

R-symmetry and supersymmetry breaking at finite temperature

E.F. Moreno^a and F.A. Schaposnik^{b,1}

^a*Department of Physics, West Virginia University,
Morgantown, West Virginia 26506-6315, U.S.A.*

^b*Departamento de Física, Universidad Nacional de La Plata,
C.C. 67, 1900 La Plata, Argentina*

E-mail: enrique.moreno@mail.wvu.edu, fidel@fisica.unlp.edu.ar

ABSTRACT: We analyze the spontaneous $U(1)_R$ symmetry breaking at finite temperature for the simple O’Raifeartaigh-type model introduced in [1] in connection with spontaneous supersymmetry breaking. We calculate the finite temperature effective potential (free energy) to one loop order and study the thermal evolution of the model. We find that the R-symmetry breaking occurs through a second order phase transition. Its associated meta-stable supersymmetry breaking vacuum is thermodynamically favored at high temperatures and the model remains trapped in this state by a potential barrier, as the temperature lowers all the way until $T = 0$.

KEYWORDS: Supersymmetry Breaking, Global Symmetries

¹Associated with CICBA.

Contents

1	Introduction	1
2	Set up of the model and the effective potential	2
3	The fate of the meta-stable vacuum	5
4	The fate of the runaway direction	7
5	Discussion	10
A	Mass matrices	11

1 Introduction

It became clear after the work of Nelson and Seiberg [2] that global R -symmetry plays a key role in connection with supersymmetry breaking. In order to have spontaneous supersymmetry breaking at the ground state of generic models there must be a global $U(1)_R$ symmetry, but in order to have non-zero gaugino masses it is necessary that this symmetry be explicitly or spontaneously broken. The work of Intriligator, Seiberg, and Shih (ISS) [3] showed how this tension between R -symmetry and supersymmetry can be exploited to find generic models with an acceptably long lived meta-stable supersymmetry breaking vacuum. Moreover, studying the Seiberg dual of $\mathcal{N} = 1$ super-QCD it has been shown that, at high temperatures, the supersymmetry breaking vacua are dynamically favored over the “supersymmetry preserving” ones¹ so that the Universe would naturally have been driven into them [4]–[9], a possibility already discussed on general grounds a long time ago in [10].

Different models with meta-stable symmetry breaking vacua and structures rather different than those discussed by ISS have been also investigated, as for example those based in gauge mediation and extraordinary gauge mediation, which cover a broad class of R -symmetric generic models with supersymmetry breaking [11]–[14].

There is a very practical mechanism proposed in [1] leading to spontaneous $U(1)_R$ breaking. It applies to O’Raifeartaigh models with a continuous space of supersymmetry breaking vacua and degenerate tree-level vacuum energy. It has been shown in that work that, due to one loop corrections, spontaneous R -symmetry breaking occurs à la Coleman-Weinberg in a very simple O’Raifeartaigh type model and for a wide range of parameters.

¹At finite temperature SUSY is always broken. With the quotation marks we mean the phase which, for zero temperature, corresponds to a supersymmetry preserving vacua.

More general models of this kind have been discussed in [15] and their thermal history has also been recently investigated [16].

It is the purpose of this work to study the question of spontaneous $U(1)_R$ symmetry breaking at finite temperature and the resulting supersymmetry breaking pattern by analyzing the thermal evolution of the O’Raifeartaigh-type model introduced in [1]. To this end we will compute the finite temperature effective potential (i.e. the free energy density) by shifting as usual the relevant background fields and use the resulting quadratic terms (the mass terms) to perform the one-loop calculation. Studying numerically the corresponding one loop effective potential we will analyze the nature of the different phase transitions, showing how parameters of the model can be chosen so as to cover the desired range of critical temperatures at which R-symmetry breaking takes place. As we shall see, our numerical results are consistent with the general analysis presented in [16] where a broad class of models for gauge mediation were considered. Indeed, in the classification of Extraordinary Gauge Mediation Models (EOGM) of [11], the model we analyze belongs to the type I class (provided one promotes the singlet messengers to fields transforming in the $5 \oplus 5$ representation of $SU(5)$). Our analysis will confirm the thermal evolution scenario advanced in [16] for type I models, in particular concerning the existence of a metastable vacuum at high temperatures with no $T = 0$ analog

In the next section we introduce the model proposed in [1] and describe its classical vacua, which includes a moduli space and a runaway direction. We then present the different terms that contribute to the one loop finite temperature effective potential V_{eff}^1 . In section 3 we calculate V_{eff}^1 along the pseudo-modulus, which is at the origin of the dynamically generated meta-stable vacuum, and analyze the R-symmetry breaking phase transition. We then extend in section 4 the calculation of V_{eff}^1 by considering a background field that interpolates between the meta-stable vacuum and the runaway direction, and discuss in detail the resulting thermal scenario. We finally summarize and discuss our results in section 5.

2 Set up of the model and the effective potential

We consider the O’Raifeartaigh model for chiral superfields considered in [1], with canonical Kähler potential and superpotential

$$W = \lambda X \phi_1 \phi_2 + m_1 \phi_1 \phi_3 + \frac{1}{2} m_2 \phi_2^2 + f X \quad (2.1)$$

This superpotential defines the underlying model which communicates supersymmetry breaking to the minimal supersymmetric Standard Model. Chiral superfields ϕ_i ($i = 1, 2, 3$) with R charges

$$R(\phi_1) = -1, \quad R(\phi_2) = 1, \quad R(\phi_3) = 3, \quad (2.2)$$

represent the messengers of supersymmetry breaking and the spurion field X generates the model’s pseudo-moduli space and has charge $R(X) = 2$. Parameters λ , f , m_1 , and m_2 will be taken, without loss of generality, as real positive numbers.

