Nuclear and particle physics aspects of Big Bang nucleo-synthesis

Osvaldo Civitarese 1 and Mercedes Mosquera 2
1 Department of Physics, University of La Plata, Argentina
2 Department of Astronomy and Geophysics, University of La Plata, Argentina
E-mail: osvaldo.civitarese@fisica.unlp.edu.ar
E-mail: mmosquera@fcaglp.unlp.edu.ar

Abstract. Big Bang Nucleo Synthesis (BBN) is the process which has occurred during the first three minutes of the Universe, that is during the time it needed to cooled down to temperatures of the order of $10^9$ K. The standard BBN model depends on one parameter, the baryonic density, and it is one of the proper tools to study the physics of the Universe at its earliest stage. In this paper we shall discuss the role of BBN as a testing ground for evidences and possible limits which could be determined from cosmological analysis based on the inclusion of sterile neutrinos, the variation of the mass of the Higgs boson and the time variation of fundamental constant.

1. Introduction

Cosmology has indeed evolved to the level of an experimental science (even showing nice error bars in its predictions, something which years ago was considered a mere speculation). Nuclear physics has a hand in cosmological studies, and it also has become an unavoidable partner in determining the limits and scope of the cosmological interpretation of data. Out of the many aspects of nuclear-physics cosmological-oriented studies, we have selected two topics: a) the variation of the mass of the Higgs boson, and b) the inclusion of sterile neutrinos in the electroweak decays, to give examples about the way in which nuclear physics and cosmology talk to each other and set a common frontier of interest.

Several observations can establish limits to the variation of different fundamental constants, such as the atomic clocks and the analysis of the spectra of quasar absorption systems. These astronomical observations suggest a possible variation of the fine structure constant and the electron-to-proton mass ratio [1]. However, other analysis of similar astronomical data gives null variation of the fine structure constant [2].

Big Bang Nucleosynthesis (BBN) is an useful tool to study time variation of fundamental constants such as the fine structure constant, $\alpha$, the Higgs vacuum expectation value, $v$, the Planck mass among others. Several theories that attempt to unify the four fundamental interactions, such as super-strings, brane world, and Kaluza-Klein theories, allow fundamental constants to vary within cosmological time scales. In a previous work we have reported on the methods which can be used to calculate it [3, 4]. If the Higgs vacuum expectation value acquires a different value during BBN than the present one, the electron mass, the proton-neutron mass difference, the Fermi constant, and the deuterium binding energy, $\epsilon_D$ will be different than the corresponding actual values [5, 6].
Here, we study the effects of a possible variation of the Higgs vacuum expectation value, considering a fixed value of the strong-coupling constant $\Lambda_{\text{QCD}}$. To perform the calculation of the primordial abundances, we use the linear dependence of $\epsilon_D$ with $v$ discussed in [3, 4]. We use observational data of D, $^4$He and $^7$Li, to obtain constraints on the variation of the participant fundamental constants. We also perform an analysis of the sensibility of these constraints upon the lithium abundance and upon the dependencies between $\epsilon_D$ and $v$.

2. Bounds from BBN

The dependence of the deuterium binding energy on the Higgs vacuum expectation value is model dependent [5]. We have calculated, for four different nucleon-nucleon potentials (Argonne $v_{18}$ potential, Bonn potential, Nijmegen potential and Reid 93 potential), the proportional constant $\kappa$ that relates $\epsilon_D$ and $v$

$$\frac{\Delta \epsilon_D}{(\epsilon_D)_0} = \kappa \frac{\Delta v}{v_0},$$

where $\Delta \epsilon_D = (\epsilon_D)_{BBN} - (\epsilon_D)_0$, and $\Delta v = v_{BBN} - v_0$. The subindexes $BBN$ and 0 indicate the value of the constant at primordial nucleo-synthesis and at the present time, respectively. In Table 1 we present the obtained values of $\kappa$. The Argonne $v_{18}$ potential includes an electromagnetic interaction, proportional to the fine structure constant. We have modified this potential to include the variation of the fine structure constant and performed the calculation of $\epsilon_D$. We obtained the relation $\Delta \epsilon_D = -0.0019 \Delta \alpha / \alpha_0$.

