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Vortices, infrared effects and Lorentz invariance violation

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Abstract

The Yang–Mills theory with non-commutative fields is constructed following Hamiltonian and Lagrangian methods. This modification of the standard Yang–Mills theory produces spatially localized solutions very similar to those of the standard non-Abelian gauge theories. This modification of the Yang–Mills theory contain in addition to the standard contribution, the term $\theta^{\mu}\epsilon_{\mu\nu\rho\lambda}(A^{\nu}F^{\rho\lambda}+\frac{2}{3}A_{\nu}A_{\rho}A_{\lambda})$ where θ_{μ} is a given space-like constant vector with canonical dimension of energy. The A_{μ} field rescaling and the choice $\theta_{\mu}=(0,0,0,\theta)$, suggest the equivalence between the Yang–Mills–Chern–Simons theory in $2+1$ dimensions and QCD in $3+1$ dimensions in the heavy fermionic excitations limit. Thus, the Yang–Mills–Chern–Simons theory in $2+1$ dimensions could be a codified way to QCD with only heavy quarks. The classical solutions of the modified Yang–Mills theory for the $SU(2)$ gauge group are explicitly studied.

1. Introduction

It has been shown that non-commutative geometry is an important mathematical ingredient that could be a clue for several important unsolved problems in theoretical physics [1]. One of the consequences of non-commutative geometry is the Lorentz invariance symmetry breaking that, as was pointed out by several authors, could happen at very high or very low energies as a consequence of the IR/UV property, implying that interesting new phenomenological possibilities could appear [2,3] and new possible extensions of the Standard Model [4].

In Ref. [5], an approach to a Lorentz invariance violating quantum field theory has been proposed, inspired in non-commutative geometry, where the fields (instead of satisfying the standard canonical commutators) obey

$$\left[\phi_i(\vec{x}),\phi_j(\vec{y})\right]=i\theta_{ij}\delta(\vec{x}-\vec{y}), \tag{1}$$

$$\left[\pi_i(\vec{x}),\pi_j(\vec{y})\right]=iB_{ij}\delta(\vec{x}-\vec{y}), \tag{2}$$

$$\left[\phi_i(\vec{x}),\pi_j(\vec{y})\right]=i\delta_{ij}\delta(\vec{x}-\vec{y}), \tag{3}$$

where $i,j,\ldots=1,2,3,\ldots$ are internal indices and θ and B are scales with dimensions of $(\text{energy})^{-1}$ and energy, respectively. These scales correspond to ultraviolet and infrared weak Lorentz invariance violations, respectively.

These small deviations of the Lorentz symmetry (ultraviolet and infrared) imply modifications to the relativity principle. In the ultraviolet sector [6], for example, it allows to describe an interesting phenomenology for UHECR, where possible new effects could be studied [7].

The present approach does not correspond to the non-commutative geometry in the true sense, where one adopts the commutator

$$[x,y]\sim\theta.$$

Rather, while the commutator (1) violates the microcausality principle imposing an ultraviolet scale, (2) affects the physics in the infrared sector. This is relevant in the infrared sector of quantum field theory [8], where other phenomena could be explained.

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Indeed, in the infrared region new windows could emerge to largely unsolved problems, such as the dark matter and energy puzzle [9], matter–antimatter asymmetry [10], primordial magnetic fields [11] and other interesting phenomena. However, important open questions concerning the meaning of the infrared scales are still unsolved [12].

Although there are no definitive general answers to these problems, one can consider particular examples which eventually could be confronted with phenomenology or experimental results.

The purpose of the present research is to extend our previous work for electrodynamics [13] to the non-Abelian case. In [13] we shown that, by deforming the canonical algebra in the infrared sector, one rediscover the Carroll–Field–Jackiw theory proposed fifteen years ago [14].

Although in the context of our approach we do not know how to treat fermionic degrees of freedom, we will consider that this modified Yang–Mills theory hide the fermions in a Chern–Simons term. From this point of view, it seems to be reasonable to think that confinement could be a phenomenon mainly due to the classical behavior of the gluonic fields.[1]

The aims of the present Letter are the following:

(1) To present a non-Abelian gauge field theory that can be understood as a Hamiltonian system where the commutators—or Poisson brackets at the classical level—are deformed in a similar way to those of a non-commutative system (although we strength that this non-commutativity is in the field space, not in the spacetime as in non-commutative geometry),

(2) To show that our approach can also be understood as a standard Yang–Mills Lagrangian plus the term

$$\theta^{\mu}\epsilon_{\mu\nu\rho\lambda}\left(A^{\nu}F^{\rho\lambda}+\frac{2}{3}A^{\nu}A^{\rho}A^{\lambda}\right), \tag{4}$$

where vectorial fields A_ν satisfy canonical commutations relations and θ_μ is a given space-like vector having dimensions of energy. One should also emphasize that this Chern–Simons term not only violate Lorentz invariance, but also C, P and T symmetries [16].

