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Double beta decay of Σ^- hyperons

C. Barbero^a, G. López Castro^b, A. Mariano^a

^a *Departamento de Física, Facultad de Ciencias Exactas, Universidad Nacional de la Plata, cc 67, 1900 La Plata, Argentina*

^b *Departamento de Física, Cinvestav del IPN, Apartado Postal 14-740, 07000 México D.F., Mexico*

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Abstract

We compute the strangeness-conserving double beta decay of Σ^- hyperons, which is the only hadronic system that can undergo such decays. We consider both, the lepton number conserving $\Sigma^- \rightarrow \Sigma^+ e^- e^- \bar{\nu} \bar{\nu}$ ($\beta\beta_{\Sigma_{2\nu}}$) and the lepton number violating $\Sigma^- \rightarrow \Sigma^+ e^- e^-$ ($\beta\beta_{\Sigma_{0\nu}}$) modes. The branching ratios of these $\beta\beta$ decays are suppressed at the level of 10^{-30} considering a light neutrino scenario in the case of the $\beta\beta_{\Sigma_{0\nu}}$ channel. The dynamical origin of such low rates and their possible enhancements are briefly discussed. Given its simplicity those decays can be used also for the purposes of illustrating the main features of double beta decays.

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1. Introduction

Neutrinoless double beta ($\beta\beta_{\Sigma_{0\nu}}$) decays would occur if a mechanism allows the violation of the total lepton number L by two units. Their observation in experiments will provide unambiguous evidence for physics beyond the Standard Model (SM). At present, the first evidences for physics beyond the SM come from the flavor oscillations in the neutrino sector that required to explain the deficit observed in solar [1] and atmospheric neutrinos [2]. Flavor oscillations of neutrinos do not require a change of the total lepton number (namely, $|\Delta L| = 0$), allow us to conclude that neutrinos are massive, but do not establish whether they are Dirac or Majorana particles. If neutrinos are Majorana particles, $\beta\beta_{0\nu}$ decays have to occur at some level; their observation will establish the Majorana nature of neutrinos beyond any doubt.

The strangeness-conserving ($\Delta S = 0$) $\beta\beta_{0\nu}$ decays have been unsuccessfully searched in nuclear transitions for several decades. An intense activity in the experimental and theoretical fronts [3–8] witness the importance of such decays as a sensitive probe of physics beyond the SM. Some examples of extensions of the SM that can induce

E-mail address: mariano@venus.fisica.unlp.edu.ar (A. Mariano).

contributions to $\beta\beta_{0\nu}$ decays are right-handed weak couplings as well as the Higgs exchange [9], right-handed weak coupling involving heavy Majorana neutrinos [10], massless Majoron emission [4, 11–14], and R-parity breaking in the supersymmetric models [15, 16]. From a theoretical point of view, $\beta\beta_{0\nu}$ decays in nuclei are limited in precision due a wide range of model-dependent predictions for the nuclear wavefunctions. In the present Letter, we study the double beta (both the lepton number-conserving $\beta\beta_{2\nu}$ and lepton number-violating $\beta\beta_{0\nu}$) strangeness-conserving decays of the Σ^- hyperon. The Σ^- hyperon is a unique system in hadron physics that can undergo strangeness-conserving double beta decays as it will be explained below. The hadronic matrix elements necessary for such calculations are well known and, therefore, can exhibit the underlying mechanisms for double beta decays in a more clean way.

As we have mentioned before, the determination of an upper bound for the effective neutrino mass in nuclear $\beta\beta_{0\nu}$ decays is limited by model-dependent evaluations of the nuclear matrix elements. Some of the difficulties we encounter in those calculations are the following:

1. The nucleus is a many body system with many degrees of freedom; in practice there is not a well defined rule to choose the most relevant components to describe an specific excitation;
2. The Hilbert space where nuclear models are worked out have a huge dimension requiring a lot of time consuming computational work; and
3. The multipole expansion for the $\beta\beta_{0\nu}$ decay amplitude is rather complex making theoretical expressions difficult to manipulate.

Despite these limitations in theoretical inputs, the large sensitivity of present experiments have been able to set strong constrains on the so-called effective Majorana mass term, $\langle m_{ee} \rangle \equiv \sum U_{e1}^2 m_{\nu_1}$, where m_{ν_i} denote neutrino mass eigenstates. By assuming that $\beta\beta_{0\nu}$ in nuclei are mediated by the exchange of light Majorana neutrinos, the experimental upper bound on $\langle m_{ee} \rangle$ is 0.2 eV [17].