In order to calculate the primordial abundances, we have modified the numerical code developed by Kawano [7] for each nucleon-nucleon potential of Table 1. For details see [3].

WMAP data are able to constraint the baryon density $\Omega_B h^2$ (related to the baryon-to-photon ratio $\eta_B$) with great accuracy, however there is still some degeneracy between the model parameters, namely: one is dealing with a parametric hypersurface defined by the values of $\Omega_B h^2$, $\Omega_{CDM} h^2$ (dark matter density in units of the critical density), $\Theta$ (gives the ratio of the co-moving sound horizon at decoupling to the angular diameter distance to the surface of last scattering), $\tau$ (re-ionization optical depth), $n_s$ (scalar spectral index), $A_s$ (amplitude of the density fluctuations). For this reason we have computed the light nuclei abundances for the following cases: i) variation of $\alpha$ and $v$ allowing $\eta_B$ vary, ii) variation of $\alpha$ and $v$ keeping $\eta_B$ fixed at WMAP value ($\eta_B^{WMAP} = (6.108 \pm 0.219) \times 10^{-10}$). In order to obtain the best fit values for the parameters, we have performed a $\chi^2$-test to compare the theoretical abundances and the observational data. We used the latest data of [8] for D, of [9] for $^4$He and, for $^7$Li we considered the data given by [10], and all available older data. Regarding the consistency of them, we have followed the treatment of [11], and increased the errors by a fixed factor: $\Theta_D = 2.37$, $\Theta_{^4\text{He}} = 2.69$ and $\Theta_{^7\text{Li}} = 1.43$ for D, $^4$He and $^7$Li, respectively.

<table>
<thead>
<tr>
<th>Potential</th>
<th>$\kappa$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Argonne</td>
<td>-1.23</td>
</tr>
<tr>
<td>Bonn</td>
<td>-0.66</td>
</tr>
<tr>
<td>Nijmegen</td>
<td>-1.66</td>
</tr>
<tr>
<td>Reid</td>
<td>-1.83</td>
</tr>
</tbody>
</table>

Table 1. Values of the coefficient $\kappa$ in the relationship $\frac{\Delta \epsilon_D}{(\epsilon_D)_0} = \kappa \frac{\Delta v}{v_0}$ [3, 4].
2.1. Variation of $\alpha$ and $\nu$ allowing $\eta_B$ to vary

We have computed the BBN abundances for different values of the fine structure constant, the Higgs vacuum expectation value and $\eta_B$, for each potential considered (see Table 1). We performed a $\chi^2$-test in order to find the best-fit-values of the parameters. In Table 2 we present the results.

Table 2. Best fit parameter values at 1$\sigma$ standard deviations, for the BBN constraints on $\eta_B$ (in units of $10^{-10}$), $\Delta \alpha / \alpha_0$ (in units of $10^{-3}$) and $\Delta \nu / \nu_0$ (in units of $10^{-3}$), for different values of $\kappa$. In all cases, the value of $\frac{\chi^2_{\text{min}}}{N-3}$ is 1.00.

<table>
<thead>
<tr>
<th>$\kappa$</th>
<th>$\eta_B \pm \sigma \left[10^{-10}\right]$</th>
<th>$\Delta \alpha / \alpha_0 \pm \sigma \left[10^{-3}\right]$</th>
<th>$\Delta \nu / \nu_0 \pm \sigma \left[10^{-3}\right]$</th>
</tr>
</thead>
<tbody>
<tr>
<td>-1.23</td>
<td>6.440$^{+0.392}_{-0.219}$</td>
<td>$-2.5 \pm 4.8$</td>
<td>29.5$^{+1.3}_{-1.1}$</td>
</tr>
<tr>
<td>-0.66</td>
<td>6.150$^{+0.365}_{-0.209}$</td>
<td>$-8.5^{+4.5}_{-4.7}$</td>
<td>29.9$^{+1.3}_{-1.4}$</td>
</tr>
<tr>
<td>-1.66</td>
<td>6.744$^{+0.317}_{-0.378}$</td>
<td>$1.5^{+4.7}_{-4.0}$</td>
<td>29.6$^{+1.1}_{-1.3}$</td>
</tr>
<tr>
<td>-1.83</td>
<td>6.901$^{+0.243}_{-0.386}$</td>
<td>$3.0 \pm 4.0$</td>
<td>29.6$^{+1.0}_{-1.4}$</td>
</tr>
</tbody>
</table>

