



Constraining the fundamental interactions and couplings with Eötvös experiments

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ABSTRACT

Upper bounds for the violation of the Weak Equivalence Principle (WEP) by the fundamental interactions have been given before. We now recompute the limits on the parameters measuring the strength of the violation with the whole set of high accuracy Eötvös experiments. Besides, limits on spatial variation of the fundamental constants α , $\sin^2 \theta_W$ and v , the vacuum expectation value of the Higgs field, are found in a model independent way. Limits on other parameters in the gauge sector are also found from the structure of the Standard Model.

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1. Introduction

The Standard Model of fundamental interactions (SM) together with General Relativity (GR) provide a consistent description of all known local low energy phenomena (i.e. low compared to the Grand Unified Theory (GUT) energy scale) in good agreement with the experiment. These theories depend on a set of parameters called the “fundamental constants”, which are supposed to be universal parameters; i.e.: time, position and reference frame invariant [1,2].

The Equivalence Principle (EP) is the physical basis of gravitational theory [3]. There are several versions of the EP [4]. The Weak Equivalence Principle (WEP) (also called Universality of Free Fall (UFF)) states that the trajectory of a freely falling test body is independent of its internal structure and composition. The Einstein Equivalence Principle (EEP) enhances the previous version imposing the equivalence between a local inertial reference frame and a freely falling one. The unrestricted validity of this very strong statement (Strong Equivalence Principle) implies that General Relativity is the unique theory of the gravitational field [5]. Thus, experimental tests of its validity probe deeply into the structure of gravitation.

The traditional model for describing a WEP breakdown is to assume that an anomalous acceleration of body A , defined as $\delta \mathbf{a}_A = \mathbf{a}_A - \mathbf{g}$ is due to a difference between its inertial mass m_A^I and its passive gravitational mass m_A^P , i.e. the coupling constant of A to the gravitational field. It is usual to parametrize δm_A in the form [4]

$$\delta m_A = m_A^P - m_A^I = \sum_K \Gamma_K \frac{E_K}{c^2}, \quad (1)$$

where E_K are the different contributions to the binding energy of A , and Γ_K are dimensionless parameters quantifying the breakdown of UFF. These parameters can be estimated from Eötvös experiments (cf. Section 2).

Most of the old published estimates have only taken into account the binding energy contribution to the nucleus mass, which is generally dominant. However, the contribution of the binding energies of nucleons is important for several problems and should be included, as it has been done in Ref. [6]. This generalization of the classical model (1) will be used in a forthcoming analysis (cf. Eq. (7)).

One of the consequences of the Equivalence Principle is that the fundamental constants must be universal parameters, because any dependence on time, position or reference frame would break the equivalence with an inertial frame [7,8]. The particular case of space dependence of dimensionless constants has been treated in those references and with weaker hypotheses (essentially energy conservation) in Ref. [9].

In short, the binding energies E_K of bodies such as nuclei are functions of the fundamental constants, and each gives a contribution $\frac{E_K}{c^2}$ to the mass. If the fundamental constants are space dependent, so is the mass of the body. In those conditions, the Lagrangian of a body in a gravitational field takes the form

$$L = - \int m(\alpha) \sqrt{g_{\mu\nu} u^\mu u^\nu} ds. \quad (2)$$

In the nonrelativistic limit one finds an anomalous acceleration

$$\delta \mathbf{a} = - \sum_j \frac{c^2}{m} \frac{\partial m}{\partial \alpha_j} \nabla \alpha_j, \quad (3)$$

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Table 1
Results of the Eötvös experiment.

| Body | Body | Source | $(\eta \pm \sigma) \times 10^{11}$ | Ref. |
|-----------|------------|--------|------------------------------------|------|
| Al | Au | Sun | 1.3 ± 1.5 | [16] |
| Al | Pt | Sun | 0.03 ± 0.04 | [17] |
| Cu | W | Sun | 0.6 ± 2.0 | [18] |
| Be | Al | Earth | -0.02 ± 0.28 | [19] |
| Be | Cu | Earth | -0.19 ± 0.25 | [19] |
| Si/Al | Cu | Sun | 0.51 ± 0.67 | [19] |
| Moon-Like | Earth-Like | Sun | 0.005 ± 0.089 | [20] |
| Be | Ti | Earth | 0.003 ± 0.018 | [21] |
| Be | Al | Earth | -0.015 ± 0.015 | [22] |
| Be | Ti | Sun | -0.031 ± 0.045 | [22] |
| Be | Al | Sun | 0.0 ± 0.042 | [22] |

where j runs over the set of fundamental constants. This anomalous acceleration is composition-dependent and its existence can be tested through Eötvös experiments (Section 2).

