Anomalous couplings

We use the world average values for the measured tau branching fractions \([1]: B_e = (17.786 \pm 0.072)\% \) and \(B_\mu = (17.356 \pm 0.064)\%\). Substituting in Eq. 3 for these and the other measured quantities we obtain \(\Delta_e = -0.0008 \pm 0.0055\) and \(\Delta_\mu = +0.0026 \pm 0.0053\) where the errors include the effects of the uncertainties on all the measured quantities appearing in Eq. 3. These results are consistent with zero which, assuming that there are no fortuitous cancellations, indicates the absence of anomalous effects within the experimental precision.

We therefore proceed to derive constraints on \(\kappa\), \(\tilde{\kappa}\), and \(\eta_{\tau\mu}\) from a combined likelihood fit to both tau decay channels. The likelihood is constructed numerically following the procedure of Ref. \([2]\) by randomly sampling all the quantities used according to their errors, conservatively assuming for each parameter that the other two parameters are zero.

We determine \(\kappa = 0.001 \pm 0.008\), where the errors correspond to one standard deviation, and constrain it to the range \(-0.014 < \kappa < 0.016\) at the 95% confidence level (C.L.). This result improves on the 95% C.L. constraint of \(|\kappa| < 0.0283\) determined by Rizzo \([2]\).

We determine \(\tilde{\kappa} = 0.00 \pm 0.16\) and constrain it to the range \(|\tilde{\kappa}| < 0.26\) at the 95% C.L. Our constraint, which is the first on this quantity, is considerably less stringent than that on \(\kappa\) due to the lack of linear terms. This also means that the likelihood for \(\tilde{\kappa}\) is symmetric by construction. Were \(\tilde{\kappa}\) to differ significantly from zero, then the likelihood distribution would have two distinct peaks either side of zero. Such structure was not, however, observed. The decay \(W \rightarrow \tau\nu\) is also sensitive to charged current dipole terms but, given that the energy scale is \(m_W\), the interpretation in terms of the static properties \(\kappa\) and \(\tilde{\kappa}\) is less clear.

We determine \(\eta_{\tau\mu} = 0.009 \pm 0.022\) and constrain it to the range \(-0.034 < \eta_{\tau\mu} < 0.053\) at the 95% C.L. The uncertainty on our measurement of \(\eta_{\tau\mu}\) is significantly smaller than that obtained by Stahl using the same technique \((\eta_{\tau\mu} = 0.01 \pm 0.05)\) \([2]\) and more recent determinations using the shape of momentum spectra of muons from \(\tau\) decays \((\eta_{\tau\mu} = 0.00 \pm 0.01)\) \([2]\).

3.2. Tau neutrino mass and mixing

We use the world average values for the measured tau branching fractions \([1]: B_e \) and \(B_\mu \) as above, \(B_\pi = (11.01 \pm 0.11)\% \) and \(B_K = (0.692 \pm 0.028)\%. Substituting in equations 4 and 5 for the measured quantities we find that both \(m_{\nu_\tau}\) and \(\sin^2 \theta\) are consistent with zero. We therefore derive constraints on \(m_{\nu_\tau}\) and \(\sin^2 \theta\) from a combined likelihood fit to the four tau decay channels. The CLEO measurement of the \(\tau\) mass was used to further constrain \(m_{\nu_\tau}\). From an analysis of \(\tau^+\tau^- \rightarrow (\pi^+ n\pi^0\nu_\tau) (\pi^- m\pi^0\nu_\tau)\) events (with \(n \leq 2, m \leq 2, 1 \leq n + m \leq 3\)), CLEO determined the \(\tau\) mass to be \(m_\tau = (1777.8 \pm 0.7 \pm 1.7)\) MeV\([2]\). The likelihood for the CLEO and BES measurements to agree, as a function of \(m_{\nu_\tau}\), is included in the global likelihood.

The fit yields upper limits of

\[
\begin{align*}
    m_{\nu_\tau} &< 38\text{ MeV} & (8) \\
    \sin^2 \theta &< 0.008 & (9)
\end{align*}
\]

at the 95% C.L. or

\[
\begin{align*}
    m_{\nu_\tau} &< 32\text{ MeV} & (10) \\
    \sin^2 \theta &< 0.007 & (11)
\end{align*}
\]

at the 90% C.L.

4. DISCUSSION

4.1. Compositeness of the tau

Derivative couplings necessarily involve the introduction of a length or mass scale. Anomalous magnetic moments due to compositeness are expected to be of order \(m_\tau / \Lambda\) where \(\Lambda\) is the compositeness scale \([?]\). We can then interpret the 95% confidence level on \(\kappa\), the quantity for which we have a more stringent bound, as a statement that the \(\tau\) appears to be a point-like Dirac particle up to an energy scale of \(\Lambda \approx m_\tau / 0.016 \approx 110\text{ GeV}\). These results are comparable to those obtained from anomalous weak neutral current couplings \([?]\) and more stringent than those obtained for anomalous electromagnetic couplings \([?, ?, ?]\).
4.2. Extended Higgs sector models

Many extensions of the Standard Model, such as supersymmetry (SUSY), involve an extended Higgs sector with more than one Higgs doublet. Such models contain charged Higgs bosons which contribute to the weak charged current with couplings which depend on the fermion masses. Of all the Michel parameters, $\eta_{\tau\mu}$ is especially sensitive to the exchange of a charged Higgs. Following Stahl [?], $\eta_{\tau\mu}$ can be written as

$$\eta_{\tau\mu} = - \left( \frac{m_{\tau} m_{\mu}}{2} \right) \left( \frac{\tan \beta}{m_H} \right)^2$$

(12)

where $\tan \beta$ is the ratio of vacuum expectation values of the two Higgs fields, and $m_H$ is the mass of the charged Higgs. This expression applies to type II extended Higgs sector models in which the up-type quarks get their masses from one doublet and the down-type quarks get their masses from the other.