The results for all potentials are similar, except for the sign of the relative variation of $\alpha$. We found good agreement, at the level of three standard deviations, between our best-fit-value of $\eta_B$ and the one obtained using WMAP data [12]. We also found null variation of $\alpha$ within two standard deviations, and variation of the Higgs vacuum expectation value, at the level of six standard deviations.

2.2. Variation of $\alpha$ and $\nu$ keeping $\eta_B$ fixed

Once again, we have computed the BBN abundances for different values of the fine structure constant and of the Higgs vacuum expectation value, for each potential considered (see Table 1). This calculation have been made keeping $\eta_B$ fixed at WMAP value $\left(\eta_B^\text{WMAP} = (6.108 \pm 0.219) \times 10^{-10}\right)$ [12]. We have performed a $\chi^2$-test in order to find the best-fit-value. In Table 3 we present the results.

Table 3. Best fit parameter values, within one standard deviation, (1$\sigma$ errors), for the BBN constraints on $\Delta \alpha / \alpha_0$ (in units of $10^{-3}$) and $\Delta \nu / \nu_0$ (in units of $10^{-3}$), with $\eta_B$ fixed at the WMAP estimation, for the different values of $\kappa$.

<table>
<thead>
<tr>
<th>$\kappa$</th>
<th>$\Delta \alpha / \alpha_0 \pm \sigma \left[10^{-3}\right]$</th>
<th>$\Delta \nu / \nu_0 \pm \sigma \left[10^{-3}\right]$</th>
<th>$\frac{\chi^2_{\text{min}}}{N-2}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>-1.23</td>
<td>$-2.0 \pm 4.0$</td>
<td>$29.0 \pm 1.5$</td>
<td>1.04</td>
</tr>
<tr>
<td>-0.66</td>
<td>$-9.0 \pm 5.0$</td>
<td>$29.5 \pm 2.0$</td>
<td>0.96</td>
</tr>
<tr>
<td>-1.66</td>
<td>$3.0 \pm 4.0$</td>
<td>$28.5 \pm 1.5$</td>
<td>1.19</td>
</tr>
<tr>
<td>-1.83</td>
<td>$5.0 \pm 4.0$</td>
<td>$28.0 \pm 2.0$</td>
<td>1.25</td>
</tr>
</tbody>
</table>

We found null variation of $\alpha$ at 2$\sigma$ level, however, the variation of $\nu$ is not-null even at 6$\sigma$. If the analysis does not include the lithium data set, we found null variation in both fundamental constants at the level of 2$\sigma$. 


In summary, we have obtained bounds on the joint variation of $\alpha$ and $v$ using the observational abundances of $D$, $^4He$ and $^7Li$, and performed the analysis for different estimations of the dependence of the deuterium binding energy on the Higgs vacuum expectation value. We have found that the four dependencies give similar results, leading to reasonable fits for the variation of $\alpha$, $v$ and $\eta_B$ for the whole data set. We only found variation of $v$ when the $^7Li$ abundance is included in the statistical analysis. If the present values of $^7Li$ abundances are correct, varying fundamental constants would be a possible candidate for solving the discrepancy between the light elements abundances and the WMAP estimates.