Recently, the detection of a spatial variation of the fine structure constant has been reported [10–12] with a gradient amplitude $(3.6 \pm 0.6) \times 10^{-6} \text{ Gpc}^{-1}$ at a $\sim 6\sigma$ level. This tantalizing result suggests that local variation of α should be tested via local experiments. And the Eötvös experiment (Section 2) offers an excellent tool for that. Indeed, Dent [13] has made such an analysis (see also, [2,14]).

The purpose of this Letter is to analyze the space variation of the fundamental constants in the SM, using the available Eötvös experiment results (cf. Table 1). We shall limit ourselves to the Gauge sector of the Standard Model, with the exception of the vacuum expectation value of the Higgs field. The organization is as follows: Section 2 summarizes the main characteristics and results of the Eötvös experiments, and describes the models we shall use for our analysis. Section 3 delineates our implementation of the structural characteristics of the test bodies such as binding energies and constitutive relations in the Standard Model. Section 4 shows our results and in Section 5 we state our conclusions.

2. A primer on Eötvös experiments

The Eötvös experiment [15], one of the most sensitive tests of the Equivalence Principle, measures the difference of acceleration between two masses A, B in the same gravitational field. It consists in suspending a pair of bodies from the arms of a torsion balance in a homogeneous gravitational field. It is easy to show that a differential acceleration would produce a torque [5,4]

$$T = LW\eta(A, B), \quad (4)$$

where L and W are the lever arm of the torsion balance and the gravitational force on the body respectively, and $\eta(A, B)$ is the Eötvös parameter. The torsion balance is rotated with a well-defined angular velocity ω with respect to the external gravitational field and only signals with the corresponding period are analyzed in order to clean the result of spurious systematic effects. Additionally, it would be possible to find any “privileged direction” defined by a gradient in the masses if a nonzero result were found.

The main result of the Eötvös experiment is the Eötvös parameter $\eta(A, B)$, defined as follows: If \mathbf{g} is the local acceleration of gravity,

$$\eta(A, B) = \frac{(\mathbf{a}_A - \mathbf{a}_B) \cdot \mathbf{n}}{|\mathbf{g}|}, \quad (5)$$

where $\mathbf{a}_{A,B}$ are the accelerations of the bodies in the gravitational field and \mathbf{n} a suitably chosen unit vector. Since the Equivalence Principle implies that $\mathbf{a}_A = \mathbf{a}_B = \mathbf{g}$, a non-null η signals its breakdown. The beautiful design of the experiment cancels many causes

of error and during the twentieth century several orders of magnitude in accuracy have been improved. Table 1 displays the results of several high accuracy Eötvös experiments.

Eq. (4) depends crucially on the homogeneity of the gravitational field \mathbf{g} and great efforts have been made to design the torsion balance so that its small inhomogeneities are canceled. Besides, due to the design, the Eötvös experiment is sensitive only to the horizontal component of the gravitational field. Thus, only the deviation of the Earth gravitational field g_\perp or the solar gravitational field g_\odot are used in the experiments. The references cited in Table 1 include many details on the design of the experiment and the analysis of experimental data.