Before we proceed with our calculation, it is interesting to take a look at other reactions that can provide information on the violation of lepton number. The upper limits available on these rare processes can be used to set upper limits on the matrix elements $\langle m_{\alpha\beta} \rangle \equiv \sum U_{\alpha i} U_{\beta i} m_{\nu_i}$, where $\alpha, \beta = e, \mu, \tau$. Thus, muon to positron conversion in nuclei $\mu^- + (A, Z) \rightarrow (A, Z - 2)e^+$ [4, 18] gives $\langle m_{e\mu} \rangle < 17(82)$ GeV depending on the spin of the initial proton pair; the production of three muons in neutrino–nucleon scattering [19] leads to the upper limit $\langle m_{\mu\mu} \rangle \lesssim 10^4$ GeV; this limit has been slightly improved at HERA through the reaction $e^+p \rightarrow \bar{\nu}_l^+ l_2^+ X$ [20], giving $\langle m_{\mu\mu} \rangle \lesssim 4 \times 10^3$ GeV and also for first time limits on the $\langle m_{l\tau} \rangle$ (connected with the τ -sector) were given; finally, the non-observation of heavy Majorana neutrinos at various colliders [21] can also be used to set limits on the effective Majorana mass. Similarly, some rare kaon decays are also useful to constrain lepton number violating interactions. For instance, present bounds on the branching ratio of the $K^+ \rightarrow \mu^+ \mu^+ \pi^-$ decay [22] translates into the upper limit $\langle m_{\mu\mu} \rangle \lesssim 4 \times 10^4$ MeV [23]. Those bounds from collision experiments and rare kaon decays are several orders of magnitude above the limit $\langle m_{\mu\mu} \rangle \leq 4.4$ eV, inferred by an analysis [24] that combines experimental constrains from atmospheric and solar neutrino oscillation and the tritium beta decay end-point experiment [25, 26].

Conversely, we can use the existing bounds from lepton number violating processes to set upper limits on the branching ratios of some rare kaon decay. Thus, the upper limit $B(K^+ \rightarrow \mu^+ \mu^+ \pi^-) \lesssim 10^{-30}$ (10^{-19}) [27] can be obtained in models with a light (heavy) neutrino scenario, while $B(K^+ \rightarrow e^+ e^+ \pi^-) \lesssim 10^{-26}$ [28] can be derived from upper bounds from nuclear $\beta\beta_{0\nu}$ decays. These values are well below the sensitivities of current experiments; for example the limits reported by the E865 experiment [23] are $B(K^+ \rightarrow e^+ e^+ \pi^-) < 6.4 \times 10^{-10}$, and $B(K^+ \rightarrow \mu^+ \mu^+ \pi^-) \lesssim 3 \times 10^{-9}$. This shows that we are far from detecting such processes, in spite of the special window in the hundred MeV region [27], and that the sensitivity of such processes are well below the previously discussed $\beta\beta_{0\nu}$ case. Nevertheless, it is important to pursue searches for $|\Delta L| = 2$ processes since they would lead to nonvanishing results even if nuclear $\beta\beta_{0\nu}$ decay turns out to vanish or becomes extremely suppressed.

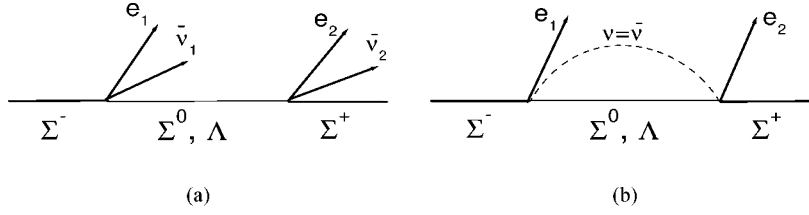


Fig. 1. Lowest order diagrams contributing to the Σ^- decay for (a) two neutrino double beta decay mode, and (b) neutrinoless double beta decay mode.

In this Letter we study the two double beta decays of the Σ^- hyperon, namely: the lepton number conserving $\Sigma^- \rightarrow \Sigma^+ e^- e^- \bar{\nu} \bar{\nu}$ ($\beta\beta_{\Sigma_{2\nu}}$) and the lepton number violating $\Sigma^- \rightarrow \Sigma^+ e^- e^-$ ($\beta\beta_{\Sigma_{0\nu}}$, $\Delta L = 2$) channels. The Feynman graphs for the $\beta\beta_{\Sigma_{2\nu}}$ and $\beta\beta_{\Sigma_{0\nu}}$ decays, indicating the two intermediate states, are shown in Fig. 1(a) and (b), respectively. To our knowledge, these decay modes have not been reported previously in the literature and even an experimental upper limit is not available. The isotriplet of Σ hyperons (Σ^+ , Σ^0 , Σ^-) is a unique system of hadrons that can undergo strangeness-conserving double beta decays. Actually, the Σ^- and the Σ^+ are not antiparticles of each other; they have a mass splitting of $m_{\Sigma^-} - m_{\Sigma^+} = 8.08$ MeV [29], which allows a sufficient phase space for $\beta\beta$ decays. An interesting feature of this system is that we can identify *only two* well defined intermediate states (Σ^0 and Λ) that can give important contributions to the $\beta\beta$ transitions under consideration. One of these states (the Σ^0) lies in the middle of the Σ^- , Σ^+ levels while the other (the Λ) is around 77 MeV below them. Namely, one of these intermediate states can be real while the other is virtual.