3. Sterile neutrinos

In recent papers [13, 14] the sensitivity of the $^4He$ primordial abundance, upon distortions of the light neutrino spectrum induced by couplings with a sterile neutrino, was analyzed. The effects due to the mixing between sterile and active neutrinos reflect upon Big Bang Nucleosynthesis (BBN) in a noticeable manner. Previous studies on this matter can be found in [15] and [16]. The results of [13] may be taken as a solid starting point for a systematic analysis of the sterile-active neutrino mixing upon cosmological observables. By the other hand, the mixing mechanism between sterile and active neutrinos has been studied in detail (see [17] and references therein), so that the calculation of neutrino distribution functions can readily be performed. The information about the neutrino distribution function, in the flavor basis and at a given temperature, is an essential element in the calculation of the neutron decay rate, which is a critical quantity entering BBN. In the framework of the standard cosmological model, sterile neutrinos would produce a faster expansion rate for the Universe and a higher yield of $^4He$. This is, indeed, a severe constraint on neutrino mixing since a higher predicted abundance of $^4He$ may be in conflict with observational data [19]. Another constraint on active-sterile neutrino mixing is the neutrino mass derived from Cosmic Microwave Background Anisotropy (CMB) [21]. The analysis of constraints presented in [22] focus on the mixing scheme at the level of the neutrino mass hierarchy, and it suggests the adequacy of the non-degenerate mass hierarchy to set limits on the mass difference between active and sterile neutrinos, $\delta m^2_{a-s}$.

In standard BBN calculations, the mixing of sterile and active neutrinos affects the leptonic fractional occupancies, which are essential quantities appearing in the expression of the weak decay rates. Thus, one needs to know, as input of the calculations, the parameters of the proposed mixing scheme, the neutrino mass hierarchy and the leptonic densities [20]. With these elements one can calculate neutron-decay-rates and neutron abundances, by assuming the freeze-out of weak interactions. The effective number of neutrino generations, $N_{\nu}$, is fixed by the analysis of CMB. Current limits on the neutrino degeneracy parameter, for light (electron) neutrinos, $\eta_l$ [14], runs from $-0.1$ to $0.3$.

Here we focus on the calculation of the abundances of $D$, $^3He$, $^4He$ and $^7Li$, in presence of sterile-active neutrino mixing in the three flavor scenario, and for the normal and inverse neutrino mass hierarchies [17]. We have compared the calculated values with data, and determined the compatibility between them by performing a $\chi^2$ statistical analysis. Since the theoretical expressions depend on the mixing angle $\sin^2 2\theta_k$, the square mass difference $\delta m^2_{14}$ (normal mass hierarchy) or $\delta m^2_{42}$ (inverse mass hierarchy), and the baryonic density $\Omega_Bh^2$, we have adopted the LSND limits on the mixing angle and the WMAP results on the baryonic density, as constraints.

The mixing between active neutrino mass eigenstates $\nu_i$ ($i = 1, 2, 3$), leading to neutrinos of a given flavor $\nu_k$ ($k = \text{light, medium, heavy}$), is described by the mixing matrix $U$ [25]

$$U = \begin{pmatrix}
  c_{13} c_{12} & s_{12} c_{13} & s_{13} \\
  -s_{12} c_{23} - s_{23} s_{13} c_{12} & c_{23} c_{12} - s_{23} s_{13} s_{12} & s_{23} s_{13} \\
  s_{23} s_{12} - s_{13} c_{23} c_{12} & -s_{23} c_{12} - s_{13} s_{12} c_{23} & c_{23} c_{13}
\end{pmatrix},$$

(2)

where $c_{ij}$ and $s_{ij}$ stand for $\cos \theta_{ij}$ and $\sin \theta_{ij}$, respectively, and CP conservation is assumed. To
this mixing we add the mixing of a sterile neutrino with: a) the neutrino mass eigenstate of lowest mass in the normal mass hierarchy, $\nu_1$, and b) to the one of the inverse mass hierarchy, $\nu_3$, by defining the mixing angle $\phi$, such that the new mixing matrix $U$ is redefined as $U(\phi)$ [17].