The resulting scalar potential (we use the same notation for superfields and their lowest components) takes the form

$$V^{tree}(X, \phi_i) = |\lambda\phi_1\phi_2 + f|^2 + |\lambda X\phi_2 + m_1\phi_3|^2 + |\lambda X\phi_1 + m_2\phi_2|^2 + |m_1\phi_1|^2 \quad (2.3)$$

and its extrema consist of:

- a moduli space

$$\phi_i^{(m)} = 0, \quad X^{(m)} \text{ arbitrary} \quad (2.4)$$

with

$$V = f^2 > 0 \quad (2.5)$$

- a runaway direction

$$\begin{aligned} \phi_1^{(r)} &= \left(\frac{f^2 m_2}{\lambda^2 m_1 \phi_3} \right)^{\frac{1}{3}}, & \phi_2^{(r)} &= - \left(\frac{f m_1 \phi_3}{\lambda m_2} \right)^{\frac{1}{3}}, & \phi_3^{(r)} &\rightarrow \infty, \\ X^{(r)} &= \left(\frac{m_1^2 m_2 \phi_3^2}{\lambda^2 f} \right)^{\frac{1}{3}} \end{aligned} \quad (2.6)$$

with

$$V \rightarrow 0. \quad (2.7)$$

The moduli space does not correspond to global minima of the potential but, as long as

$$|X| < \frac{m_1}{\lambda} \frac{1 - y^2}{2y} \quad (2.8)$$

where

$$y = \frac{\lambda f}{m_1 m_2} \quad (2.9)$$

it leads to local minima of the potential. Since the X field is R -charged, such flat direction breaks the global $U(1)_R$ symmetry for any $X \neq 0$ in the range (2.8). It is clear now that if quantum corrections produce a minimum at some point $\langle X \rangle \neq 0$ of this flat direction, which then corresponds to a pseudo-moduli, the associated vacuum expectation value will spontaneously break the R -symmetry. This was shown at $T = 0$ in [1] by computing the one-loop effective potential. We will now extend the analysis to include thermal effects by computing the finite temperature effective potential up to one loop, which takes the form [17]

$$V_1^{eff}(X^{cl}, \phi_i^{cl}) = V^{tree}(X^{cl}, \phi_i^{cl}) + V_1^0(X^{cl}, \phi_i^{cl}) + V_1^T(X^{cl}, \phi_i^{cl}). \quad (2.10)$$

The original fields are written in the form

$$\begin{aligned} X &= X^{cl} + x \\ \phi_i &= \phi_i^{cl} + \varphi_i \end{aligned} \quad (2.11)$$

to proceed to compute the one-loop contribution by integrating terms quadratic in the fluctuations x, φ_i . The zero temperature piece V_1^0 of the effective potential is given by the usual supersymmetric generalization of the Coleman-Weinberg formula

$$V_1^0 = \frac{1}{64\pi^2} \mathcal{S}\text{Tr} \mathcal{M}^4 \log \frac{\mathcal{M}^2}{\Lambda^2} \quad (2.12)$$

where $\mathcal{S}\text{Tr}$ is the supertrace including a negative sign for fermions, \mathcal{M} stands for the full mass matrix resulting from the shift (2.11), $\mathcal{M} = \mathcal{M}(X^{\text{cla}}, \phi_i^{\text{cla}})$, and Λ is a mass scale. Concerning the finite temperature contribution, one has [17]

$$V_1^T = \frac{T^4}{2\pi^2} \sum_i \pm n_i \int_0^\infty ds s^2 \log \left(1 \mp e^{-\sqrt{s^2 + \mathcal{M}_i^2/T^2}} \right) \quad (2.13)$$

where the sum is over all degrees of freedom ($\{n_i\}$ denotes the number of degrees of freedom, $n = 2$ for complex scalars and Weyl fermion and the upper (lower) sign is for bosons (fermions)). Finally, \mathcal{M}_i denotes the eigenvalues of the \mathcal{M} -matrix.

In order to make contact between the parameters of the model with scalar potential (2.3) and those of the Minimal Supersymmetric Standard Model (MSSM) one has to consider masses of the observable fields. It should be mentioned that a superpotential of the type (2.1) should be in principle supplemented with a minimal gauge mediation (MGM) messenger ϕ_4 which, coupled to the spurion field X through a term of the form $X\phi_4^2$, will effectively give a mass to the otherwise massless gaugino [11]. Note that the introduction of this additional messenger would promote our model to a type III one, for which, instead of a condition of the form (2.8) stability requires an upper and a lower bound for X , $X_{\text{max}} > |X| > X_{\text{min}}$, as noted in [11] for $T = 0$ and discussed in [16] for finite T . In the case of the model we consider one should adjust the parameters so that such bounds hold at all temperatures and as the temperature grows $X_{\text{min}}(T)$ approaches the origin faster than the pseudomodulus minimum. We leave for a future work a detailed analysis of this issue and proceed to determine the orders of magnitude of the different superpotential parameters by analyzing sfermion masses.

