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# Bose–Einstein condensation in helium white dwarf stars. I

M.E. Mosquera [a,b], O. Civitarese [b,*], O.G. Benvenuto [a,c], M.A. De Vito [a,c]

[a] Faculty of Astronomy and Geophysics, University of La Plata, Paseo del Bosque s.n., La Plata, Argentina
[b] Department of Physics, University of La Plata, c.c. 67 1900, La Plata, Argentina
[c] Instituto de Astrofisica La Plata, CCT, Argentina

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ABSTRACT

The formation of a Bose–Einstein condensate in the interior of helium white dwarfs stars is discussed. Following the proposal made by Gabadadze and Rosen, we have explored the consequences of such a mechanism by calculating the cooling time of the stars. We have found that it is shorter than the value predicted by the standard model.



## 1. Introduction

The existence of a charged condensate, as a possible state of matter in the interior of white dwarf (WD) stars, has been suggested in Refs. [1–5]. In the simplest scenario, WD stars are described as compact objects made of $^4$He-nuclei and electrons. If the temperature is low enough, helium nuclei could form a condensate, e.g. a quantum state of macroscopic size with all the nuclei in their ground state and in presence of background fermions (electrons). The formation of such a condensate depends upon the inter-particle distance and, subsequently, of the correlation length between pairs of interacting nuclei. Under normal conditions [6], a WD has nuclear densities of the order of $6 \times 10^{-4}$ MeV$^3$ up to $6 \times 10^{-1}$ MeV$^3$, to which we associate inter-particle distances of the order of 200 to 2000 fm, that is in the range of 20 to 200 times the diameter of a helium nucleus. This condition seemingly excludes strong interactions between nuclei of $^4$He. Therefore, if the temperature is low enough, one would expect to find signals of Bose–Einstein condensation (BEC) of a non-interacting system of helium nuclei [1–5]. In the present work, we explore the consequences of the formation of a BEC regime in a system of $^4$He-nuclei interacting with photons at temperatures of the order of $T \sim 10^{5.5}$–$10^8$ K and at WD densities. In a subsequent paper (Ref. [7]) we investigate the thermal evolution of the WD-luminosity in a detailed manner. The Letter is organized as follows. In Section 2 we present the formalism of BEC by adapting the conventional quantum statistical mechanics framework to the description of extended media. After solving the equations which yield the spectrum of vector transversal, longitudinal and scalar bosons, we construct the thermodynamical functions of the BEC regime. As a first step we perform the calculation of the specific heat. In Section 3 we discuss the cooling of helium WD in presence of BEC. Finally, in Section 4 we draw our conclusions.

## 2. The physics of a charged condensate

In a series of recent papers [1–5], the interior a WD star was described as an interacting system of $^4$He-nuclei. The formation of a condensate in a system composed of photons, electrons and $^4$He-nuclei was discussed, by applying the Lagrangian formalism. We shall review, in the following, the relevant features of such a formalism, which is a proper tool to account for interactions in an extended media.

The starting point, for the formalism of Ref. [3], consists of the solution of the bosonic excitations described by the Lagrangian

$$\mathcal{L} = \mathcal{L}_{\text{photon}} + \mathcal{L}_{\text{elec}} + \mathcal{L}_{\text{He}}^{\text{free}} + \mathcal{L}_{\text{He,photon}}^{\text{int}}, \tag{1}$$

where

$$\mathcal{L}_{\text{photon}} = -\frac{1}{4} F_{\mu\nu} F^{\mu\mu},$$
$$\mathcal{L}_{\text{elec}} = \overline{\Psi}(i\gamma^{\mu} D_{\mu} - m_e)\Psi + \mu_f \overline{\Psi}\Psi,$$
$$\mathcal{L}_{\text{He}}^{\text{free}} = \frac{1}{2}(\partial_{\mu}\sigma)^2 - \frac{1}{2} m_{He}^2 \sigma^2,$$
$$\mathcal{L}_{\text{He,photon}}^{\text{int}} = 2e^2 \widetilde{A}_{\mu}^2 \sigma^2. \tag{2}$$

* Corresponding author.
*E-mail address:* osvaldo.civitarese@fisica.unlp.edu.ar (O. Civitarese).

