

Possible test of local Lorentz invariance from τ decays

L. Anchordoqui, M. T. Dova, D. Gómez Dumm, P. Lacentre

Laboratorio de Física Teórica, Departamento de Física, U. N. L. P., c. c. 67, 1900 La Plata, Argentina

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Abstract. We analyze the possibility of testing local Lorentz invariance from the observation of tau decays. Future prospects of probing distances below the electroweak characteristic scale are discussed.

Introduction

The question of the existence of a universal length, below which our present notion of flat spacetime geometry is not valid, is of great theoretical importance [1]. In this respect, it is important to point out that the current success of the Standard Model (SM) in describing the electroweak processes serves as a constraint for any observable breakdown of local Lorentz invariance (LLI), which should appear as a small deviation from the SM predictions. In this way, any search for LLI nonconservation effects will require to test distances below the electroweak scale, i.e. of order 10^{-16} cm or less.

In order to get some insights into this subject, one possible way is to consider the experimental information on the decays of particles which move at high velocities in the lab system. The comparison between the measured lifetimes at these energies and the corresponding values in the rest frame would provide a direct test of the time dilation formula. In this direction, a proposal has been recently presented by Freeman et al. [2] to measure the lifetimes of 530 GeV charged pions at Fermilab.

In this letter, we concentrate on the possibilities of testing LLI from the observation of tau lepton decays. In particular, we consider the strongly relativistic tau pairs produced at LEP-I, taking profit of the increasing statistic and excellent experimental precision of the tau lifetime measurement. By comparing this lifetime value with the low energy results, it is possible to determine an upper bound for the LLI-breaking characteristic length. In addition, we consider the possibility of testing LLI from the measurement of particular τ decay branching ratios, instead of dealing with the total decay

width. The future prospects of probing distances below the electroweak characteristic scale are also discussed.

Framework

The idea of testing LLI from particle decay rates is not new. Indeed, this possibility was suggested many years ago by Rédei [3], who studied possible LLI breakdown effects on the decay of charged pions and muons. Now, considering the extraordinary evolution of the experimental knowledge, it is worth to recover this proposal including present data on particle lifetimes. We will use this framework extended to the τ lepton decays.

Following the scheme of Rédei, we begin by introducing into the theory a timelike unit vector n which characterizes the chosen reference frame [4]. As a consequence, the transition amplitudes become (in general) not invariant under a Lorentz transformation of the particle states if the reference system is kept unchanged. The theory does not exhibit Lorentz invariance in the so-called “active” sense. However, one still has Lorentz invariance in the “passive” sense, i.e., under a simultaneous transformation of the particle variables and the vector n (this is necessary in order to guarantee that the probabilities remain independent of the velocity of the observer).

We will study the main tau decay partial widths considering in each case a noncausal effective Hamiltonian of the type proposed by Rédei,

$$\int \mathcal{H}_{\text{int}} d^4x = \frac{G_F}{\sqrt{2}} \int \int d^4x d^4y \bar{\nu}_\tau(x) \gamma^\mu (1 - \gamma_5) \tau(x) \times F(x - y) J_\mu(y), \quad (1)$$

where the current J_μ depends on the decay channel, and the noncausality is introduced through the form factor $F(x)$. The latter is given by

$$F(x) = \frac{3}{4\pi\alpha^3} \delta(n \cdot x) \Theta(\alpha^2 + x^2 - (n \cdot x)^2)$$

and satisfies $F(x) \rightarrow \delta(x)$ in the limit $\alpha \rightarrow 0$. In the lab system (which can be considered at rest with respect to the

surrounding macroscopic bodies¹), we may take $n = (1, \mathbf{0})$. Then the form factor reduces to

$$F(x) = \frac{3}{4\pi\alpha^3} \delta(x_0) \Theta(\alpha^2 - |\mathbf{x}|^2), \quad (2)$$

where it becomes clear that α represents a characteristic length below which the noncausality effects will take place.

We are interested in computing the decay amplitudes in the lab system for strongly relativistic tau leptons. The τ polarization will be also included, since it can in principle contribute non-negligibly to the LLI breakdown effects we are looking for (in the case of LEP-I, the taus are produced with a polarization of approximately 14%)².