On another hand, the hadronic matrix elements required for the calculations are dominated by a couple of axial and vector form factors, which have been computed by several authors in the literature [30,31] with good agreement among them. This is an important advantage over the evaluation of nuclear transition matrix elements. All these characteristics of the Σ hyperon system make very interesting the study of their double beta decays, even as a possible textbook example to learn the formalism of $\beta\beta$ transitions applied to free baryons. In the following we consider each case separately.

2. Lepton number-conserving $\beta\beta$ decay

One of the four Feynman graphs corresponding to the $\beta\beta_{\Sigma_{2\nu}}$ decay is shown in Fig. 1(a) (the other three contributions are obtained under proper antisymmetrization with respect to final state electrons and antineutrinos).

The effective four-fermion weak Hamiltonian acting at each vertex has the usual current–current form [8,31]

$$H_W = \frac{G}{\sqrt{2}} J_\mu j^\mu + \text{h.c.}, \quad (1)$$

where

$$J_\mu = \bar{\psi}_{B'} \gamma_\mu (f_1 + g_1 \gamma_5) \psi_B, \quad (2)$$

is the baryonic current operator underlying the $B \rightarrow B'$ transition and

$$j_\mu = \bar{\psi}_e \gamma_\mu (1 - \gamma_5) \psi_\nu, \quad (3)$$

is the $V - A$ leptonic current operator.¹ In Eq. (2), f_1 and g_1 are dimensionless vector and axial-vector form factors for the $B \rightarrow B'$ transition. In our approximation we have neglected their momentum transfer dependence and we

¹ We do not consider here the possible existence of right-handed neutrinos, since their contribution to the nuclear double beta decay is negligible [4,8].

Table 1
Hyperon form factors at zero momentum transfer (see Table III in Ref. [31])

η	$f_{A\eta}$	$g_{A\eta}$	$f_{B\eta}$	$g_{B\eta}$
Λ	0	-0.60	0	-0.60
Σ^0	1.41	-0.69	-1.41	0.69

have also neglected the small contributions of the induced magnetic and scalar form factors for the vector and axial currents. The effective weak coupling constant is $G = G_F V_{ud}$, where $G_F = (1.16639 \pm 0.00003) \times 10^{-11} \text{ MeV}^{-2}$ is the Fermi constant and V_{ud} the relevant element of the Cabibbo–Kobayashi–Maskawa mixing matrix.

The rate for the $\beta\beta_{\Sigma_{2\nu}}$ decay from the initial $\Sigma^- \equiv A$ to the final $\Sigma^+ \equiv B$ hyperon is given in the rest frame of A by (we use natural units, i.e., $\hbar = c = m_e = 1$)

$$d\Gamma_{\Sigma_{2\nu}} = \pi \sum_{\text{spin}} |\mathcal{M}_{2\nu}|^2 \delta(m_A - \epsilon_B - \epsilon_1 - \epsilon_2 - \omega_1 - \omega_2) \frac{d\mathbf{p}_1}{(2\pi)^3} \frac{d\mathbf{p}_2}{(2\pi)^3} \frac{d\mathbf{q}_1}{(2\pi)^3} \frac{d\mathbf{q}_2}{(2\pi)^3}, \quad (4)$$

where m_A (ϵ_B) is the energy of the initial (final) hyperon, and ϵ_i and \mathbf{p}_i (ω_i and \mathbf{q}_i) denote the energy and momentum of the electron (neutrino). The decay amplitude reads

$$\mathcal{M}_{2\nu} = [1 - P(e_1 e_2)][1 - P(\nu_1 \nu_2)] \sum_{\eta=\Sigma^0, \Lambda} \frac{\langle p_B; e_1 e_2 \tilde{\nu}_1 \tilde{\nu}_2 | H_W | \eta, Q; e_1 \tilde{\nu}_1 \rangle \langle \eta, Q; e_1 \tilde{\nu}_1 | H_W | p_A \rangle}{m_A - \epsilon_Q(\eta) - \epsilon_1 - \omega_1}, \quad (5)$$