Let

$$M(A, Z) = m_p Z + m_n N + m_e Z - \frac{B(Z, A)}{c^2} \quad (6)$$

be the atomic mass of a body of mass number A , atomic number Z , neutron number $N = (A - Z)$ and binding energy $B(Z, A)$. The difference between inertial and passive gravitational mass of the above body will be [6]

$$\begin{aligned} \delta M &= \delta m_p Z + \delta m_n N + \delta m_e Z - \delta \frac{B(Z, A)}{c^2} \\ &= \frac{\delta(m_p + m_n + m_e)}{2} A + \delta Q \frac{(N - Z)}{2} - \delta \frac{B(Z, A)}{c^2}, \end{aligned} \quad (7)$$

where

$$Q = m_n - m_p - m_e \quad (8)$$

is the decay energy of the neutron. The relative mass difference will be

$$\frac{\delta M}{M} \simeq \frac{\delta(m_p + m_n + m_e)}{2m_p} + \frac{N - Z}{2A} \frac{\delta Q}{m_p} - \delta \frac{B(Z, A)}{Am_p c^2}, \quad (9)$$

an expression which includes both the nuclear binding energy $B(Z, A)$ and the contribution of the particle rest masses δm_k . This model is equivalent to work with constant masses for the nucleons in some suitable system of units.

With model (9) (which generalizes model (1)), the Eötvös parameter reads

$$\eta(X, Y) = \frac{\delta m_X}{m_X} - \frac{\delta m_Y}{m_Y} = \sum_K \Gamma_K \left(\frac{\hat{E}_K}{Mc^2} \Big|_X - \frac{\hat{E}_K}{Mc^2} \Big|_Y \right), \quad (10)$$

where

$$\frac{\hat{E}_K}{Mc^2} = \frac{N - Z}{2A} \frac{\delta Q_K}{m_p} - \delta \frac{B(Z, A)_K}{Am_p c^2}, \quad (11)$$

includes the contribution of each form of energy to the binding energies of neutron and proton. A set of experiments with bodies of different compositions permits in principle the measurement of the Γ_K parameters.

Finally, we shall parametrize $\frac{\hat{E}_K}{Mc^2}$ either in the generalized “classical” form (10) for a test of the Equivalence Principle, or in the form (3) for testing the position dependence of the fundamental constants. In the last case, the Eötvös parameter will read, after some algebra,

$$\eta(A, B) = \frac{c^2}{g} \sum_j \frac{\partial \ln \frac{M^{(A)}}{M^{(B)}}}{\partial \ln \frac{\alpha_j}{\Lambda_j}} \mathbf{n} \cdot \nabla \ln \frac{\alpha_j}{\Lambda_j} \quad (12)$$

where Λ_j are suitable normalization constants.

3. Binding energies and fundamental constants

The main ingredients for our analysis are the binding energies E_K and their dependence on the fundamental constants. We shall discuss separately nuclear binding energies and neutron–proton mass differences.

3.1. Nuclear binding energies

The largest contribution to the binding energy of an atom comes from the nuclear binding, which has been discussed for a long time. The simplest approach is to use the semi-empirical mass formula [23,24] complemented with the estimate of the weak interactions contribution to the binding energy [25,26]. There are simple analytic approximations for the strong, Coulomb and weak contributions to the binding energy B , namely

$$\frac{E_S}{Mc^2} = a_V - a_S A^{-1/3} - a_A \frac{(N-Z)^2}{A^2}, \quad (13a)$$

$$\frac{E_C}{Mc^2} = \frac{3e^2 Z(Z-1)}{5r_0 A^{4/3}}, \quad (13b)$$

$$\frac{E_W}{Mc^2} = \mathcal{G} G_F 2^{-2/3} V^{-1} \left\{ NZ \left[(3\alpha_\beta^2 - 1) - 4 \left(\frac{1}{2} - 2 * \sin^2 \theta_W \right) \right] + \frac{N^2}{2} + \frac{Z^2}{2} (4 \sin^4 \theta_W - 2 \sin^2 \theta_W + 1) \right\}. \quad (13c)$$

In the above equations $r_0 A^{1/3}$ is the nuclear radius, $V = \frac{4\pi}{3} A r_0^3$ the nuclear volume and $N = A - Z$ the neutron number. α_β is the G_A/G_V ratio for neutron decay, and G_F and θ_W are the Fermi constant and the Weinberg angle respectively. \mathcal{G} is an enhancement factor of the weak interactions due to the strong ones [26]. Besides, the “strong constants” a_V , a_S , a_A as well as m_p , m_n are all proportional to Λ_{QCD} in the chiral limit.