Sfermion masses m_f^2 can be extracted from the matter wave function renormalization through the formula [18],

$$m_f^2 \sim \frac{\alpha^2}{(4\pi)^2} \left(\frac{f}{m} \right)^2 \tilde{N} \quad (2.14)$$

where α is the running coupling constant of the underlying gauge theory (evaluated at the messenger scale, $\alpha/4\pi \sim 10^{-2}$) and

$$\tilde{N} = \lambda^2 \frac{\partial^2}{\partial x \partial x^*} \sum_{i=1}^3 \log^2 |\mathcal{M}_{F_i}|^2. \quad (2.15)$$

Here \mathcal{M}_{F_i} are the eigenvalues of the fermion mass matrix resulting from superpotential (2.1) and for simplicity we have set $m_1 = m_2 = m$ and defined $x = \lambda X/m$. Given configuration (2.4), \mathcal{M}_F can be written in the form

$$\mathcal{M}_F^2 = m^2 \begin{pmatrix} xx^* + 1 & x & 0 \\ x^* & xx^* + 1 & x \\ 0 & x^* & 1 \end{pmatrix} \quad (2.16)$$

Formula (2.14) is valid in the regime $f \ll m^2$ for which supersymmetry is broken only in the effective field theory below the messenger scale by soft terms.

Now one can check that

$$\bar{N}(x \rightarrow 0) = \lambda^2, \quad \bar{N}(x \rightarrow \infty) = 0 \quad (2.17)$$

Moreover, had we added the MGM messenger, the \bar{N} behavior at infinity would have raised to λ^2 so that we can take $\bar{N} \sim \lambda^2$ in the whole range. In fact, if one scales $X \rightarrow X/\lambda$ and $f \rightarrow \lambda f$ the coupling λ completely disappears from superpotential (2.1) so that we can just set $\bar{N} \sim 1$ in (2.14).

Since one expects that the sfermion mass should be in the TeV scale, one infers from (2.14) that $f/m \sim 100$ TeV, this in turn implying that $m \gg 100$ TeV. The estimate would remain nearly unchanged if instead of the assumption $f \ll m^2$ we consider the case $f \sim m^2$. We conclude that for the analysis of the thermal evolution of the system, high temperatures will correspond to $T \gg 100$ TeV.

3 The fate of the meta-stable vacuum

We start by considering the effective potential for configuration (2.4), that is, we take $\phi_i^{\text{cl}} = \phi_i^{(m)} = 0$ and $X^{\text{cl}} = X^{(m)} = X$ in formula (2.10). In this case the boson mass matrix takes the form (we omit the superscript m)

$$\mathcal{M}_B^2 = \begin{pmatrix} m_1^2 + \lambda^2 X^2 & m_2 \lambda X & 0 & 0 & f \lambda & 0 \\ m_2 \lambda X & m_2^2 + \lambda^2 X^2 & m_1 X \lambda & f \lambda & 0 & 0 \\ 0 & m_1 \lambda X & m_1^2 & 0 & 0 & 0 \\ 0 & f \lambda & 0 & m_1^2 + \lambda^2 X^2 & m_2 \lambda X & 0 \\ f \lambda & 0 & 0 & m_2 \lambda X & m_2^2 + \lambda^2 X^2 & m_1 \lambda X \\ 0 & 0 & 0 & 0 & m_1 \lambda X & m_1^2 \end{pmatrix} \quad (3.1)$$

while the fermion mass matrix reads

$$\mathcal{M}_F^2 = \begin{pmatrix} m_1^2 + \lambda^2 X^2 & m_2 \lambda X & 0 & 0 & 0 & 0 \\ m_2 \lambda X & m_2^2 + \lambda^2 X^2 & m_1 X \lambda & 0 & 0 & 0 \\ 0 & m_1 \lambda X & m_1^2 & 0 & 0 & 0 \\ 0 & 0 & 0 & m_1^2 + \lambda^2 X^2 & m_2 \lambda X & 0 \\ 0 & 0 & 0 & m_2 \lambda X & m_2^2 + \lambda^2 X^2 & m_1 \lambda X \\ 0 & 0 & 0 & 0 & m_1 \lambda X & m_1^2 \end{pmatrix} \quad (3.2)$$

Using this result, one can compute the zero-temperature one-loop contribution (2.12), as originally calculated in [1],

$$V_1^0 = \frac{1}{64\pi^2} \text{Tr} \left(\mathcal{M}_B^4 \log \frac{\mathcal{M}_B^2}{\Lambda^2} - \mathcal{M}_F^4 \log \frac{\mathcal{M}_F^2}{\Lambda^2} \right) \quad (3.3)$$

as well as the finite temperature one, eq. (2.13), which can be rewritten in the form

$$V_1^T = \frac{T^4}{2\pi^2} \sum_{i=1}^6 \int_0^\infty ds s^2 \left(\log \left(1 - e^{-\sqrt{s^2 + \mathcal{M}_{\tilde{B}i}^2}/T} \right) - \log \left(1 + e^{-\sqrt{s^2 + \mathcal{M}_{\tilde{F}i}^2}/T} \right) \right) \quad (3.4)$$