with $\epsilon_Q(\eta) = \sqrt{\mathbf{Q}^2 + m_\eta^2}$ and $\mathbf{Q} \equiv -\mathbf{p}_1 - \mathbf{q}_1 = \mathbf{p}_B + \mathbf{p}_2 + \mathbf{q}_2$ being the energy and momentum of the intermediate baryon state, and $P(x_1 x_2)$ the operator that exchanges x_1 with x_2 . Thus, after introducing Eqs. (1)–(3) in Eq. (5) we get

$$\mathcal{M}_{2\nu} = \frac{G^2}{2} [1 - P(e_1 e_2)][1 - P(\nu_1 \nu_2)] \sum_{\eta=\Sigma^0, \Lambda} \frac{B_\mu(p_A, Q; A) B_\nu(Q, p_B; B) L^{\mu\nu}(p_1, q_1, p_2, q_2)}{m_A - \epsilon_Q(\eta) - \epsilon_1 - \omega_1}, \quad (6)$$

where

$$B_\mu(p_I, p_F; X) = \bar{u}(p_F) \gamma_\mu (f_{X_\eta} + g_{X_\eta} \gamma_5) u(p_I), \quad (7)$$

is the baryonic matrix element between states with momentum p_I and p_F , and

$$L_{\mu\nu}(p_1, q_1, p_2, q_2) = \bar{u}_e(p_2) \gamma_\mu (1 - \gamma_5) u_\nu(q_2) \bar{u}_e(p_1) \gamma_\nu (1 - \gamma_5) u_\nu(q_1), \quad (8)$$

is the leptonic tensor. The values for the form factors $f_{A\eta} \equiv f_1(\Sigma^- \eta)$, $g_{A\eta} \equiv g_1(\Sigma^- \eta)$, $f_{B\eta} \equiv f_1(\eta \Sigma^+)$ and $g_{B\eta} \equiv g_1(\eta \Sigma^+)$ at zero momentum transfer are summarized in Table 1.

Given the small mass difference between the relevant hyperon states, we can use the non-relativistic impulse approximation for the baryonic current [8,32]. The small mass difference between the Σ hyperon states is responsible of the suppression of the decay via the real intermediate Σ^0 particle, while the decay through an intermediate on-shell Λ state is forbidden since $m_\Lambda < m_\Sigma^+$. Keeping only the usually called Fermi and Gamow–Teller operators we have

$$B_\mu(p_I, p_F; X) = \chi_{m_{s_F}}^\dagger (f_{X_\eta} g_{\mu 0} - g_{X_\eta} \sigma_k g_{\mu k}) \chi_{m_{s_I}}, \quad (9)$$

where s_I and s_F denote the spin of the initial and final baryons in the $I \rightarrow F$ transition. In the spirit of the non-relativistic approximation the energy denominator in Eq. (6) can be also simplified making $\epsilon_Q(\eta) \cong m_\eta$. Note that in the case of the Σ^0 intermediate state, Eq. (6) exhibits a singularity when $\epsilon_1 + \omega_1 = m_A - m_{\Sigma^0} = 4.8 \text{ MeV}$ (note that this singularity does not appear for the Λ intermediate state because $m_A - m_\Lambda = 81.7 \text{ MeV} \gg \epsilon_1 + \omega_1$). This singularity can be cured by taking into account the finite width ($\Gamma_{\Sigma^0} = 8.89 \text{ keV}$ [29]) of the Σ^0 intermediate state. Therefore, we will define:

- In the case $\eta = \Sigma^0$:

$$\frac{1}{m_A - m_{\Sigma^0} - \epsilon_1 - \omega_1} \rightarrow \frac{1}{m_A - m_{\Sigma^0} - \epsilon_1 - \omega_1 + i \frac{\Gamma_{\Sigma^0}}{2}} \equiv h_{\Sigma^0}(\epsilon_1, \omega_1). \quad (10)$$

- In the case $\eta = \Lambda$:

$$\frac{1}{m_A - m_{\Lambda} - \epsilon_1 - \omega_1} \equiv h_{\Lambda}(\epsilon_1, \omega_1). \quad (11)$$

After a lengthy but straightforward calculation we can write the decay rate as follows:

$$\Gamma_{\Sigma_{2\nu}} = \frac{G^4}{8\pi^7} \sum_{j=1}^3 \sum_{\eta\eta'} C_j(\eta\eta') \mathcal{I}_j(\eta\eta'). \quad (12)$$