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In these expressions $\sigma$ is the scalar field, $\widetilde{A}_\mu$ is the photon field ($\widetilde{A}_\mu = A_\mu + \frac{\mu_s}{2e}\delta_{\mu 0}$), $\mu_s$ is the chemical potential for the $^4$He-nuclei, and $F_{\mu\nu} = \partial_\mu \widetilde{A}_\nu - \partial_\nu \widetilde{A}_\mu$. In the fermion sector $\Psi$ is the electron field, $D_\mu = \partial_\mu + ieA_\mu$, and $\mu_f$ is the chemical potential of the electrons. In these expressions we have adopted the convention $\hbar = c = 1$.

According to Ref. [3], to obtain the spectrum of small perturbations, one can write the fields in terms of small perturbations:

$$\sigma = \sqrt{\frac{J_0}{2m_{\mathrm{He}}}} + \delta\sigma,$$
$$\widetilde{A}_0 = \frac{m_{\mathrm{He}}}{2e} + \delta A_0,$$
$$\widetilde{A}_j = \delta A_j \tag{3}$$

In the above equation $m_{\mathrm{He}}$ is the rest-mass of $^4$He and $J_0$ is the electron number-density. After replacing these expression in the Lagrangian of Eq. (1), and after some algebra, one obtains the following Lagrangian for bosonic perturbations

$$\mathcal{L} = -\frac{1}{4} f_{\mu\nu}^2 + \frac{1}{2}(\partial_\mu \delta\sigma)^2 + 2M^2 \delta\sigma\,\delta A_0 + \frac{1}{2} m_0^2 (\delta A_0)^2 - \frac{1}{2} m_\gamma^2 (\delta A_j)^2, \tag{4}$$

where

$$f_{\mu\nu} = \partial_\mu \delta A_\nu - \partial_\nu \delta A_\mu,$$
$$m_\gamma = 2e\sqrt{\frac{J_0}{2m_{\mathrm{He}}}},$$
$$M^2 = m_{\mathrm{He}} m_\gamma,$$
$$m_0^2 = m_\gamma^2 + 2e^2 \frac{(3\pi^2 J_0)^{2/3}}{\pi^2}. \tag{5}$$

From the above Lagrangian one can calculate the spectrum of small perturbations, and obtain the energies of the boson fields, which are written

$$(\Omega^{\mathrm{Tr}})^2 = \mathbf{k}^2 + m_\gamma^2, \tag{6}$$

$$(\Omega^{\mathrm{L}})^2 = \frac{m_0^2 + m_\gamma^2}{2m_0^2}\mathbf{k}^2 + \frac{m_\gamma^2}{2} + 2\frac{M^4}{m_0^2} - \left\{\left[2\frac{M^4}{m_0^2} - \frac{m_\gamma^2}{2} + \mathbf{k}^2\left(\frac{m_0^2 - m_\gamma^2}{2m_0^2}\right)\right]^2 + 4M^4\frac{m_\gamma^2}{m_0^4}\mathbf{k}^2\right\}^{1/2}, \tag{7}$$

$$(\Omega^{\mathrm{S}})^2 = \frac{m_0^2 + m_\gamma^2}{2m_0^2}\mathbf{k}^2 + \frac{m_\gamma^2}{2} + 2\frac{M^4}{m_0^2} + \left\{\left[2\frac{M^4}{m_0^2} - \frac{m_\gamma^2}{2} + \mathbf{k}^2\left(\frac{m_0^2 - m_\gamma^2}{2m_0^2}\right)\right]^2 + 4M^4\frac{m_\gamma^2}{m_0^4}\mathbf{k}^2\right\}^{1/2}, \tag{8}$$

where the factors $\Omega^\alpha$ ($\alpha = \mathrm{Tr}$, L, S), are the energies of vector transversal (Tr), longitudinal (L) and scalar (S) bosonic excitations [3]. The mass for the scalar mode is