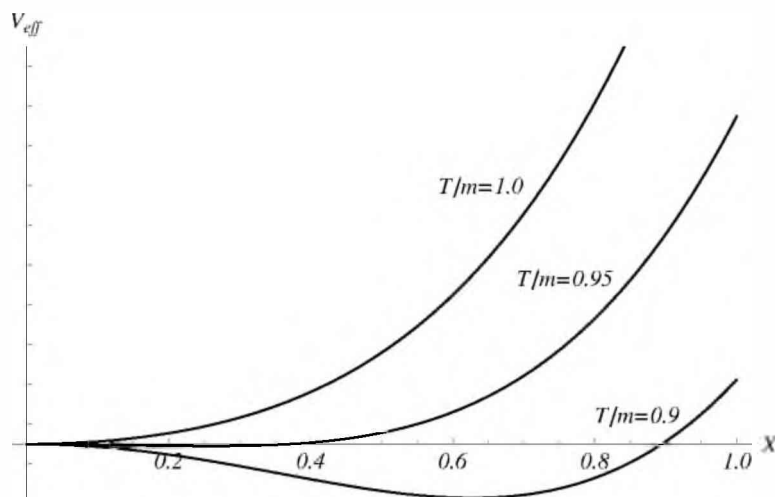


Figure 1. The effective potential as a function of $|X|$ showing the second order phase transition (we have taken $r = 4$ and $y = 0.2$). The curve in the middle corresponds to the critical temperature which for the chosen parameters takes the value $T_R/m = 0.95$.

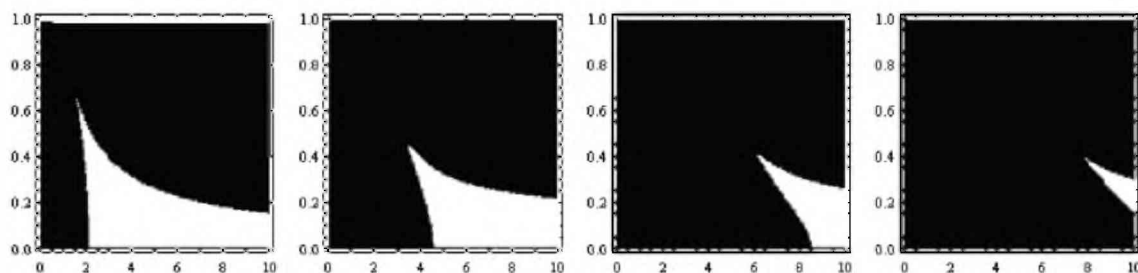


Figure 2. Plot of $y = \lambda f / (m_1 m_2)$ as a function of $r = m_2 / m_1$ for $T = 0, 1, 1.5,$ and 1.8 (from left to right). The white region corresponds to a local (R -symmetry breaking) minimum (with no tachyons).

One can scale $X \rightarrow m_1 X / \lambda$ and masses so that the effective potential only depends on the rescaled X and on two parameters: y , defined in eq. (2.9), and r , given by

$$r = \frac{m_2}{m_1} \tag{3.5}$$

so that $V_1^{eff} = V_1^{eff}(X; r, y)$ with m_1 giving the mass scale.

Eigenvalues \mathcal{M}_{B_i} and \mathcal{M}_{F_i} (with $i = 1, \dots, 6$) of mass matrices \mathcal{M}_B and \mathcal{M}_F have to be computed numerically. Of course, at $T = 0$ one reproduces the results in [1] thus finding that, for a wide range of parameters, there is a meta-stable vacuum where $U(1)_R$ is spontaneously broken. Concerning the thermal evolution we show in figure 1 the plot of V_1^{eff} as a function of X for different temperatures. In figure 2 we represent the change with temperature of the region (shown in white) in the r, y plane where there is a $U(1)_R$ symmetry breaking local minimum of the potential satisfying (2.8).

Using different pairs of values (r, y) in the range where R symmetry breaking occurs

(white region in figure 2) we have then found a second order phase transition at a certain critical temperature T_R , so that for $T < T_R$ there is a minimum away from the origin, i.e. at $X = \langle X \rangle \neq 0$.

Interestingly enough, changing parameters one can make the critical temperature vary in a wide range. For example, for the choice of parameters corresponding to figure 1, ($y = 0.2, r = 4$) the critical temperature is $T_R/m = 0.95$ while for $y = 0.2, r = 2.07$ it becomes $T_R/m \sim 10^{-3}$. In fact, by choosing parameters (r, y) closer and closer to the left frontier of the white region in figure 2 one can lower the critical temperature as much as wanted. Taking into account the condition $m \gg 100$ TeV previously found from the requirement that $m_{\text{sf}} \sim 1$ TeV, we see that the critical temperature at which R-symmetry is broken can be adjusted in a wide range going for the two choices we have used as example, from $T_R \gg 100$ TeV to $T_R \sim 1$ TeV. It should be noted that as the value of the critical temperature lowers the R-symmetry breaking VEV $\langle X \rangle$ gets closer to the origin.