Let us first consider the leptonic decays $\tau^- \rightarrow l \bar{\nu}_l \nu_\tau$ ($l = \mu^-, e^-$) for a τ lepton with polarization P_τ . The corresponding decay amplitude is found to be

$$|\mathcal{M}|^2 = 64 G_F^2 (q_\tau \cdot q_{\nu_l} - m_\tau s \cdot q_{\nu_l}) \times (q_\nu \cdot q_l) [g(n, q_\tau - q_{\nu_\tau})]^2, \quad (3)$$

where s stands for the tau polarization 4-vector ($q_\tau \cdot s = 0$, $s^2 = -P_\tau^2$) and we have neglected the lepton mass m_l . The function $g(n, Q)$ is the Fourier transform of $F(x)$,

$$g(n, Q) = 1 - \frac{1}{10} \alpha^2 [(Q \cdot n)^2 - Q^2] + \mathcal{O}(\alpha^4). \quad (4)$$

In order to calculate the partial widths $\Gamma_l(E_\tau)$, it is convenient to perform a transformation to the tau rest frame. However, notice that one has to boost both the particle variables and the vector n to get the right result. The Fourier transform of $F(x)$ in the new reference system reads

$$g(n, Q)^{(rest)} = 1 - \frac{1}{10} \alpha^2 [\gamma^2 v^2 (E_{\nu_\tau} - m_\tau)^2 + E_{\nu_\tau}^2 + 2\gamma^2 v (E_{\nu_\tau} - m_\tau) E_{\nu_\tau} \cos \theta + \gamma^2 v^2 E_{\nu_\tau}^2 \cos^2 \theta], \quad (5)$$

where θ is the angle between the boost direction and the 3-momentum \mathbf{q}_{ν_τ} and we have used $\gamma^2 = (1 - v^2)^{-1}$, being v the tau velocity in the lab system. After integration over the phase space, we find for the leptonic partial widths

$$\Gamma_l(E_\tau) = \frac{G_F^2 m_\tau^5}{192 \pi^3} \gamma^{-1} [1 - \alpha^2 \delta_l(E_\tau)], \quad l = e, \mu, \quad (6)$$

with

$$\delta_l(E_\tau) = \frac{1}{5} E_\tau^2 \left(\frac{2}{15} + \frac{31}{90} v^2 + \frac{1}{18} P_\tau v \right). \quad (7)$$

In the limit $\alpha \rightarrow 0$, we get the relation

$$\Gamma_l(E_\tau) = \gamma^{-1} \Gamma_l(m_\tau), \quad (8)$$

i.e. we recover the usual time dilation formula.

We consider now the decays $\tau^- \rightarrow h \nu_\tau$, with $h = \pi^-, \rho^-, a_1^-$. The contribution of these processes, together with the leptonic channels treated above, add up to approx. 90% of the total tau decay width. In analogy with (6), the

corresponding partial widths for τ leptons with energy E_τ can be written as

$$\Gamma_h(E_\tau) = \Gamma_h(m_\tau) \gamma^{-1} [1 - \alpha^2 \delta_h(E_\tau)], \quad h = \pi^-, \rho^-, a_1^-, \quad (9)$$

where we have assumed $\delta_h(m_\tau) \ll \delta_h(E_\tau)$. This is justified when we deal with ultrarelativistic taus. The deviation factors δ_h can be easily calculated, resulting

$$\delta_h(E_\tau) = \frac{1}{20} E_\tau^2 \left[(1 - x_h)^2 + \frac{1}{3} (1 + 10 x_h + x_h^2) v^2 + \frac{2}{3} P_\tau \lambda_h (1 - x_h^2) v \right], \quad (10)$$

where $x_h \equiv m_h^2/m_\tau^2$, and λ_h is given by

$$\lambda_h \equiv \begin{cases} 1 & \text{for } h = \pi^- \\ \frac{1-2x_h}{1+2x_h} & \text{for } h = \rho^-, a_1^- \end{cases}. \quad (11)$$

In the same way, the corrected expression for the total τ decay rate will read

$$\Gamma(E_\tau) = \sum_i \Gamma_i(E_\tau) = \gamma^{-1} \Gamma(m_\tau) (1 - \alpha^2 \Delta(E_\tau)) \quad (12)$$

with

$$\Delta(E_\tau) \equiv \sum_i \frac{\Gamma_i}{\Gamma} \delta_i(E_\tau), \quad (13)$$

where the sum extends in principle to all τ decay channels. We will neglect here the deviations corresponding to the less significant branching ratios, restricting ourselves to the δ_i contributions given by (7) and (10). Notice that the energy dependence of Γ_i and Γ in (13) has been omitted, since it would imply a correction of order α^4 to the total decay rate (12).