(3) To discuss how the modified Yang–Mills equations in four dimensions can be understood in three dimensions as a Yang–Mills–Chern–Simons system and how the behavior of the gauge fields is very similar to the Nielsen–Olesen vortices.

(4) To give arguments that suggest that this modified Yang–Mills–Chern–Simons theory could be understood as a kind of "bosonized" QCD theory at low energy.

2. Yang–Mills theory with non-commutative fields

In this section we will construct the Yang–Mills theory with non-commutative fields following similar arguments to those given in Ref. [13] (in other context see also Ref. [19]). In this Letter, Latin indices denote spatial components and the metric is taken as $\mathrm{diag}(-1,1,1,1)$.

[1] In another context, this problem was studied in [15].

Essentially, one starts considering the following modified Poisson brackets

$$\{A_i^a(x),A_j^b(x')\}_{P.B}=0, \tag{5}$$

$$\{A_i^a(x),\Pi_j^b(x')\}_{P.B}=\delta^{ab}\delta_{ij}\delta^3(x-x'), \tag{6}$$

$$\{\Pi_i^a(x),\Pi_j^b(x')\}_{P.B}=\epsilon_{ijk}\theta^k\delta^{ab}\delta^3(x-x'), \tag{7}$$

where the parameter θ_k ($k=1,2,3$) is a vector in the space that is responsible for the Lorentz invariance violation. Also, the indices $a,b,c,\ldots$ represent internal indices, corresponding to the gauge group. In our analysis we will take the $SU(2)$ group as an example whose structure constants are just $i\epsilon^{abc}$, i.e., the total antisymmetric tensor. One should note that a term like $\epsilon^{abc}\gamma_c\delta_{ij}\delta^3(x-x')$ could also be added to the right-hand side of (7), but since we are looking for terms which violate the Lorentz symmetry, this contribution would be irrelevant in the following analysis.

Following [13], we will keep the gauge symmetry exact[2] while breaking the Lorentz invariance. Therefore, the first step is to find in this context the correct generators for the gauge transformations in terms of A and Π fields.

We then modify the standard Gauss law,

$$\Psi_a(x)\equiv\vec{\nabla}\cdot\vec{\Pi}_a+g\epsilon^{abc}\vec{\Pi}_b\cdot\vec{A}_c=0, \tag{8}$$

by adding a new term, $\Lambda(x)$, to get

$$\Psi'(x)=\Psi(x)+\Lambda(x). \tag{9}$$

This new term should depend only on the gauge potentials in order to reproduce the usual gauge transformation on A.

Taking into account the commutation relations

$$\{\Pi_i^a(x),B_k^b(x')\} =-\epsilon_{ijk}\left(\delta^{ab}\partial^j\delta^3(x-x')+g\epsilon^{abc}A_c^j(x)\delta^3(x-x')\right),$$

and

$$\begin{aligned}\{\Pi_i^a(x),\Psi^b(x)\} &=-g\epsilon^{abc}\Pi_{ic}\delta^3(x-x')\\ &\quad-\epsilon_{ijk}\theta^k\left(\partial^j\delta^3(x-x')\delta^{ab}+g\epsilon^{abc}A_c^j\delta^3(x-x')\right),\end{aligned} \tag{10}$$

where $B_k^b=\frac{1}{2}\epsilon_{ijk}F^{bij}$, one finds that the commutator between Π_i^a and $\Psi^a(x)$ is given by

$$\{\Pi_i^a(x),\Psi^b(x')-\vec{\theta}\cdot\vec{B}^b(x')\}=-g\epsilon^{abc}\Pi_{ci}(x)\delta^3(x-x').$$

Therefore, the correct generator of the gauge transformations in this theory becomes

$$\Xi'_\Omega=-\int d^3x\,\Omega^a(x)\left(\vec{\nabla}\cdot\vec{\Pi}^a+g\epsilon^{abc}\vec{\Pi}_b\cdot\vec{A}_c-\vec{\theta}\cdot\vec{B}^a\right), \tag{11}$$

or equivalently

$$\Xi'_\Omega=-\int d^3x\,\Omega^a(x)\left((\vec{D}\cdot\vec{\Pi})^a-\vec{\theta}\cdot B^a\right), \tag{12}$$

where $\vec{D}$ is the covariant gradient.

[2] This condition allows us to discard in (7) the term previously mentioned, otherwise we would be forced to include it.