The $C_j(\eta\eta')$ terms denote quartic combinations of form factors,

$$\begin{aligned} C_1(\eta\eta') &= f_{A\eta} f_{B\eta} f_{A\eta'} f_{B\eta'} + 3g_{A\eta} f_{B\eta} g_{A\eta'} f_{B\eta'} + 3f_{A\eta} g_{B\eta} f_{A\eta'} g_{B\eta'} + 9g_{A\eta} g_{B\eta} g_{A\eta'} g_{B\eta'}, \\ C_2(\eta\eta') &= f_{A\eta} f_{B\eta} f_{A\eta'} f_{B\eta'} + 3g_{A\eta} f_{B\eta} f_{A\eta'} g_{B\eta'} + 3f_{A\eta} g_{B\eta} g_{A\eta'} f_{B\eta'} - 3g_{A\eta} g_{B\eta} g_{A\eta'} g_{B\eta'}, \\ C_3(\eta\eta') &= f_{A\eta} f_{B\eta} f_{A\eta'} f_{B\eta'} + 3f_{A\eta} f_{B\eta} g_{A\eta'} g_{B\eta'} + 3f_{A\eta'} f_{B\eta'} g_{A\eta} g_{B\eta} + 3g_{A\eta} f_{B\eta} f_{A\eta'} g_{B\eta'} \\ &\quad + 3g_{A\eta'} f_{B\eta'} f_{A\eta} g_{B\eta} + 3g_{A\eta} f_{B\eta} g_{A\eta'} f_{B\eta'} + 3f_{A\eta} g_{B\eta} f_{A\eta'} g_{B\eta'} - 3g_{A\eta} g_{B\eta} g_{A\eta'} g_{B\eta'}, \end{aligned} \quad (13)$$

and $\mathcal{I}_j(\eta\eta')$ are the phase space factors defined as follows:

$$\begin{aligned} \mathcal{I}_1(\eta\eta') &= \int_1^{\epsilon_0-1} p_1^2 dp_1 \int_1^{\epsilon_0-\epsilon_1} p_2^2 dp_2 \int_0^{\epsilon_0-\epsilon_1-\epsilon_2} q_1^2 (\epsilon_0 - \epsilon_1 - \epsilon_2 - q_1)^2 dq_1 \\ &\quad \times h_{\eta}(\epsilon_2, \epsilon_0 - \epsilon_1 - \epsilon_2 - q_1) h_{\eta'}^*(\epsilon_2, \epsilon_0 - \epsilon_1 - \epsilon_2 - q_1), \\ \mathcal{I}_2(\eta\eta') &= \int_1^{\epsilon_0-1} p_1^2 dp_1 \int_1^{\epsilon_0-\epsilon_1} p_2^2 dp_2 \int_0^{\epsilon_0-\epsilon_1-\epsilon_2} q_1^2 (\epsilon_0 - \epsilon_1 - \epsilon_2 - q_1)^2 dq_1 \\ &\quad \times \text{Re}[h_{\eta}(\epsilon_2, \epsilon_0 - \epsilon_1 - \epsilon_2 - q_1) h_{\eta'}^*(\epsilon_1, q_1)], \\ \mathcal{I}_3(\eta\eta') &= \int_1^{\epsilon_0-1} p_1^2 dp_1 \int_1^{\epsilon_0-\epsilon_1} p_2^2 dp_2 \int_0^{\epsilon_0-\epsilon_1-\epsilon_2} q_1^2 (\epsilon_0 - \epsilon_1 - \epsilon_2 - q_1)^2 dq_1 \\ &\quad \times \text{Re}[h_{\eta}(\epsilon_2, \epsilon_0 - \epsilon_1 - \epsilon_2 - q_1) h_{\eta'}^*(\epsilon_1, \epsilon_0 - \epsilon_1 - \epsilon_2 - q_1)], \end{aligned} \quad (14)$$

where $\epsilon_0 \equiv m_A - m_B$.

The numerical values for the factors entering in the expression of the decay rate, Eq. (12), are given in Table 2. We can check that the main contribution comes from the term $C_1(\Sigma^0 \Sigma^0) \mathcal{I}_1(\Sigma^0 \Sigma^0)$, which includes the contribution of a real Σ^0 hyperon intermediate state.

Using the values obtained in Table 2, we can compute the branching ratio from the rate in Eq. (12). We obtain:

$$B(\beta\beta_{\Sigma_{2\nu}}) = 1.38 \times 10^{-30} (1.36 \times 10^{-30}). \quad (15)$$

Just for comparison we have shown within parenthesis the value corresponding to the contribution of the Σ^0 intermediate state. As expected, this contribution dominates almost completely the decay rate. The branching ratio given above turns out to be very suppressed due essentially to the large decay width of the Σ^0 hyperon appearing in Eq. (10). As we know, the decay rate of the Σ^0 is 10 orders of magnitude larger than those of the charged Σ hyperons because it can undergo the electromagnetic decay $\Sigma^0 \rightarrow \Lambda\gamma$.