The mixing between neutrino mass eigenstates, and particularly the inclusion of the sterile neutrino as a partner of the light neutrino, affects the statistical occupation factors of neutrinos of a given flavor. The equation which determines the structure of the neutrino occupation factors, in the basis of mass eigenstates and for an expanding Universe, can be written:

$$\frac{\partial f}{\partial t} - H E_\nu \frac{\partial f}{\partial E_\nu} = i [H_0, f],$$

where $t$ is time, $H$ is the expansion rate of the Universe, defined as $H = \sqrt{\frac{8\pi^2 N}{45 M_{Planck}^2}} T^2 = \mu_F T^2$, $T$ is the temperature, $E_\nu$ is the energy of the neutrino, and $H_0$ is the unperturbed mass term of the neutrino’s Hamiltonian in the rest frame. The initial condition is fixed by defining the occupation numbers at the temperature $T_0 = 3$ MeV [26],

$$\left( \begin{array}{cccc} f_{11} & f_{12} & f_{13} & f_{14} \\ f_{21} & f_{22} & f_{23} & f_{24} \\ f_{31} & f_{32} & f_{33} & f_{34} \\ f_{41} & f_{42} & f_{43} & f_{44} \end{array} \right) |_{T_0} = \frac{1}{1 + e^{E_\nu/T_0 - \eta}} \left( \begin{array}{cccc} \cos^2 \phi & 0 & 0 & \sin \phi \cos \phi \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ \sin \phi \cos \phi & 0 & 0 & \sin^2 \phi \end{array} \right),$$

for the normal mass hierarchy, and

$$\left( \begin{array}{cccc} f_{11} & f_{12} & f_{13} & f_{14} \\ f_{21} & f_{22} & f_{23} & f_{24} \\ f_{31} & f_{32} & f_{33} & f_{34} \\ f_{41} & f_{42} & f_{43} & f_{44} \end{array} \right) |_{T_0} = \frac{1}{1 + e^{E_\nu/T_0 - \eta}} \left( \begin{array}{cccc} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \cos^2 \phi & \sin \phi \cos \phi \\ 0 & 0 & \sin \phi \cos \phi & \sin^2 \phi \end{array} \right),$$

for the inverse mass hierarchy.

To obtain the solutions of Eq.(3) we have written the commutator in the r.h.s of Eq.(3), in terms of the square mass differences, $\delta m_{ij}^2 = m_i^2 - m_j^2$:

$$[H_0, f] = \frac{1}{2p} \begin{bmatrix} 0 & \delta m_{12}^2 f_{12} & \delta m_{13}^2 f_{13} & \delta m_{14}^2 f_{14} \\ -\delta m_{12}^2 f_{21} & 0 & \delta m_{13}^2 f_{23} & \delta m_{14}^2 f_{24} \\ -\delta m_{13}^2 f_{31} & -\delta m_{23}^2 f_{23} & 0 & \delta m_{24}^2 f_{34} \\ -\delta m_{14}^2 f_{41} & -\delta m_{24}^2 f_{24} & -\delta m_{34}^2 f_{34} & 0 \end{bmatrix}. \quad (6)$$

The value of the mixing angle $\theta_{13}$ is constrained by the upper limit given by [27] so that $\tan \theta_{13} \leq 10^{-3}$. The solution in the basis of mass eigenstates reads

$$f_{ii} = \frac{\text{const}}{1 + e^{E_\nu/T_0 - \eta}},$$

$$f_{ij} = \frac{\text{const}}{1 + e^{E_\nu/T_0 - \eta}} \exp \left[ i \delta m_{ij}^2 T \left( \frac{1}{T^3} - \frac{1}{T_0^3} \right) \right], \quad (7)$$

where the normalization constants are fixed by the initial conditions ($T = T_0$). In the above expressions, $\eta$ is the ratio between the neutrino chemical potential and the temperature. This parameter depends on the adopted value of the leptonic number L [14, 13]. Explicit expressions of $\eta$ versus $L$ can be found in [14]. In the present context we have taken $\eta$ as an input for the calculations.
3.1. Decay Rates and neutron abundance

In the following we shall outline the main steps of the calculation of neutron decay rates, for the
electroweak processes $n + e^+ \rightarrow p + \nu$ and $n + \nu \rightarrow p + e^-$. The starting point is the calculation of the reduced rates $\lambda_{\pm}$

$$
\lambda (n + \nu \rightarrow p + e^-) = \lambda_- = \lambda_0 \int_0^{\infty} dp_\nu p_\nu E_\nu p_e E_e \left(1 - f_e\right) f_\nu,
$$