Similarly to the commutative case, one can propose the Hamiltonian

$$H = \int d^3x \, \frac{1}{2}\left(\vec{\Pi}^a \cdot \vec{\Pi}^a + \vec{B}^a \cdot \vec{B}^a\right) + \int d^3x \, A^{a0}\left(\vec{\nabla} \cdot \vec{\Pi}^a + g\epsilon^{abc}\vec{\Pi}^b \cdot \vec{A}^c - \vec{\theta} \cdot \vec{B}^a\right). \quad (13)$$

Therefore, the equations of motion becomes

$$\vec{\Pi}^a = \dot{\vec{A}}^a + \vec{\nabla} A^{a0} + g\epsilon^{abc} A_{b0} \vec{A}_c,$$
$$\left\{\Pi_i^a(x), H\right\} = \epsilon_{ijk}\left(D^j B^k\right)^a - g\epsilon^{abc} A^{b0} \Pi_c^i - \vec{\theta} \times \vec{\Pi}^a. \quad (14)$$

The first equation establishes that $\Pi_i^a = -F_i^{a0}$. The second equation can be written as

$$(D_0 \Pi_i)^a = \epsilon_{ijk}\left(D^j B^k\right)^a - \vec{\theta} \times \vec{\Pi}^a,$$

where the last term introduces a modification with respect to the commutative case.

Finally, one can find an equivalent Lagrangian which reproduces the same equations of motion by identifying a set of commuting fields $\mathcal{P}$ [18] such that the A's and $\mathcal{P}$'s be canonically conjugate variables. These linear combination of Π's and A's can be identified as

$$P_i^a \equiv \Pi_i^a - \frac{1}{2}\epsilon_{ijk} A^{ja} \theta^k,$$

which satisfy the canonical algebra

$$\left\{A_i^a(x), P_j^b(x')\right\} = \delta_{ij}\delta^{ab}\delta^3(x - x'), \quad (15)$$
$$\left\{P_i^a(x), P_j^b(x')\right\} = 0. \quad (16)$$

In terms of these new variables the Lagrangian becomes

$$L = \int d^3x \, P_i^a \dot{A}_i^a - H,$$

where the only modifications comes from the terms proportional to $\vec{\theta}$.

Thus, the equivalent Lagrangian density becomes

$$L = L_0 + \frac{1}{2}\theta_k C_k, \quad (17)$$

where L_0 is the Yang–Mills Lagrangian density,

$$\mathcal{L}_0 = -\frac{1}{4} F_{\mu\nu}^a F^{a\mu\nu},$$

with

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + g[A_\mu, A_\nu],$$

and C_k given by,

$$C_k = -\epsilon_{ijk}\left(-\dot{A}_i^a A_j^a + A^{a0} F_{ij}^a\right). \quad (18)$$

Using $\dot{A}_i = \partial_0 A_i$ and the definition of F_{ij}^a, one finds,

$$\int d^3x \, C_k = \int d^3x \, \epsilon_{ijk}\left(-A_i^a \partial_0 A_j^a - A_0^a \partial_i A_j^a + A_0^a \partial_j A_i^a - g\epsilon^{abc} A_0^a A_i^b A_j^c\right) = \int d^3x \, 2\epsilon_{k\nu\rho\sigma} \operatorname{tr}\left(A^\nu F^{\rho\sigma} + \frac{2}{3} g A^\nu A^\rho A^\sigma\right). \quad (19)$$

Collecting all the terms one finds that (17) can be written as

$$L = -\frac{1}{2}\operatorname{tr}\left\{F_{\mu\nu}F^{\mu\nu}\right\} + 2\theta^\mu \epsilon_{\mu\nu\rho\sigma} \operatorname{tr}\left(A^\nu F^{\rho\sigma} + \frac{2}{3} g A^\nu A^\rho A^\sigma\right), \quad (20)$$

where the AF and A^3 non-Abelian contributions coincides with the Chern–Simons term and θ^μ is a space-like vector [19].

Thus, (20) shows the equivalence between a Hamiltonian formulation with deformed commutators (or Poisson brackets) and a standard Lagrangian formulation which explicitly breaks down Lorentz invariance.

Finally, we note that the Chern–Simons term should be treated as a non-perturbative contribution. Indeed, if one rescales $A_\mu^a \to g^{-1} A_\mu^a$ then the action becomes

$$S = \int d^4x \left[\frac{1}{2g^2}\operatorname{tr}\left(F^2\right) + \frac{\theta_\mu}{g^2}\epsilon^{\mu\nu\rho\sigma}\operatorname{tr}\left(A_\nu F_{\rho\sigma} + \frac{2}{3} A_\nu A_\rho A_\sigma\right)\right].$$

Therefore, the Chern–Simons term must be considered at the same foot as the standard kinetic F^2 part in the g expansion.