Table 2

Numerical values for $\mathcal{C}_j(\eta\eta')$ and $\mathcal{I}_j(\eta\eta')$ (in units of MeV^9) for $\beta\beta_{\Sigma_{2\nu}}$ decay

j	$\mathcal{C}_j(\Lambda\Lambda)$	$\mathcal{I}_j(\Lambda\Lambda)$	$\mathcal{C}_j(\Sigma^0\Sigma^0)$	$\mathcal{I}_j(\Sigma^0\Sigma^0)$	$\mathcal{C}_j(\Lambda\Sigma^0) = \mathcal{C}_j(\Sigma^0\Lambda)$	$\mathcal{I}_j(\Lambda\Sigma^0) + \mathcal{I}_j(\Sigma^0\Lambda)$
1	1.166	5.46×10^{-1}	11.672	6.740×10^5	-1.543	4.106×10
2	-0.389	5.46×10^{-1}	8.952	2.162×10^3	0.514	3.252×10^2
3	-0.389	5.46×10^{-1}	20.3101	4.598×10^3	-1.633	4.170×10

3. Neutrinoless double beta decay

The decay rate for the (three-body) neutrinoless $\beta\beta_{\Sigma_{0\nu}}$ mode reads

$$d\Gamma_{\Sigma_{0\nu}} = \pi \sum_{\text{spin}} |\mathcal{M}_{0\nu}|^2 \delta(m_A - \epsilon_B - \epsilon_1 - \epsilon_2) \frac{d\mathbf{p}_1}{(2\pi)^3} \frac{d\mathbf{p}_2}{(2\pi)^3}. \quad (16)$$

The decay amplitude corresponding to the diagram in Fig. 1(b) (after proper anti-symmetrization with respect to identical electrons) is

$$\mathcal{M}_{0\nu} = [1 - P(e_1 e_2)] \sum_{\eta=\Sigma^0, \Lambda} \sum_{s_\nu} \int \frac{d^4 q}{(2\pi)^4} \frac{\langle p_B; e_1 e_2 | H_W | \eta, Q(q); e_1 \tilde{\nu} \rangle \langle \eta, Q(q); e_1 \tilde{\nu} | H_W | p_A \rangle}{m_A - \epsilon_1 - q - \epsilon_Q(\eta)}. \quad (17)$$

The four-momentum of the intermediate state η is $Q(q) \equiv p_A - p_1 - q = p_B + p_2 - q$ and its energy is $\epsilon_Q(\eta) = \sqrt{\mathbf{Q}^2(\mathbf{q}) + m_\eta^2}$. Introducing (1) into (17), and expanding the weak neutrino eigenstate as a mixture of light massive Majorana neutrino states, i.e., $u_{\tilde{\nu}}(q) = \sum_l U_{e1} U_{\nu l}(q)$, we get

$$\mathcal{M}_{0\nu} = G^2 [1 - P(e_1 e_2)] \sum_l m_{\nu_l} U_{e1}^2 \sum_\eta \bar{u}(p_B) \gamma_\mu (f_{B\eta} + g_{B\eta} \gamma_5) I_\eta(p_1) \gamma_\nu (f_{A\eta} + g_{A\eta} \gamma_5) u(p_A) L^{\mu\nu}(p_1, p_2), \quad (18)$$

with the leptonic factor

$$L_{\mu\nu}(p_1, p_2) = \bar{u}_e(p_1) \gamma_\mu (1 - \gamma_5) \gamma_\nu u_e^C(p_2). \quad (19)$$

The factor $I_\eta(p_1)$ in Eq. (18) corresponds to the following loop integral ($Q(q) = p_A - p_1 - q$):

$$I_\eta(p_1) = \frac{d^4 q}{(2\pi)^4} \frac{\not{Q}(q) + m_\eta}{(q^2 - m_\eta^2)(Q^2(q) - m_\eta^2)}, \quad (20)$$

which has a logarithmic divergence that we will manipulate in a simple cutoff procedure. After using Feynman parametrization techniques [33], we get

$$I_\eta(p_1) = \frac{i}{8\pi^2} \int_0^1 dx [(p_A - p_1)x + m_\eta] \int_0^{\Lambda_c} \frac{k^3 dk}{(k^2 + M^2)^2}, \quad (21)$$

where Λ_c is the cutoff energy, and we have defined

$$M^2 = m_\eta^2(1-x) - (p_A - p_1)^2(1-x)x + m_\nu^2 x. \quad (22)$$

We stress here that the origin of this logarithmic divergence is related to the effective vertices we are using for the hadronic form factors. This divergence can in principle be cured by including the weak form factors which are expected to fall as $1/[(p_A - Q(q))^2]$ with $q^2 \rightarrow \infty$ in the dipole approximation, but their real behavior in the large q^2 limit are actually determined by QCD. In the case of the nuclear $\beta\beta_{0\nu}$ decays it is usual to assume that

the divergence is driven by the internucleon distance, which sets $\Lambda_c \sim 1$ GeV. This value can be identified with the largest momenta that the neutrino can carry which in turn is fixed by the lowest distance between two nucleons in the nucleus ($\Lambda_c \sim (2d)^{-1}$, where d is the nucleon radius). We will assume here that the average distance between the quarks within the hyperons are of the order of a typical hyperon (or nucleon) radius. Here, we adopt the point of view that our results are stable as far as they do not depend strongly on the specific value of the cutoff Λ_c .