4 The fate of the runaway direction

We will now study the behavior of the runaway direction as the temperature changes. To this end, we will follow an approach similar to that used in [4] in the case of the ISS and consider a path $(X^{\text{int}}, \phi_i^{\text{int}})$ interpolating between the meta-stable supersymmetry vacua and the supersymmetric runaway direction. A convenient choice of path is

$$\begin{aligned}
 X^{\text{int}} &= \left(\frac{m_1^2 m_2 \phi_3^2}{\lambda^2 f} \right)^{\frac{1}{3}} + (1 - h(\phi_3)) \langle X \rangle, \\
 \phi_1^{\text{int}} &= h(\phi_3) \left(\frac{f^2 m_2}{\lambda^2 m_1 \phi_3} \right)^{\frac{1}{3}}, \quad \phi_2^{\text{int}} = - \left(\frac{f m_1 \phi_3}{\lambda m_2} \right)^{\frac{1}{3}}, \quad \phi_3^{\text{int}} = \phi_3,
 \end{aligned}
 \tag{4.1}$$

The function $h(\phi_3)$ should be chosen so as to conciliate the behavior of X and ϕ_1 at the two-endpoints. An appropriate election is

$$h(y) = \frac{2}{\pi} \arctan cy
 \tag{4.2}$$

where c is a parameter to be chosen so that the path, which goes from the zero temperature meta-stable local minima ($\phi_3 = 0$) at $X = \langle X \rangle$ to the runaway value ($\phi_3 \rightarrow \infty$) does not have modes with negative square masses.

We present in an appendix the explicit form of boson and fermion masses for the path (4.1). From their explicit form one can numerically study the effective potential as a function of ϕ_3 and the temperature, $V_1^{\text{eff}} = V_1^{\text{eff}}(\phi_3, T)$, and determine the resulting minima landscape. First, one has to numerically compute the mass eigenvalues and then evaluate the zero temperature one-loop contribution to the effective potential (eq. (3.3)) as well as the finite temperature one, V_1^T , given by eq. (3.4).

One should note that at very high temperatures V_1^T , as given by formula (2.13), becomes

$$V_1^T \sim -\frac{\pi^2}{8} T^4 \quad \text{for } T \rightarrow \infty
 \tag{4.3}$$

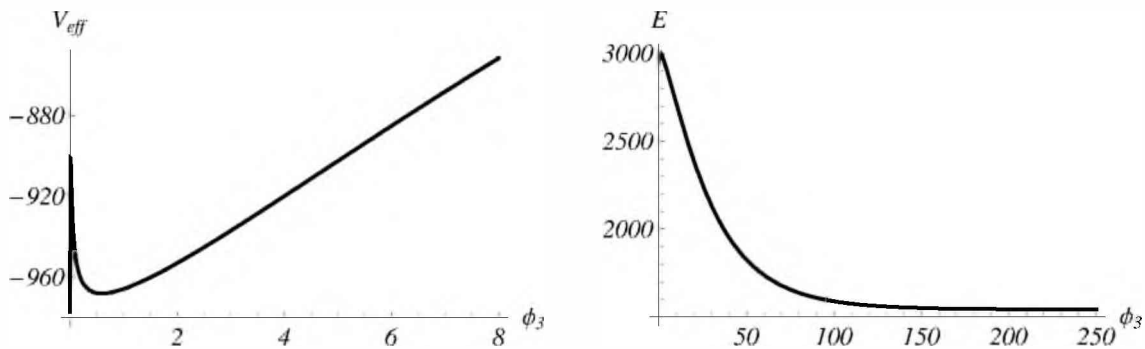


Figure 3. Free energy vs. total energy for $T/m = 5$.

Note that the negative sign in the effective potential is harmless since at finite temperature V_1^{eff} should be identified with the free energy as a function of the order parameter while the total energy is given by

$$E = V_1^{eff} - T \frac{\partial V_1^{eff}}{\partial T} \tag{4.4}$$

which is indeed positive for all temperatures. We show in figure 3 the free energy V_1^{eff} (left) and the total energy E (right) at very high temperatures. The figure clearly shows that although the energy is lower in what will become at zero temperature the runaway direction, the entropy contribution favors the non supersymmetric free energy minimum near the origin

From the numerical analysis of the complete effective potential $V_1^{eff}(\phi_3, T)$ one infers the following scenario for the thermal evolution of the effective potential:

- For $T/m \gg 1$ the potential has an absolute minimum at the origin in field space and it grows without bound for large values of ϕ_3 . The zero-temperature meta-stable vacuum in the pseudomoduli direction has not yet started to develop and one finds, in addition, a local minimum at a finite value ϕ_3^* (i.e. $V_1^{eff*}(\phi_3^*, T^*) > V_1^{eff*}(0, T^*)$)
- As the temperature lowers, the slope of the potential at infinity decreases until it becomes negative. The change of sign takes place at a temperature T_h at which the absolute minimum of the potential is still at the origin.
- At a lower temperature T_b the local minimum V_1^{eff*} disappears.
- At a lower temperature T_{ra} , $V_1^{eff}(\phi_3 \rightarrow \infty, T_{ra}) = V_1^{eff}(0, T_{ra})$ so that the runaway minimum appears and a first order phase transition starts.
- As already discussed, at a lower temperature T_R the R -symmetry breaking meta-stable vacuum arises.