These analytic expressions, which are reasonably accurate, display the dependence of the nuclear binding energies on the fundamental constants.

3.2. Neutron–proton mass difference

The other contribution to the mass is the neutron–proton mass difference which contributes to the neutron decay energy Q .

Model independent contribution of the strong, electromagnetic and weak forces to the neutron–proton mass difference ΔM can be computed with the Cottingham formula [27] and its generalizations for the strong [28] and weak [6] interactions. Their calculated values are:

$$\begin{aligned} \left. \frac{\Delta M}{M} \right|_S &= 2.22 \times 10^{-3}, \\ \left. \frac{\Delta M}{M} \right|_E &= -0.83 \times 10^{-3}, \\ \left. \frac{\Delta M}{M} \right|_W &= -5.0 \times 10^{-9}. \end{aligned} \quad (14)$$

However, the explicit dependence on the fundamental constants is not obvious. A careful analysis of the respective expressions shows that the electromagnetic contribution is proportional to α and the weak one to G_F . Besides, the weak contribution has a dependence on $\sin^2 \theta_W$, which must be numerically computed with Ref. [6] method. The result is

$$\frac{\sin^2 \theta_W}{M} \frac{\partial \Delta M}{\partial \sin^2 \theta_W} \simeq 2.0 \times 10^{-8}. \quad (15)$$

Finally, an important result is that the “strong” contribution to ΔM is not proportional to Λ_{QCD} near the chiral limit but to the $u - d$ quarks mass difference, a result that can be derived in an elementary way from Chiral Perturbation Theory [29] and that is quantitatively confirmed in lattice calculations (see, for instance, [30]).

Since quark and electron masses are proportional to the vacuum expectation value of the Higgs field v , $m_i = y_i v$, so is Q . The available Eötvös experiments are not enough to separate the Yukawa coupling parameters and v . So in this Letter we limit ourselves to the analysis of the gauge sector plus the single Higgs sector parameter v . With this limitation, we find the following expression for Q as a function of the fundamental constants α , v and $\sin^2 \theta_W$:

$$\frac{\delta Q}{M} = \frac{\delta \alpha}{\alpha} \frac{\Delta M}{M} \Big|_E + \frac{\delta v}{v} \frac{Q}{M} + \frac{\sin^2 \theta_W}{M} \frac{\partial \Delta M}{\partial \sin^2 \theta_W} \frac{\delta \sin^2 \theta_W}{\sin^2 \theta_W}. \quad (16)$$

If for each of the fundamental constants α_i we replace

$$\delta \alpha_i = \nabla \alpha_i \cdot \delta \mathbf{r} \quad (17)$$

we obtain the contribution of Q to the Eötvös parameter.

3.3. Constitutive relations

In this subsection we shall use the fine structure constant α , the vacuum expectation value of the Higgs field v and the squared sine of Weinberg’s angle θ_W as our basic variables. Other fundamental constants from the gauge sector are related to our basic constants in the form

$$\alpha = \alpha_1 \sin^2 \theta_W, \quad \tan^2 \theta_W = \frac{\alpha_2}{\alpha_1}, \quad G_F = \frac{1}{\sqrt{8} v^2}, \quad (18a)$$

$$M_W^2 = \frac{\alpha_1}{2} v^2, \quad M_Z^2 = \frac{M_W^2}{\cos^2 \theta_W}, \quad \alpha_3 = \frac{\beta^{-1}}{\ln \frac{\mu^2}{\Lambda_{\text{QCD}}^2}}. \quad (18b)$$

The last equation shows that in QCD system of units, α_3 is automatically constant.

3.4. Scaling and systems of units

The need of working with nondimensional quantities when studying the variation of fundamental constants it has been discussed in many papers (see, for instance, [1,2]) since a suitable choice of units may cancel its variation. Many measurements, however, are carried out on dimensional quantities and its analysis must be done starting with these data. This problem may be solved either by transforming the dimensional quantities to a standard system of units [31] or transforming these dimensional quantities into dimensionless ones through division by a suitably chosen constant.