The integration in Eq. (21) can be simplified by neglecting in Eq. (22) all lepton masses and momenta, which is consistent for the neutrino since we are assuming a light neutrino scenario. We obtain

$$I_\eta(p_1) = \frac{i}{(4\pi)^2} [(\not{p}_A - \not{p}_1)F_\eta + m_\eta G_\eta], \quad (23)$$

where

$$\begin{aligned} F_\eta &= \frac{1}{4}(1+m^2)\ln(m+m') - \frac{1}{2}(1-m^2)\ln(1-m) - \frac{1}{2}m^2\ln(m) + \frac{1}{4}(1-m^2)\ln(m') \\ &\quad - i\frac{\pi}{2}(1+m^2) + \frac{[2m' - (1-m)^2](1+m)}{2D} \left[\arctg\left(\frac{1-m}{D}\right) + \arctg\left(\frac{1+m}{D}\right) \right], \\ G_\eta &= \frac{1}{2}(1+m)\ln(m+m') - (1-m)\ln(1-m) - m\ln(m) + \frac{1}{2}(1-m)\ln(m') \\ &\quad - i\pi(1+m) + \frac{2m' - (1-m)^2}{D} \left[\arctg\left(\frac{1-m}{D}\right) + \arctg\left(\frac{1+m}{D}\right) \right]. \end{aligned} \quad (24)$$

In the above expressions, we have introduced the following dimensionless constants: $m = m_\eta^2/m_A^2$, $m' = \Lambda_c^2/m_A^2$ and $D = \sqrt{4m' - (1-m)^2}$.

In order to evaluate the unpolarized squared amplitude we will use the non-relativistic impulse approximation for the final baryon and take $p_B \simeq (m_B, \mathbf{0})$ (we cannot do the same approximation for the intermediate baryon state in this case). The decay rate for the $\beta\beta_{0\nu}$ transition becomes:

$$\begin{aligned} \Gamma_{\Sigma_{0\nu}} &= \langle m_{ee} \rangle^2 \frac{G^4}{4\pi^7} \mathcal{I}_{0\nu} \\ &\quad \times \sum_{\eta\eta'} \left[m_A^2 F_\eta F_{\eta'}^* \mathcal{D}_1(\eta\eta') - 2m_{A\eta} m_\eta (G_\eta F_{\eta'}^* + G_{\eta'}^* F_\eta) \mathcal{D}_2(\eta\eta') + 4m_\eta m_{\eta'} G_\eta G_{\eta'}^* \mathcal{D}_3(\eta\eta') \right], \end{aligned} \quad (25)$$

where $\langle m_{ee} \rangle = \sum_l m_{\nu_l} U_{el}^2$ is the effective neutrino mass. The other factors appearing in Eq. (25) are defined as follows:

$$\begin{aligned} \mathcal{D}_1(\eta\eta') &= (f_{A\eta} f_{B\eta} + g_{A\eta} g_{B\eta})(f_{A\eta'} f_{B\eta'} + g_{A\eta'} g_{B\eta'}), \\ \mathcal{D}_2(\eta\eta') &= (f_{A\eta} f_{B\eta} - g_{A\eta} g_{B\eta})(f_{A\eta'} f_{B\eta'} + g_{A\eta'} g_{B\eta'}), \\ \mathcal{D}_3(\eta\eta') &= (f_{A\eta} f_{B\eta} - g_{A\eta} g_{B\eta})(f_{A\eta'} f_{B\eta'} - g_{A\eta'} g_{B\eta'}), \end{aligned} \quad (26)$$

for the product of form factors, and

$$\mathcal{I}_{0\nu} = \int_1^{\epsilon_0-1} \epsilon_1 (\epsilon_0 - \epsilon_1) \sqrt{(\epsilon_1^2 - 1)[(\epsilon_0 - \epsilon_1)^2 - 1]} d\epsilon_1 \quad (27)$$

for the phase space integral.