It is important to remark that, within the scheme under consideration, the branching ratios are no longer fixed numbers. They depend in general on the energy of the decaying particle. In the case of the τ leptons, the above equations conduce to the (order α^2) relations

$$\frac{\Gamma_i(E_\tau)}{\Gamma(E_\tau)} = \frac{\Gamma_i(m_\tau)}{\Gamma(m_\tau)} [1 + \alpha^2 (\Delta(E_\tau) - \delta_i(E_\tau))]. \quad (14)$$

In this way, the sole measurement of branching ratios provides another possible test of LLI.

Numerical analysis

As stated above, the experimental data on the τ lepton lifetime at both high and low energies provide an upper bound for the characteristic LLI breakdown length α . In addition, it is possible to estimate how much energy would be necessary to probe distances below the electroweak scale for a given accuracy in the τ lifetime measurements.

Let us first consider the τ leptons which are produced at approximately 45 GeV in LEP-I and SLD. From the corresponding experimental data, the weighted average value of the τ lifetime is found to be [6]

$$\tau_\tau = 291.4 \pm 1.6 \text{ fs}. \quad (15)$$

For low energy τ 's, the lifetime value is less accurate. The present measurements include a 2.5% error, which is indeed

¹ Most of the low velocity experiments which test LLI consider a preferential reference frame comoving at 350 km/s with the center of mass of the Universe (as seen from the dipole asymmetry of the 3 K background radiation [5]). However, this cosmic motion can be neglected for our purposes in view of the high LEP-I velocities

² The spatial component of n is not zero in the rest frame of the decaying tau. Therefore, n introduces a spatial anisotropy and the decay probability is in general a function of the tau polarization direction

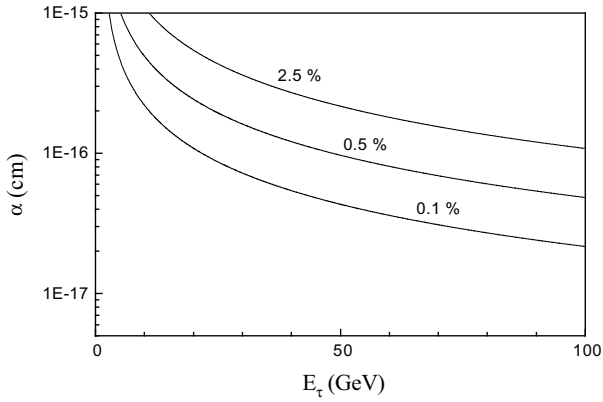


Fig. 1. Distance that is probed by the LLI-test, as a function of the τ lepton energy. Different accuracies in the τ lifetime measurements are considered

the dominant one when determining the bound for α . By combination of the CLEO [7] and ARGUS [8] results, we obtain

$$\tau_\tau = 292 \pm 7 \text{ fs} . \quad (16)$$

As can be seen, the data in (15) and (16) are compatible with a null value of α . In order to obtain an upper bound, we consider the modified time dilation formula in (12), with the δ_i corrections given by (7) and (10). With a 95% confidence level, we find

$$\alpha \leq 3.3 \times 10^{-16} \text{ cm} , \quad (17)$$

where the numerical evaluation of the total deviation factor Δ has been performed taking the experimental values for the particle masses and τ decay branching ratios from Ref. [9].

From the expression (12), one can also examine the future possibilities of probing even smaller distances. In Fig. 1, we plot the value of α that would be sensitive to this test for a given energy of the decaying tau. The accuracy in the tau lifetime measurement (this means, for both high and low energies) is included as a parameter. If we take into account τ -leptons of ~ 50 GeV, it can be seen that this accuracy should be increased up to a 0.5% in order to reach a sensitivity to α below the electroweak characteristic length of 10^{-16} cm. For energies of ~ 100 GeV, however, distances beyond this limit could be probed with a τ lifetime precision of about 2%.