As an example, for the parameter choice $r = 4, y = 0.2$ already used to discuss the meta-stable vacuum evolution, the temperatures defined above take the values

$$T_h/m = 2.96, \quad T_b/m = 1.29, \quad T_{ra}/m = 1.14, \quad T_R/m = 0.95 \tag{4.5}$$

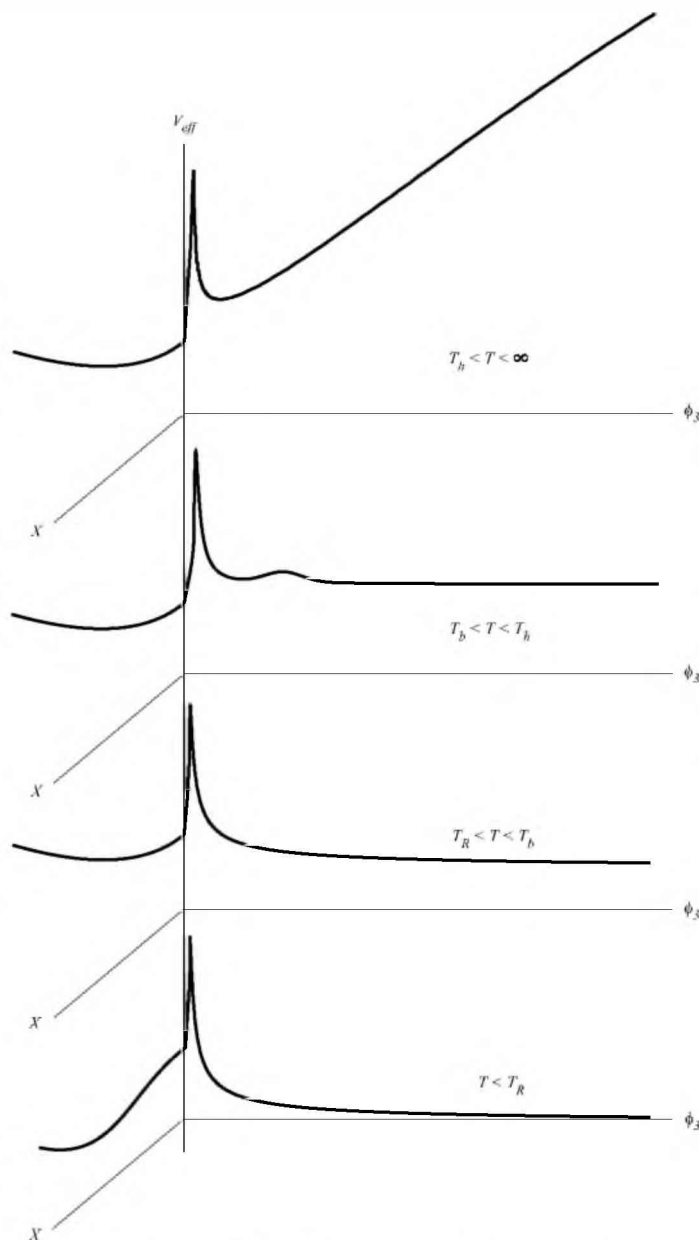


Figure 4. Evolution of the effective potential with temperature.

We have already described how changing parameters (r, y) towards the left border of the R -symmetry braking region (white region in figure 2) lowers the critical temperature at which the transition to the meta-stable vacuum takes place. All other temperatures lower but their change is not so marked. As an example, for $(r = 2.7, y = 0.2)$ one has

$$T_h/m = 1.5, \quad T_b/m = 0.99, \quad T_{ra}/m = 0.81, \quad T_R/m = 1 \times 10^{-3} \quad (4.6)$$

Figure 4 shows a qualitative representation of the above scenario.

In order to exclude the possibility that the system escapes towards the runaway direction instead of decaying into the meta-stable vacuum let us note that for $T > T_b$ the

effective potential has an absolute minimum at the origin. Only for temperatures $T \leq T_b$ the runaway direction corresponds to an (asymptotic) global minimum of the effective potential. Since such temperatures are sufficiently low as to neglect thermal corrections, one can see [1] that the barrier preventing the system to roll-down along the runaway direction has a width of order y^{-1} while its height is of order y^0 . Hence, by taking y sufficiently small the system will remain in the vacuum at the origin while $T_b < T < T_R$ and then smoothly evolve towards the meta-stable vacuum for $T < T_R$.

5 Discussion

We have analyzed the thermal evolution of the simplest O’Raifeartaigh-type model in which spontaneous R-symmetry breaking occurs dynamically, leading to a runaway behavior at large fields and a meta-stable vacuum which, at zero temperature, spontaneously breaks supersymmetry. Studying the effective potential at finite temperature we have shown that the $U(1)_R$ breaking arises through a second order phase transition. Remarkably, the critical temperature at which the R-symmetry breaking phase starts can be lowered by an appropriate choice of parameters and this also implies that the VEV of the spurion field X also decreases.

We also analyzed the thermal evolution of the runaway direction finding, as expected, that high temperature contributions rise the asymptotic directions of the effective potential. Remarkably, we found that at high temperatures there is an extra local minimum of the effective potential, though energetically unfavored with respect to the meta-stable vacuum. At some temperature (T_b) this local minimum disappears.

The whole thermal evolution sequence is as follows: At high temperatures the model is driven to the meta-stable SUSY-breaking vacuum. As the temperature decreases, the SUSY runaway direction becomes energetically favored but the transition between phases is long lived, so the system remains in the meta-stable vacuum. There is also an extra local minimum but with higher effective potential than the meta-stable vacuum. As the temperature decreases this extra minimum fades away. Finally, at an even lower temperature (T_R), the R symmetry is broken and a second-order phase transition occurs. This sequence, with the exception of the existence and eventual disappearance of the extra local minimum, is similar to the one described in [4]–[8] for the magnetic dual of SuperQCD. As stated in the introduction, the model studied here can be extended to the form of a type I model in the classification of ref. [11]. The general properties of the thermal evolution of these models was discussed in [16] and our numerical analysis of the vacuum structure at different temperatures is consistent with them. In particular our results confirm the existence of an extra vacuum at high T in addition to the one at the origin, with no analog at $T = 0$. This extra vacuum disappears as the temperature lowers below T_b .