(8)

and the total neutron to proton decay rate

$$
\lambda_{np}(y) = \lambda_-(y) + \lambda_+(y).
$$

(10)

The final expression for the neutron to proton decay rate is obtained by fixing the normalization $\lambda_0$, of Eq.(10), from the neutron half-life

$$
\frac{1}{\tau} = \frac{4\lambda_0 \Delta m^2_{np}}{255}.
$$

(11)

The neutron abundance, until the freeze-out of weak interactions, is expressed in terms of the neutron to proton decay rate, $\lambda_{np}$ of Eq.(10) as

$$
X_{\text{neutrons}} = \int_{0}^{\infty} dw\ e^{w+\eta} \left(\frac{1}{1 + e^{w+\eta}}\right)^2 e^{-(\mu F\Delta m^2_{\text{np}})^{-1}} \int_{w}^{\infty} du \left(1 + e^{-u-\eta}\right) \lambda_{np}(u).
$$

(12)

The quantity $X_{\text{neutrons}}$ is, therefore, a function of $\lambda_{np}$ and, consequently, of the occupation factors $f_\nu$, which contain the information about the mixing between active and sterile neutrinos. The next step consists on the calculation of primordial nuclear abundances. It is a semi-analytic approach based on the balance between production and destruction of a given nuclear element, which requires the knowledge of $X_{\text{neutrons}}$. The above presented framework shows that the calculation of primordial abundances may indeed be taken as a tool to test leptonic mechanisms, like the mixing between sterile and active neutrinos, as it has been pointed out by Kishimoto et al. [13].

To perform the calculations we have adopted the oscillation parameters determined from SNO, SK and CHOOZ measurements [27]. The mixing with the sterile neutrino, represented by the mixing angle $\phi$, is taken as an unknown variable, within the limits fixed by the LSND data [23]. The mass splitting $\delta m^2_{14}$ (or $\delta m^2_{34}$) was taken from the analysis given by Keränen et al. [17]. The actual value is fixed at $\delta m^2 = 10^{-11} \text{ eV}^2$. We have then calculated the neutron abundance, by applying the formalism of the previous section. The baryonic density $\Omega_B h^2$ (see Ref.[29]) was varied within the limits $0.010 < \Omega_B h^2 < 0.035$. Concerning the value of $\eta$ we have varied it in the interval determined by the allowed values of the potential lepton number, $\mathcal{L} = 2L_{u_x} + L_{\nu_x} + L_{\nu_x}$, that is $0.0 \leq \mathcal{L} \leq 0.4$ [13, 14]. In the present calculations we have adopted the values $0.0 \leq \eta \leq 0.07$ which are consistent with the densities $0.0 \leq L_{\nu_x} \leq 0.05$ [14].

To determine the allowed values of the mixing angle $\phi$ we have performed a $\chi^2$-minimization, after computing the primordial abundances. The absolute minimum is located at $\sin^2 2\phi = 0.000 \pm 0.026$, and $\Omega_B h^2 = 0.0253 \pm 0.0015$, both set of results have been obtained by using the solution (7) for the occupations. The smallness of the mixing angle does not contradict LSND results but the value of the baryonic density is outside the limits determined by WMAP [24], that is: $(\Omega_B h^2)_{\text{WMAP}} = 0.0223 \pm 0.0008$. This disagreement between theory and data may be caused by large uncertainties in the $^7\text{Li}$-data. As pointed out by Richard et al. [30], the validity of the data on $^7\text{Li}$ may be questioned by the uncertainties inherent to the physics of $^7\text{Li}$ in
the interior of the stars, i.e; the turbulent transport in the radiative zone of stars. In contrast, the situation improves if the data on $^7\text{Li}$ are removed at the time of performing the statistical analysis. For this case, the best value of the mixing angle is $\sin^2 2\phi = 0.018 \pm 0.098$, and the baryonic density corresponding to the minimum, $\Omega_Bh^2 = 0.0216 \pm 0.0017$, is indeed consistent with the WMAP data. The anomalous feature associated with the inclusion of $^7\text{Li}$ in the set of data persists if other elements are removed from the data. We have verified it by systematically removing, one at the time, the abundances of D, $^3\text{He}$, and $^4\text{He}$, and keeping the data on $^7\text{Li}$.