We determine the one-sided constraint $\eta_{\tau\mu} > -0.0186$ at the 95% C.L. which rules out the region $m_H < (1.86 \tan \beta) \, \text{GeV}$ at the 95% C.L. as shown in Fig. 1. An almost identical constraint on the high $\tan \beta$ region of type II models may be obtained from the process $B \rightarrow \tau \nu$ [?]. The most stringent constraint, from the L3 experiment, rules out the region $m_H < (2.09 \tan \beta) \, \text{GeV}$ at the 95% C.L. [?]. Within the specific framework of the minimal supersymmetric standard model, the process $B \rightarrow \tau \nu X$ rules out the region $m_H < (2.33 \tan \beta) \, \text{GeV}$ at the 95% C.L. [?]. This limit, however, depends on the value of the Higgsino mixing parameter $\mu$ and can be evaded completely for $\mu > 0$. The non-observation of proton decay also tends to rule out the large $\tan \beta$ region but these constraints are particularly model-dependent. The very low $\tan \beta$ region is ruled out by measurements of the partial width $\Gamma(Z \rightarrow bb)$. For type II models the approximate region excluded is $\tan \beta < 0.7$ at the 2.5$\sigma$ C.L. for any value of $M_H$ [?]. Complementary bounds for the full $\tan \beta$ region are derived from the CLEO measurement of $BR(b \rightarrow s\gamma) = (2.32 \pm 0.57 \pm 0.35) \times 10^{-4}$ which rules out, for type II models, the region $M_H < 244 + 63/(\tan \beta)^{1.3}$ [?]. This constraint can, however, be circumvented in SUSY models where other particles in the loops can cancel out the effect of the charged Higgs. Direct searches at LEP II exclude the region $m_H < 54.5 \, \text{GeV}$ for all values of $\tan \beta$ [?]. The CDF search for charged Higgs bosons in the process $t \rightarrow bH^+$ rules out the region of low $m_H$ and high $\tan \beta$ [?].

The 95% C.L. constraints in the $m_H$ vs. $\tan \beta$ plane, from this and other analyses, are shown in Fig. 1. We anticipate that the constraints from $Z \rightarrow bb$ and $b \rightarrow s\gamma$ will improve significantly in the near future due to new measurements from the LEP and CLEO collaborations and from refinements in the theoretical treatment [?]. Some caution is advised in the interpretation of the large $\tan \beta$ regime which becomes non-perturbative for $\tan \beta > O(70)$. Future improved measurements of the $\tan \beta$ branching fractions and lifetime will, however, extend the constraints on $\tan \beta$ towards lower values, where perturbative calculations are more applicable.

![Figure 1. Constraints on $m_H$ as a function of $\tan \beta$ at the 95% C.L., from this analysis of $\eta_{\tau\mu}$ and the other analyses described in the text.](image-url)
4.3. Tau neutrino mass and mixing

The limit on $m_{\nu_3}$ can be reasonably interpreted as a limit on $m_{\nu_\tau}$, since the mixing of $m_{\nu_3}$ with lighter neutrinos is also small [?]. The best direct experimental constraint on the tau neutrino mass is $m_{\nu_\tau} < 18.2$ MeV at the 95% confidence level[?] which was obtained using many-body hadronic decays of the $\tau$. While our constraint is less stringent, it is statistically independent. Moreover, it is insensitive to fortuitous or pathological events close to the kinematic limits, the absolute energy scale of the detectors, and the details of the resonant structure of multi-hadron $\tau$ decays [?]. Since LEP has completed running at the $Z$ it is unlikely that significantly improved constraints on $m_{\nu_\tau}$, using multi-hadron final states, will be forthcoming in the foreseeable future.

Future improved measurements of the tau branching fractions, lifetime, and the tau mass from direct reconstruction would enable significant improvements to be made in the determinations of both $m_{\nu_\tau}$ and $\sin^2 \theta$. If CLEO and the $b$-factory experiments were to reduce the uncertainties on the experimental quantities by a factor of approximately 2, then the constraints on $m_{\nu_\tau}$ from the technique we have described would become the most competitive. Were a tau-charm factory to be built, then the determination of $m_{\nu_\tau}$ by direct reconstruction would again become the most sensitive technique. Our upper limit on $\sin^2 \theta$ is already the most stringent experimental constraint on mixing of the third and fourth neutrino generations.

5. SUMMARY

From an analysis of tau leptonic branching fractions we determine

$$\kappa = 0.001 \pm 0.008; \quad (13)$$

$$\tilde{\kappa} = 0.00 \pm 0.16; \quad (14)$$

$$\eta_{\tau\mu} = 0.009 \pm 0.022. \quad (15)$$

Each of these results is the most precise determination to date. The result for $\kappa$ indicates that the tau is point-like up to an energy scale of approximately 110 GeV. The result for $\eta_{\tau\mu}$ constrains the charged Higgs of type II two-Higgs doublet models, such that the region

$$m_H < (1.86 \tan \beta) \text{ GeV} \quad (16)$$

is excluded at the 95% C.L. The fit for tau neutrino mass and mixing yields upper limits of

$$m_{\nu_3} < 38 \text{ MeV} \quad (17)$$

$$\sin^2 \theta < 0.008 \quad (18)$$

at the 95% confidence level.

Acknowledgements

M.T.D. acknowledges the support of CONICET, Argentina. J.S. and L.T. would like to thank the Department of Physics, Universidad Nacional de La Plata for their generous hospitality and the National Science Foundation for financial support. J.S. gratefully acknowledges the support of the International Centre for Theoretical Physics, Trieste.