$$\Omega^{\mathrm{S}}(\mathbf{k} = 0) = 2\frac{M^2}{m_0} = m_s.$$

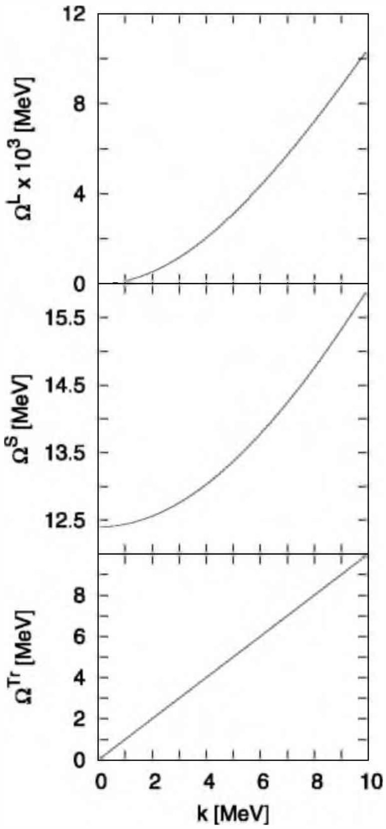


**Fig. 1.** Energy spectrum for the scalar, longitudinal and transversal modes of Eqs. (6), (7) and (8), for $J_0 = 10^{-6}$ MeV$^3$.

This mass depends on the electron number density ($J_0$) (see Eqs. (5)). For a WD, this density is in the interval $J_0 \approx (10^{-6}$–$10^{-3})$ MeV$^3$, given a mass $m_s$ in the range 12 MeV–40 MeV, and a mass $m_\gamma$ of the order of $2 \times 10^{-6}$–$6 \times 10^{-5}$ MeV. In Fig. 1 we present the spectrum of the vector and scalar fields for a given value of the electron density $J_0$. We shall now turn the attention to the calculation of the heat capacity for an interacting system of $^4$He-nuclei. The bosonic contribution for the grand canonical potential $\Omega_{bos}$ ($k_B = 1$) is written

$$-\Omega_{bos}(z, V, T) = -T \sum_{\alpha(\mathrm{Tr,L,S})} g_\alpha V \int \frac{d^3k}{(2\pi)^3} \ln\left(1 - z_\alpha e^{-\Omega_k^\alpha / T}\right), \tag{9}$$

where $T$ is the temperature, $z_\alpha$ is the fugacity and $g_\alpha$ is the boson degeneration factor. Following the canonical prescription of quantum statistical mechanics one can calculate the particle-number density, $\rho$, and the energy from the grand potential $\Omega_{bos}$ [8], and the heat capacity $C_V$. In the interior of a helium-WD, the electrons are strongly degenerate. Thus, the contribution to the heat capacity due to electrons is negligible when compared to the contribution due to the non-degenerate nuclei. For this reason we did not include electrons in the present treatment of the cooling of helium-WD.

The Bose–Einstein condensation occurs when a macroscopically large number of particles occupy the zeroth-momentum quantum state. The fugacity in the BEC is constant and $z = 1$ for $T < T_c$, where $T_c$ is the temperature of condensation. This temperature is calculated by fixing the density $\rho$ and solving the equation

$$T_c = \left(\frac{4\pi^2 \rho}{g}\right)^{1/3} \frac{1}{[F(\phi_c)]^{1/3}}. \tag{10}$$

The structure of the function $F(\phi_c)$ of the above equation is rather involved. For the sake of completeness we show it for the scalar mode, namely

$$F(\phi) = \int_0^\infty \mathrm{d}y \, \frac{(y+\phi)\omega(y,\phi)\alpha(y,\phi)}{e^y - 1},$$

where

$$\alpha(y,\phi) = \Bigg\{ \frac{m_0^2 + m_\gamma^2}{2m_\gamma^2}(y+\phi)^2 - \frac{m_0^4}{8M^4}\phi^2 - \left[ \left( \frac{m_0^2 - m_\gamma^2}{2m_\gamma^2}(y+\phi)^2 - \frac{m_0^4}{8M^4}\phi^2 \right)^2 + \frac{m_0^2}{m_\gamma^2}\phi^2(y+\phi)^2 - \frac{m_0^4}{4M^4}\phi^4 \right]^{1/2} \Bigg\}^{1/2},$$