One of the beauties of the Eötvös experiment is that it has a “natural” way of defining η as nondimensional parameter, and Eq. (10) is already in dimensionless form. Besides, since the anomalous acceleration (3) can be written as

$$\delta \mathbf{a} = - \sum_j \frac{c^2}{m} \frac{\partial m}{\partial \alpha_j} \nabla \ln \alpha_j. \quad (19)$$

Any normalization constant will not contribute to the differential acceleration.

In this Letter we shall use “QCD units”: that is, we assume that

$$\nabla \Lambda_{\text{QCD}} = 0. \quad (20)$$

Table 2
Test of the Equivalence Principle.

| Param. | $\Gamma \pm \sigma(\Gamma)$ | Correlations | | | Up. bound |
|------------|----------------------------------|--------------|------|------|----------------------|
| Γ_S | $(-1.3 \pm 1.6) \times 10^{-10}$ | 1.00 | 0.98 | 0.84 | 5×10^{-10} |
| Γ_C | $(-3.0 \pm 4.0) \times 10^{-10}$ | 0.98 | 1.00 | 0.87 | 1.2×10^{-9} |
| Γ_W | $(-2.7 \pm 8.6) \times 10^{-4}$ | 0.85 | 0.87 | 1.00 | 2.6×10^{-3} |
| Γ_C | $(1.5 \pm 6.7) \times 10^{-11}$ | | 1.00 | 0.00 | 2×10^{-10} |
| Γ_W | $(2.2 \pm 2.0) \times 10^{-4}$ | | 0.00 | 1.00 | 6×10^{-4} |

In the nuclear binding energies the dependence of the strong binding energy on the (u , d , s) quark masses can be neglected. This implies that $\Gamma_S = 0$ since we are working near the chiral limit.

However, above approach is not always correct. Indeed, the logarithmic derivatives of the binding energy parameters a_V , a_S , a_A with respect to the quark masses could be large in some cases. Refs. [32–34] make a detailed analysis on the subject. In our case, the only important contribution from the quark masses is in the “strong” contribution to the $p - n$ mass difference.

4. Results

We have performed the above sketched calculations both to test the Equivalence Principle and the existence of gradients of the fundamental constants in the Standard Model. A weighted least squares procedure was applied to the values of η in Table 1, the conditional equation being given by either Eq. (10) or in the form corresponding to spatial variation (12).

4.1. Test of the Equivalence Principle

Table 2 shows our results for the test of the Equivalence Principle. We have used the expressions (13) with the contributions (14) from the neutron–proton mass difference. The first three lines of the table show the result assuming that the three interactions break the Equivalence Principle. The last two lines assume that the strong contribution satisfies the Equivalence Principle ($\Gamma_S = 0$). An enhancement factor $\mathcal{G} = 8$ for the Weak Interactions has been assumed [6]. Our results are similar to those of Ref. [6], but the new bounds are smaller due to the inclusion of higher accuracy results [20–22].

The large correlations in the first three lines suggest that either the breakdown of the Equivalence Principle should be analyzed simultaneously or that a constraint such as ($\Gamma_S = 0$) should be imposed and in this case only violation parameters relative to the strong interactions will be found.

4.2. Spatial variation of fundamental constants

Turning to the spatial variation problem, it is convenient to work with the nondimensional quantity [14]

$$\Theta_j = \frac{c^2}{g_\odot} \frac{|\nabla \alpha_j|}{\alpha_j}, \quad (21)$$

which is the “natural” nondimensional parameter for this problem. As explained before, we use as basic variables α , ν and $\sin^2 \theta_W$. Again, the values of the Θ parameters were found by least squares adjustment and upper bounds were obtained as 3σ values. The results of the adjustment are displayed in Table 3 in the same format as the one in Table 2.