The neutrinoless double beta decay rate depends on the cutoff Λ_c and the neutrino effective mass $\langle m_{ee} \rangle$, which are free parameters in our model. Based on present bounds on electron neutrino mass \sim eV [17], we show in Fig. 2 the decay rate $\Gamma_{\Sigma_{0\nu}}$ as a function of the cutoff Λ_c , for a fixed neutrino mass of 10 eV. From this figure we can

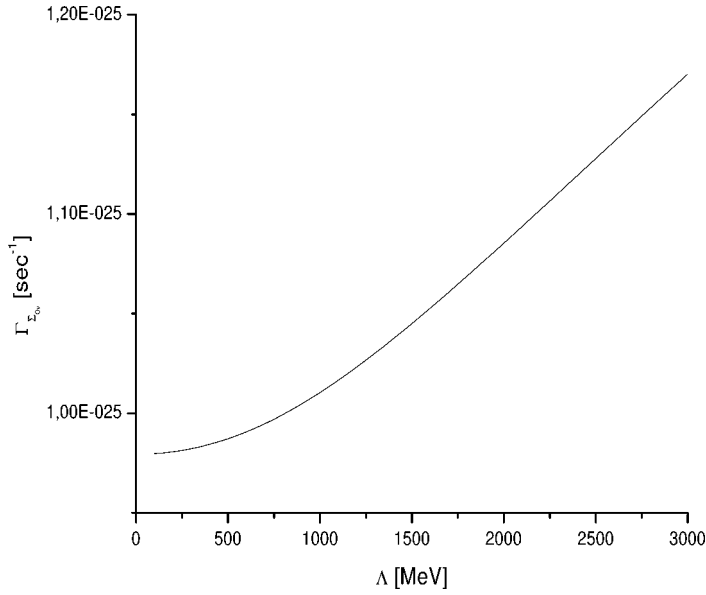


Fig. 2. $\beta\beta_{\Sigma_{0\nu}}$ decay rate as a function of the cutoff Λ_c , for $\langle m_{ee} \rangle = 10$ eV.

clearly see that our results are not very sensitive to the specific cutoff value in the region under consideration, which gives further support to the method employed for the logarithmic divergence.

For illustrative purposes let us use $\Lambda_c = 1$ GeV, which corresponds to a rough estimate of the inverse of the size of hyperons. The branching ratio in this case becomes

$$B(\beta\beta_{\Sigma_{0\nu}}) = 1.49 \times 10^{-35}. \quad (28)$$

Thus, the neutrinoless double beta decay of the Σ^- hyperon is very suppressed and even smaller than the branching ratios of $\beta\beta_{0\nu}$ decays of kaons. Note, however, that in the case of Σ hyperons we do not have a single neutrino (tree-level) intermediate state contribution as in the case of neutrinoless double beta kaon decays [27].

4. Summary and conclusions

We have considered the strangeness-conserving double beta decays of Σ^- hyperons. Several characteristics make this particle a unique and interesting system to study (and to learn) such decays as an alternative to the corresponding decays in nuclei. First, the isotriplet of Σ hyperons is the only system of hadrons that can undergo double beta decays since the $I_3 = +1, -1$ components have different masses. Second, we can identify a few intermediate states that dominate such decays where one state is in the middle and the other is below the initial and final hadronic levels. Finally, the uncertainties in hadronic matrix elements are much smaller than in the nuclear case.

We study both the lepton number-conserving $\beta\beta_{\Sigma_{2\nu}}$ and the lepton number-violating $\beta\beta_{\Sigma_{0\nu}}$ decays. For the lepton number conserving decay we obtain $B(\beta\beta_{\Sigma_{2\nu}}) = 1.38 \times 10^{-30}$ and for the neutrinoless decay we get $B(\beta\beta_{\Sigma_{0\nu}}) = 1.49 \times 10^{-35}$, standing in the light neutrino scenario in the last case. The suppression of the lepton number-conserving decay is due to the large decay width of the Σ^0 intermediate state. Therefore, it would be nice to find an analogue system to the Σ hyperons (the Σ_b baryons?) where the mechanism under consideration could produce an enhancement of the decay rate if the corresponding isotriplet is similar to the spectra of Σ hyperons.

Our result for the neutrinoless double beta decay is almost insensitive to the specific value of the cutoff parameter used to regulate the divergent integrals.

Our numerical results for the branching ratios may look discouraging. However, let us assume an hypothetical model where $B(\beta\beta_{\Sigma_{0\nu}}) = (10^{-20} \sim 10^{-25}) \langle m_{ee} \rangle^2$; in this very optimistic scenario, an experimental upper limit of 10^{-8} would translate into the interesting bound $\langle m_{ee} \rangle \leq 1 \sim 300$ MeV. On another hand, if there exists such a model that causes Σ^- to have faster neutrinoless double beta decays, the SM background due to the lepton number-conserving double beta decays would certainly be very small.

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