As an example of this last fact, we think it is instructive to check a simple case. Let us consider the plane wave solutions given by Coleman [20].

This solution of the Yang–Mills theory is given by the ansatz,

$$A_+^a = f^a\left(x^+\right)x^1 + g^a\left(x^+\right)x^2 + h^a\left(x^+\right).$$

Here we are using light-cone coordinates, $A_\pm^a = A_0^a \pm A_3^a$ and $x^\pm = x^0 \pm x^3$. The functions f^a, g^a and h^a are arbitrary but decreasing like $|x|^\alpha$, with α a negative constant, for large arguments $|x|$. So, the strength tensor becomes $F_{+1}^a = f^a$ and $F_{+2}^a = g^a$.

Then, we can consider a correction of this solution depending on θ perturbatively. This will give a perturbative ansatz for F,

$$F_{\mu\nu}^a = F^{(0)a}_{\ \ \mu\nu} + F^{(1)a}_{\ \ \mu\nu} + O\left(\theta^2\right),$$

where $F^{(0)}$ is the Coleman solution and $F^{(1)}$ is the first order correction in θ. Then, the equations of motion up to first order are,

$$\partial_\mu F^{(1)a\mu\nu} + g\epsilon^{abc}\left(A^{(0)b}_{\ \ \mu} F^{(1)c\mu\nu} + A^{(1)b}_{\ \ \mu} F^{(0)c\mu\nu}\right) - \frac{1}{2}\theta_\mu \epsilon^{\mu\nu\sigma\rho} F^{(0)a}_{\ \ \sigma\rho} = 0. \quad (21)$$

It is easy to see that, for large $|x^+|$, the perturbation $F^{(1)}$ goes as $|x^+|^{\alpha+1}$ and then, for large distances, it is bigger than the zeroth order contribution to the perturbative solution. Hence, it is not justified to take the non-commutative contribution as a perturbation to the Yang–Mills equations.

In the next section, we will deal with an exact solution for the complete (sourceless) equations of motion on $\mathbb{R}^2 \backslash \{0\}$. We will find that it corresponds to non-perturbative vortex configurations.

3. Vortex solutions of the modified Yang–Mills theory

Generally speaking one should note that the modification in (20) breaks rotational invariance, and the equations of motion become equivalent to a coupled Yang–Mills–Chern–Simons system if the space-like vector θ_μ is chosen in a particular spatial direction.

This last fact is quite interesting. Indeed, if one choose $\theta_\mu = (0,0,0,\theta)$, one finds an almost Yang–Mills–Chern–Simons theory with non-commutative gauge fields in $2+1$ dimensions after to use a suitable rescaling of fields. The difference, however, is that the A_μ field depend on (x_0, x_1, x_2, x_3) instead of (x_0, x_1, x_2) as usual. This result seems to be completely general.

One should also note that, in analogy with the quantum Hall effect, physical excitations like quarks in a Yang–Mills Lorentz symmetry breaking theory necessarily must live in $2+1$ dimensions (although the A_μ field is four-dimensional).

The above discussion can also be extended to any gauge group.

Now, we will discuss an exact solution for the non-commutative $SU(2)$ Yang–Mills theory with a vortex behavior.

The modified Yang–Mills equations are,

$$\left(D_\nu F^{\mu\nu}\right)^a - \frac{\theta_\nu}{2}\epsilon^{\mu\nu\rho\sigma} F^a_{\rho\sigma} = 0. \tag{22}$$

The solutions for these equations have been extensively discussed in the literature (for a review see, e.g. Ref. [17]) and, in particular, the vortex-like solutions for Yang–Mills–Chern–Simons are well known.

However, we emphasize here that, although these vortex solutions fit perfectly in our problem, they are also mandatory if the canonical commutators are modified as in (5)–(7). Indeed, the θ parameter imply the choice of a particular plane and—as we are interested in the infrared limit—one could neglect *mutatis mutandis* the short distances effects.

In order to solve (22), let us consider a set of coordinates in $\mathbb{R}^3$, and the unitary vectors in the plane $x_3 =$ const, $\hat{\phi}_i = \epsilon_{ij}x^j/\rho$, $\hat{\rho}_i = x_i/\rho$, where ρ is the standard radial coordinate in the plane. Let us consider the following axially symmetric ansatz for the gauge fields

$$A^a_0 = \hat{\phi}^a \psi_2(x_0, \vec{r}), \qquad A^a_i = \hat{\phi}^a \hat{\phi}_i \psi_1(x_0, \vec{r}) + \delta^a_3 \hat{\phi}_i \frac{1}{\rho}, \tag{23}$$

and $A^0_3 = 0$.