Another possible test of the presence of LLI violation can be obtained by comparing the τ and μ leptonic partial widths [10]. Let us consider the ratio

$$R(E_\tau) \equiv \frac{\Gamma(\tau \rightarrow \nu_\tau e \bar{\nu}_e)}{\Gamma(\mu \rightarrow \nu_\mu e \bar{\nu}_e)} = B_e(E_\tau) \frac{\tau_\mu(E_\mu)}{\tau_\tau(E_\tau)} , \quad (18)$$

where B_e is the branching ratio for the $\tau \rightarrow \nu_\tau e \bar{\nu}_e$ decay and E_τ is the energy of the decaying τ in the lab system. Within our framework, it is easy to show that

$$R(E_\tau) = \left(\frac{g_\tau}{g_\mu} \right)^2 \left(\frac{m_\tau}{m_\mu} \right)^5 (\Delta_W \Delta_\gamma)^{-1} (1 - \alpha^2 \delta_l(E_\tau)) , \quad (19)$$

where g_l stands for the coupling constant corresponding to the weak charged current of a lepton l . The factors $\Delta_\gamma \simeq 1 +$

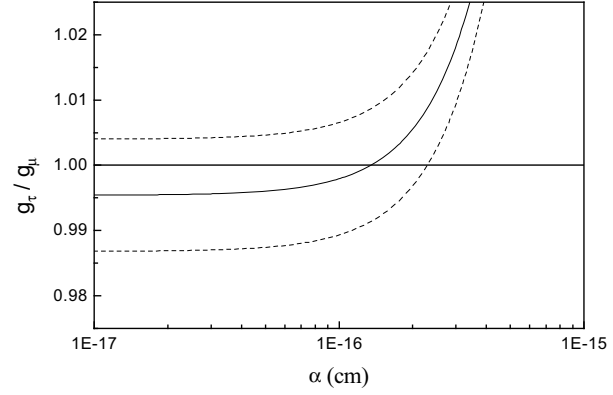


Fig. 2. Ratio g_τ/g_μ calculated from measured values of τ and μ masses and leptonic decays. *Solid and dashed lines* represent average and 2σ values respectively

8.6×10^{-5} and $\Delta_W \simeq 1 - 3.0 \times 10^{-4}$ come from the inclusion of radiative corrections to the tree level decay amplitudes. We have not included the correction $\delta_l(E_\mu)$, which has been assumed to be much smaller than $\delta_l(E_\tau)$. From (18) and (19), we finally have

$$\left(\frac{g_\tau}{g_\mu} \right)^2 (1 - \alpha^2 \delta_l(E_\tau)) = \left(\frac{m_\mu}{m_\tau} \right)^5 \frac{\tau_\mu}{\gamma^{-1} \tau_\tau(E_\tau)} B_e(E_\tau) \Delta_W \Delta_\gamma , \quad (20)$$

i.e. the presence of the LLI-breakdown parameter α would be observed as a violation of the $\tau - \mu$ universality.

In reference [10], the right hand side of (20) has been evaluated using the experimental values of the masses and decay rates. Setting $\alpha = 0$, the LEP measurements for τ decays conduce to

$$\frac{g_\tau}{g_\mu} = 0.9954 \pm 0.0043 . \quad (21)$$

Within the LLI-violating scheme we are considering, the accurate value in the above expression, together with the requirement of universality $g_\tau = g_\mu$, provide a new upper bound for the parameter α . In Fig. 2, we plot the value of the ratio g_τ/g_μ that is obtained from eq. (20) for values of α from 10^{-17} to 10^{-15} cm. The solid curve corresponds to the average experimental values of the parameters entering the right hand side, while the dashed lines take into account a 2σ deviation. Within this confidence level, the agreement with universality conduces to

$$\alpha \leq 2.3 \times 10^{-16} \text{ cm} . \quad (22)$$

Let us finally stress the fact that measurements of branching ratios at high and low energies can provide accurate tests of LLI. In this sense, the τ leptons represent an ideal laboratory, in view of the various significant decay channels which can be observed. The prospects are also encouraged by the present improvement in these measurements at LEP [11] and the proposal of building a Tau-Charm factory [12], where the expected precision on the values of branching ratios will be far below 1% in one-year data sample.

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