An implicit assumption necessary to apply our results in a cosmological context is that the reheat temperature T_{reheat} is larger enough (with respect to the supersymmetry breaking scale) as to guarantee that the supersymmetry breaking history develops quasi-statically, in a situation of thermal equilibrium. This justifies to look for the minima of the free energy not taking into account possible interaction between fields and the heat bath.

Ignoring the possibility of non-equilibrium situations our results suggest that although the runaway direction starts to develop before the R-symmetry breaking meta-stable minimum appears, the system will not roll-down from the minimum at the origin because of the existence of a very high barrier so that when the R-symmetry breaking meta-stable vacuum is available, it will evolve to it and remain there for a sufficiently large time as to ensure that the Universe is still trapped there.

We would like to end this work by pointing out two directions in which we hope to continue our investigation on R-symmetry breaking and supersymmetry breaking at finite temperature. One concerns the analysis of models with explicit R-symmetry breaking which, under certain conditions, have supersymmetric vacua, runaway directions and meta-stable vacua [19]. As discussed in [20], the way in which R-symmetry is broken (spontaneously or explicitly) leaves a clear imprint on the phenomenology of the MSSM and it is then worthwhile to study broad classes of such models so as to compare the resulting thermal patterns. The other direction is related to the analysis in [21] on how pseudomoduli arising in generalized O’Raifeartaigh models from additional global symmetries can be candidates to dark matter (see also [22]). In this context it would be of interest to investigate the thermal evolution of such models along the lines developed here. We hope to analyze these issues in a future work.

Acknowledgments

We would like to thank Diego Marqués for his criticism and helpful comments. This work was partially supported by PIP6160-CONICET, BID 1728OC/AR PICT20204-ANPCYT grants and by CIC and UNLP, Argentina.

A Mass matrices

We write the boson and fermion mass matrices corresponding to the path (4.1) in the form

$$\mathcal{M}_F^2 = \begin{pmatrix} A & 0 \\ 0 & A \end{pmatrix}, \quad \mathcal{M}_B^2 = \begin{pmatrix} A & B \\ B & A \end{pmatrix} \tag{A.1}$$

where A and B are symmetric 4×4 matrices with nonzero elements

$$A_{11} = \frac{r^2 y^{4/3} h(\phi_3)^2 m_1^{8/3}}{\phi_3^{2/3} \lambda^{2/3}} + y^{2/3} \phi_3^{2/3} \lambda^{2/3} m_1^{4/3}$$

$$A_{12} = A_{21} = \frac{m_1^{4/3} r y^{2/3} \lambda^{2/3} h(\phi_3) \left(-h(\phi_3) x_0 + x_0 + \frac{\sqrt[3]{m_1 \phi_3^{2/3}}}{\sqrt[3]{y \sqrt[3]{\lambda}}} \right)}{\sqrt[3]{\phi_3}}$$

$$A_{13} = A_{31} = -m_1^{2/3} x_0 \sqrt[3]{y} \sqrt[3]{\phi_3} \lambda^{4/3} - m_1 \phi_3 \lambda + \frac{\left(r^2 y^{2/3} m_1^{7/3} + x_0 \sqrt[3]{y} \phi_3^{2/3} \lambda^{5/3} m_1^{2/3} \right) h(\phi_3)}{\sqrt[3]{\phi_3} \sqrt[3]{\lambda}}$$

$$A_{14} = A_{41} = -m_1^{5/3} \sqrt[3]{y} \sqrt[3]{\phi_3} \sqrt[3]{\lambda}$$

$$\begin{aligned}
A_{22} &= m_1^2 + y^{2/3} \phi_3^{2/3} \lambda^{2/3} m_1^{4/3} + \left(x_0 \lambda - x_0 h(\phi_3) \lambda + \frac{\sqrt[3]{m_1} \phi_3^{2/3} \lambda^{2/3}}{\sqrt[3]{y}} \right)^2 \\
A_{23} &= A_{32} = m_1 r \left(x_0 \lambda + \frac{\sqrt[3]{m_1} \phi_3^{2/3} \lambda^{2/3}}{\sqrt[3]{y}} - (m_1 y + x_0 \lambda) h(\phi_3) \right) \\
A_{33} &= \frac{r^2 y^{4/3} h(\phi_3)^2 m_1^{8/3}}{\phi_3^{2/3} \lambda^{2/3}} + r^2 m_1^2 + \left(x_0 \lambda - x_0 h(\phi_3) \lambda + \frac{\sqrt[3]{m_1} \phi_3^{2/3} \lambda^{2/3}}{\sqrt[3]{y}} \right)^2 \\
A_{34} &= A_{43} = \frac{\phi_3^{2/3} \lambda^{2/3} m_1^{4/3}}{\sqrt[3]{y}} + x_0 \lambda m_1 - x_0 \lambda h(\phi_3) m_1 \\
A_{44} &= m_1^2 \\
B_{12} &= B_{21} = \frac{m_1^{4/3} r \sqrt[3]{y} \sqrt[3]{\lambda} (h(\phi_3) - 1) \left(\sqrt[3]{m_1} \phi_3^{2/3} - x_0 \sqrt[3]{y} \sqrt[3]{\lambda} h(\phi_3) \right)}{\sqrt[3]{\phi_3}} \\
B_{13} &= B_{31} = m_1^{2/3} x_0 \sqrt[3]{y} \sqrt[3]{\phi_3} \lambda^{4/3} (h(\phi_3) - 1) \\
B_{23} &= B_{32} = -m_1^2 r y (h(\phi_3) - 1)
\end{aligned} \tag{A.2}$$