$$\omega(y,\phi) = \left[ \left( \frac{m_0^4}{8M^4}\phi^2 - \frac{m_0^2 - m_\gamma^2}{2m_\gamma^2}(y+\phi)^2 \right) \times \frac{m_0^2 - m_\gamma^2}{m_\gamma^2} - \frac{m_0^2}{m_\gamma^2}\phi^2 \right] \times \left[ \left( \frac{m_0^2 - m_\gamma^2}{2m_\gamma^2}(y+\phi)^2 - \frac{m_0^4}{8M^4}\phi^2 \right)^2 + \frac{m_0^2}{m_\gamma^2}\phi^2(y+\phi)^2 - \frac{m_0^4}{4M^4}\phi^4 \right]^{-1/2} + \frac{m_0^2 + m_\gamma^2}{m_\gamma^2},$$

and $\phi = \frac{2M^2}{m_0 T}$. The condensation temperatures, $T_c$, for the longitudinal and transversal modes are, for the considered densities, at least one or two orders of magnitude smaller than the lowest limit for $T_c$ computed for the scalar mode. In other words, the contribution of these modes yields values of the condensation temperature much outside the range allowed by helium-WD conditions. The heat capacity is written

$$C_V = \frac{g}{4\pi^2}T^3 \left[ \frac{1}{2}Q_{2,2}^{2,-1,0}(y,\phi,\lambda=1) + Q_{2,1}^{0,1,1}(y,\phi,\lambda=1) + 2Q_{1,1}^{1,1,0}(y,\phi,\lambda=1) + Q_{2,0}^{1,1,0}(y,\phi,\lambda=1) \right], \tag{11}$$

for $T < T_c$, and

$$C_V = \frac{g}{4\pi^2}T^3 \Bigg[ \frac{1}{2}Q_{2,2}^{2,-1,0}(y,\phi,\lambda) + Q_{2,1}^{0,1,1}(y,\phi,\lambda) + 2Q_{1,1}^{1,1,0}(y,\phi,\lambda) + Q_{2,0}^{1,1,0}(y,\phi,\lambda) - \left( \frac{1}{2}Q_{1,2}^{2,-1,0}(y,\phi,\lambda) + Q_{2,1}^{0,1,1}(y,\phi,\lambda) + Q_{1,0}^{1,1,0}(y,\phi,\lambda) + Q_{0,1}^{1,1,0}(y,\phi,\lambda) \right)^2 \times \left( \frac{1}{2}Q_{0,2}^{2,-1,0}(y,\phi,\lambda) + Q_{0,1}^{0,1,1}(y,\phi,\lambda) + Q_{0,0}^{1,1,0}(y,\phi,\lambda) \right)^{-1} \Bigg], \tag{12}$$

for $T > T_c$. In Eqs. (11) and (12)

$$Q_{n,m}^{a,b,c}(y,\phi,\lambda) = \int_0^\infty \mathrm{d}y \left[ \left( e^y \lambda^{-1} - 1 \right)^{-1} y^n (y+\phi)^m \times \left[\omega(y,\phi)\right]^a \left[\alpha(y,\phi)\right]^b \left[\delta(y,\phi)\right]^c \right],$$

and

$$\delta(y,\phi) = \frac{m_0^4}{m_\gamma^2}(y+\phi)\phi^4 \frac{4M^4 + m_\gamma^4 - m_0^2 m_\gamma^2}{4M^4} \times \left[ \left( \frac{m_0^2 - m_\gamma^2}{2m_\gamma^2}(y+\phi)^2 - \frac{m_0^4}{8M^4}\phi^2 \right)^2 + \frac{m_0^2}{m_\gamma^2}\phi^2(y+\phi)^2 - \frac{m_0^4}{4M^4}\phi^4 \right]^{-3/2}.$$

In the absence of the condensation (no-BEC regime) the equation for the heat capacity yields $C_V = \frac{3}{2}$.