Using (22), one finds that

$$\psi_1 = c e^{-\alpha x_0} K_1(M\rho), \tag{24}$$

$$\psi_2' = \theta \psi_1, \tag{25}$$

where $\psi_2' = d\psi_2/d\rho$ and $K_1(x)$ is the Bessel function of the second kind.

The coefficient M is defined as

$$M = \sqrt{\theta^2 + \alpha^2},$$

and c and α are constant with dimensions of energy. The gauge potential given in (23) fall exponentially to zero when $\rho \to \infty$.

For the configuration discussed above, the chromomagnetic energy is finite

$$E_m = \frac{1}{2}\int d^2x\, B_a \cdot B_a = \frac{\pi c^2}{2}. \tag{26}$$

But the chromoelectric energy is logarithmically divergent at the ultraviolet region:

$$E_e = \frac{1}{2}\int d^2x\, \vec{E}^a \cdot \vec{E}^a = \pi \int_\Lambda^\infty d\rho\, \rho\left[(\Psi_2')^2 + (\dot{\Psi}_1)^2\right]$$
$$= \pi c^2 \int_\Lambda^\infty du\, u K_1^2(u), \tag{27}$$

where Λ is a given cutoff. However, one should notice that it is at the infrared region where θ is relevant.

Thus, we see that in four dimensions one finds that the energy of these solutions increases linearly with L for large distances and, therefore, the gluonic fields would appear as confined along the z direction.

Finally, we would like to sketch a possible origin of the "four-dimensional" Chern–Simons term. In so doing, let us consider massless QCD in four dimensions, described by the Lagrangian

$$L = -\frac{1}{4}F^2 + \bar{\psi}(i\not{D})\psi, \tag{28}$$

where a sum over flavor indices is assumed.

Naively, one could expect that, by integrating the quark fields to find an effective action at low energies, one could obtain contributions different from a Chern–Simons term. However, one also can argue the following: at very low energy, considering only heavy quarks, let us suppose that the space–time is compactified so that the quark field could be written as

$$\psi(x_0, x_1, x_2, x_3) = e^{i\frac{x_3}{\ell}} \varphi(x_0, x_1, x_2),$$

where ℓ is the compactification radius[3] (for a more detailed discussion see [21]).

Once this compactification is assumed, the "heavy quark" acquires an effective mass $m = 1/\ell$, and Eq. (28) in this effective description becomes

$$L = -\frac{1}{4}F^2 + \bar{\varphi}(i\not{D} - m)\varphi, \tag{29}$$

where $\bar{\varphi}(i\not{D} - m)\varphi$ is a fermionic three-dimensional Lagrangian. Here, the fermionic determinant can be calculated and the result at the lowest order in $1/m = \ell$ is the Chern–Simons term [22].

It is worthwile to notice that this kind of topological terms coming from integrated-out fermions appear also in different contexts. For example, D'Hoker and Farhi consider fermions getting large masses through Yukawa couplings to Higgs fields [23], obtaining at low energies a Wess–Zumino–Witten term as a relic of the quark degrees of freedom in the heavy mass limit.

Another interesting point is how this $(2+1)$-dimensional case is related to the $(3+1)$-dimensional one. The answer to

[3] This compactification occur when we consider, for example, a membrane vibrating in the space. If the transversal amplitude is small enough, then the phonons propagate only on the surface of the membrane, and the compactification is a good approximation.

this question is quite simple: the connection between the gauge field in three and four dimensions is

$$A_\mu^{(3)} \rightarrow \sqrt{\ell} A_\mu,$$

then with this rescaling the four-dimensional measure become

$$\frac{d^4x}{\ell} \rightarrow \tilde{d}^3x.$$

We conclude this section emphasizing that the analysis here presented could be a new route to understand some non-perturbative aspects of QCD.

4. Conclusions

In this Letter we have shown that deforming the Poisson brackets for the canonical momenta in a Yang–Mills theory, the resulting theory is equivalent to a Yang–Mills–Chern–Simons system. The classical theory—taking the $SU(2)$ group—has vortex-like solutions similar to the Nielsen–Olesen ones. The difference, however, is that in our case they appear as a consequence of Lorentz invariance violation.

However, we emphasize that our result does not imply that quarks fields are absent in our approach. Rather, the Chern–Simons term contains the information about the fermionic degrees of freedom and, in this sense, our procedure could provide an alternative route to study non-perturbative effects.

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