Logarithmic differentiation of the Standard Model relations in (18), after normalization by division by suitable powers of Λ_{QCD} , yields a system of linear equations for the gradients of the parameters from which the upper bounds of Table 4 are found. The quantities

Table 3
Bounds on the space variation of fundamental constants: Basic variables.

| Param. | $\Gamma \pm \sigma(\Gamma)$ | Correlations | | | Up. bound |
|----------------------------|---------------------------------|--------------|------|------|-----------------------|
| Θ_α | $(0.5 \pm 1.6) \times 10^{-10}$ | 1.00 | 0.74 | 0.71 | 4.8×10^{-10} |
| Θ_ν | $(-0.5 \pm 2.5) \times 10^{-8}$ | 0.74 | 1.00 | 0.99 | 1.1×10^{-7} |
| $\Theta_{\sin^2 \theta_W}$ | $(0.1 \pm 1.1) \times 10^{-2}$ | 0.71 | 0.99 | 1.00 | 3.3×10^{-2} |

Table 4
Bounds on the space variation of fundamental constants: Gauge sector.

| j | Θ_j | ℓ_j pc | j | Θ_j | ℓ_j pc |
|-------------------|-----------------------|----------------------|------------|----------------------|-------------------|
| α | 4.8×10^{-10} | 1.1×10^{12} | ν | 7.5×10^{-8} | 6.6×10^9 |
| $\sin^2 \theta_W$ | 3.4×10^{-2} | 1.4×10^4 | α_1 | 3.4×10^{-2} | 1.4×10^4 |
| α_2 | 1.0×10^{-2} | 4.8×10^4 | G_F | 1.5×10^{-7} | 3.4×10^9 |
| M_W^2 | 3.4×10^{-2} | 1.4×10^4 | M_Z^2 | 2.4×10^{-2} | 2.0×10^4 |

$$\ell_j = \frac{\alpha_j}{|\nabla \alpha_j|} \quad (22)$$

define distance scales where the spatial variation of a given fundamental constant becomes important.

The results summarized in Tables 2 to 4 are the main results of this Letter.

5. Conclusion

The results stated in Sections 2 and 4 show that no violation of the Equivalence Principle is observable in laboratory experiments down to the 10^{-13} level. The classical model decomposition of the Eötvös parameter (10) shows that the contributions of the fundamental interactions to such a violation are extremely small.

On the other hand, the order of magnitude of the upper bounds for the gradients of the fundamental constants are very variable, from $\sim 10^{-12}$ pc $^{-1}$ for α to $\sim 10^{-4}$ pc $^{-1}$ for α_2 . These extremely small gradients of galactic or cosmological scale, are the best available bounds on the spatial variation of fundamental constants.

The results of this Letter are in a certain sense complementary to those of Ref. [13] where the analysis was focused mainly on the Higgs sector of the Standard Model and on the sensitivity to the Newtonian potential. See also Ref. [2] for a more complete analysis of that sector.

Our upper bounds, however, are too big for an independent test of the reported cosmological gradient of α . Our smallest bound is obtained assuming that only α has a sensible variation

$$\frac{|\nabla \alpha_j|}{\alpha_j} < 2 \times 10^{-4} \text{ Gpc}^{-1}, \quad (23)$$

and it is about 60 times greater than the detected one. This is not far from the needed sensitivity and the proposed MICROSCOPE [35] or STEP [36] experiments, whose accuracy is about a thousand times greater should be able to detect it.

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References

- [1] Jean-Philippe Uzan, Rev. Mod. Phys. 75 (2003) 403.
- [2] Jean-Philippe Uzan, Liv. Rev. Rel. 14 (2) (2011), 145 pp., 10 figures, review for Living Reviews in Relativity.
- [3] A. Einstein, Ann. Phys. (German) 35 (1911) 898.
- [4] C.M. Will, Theory and Experiment in Gravitational Physics, Cambridge University Press, March 1993.
- [5] S. Weinberg, Gravitation and Cosmology, Wiley, 1972.