## 3. Cooling of helium-WD considering charged condensation

We shall now focus the attention on the calculation of the cooling time of a He-WD, in presence of the BEC phenomenon. The luminosity of a star can be written as [9]

$$L = -c_v \frac{\mathrm{d}T_*}{\mathrm{d}t}, \tag{13}$$

where $t$ is the time. Considering the Rosseland opacity, as given in terms of the Kramers approximation, and integrating the equations of hydrostatic equilibrium and radiative transport, we find the luminosity of the WD as a function of the temperature $T_*$ of the degenerate interior (see Eq. (4.1.11) of [9]). We can express the luminosity as a function of the temperature

$$L = \theta T_*^{3.5}\ \mathrm{erg/s}, \tag{14}$$

$$\theta = 5.7 \times 10^5 \frac{\mu}{\mu_e^2} \frac{1}{Z(1+X)} \frac{M}{M_\odot}. \tag{15}$$

Using the previous equations, and considering the specific heat per ion as the only quantity that depends on the temperature, we can determine the cooling time $\Delta t$ of the WD by

$$\Delta t = \frac{1}{\theta} \int_{T_f}^{T_0} \frac{c_V(T_*)}{T_*^{3.5}} \, \mathrm{d}T_*. \tag{16}$$

One can then perform the integral by using the expression for the specific heat per particle $c_v$ (see Section 2), and obtain the ratio between the cooling time in presence of BEC, and the cooling time without the BEC-mechanism as a function of the final temperature. In Fig. 2 we show the results of this ratio for the density $\rho = 10^6 \frac{\mathrm{g}}{\mathrm{cm}^3}$. For a fixed density the cooling time in presence of BEC, near the critical temperature, is greater than the one predicted by the standard model (no BEC situation). However, once the condensate is formed, the ratio between them goes to zero, since the heat capacity of the standard model is constant. Because the specific heat content of the WD interior falls with its temperature, the WD cools *faster* than predicted by the standard theory.

So far, we have not included crystallization as a possible mechanism in the cooling of a helium-WD, although it is true, indeed, that Coulomb interactions would favor a crystal structure. Following the arguments of Van Horn [10] this should be the case if the quantity $\Gamma$

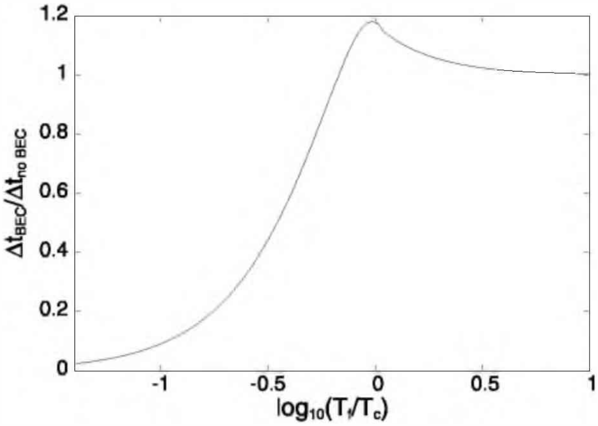


**Fig. 2.** Ratio between the cooling time in presence of BEC, $\Delta_{BEC}$, and the standard cooling time with no BEC, $\Delta_{noBEC}$, as a function of the final temperature and for the density, $\rho = 10^6 \frac{g}{cm^3}$. The value of $T_c$ is given by Eq. (10) at the given density: $T_c = 9.2 \times 10^6$ K.

$$\Gamma = \frac{3E_{\text{Coulomb}}}{2E_{\text{Thermal}}} = \frac{(Ze)^2}{4\pi d k_B T} \tag{17}$$

(where $d$ is the average distance between nuclei) satisfies the condition $\Gamma > 170$ [10]. For the case of C–O WD the densities and temperatures allow for this condition to be fulfilled. However, for the case of helium-WD Coulomb effects are much lower ($\Gamma < 170$). To achieve a value of $\Gamma$ consistent with crystallization, a helium-WD must be much cooler, a process which needs a time scale larger than the age of the Universe.

## 4. Conclusions

In this Letter we have discussed the mechanism of Bose–Einstein condensation in the interior of helium white dwarfs stars, by following the suggestion of Gabadadze and Rosen [1,2]. We have explicitly adopted the BEC hypothesis to calculate the cooling time of the star. As a consequence of the strong departure of the temperature dependence of the specific heat, due to the BEC phenomenon, the calculated cooling time is much shorter than the one predicted by standard model. It can, therefore, be concluded that the observation of a sharp variation of the cooling time may indeed confirm the predictions of [1,2]. Work concerning the effects of BEC condensation on the luminosity curve of helium-WD is in progress and the results will be published elsewhere [7].

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