References

- [1] D. Shih, *Spontaneous R-symmetry breaking in O’Raifeartaigh models*, *JHEP* **02** (2008) 091 [[hep-th/0703196](#)] [[SPIRES](#)].
- [2] A.E. Nelson and N. Seiberg, *R symmetry breaking versus supersymmetry breaking*, *Nucl. Phys. B* **416** (1994) 46 [[hep-ph/9309299](#)] [[SPIRES](#)].
- [3] K.A. Intriligator, N. Seiberg and D. Shih, *Dynamical SUSY breaking in meta-stable vacua*, *JHEP* **04** (2006) 021 [[hep-th/0602239](#)] [[SPIRES](#)].
- [4] S.A. Abel, C.-S. Chu, J. Jaeckel and V.V. Khoze, *SUSY breaking by a metastable ground state: why the early universe preferred the non-supersymmetric vacuum*, *JHEP* **01** (2007) 089 [[hep-th/0610334](#)] [[SPIRES](#)].
- [5] N.J. Craig, P.J. Fox and J.G. Wacker, *Reheating metastable O’Raifeartaigh models*, *Phys. Rev. D* **75** (2007) 085006 [[hep-th/0611006](#)] [[SPIRES](#)].
- [6] W. Fischler, V. Kaplunovsky, C. Krishnan, L. Mannelli and M.A.C. Torres, *Meta-stable supersymmetry breaking in a cooling universe*, *JHEP* **03** (2007) 107 [[hep-th/0611018](#)] [[SPIRES](#)].
- [7] S.A. Abel, J. Jaeckel and V.V. Khoze, *Why the early universe preferred the non-supersymmetric vacuum. II*, *JHEP* **01** (2007) 015 [[hep-th/0611130](#)] [[SPIRES](#)].
- [8] V.S. Kaplunovsky, *Metastable supersymmetry breaking in a cooling universe*, *AIP Conf. Proc.* **957** (2007) 99 [[arXiv:0711.0031](#)] [[SPIRES](#)].
- [9] J. Jaeckel, *Metastable SUSY breaking-predicting the fate of the universe*, *Nucl. Phys. A* **820** (2009) 83c–90c [[arXiv:0810.5307](#)] [[SPIRES](#)].
- [10] J.R. Ellis, C.H. Llewellyn Smith and G.G. Ross, *Will the universe become supersymmetric?*, *Phys. Lett. B* **114** (1982) 227 [[SPIRES](#)].

- [11] C. Cheung, A.L. Fitzpatrick and D. Shih, *(Extra)ordinary gauge mediation*, *JHEP* **07** (2008) 054 [[arXiv:0710.3585](#)] [[SPIRES](#)].
- [12] R. Kitano, H. Ooguri and Y. Ookouchi, *Direct mediation of meta-stable supersymmetry breaking*, *Phys. Rev. D* **75** (2007) 045022 [[hep-ph/0612139](#)] [[SPIRES](#)].
- [13] N. Haba and N. Maru, *A Simple Model of Direct Gauge Mediation of Metastable Supersymmetry Breaking*, *Phys. Rev. D* **76** (2007) 115019 [[arXiv:0709.2945](#)] [[SPIRES](#)].
- [14] R. Essig, J.-F. Fortin, K. Sinha, G. Torroba and M.J. Strassler, *Metastable supersymmetry breaking and multitrace deformations of SQCD*, *JHEP* **03** (2009) 043 [[arXiv:0812.3213](#)] [[SPIRES](#)].
- [15] Z. Komargodski and D. Shih, *Notes on SUSY and R-symmetry breaking in Wess-Zumino models*, *JHEP* **04** (2009) 093 [[arXiv:0902.0030](#)] [[SPIRES](#)].
- [16] A. Katz, *On the thermal history of calculable gauge mediation*, [arXiv:0907.3930](#) [[SPIRES](#)].
- [17] L. Dolan and R. Jackiw, *Symmetry behavior at finite temperature*, *Phys. Rev. D* **9** (1974) 3320 [[SPIRES](#)].
- [18] G.F. Giudice and R. Rattazzi, *Extracting supersymmetry-breaking effects from wave-function renormalization*, *Nucl. Phys. B* **511** (1998) 25 [[hep-ph/9706540](#)] [[SPIRES](#)].
- [19] D. Marques and F.A. Schaposnik, *Explicit R-symmetry breaking and metastable vacua*, *JHEP* **11** (2008) 077 [[arXiv:0809.4618](#)] [[SPIRES](#)].
- [20] S.A. Abel, C. Durnford, J. Jaeckel and V.V. Khoze, *Patterns of gauge mediation in metastable SUSY breaking*, *JHEP* **02** (2008) 074 [[arXiv:0712.1812](#)] [[SPIRES](#)].
- [21] D. Shih, *Pseudomoduli dark matter*, [arXiv:0906.3346](#) [[SPIRES](#)].
- [22] B. Keren-Zur, L. Mazzucato and Y. Oz, *Dark matter and pseudo-flat directions in weakly coupled SUSY breaking sectors*, [arXiv:0906.5586](#) [[SPIRES](#)].