- [6] N. Chamoun, H. Vucetich, *Phys. Lett. B* 541 (2002) 291.
- [7] P.J. Peebles, R.H. Dicke, *Phys. Rev.* 127 (July 1962) 629.
- [8] R.H. Dicke, in: C. deWitt, B. deWitt (Eds.), *Relativity, Groups and Topology*, Gordon and Breach, 1964.
- [9] M.P. Haugan, *Ann. Phys.* 118 (March 1979) 156.
- [10] J.K. Webb, J.A. King, M.T. Murphy, V.V. Flambaum, R.F. Carswell, M.B. Bainbridge, *Phys. Rev. Lett.* 107 (19) (November 2011) 191101.
- [11] J.A. King, J.K. Webb, M.T. Murphy, V.V. Flambaum, R.F. Carswell, M.B. Bainbridge, M.R. Wilczynska, F. Elliot Koch, *MNRAS* 422 (2012) 3370.
- [12] J.C. Berengut, E.M. Kava, V.V. Flambaum, *Astronom. Astrophys.* 542 (June 2012) A118.
- [13] Thomas Dent, *Phys. Rev. Lett.* 101 (2008) 041102.
- [14] H. Vucetich, in: U. Cotti, M. Mondragón, G. Tavares-Velasco (Eds.), *Particles and Fields*, in: American Institute of Physics Conference Series, vol. 670, June 2003, pp. 298–305.
- [15] R.V. Eötvös, D. Pekár, E. Fekete, *Ann. Phys.* 373 (1922) 11.
- [16] P.G. Roll, R. Krotkov, R.H. Dicke, *Ann. Phys. (N. Y.)* 26 (1964) 442.
- [17] V.B. Braginski, V.I. Panov, *Sov. Phys. J. E. T. P.* 34 (1972) 463.
- [18] G.M. Keiser, J.E. Faller, in: R. Ruffini (Ed.), *Proc. of the 2nd Marcel Grossman Meeting on General Relativity*, 1982, p. 969.
- [19] Y. Su, B.R. Heckel, E.G. Adelberger, J.H. Gundlach, M. Harris, G.L. Smith, H.E. Swanson, *Phys. Rev. D* 50 (September 1994) 3614.
- [20] S. Baeßler, B.R. Heckel, E.G. Adelberger, J.H. Gundlach, U. Schmidt, H.E. Swanson, *Phys. Rev. Lett.* 83 (November 1999) 3585.
- [21] S. Schlamminger, K.-Y. Choi, T.A. Wagner, J.H. Gundlach, E.G. Adelberger, *Phys. Rev. Lett.* 100 (4) (February 2008) 041101.
- [22] E.G. Adelberger, J.H. Gundlach, B.R. Heckel, S. Hoedl, S. Schlamminger, *Prog. Part. Nucl. Phys.* 62 (2009) 102.
- [23] C.F. von Weizsäcker, *Z. Phys.* 96 (1935) 35.
- [24] B. Martin, *Nuclear and Particle Physics: An Introduction*, 2nd edition, Wiley, New York, 2009.
- [25] Mark P. Haugan, Clifford M. Will, *Phys. Rev. Lett.* 37 (1) (1976).
- [26] Ephraim Fishbach, Mark P. Haugan, Dubravko Tadić, Hai-Yang Cheng, *Phys. Rev. D* 32 (1985) 154.
- [27] W.N. Cottingham, *Ann. Phys.* 25 (1963) 424.
- [28] L.N. Epele, H. Fanchiotti, C.A. García Canal, R. Méndez Galain, in: H.M. Nussenzweig, S. MacDowell, R.A. Salmeron (Eds.), *Frontier Physics: Essays in Honour of Jayme Tiomno*, 1991, p. 57.
- [29] J. Gasser, H. Leutwyler, *Phys. Rep.* 87 (July 1982) 77.
- [30] Silas R. Beane, Kostas Orginos, Martin J. Savage, *Nucl. Phys. B* 768 (2007) 38.
- [31] P. Sisterna, H. Vucetich, *Phys. Rev. D* 41 (February 1990) 1034.
- [32] T. Damour, J.F. Donoghue, *Phys. Rev. D* 82 (8) (October 2010) 084033.
- [33] T. Damour, J.F. Donoghue, *Classical Quantum Gravity* 27 (20) (October 2010) 202001.
- [34] T. Damour, J.F. Donoghue, *Classical Quantum Gravity* 28 (16) (August 2011) 162001.
- [35] Microscope: Microsatellite for the test of the equivalence principle, <http://smc.cnes.fr/MICROSCOPE/index.htm>.
- [36] Step: Satellite test of the equivalence principle, <http://einstein.stanford.edu/STEP/>.