For the case of inverse mass hierarchy the occupation factor is strongly constrained by the value of $\theta_{13}$ and the difference with respect to the thermal occupation factor vanishes. For both the normal and inverse hierarchy solutions, particle number conservation was enforced, on the average, by the factor $\mu_P$ (see its definition following Eq.(3)). Because of the high temperature we have not included collision terms in Eq. (3).

Similar results, related to the abundance of $^7\text{Li}$, have been obtained in the calculations of nuclear abundances in the context of cosmological models [31], and also in the case of a two neutrino mixing [28].

Finally, the best values of the baryon density and the mixing angle, both with and without including $^7\text{Li}$ in the analysis, are shown in Table 4 as functions of $\eta$. In agreement with the expectations of [18], and with our own, both sets of results do not differ much, or at least they do not show a pronounced dependence, with respect to the chemical potential.

### Table 4. Best values of the mixing angle and of the baryonic density, determined from the $\chi^2$ analysis of the calculated abundances, as functions of the parameter $\eta$. Left and right sides of the table show the results obtained with and without considering the data on $^7\text{Li}$, respectively.

<table>
<thead>
<tr>
<th>$\eta$</th>
<th>$\Omega_Bh^2$</th>
<th>$\sin^2 2\phi$</th>
<th>$\Omega_Bh^2$</th>
<th>$\sin^2 2\phi$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.00</td>
<td>0.0253 ± 0.0015</td>
<td>0.000 ± 0.026</td>
<td>0.0216 ± 0.0017</td>
<td>0.018 ± 0.098</td>
</tr>
<tr>
<td>0.01</td>
<td>0.0250 ± 0.0014</td>
<td>0.000 ± 0.010</td>
<td>0.0216 ± 0.0020</td>
<td>0.012 ± 0.022</td>
</tr>
<tr>
<td>0.02</td>
<td>0.0248 ± 0.0014</td>
<td>0.000 ± 0.015</td>
<td>0.0218 ± 0.0020</td>
<td>0.004 ± 0.030</td>
</tr>
<tr>
<td>0.03</td>
<td>0.0246 ± 0.0012</td>
<td>0.000 ± 0.034</td>
<td>0.0216 ± 0.0018</td>
<td>0.008 ± 0.080</td>
</tr>
<tr>
<td>0.04</td>
<td>0.0244 ± 0.0016</td>
<td>0.000 ± 0.039</td>
<td>0.0216 ± 0.0019</td>
<td>0.018 ± 0.090</td>
</tr>
<tr>
<td>0.05</td>
<td>0.0244 ± 0.0016</td>
<td>0.000 ± 0.056</td>
<td>0.0216 ± 0.0017</td>
<td>0.052 ± 0.090</td>
</tr>
<tr>
<td>0.06</td>
<td>0.0244 ± 0.0016</td>
<td>0.000 ± 0.101</td>
<td>0.0216 ± 0.0018</td>
<td>0.108 ± 0.090</td>
</tr>
</tbody>
</table>

### 4. Conclusions
So far, the results presented in the previous sections show that it is indeed possible to combine nuclear, neutrino and cosmological models to analyze astrophysical data. The analysis of BBN results shows that there is a clear sensitivity of the data respect to neutrino mass hierarchies and to the mixing with sterile neutrinos. The calculated dependence of the baryon density $\Omega_Bh^2$, respect to the mixing angle $\sin^2 2\phi$, is in good agreement with the limits extracted from the WMAP and LSND measurements, only if data on the abundance of $^7\text{Li}$ are excluded from the analysis. Changes in the vacuum expectation value of the Higgs field could explained for the discrepancies between the theoretical and observed BBN abundances.

### Acknowledgments
This work was partly supported by the CONICET and by the ANPCYT (Argentina).
References