Conventional vs gauge couplings in elastic πN scattering

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Abstract. We perform a review on the importance of the $\Delta(1232\ \mathrm{MeV})$ resonance in different processes, analyze the existing problems with the fields used to describe it theoretically and finally we make an analysis on the use of $\pi N\Delta$ alternative couplings in πN scattering. We compare numerically results for the elastic cross section obtained by using the conventional couplings, already adopted in several reaction calculations, with those obtained with the so called "spin-3/2 gauge invariant" vertexes suggested recently. Confronting with experimental cross section data, we see that these results are by no means equivalent and that the differences between them cannot be compensated by a readjustment of free parameters of background contributions. We find that the conventional couplings work better than the gauge ones in describing the data.

1. Introduction

We can ask ourselves, why to put attention on the isobar $\Delta(1232\ \mathrm{MeV})$ resonance? The immediate answer would be: since it is the first excited state of the nucleon(N) and has a prominent role in strong interactions. It dominates the pion-production phenomena being excited in $\pi N \to \pi'N'$, $\pi N \to \pi'N'\gamma$[11] scattering, which are useful reactions in order to check different isobar models regards Lorentz invariance, unitarity, and gauge invariance. It is also present in the $\gamma N \to \pi N$ photoproduction process[2], important to determine the deformation in the 3 quark(q) constitutive wave function. Also, it appears in $eN \to e'N'\pi$ electroproduction scattering [3] that is a reaction important to get vector form factors and participates in the $\nu N \to lN'\pi$ weak production [4], this being the main experimental background in neutrino oscillation experiments. On the other side, the Δ resonance dominates many nuclear phenomena at energies above the π-production threshold. In cosmology, it is largely responsible for the "GZK cut-off" [5], drooping of cosmic ray rate due excitation of Δ by scattering of CMB photons. High precision measurements of the $N \to \Delta$ transition was possible at LEGS, BATES, ELSA, MAMI and JLAB (several experiments on scattering by electromagnetic probes). The experimental effort, has been accompanied by exciting developments in dynamical models, lattice QCD and chiral effective-field theories.

The $\Delta(1232)$ was discovered 60 years ago by Fermi and Colaborators at the Chicago cyclotron (now Fermilab), being it mass, width, spin and isospin, $m = 1232\ \mathrm{MeV} \simeq m_N + 300\ \mathrm{MeV}$, $\Gamma = 120$

MeV ($\tau = 10^{-23}$ sec), $S = T = 3/2$, respectively. Within the SU(6) quark model the N appears as the ground state of a 3q system in a confining potential. In the same way the Δ excitation in πN scattering, for example $\pi(u\bar{d})p(uud) \rightarrow \Delta^{++}(uuu) \rightarrow \pi'(u\bar{d})p'(uud)$, can be seen as a $d\bar{d}$ annihilation. It then decays 99% of time in πN channel and 1% in the $\gamma\pi$ electromagnetic one, being this excitation essentially a spin flip magnetic dipole one (see Figure 1). Nevertheless, if we considered D admixtures in the N or Δ wave functions, a quadrupole transition is present.
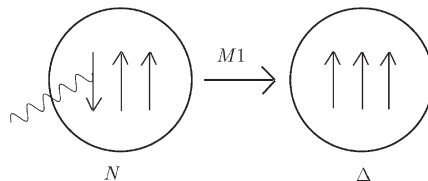

Figure 1. $M1$ Excitation of the Δ.

From the theoretical point of view, we know that pions are the Goldstone bosons of the spontaneously broken chiral symmetry of QCD and as the coupling with them goes as p_π, at low energies perturbative expansion in powers of it is possible, we having the Chiral Perturbation Theory [7]. On the other hand the resonance parameters can be extracted from reactions by using unitary isobar models: unitarized tree-level calculations based on phenomenological Lagrangians. Nevertheless "pion cloud" effects, which represents the coupling of a pion to quarks, could only be comprehensively studied within dynamical models based in T-matrix calculations.

2. The Rarita-Schwinger field

For implementing any of the mentioned approaches we need the kinetic $\mathcal{L}_\Delta$ and interacting $\mathcal{L}_{\pi N\Delta}$ Lagrangians. The Rarita Schwinger (RS) spinor ψ_μ, which represents mathematically the Δ, is taken as an element of the non-unitary representation $[(1/2, 0) \oplus (0, 1/2)] \otimes [(1/2, 1/2) = (1/2, 0) \otimes (0, 1/2)]$ of the Lorentz group. That is, $\psi_\mu \equiv \psi \otimes W_\mu$, being ψ a Dirac spinor and W_μ is a 4-vector with spin $S = 0, 1$ in the rest or helicity frames. Omitting for a moment the Lorentz structure and putting attention only in the angular momentum we have representations $D(1/2) \otimes D(0) = D(1/2)$ and $D(1/2) \otimes D(1) = D(3/2) \oplus D(1/2)$, what enclose $2(S = 1/2) + 4(S = 3/2) + 2(S = 1/2) + antiparticles = 16$ degrees of freedom (dof). $\psi_\mu(x)$ satisfies the Dirac and Klein Gordon equations and additionally those states built with $W_\mu(S = 1)$, satisfy the Proca one. From the 16 states only 8 satisfy the subsidiary conditions $\partial^\mu\psi_\mu = \gamma^\mu\psi_\mu = 0$, what are seen as making zero "projections" of ψ_μ on spin-1/2 subspaces to get right dof counting for the free case.

The RS field is a "constrained" dynamical system supplemented by constraints or subsidiary conditions , being the Lagrangian from which we can get the equation of motion plus the constraints

$$\mathcal{L}_{free} = \overline{\psi}_\mu(x)\left\{i\partial_\alpha\Gamma^\alpha_{\mu\nu} - mB_{\mu\nu}\right\}\psi^\nu(x), \tag{1}$$

where

$$\Gamma^\alpha_{\nu\mu} = g_{\mu\nu}\gamma^\alpha + \frac{1}{3}\gamma_\mu\gamma^\alpha\gamma_\nu - \frac{1}{3}(\gamma_\mu g^\alpha_\nu + g^\alpha_\mu\gamma_\nu),$$

$$B_{\nu\mu} = g_{\mu\nu} - \frac{1}{3}\gamma_\mu\gamma_\nu , \tag{2}$$

proposed by Rarita and Schwinger in 1941. $\mathcal{L}_{free}$ includes the constraints and Γ, B do not mix $3/2$ with $1/2$ states, then it only fix $3/2$ component of ψ_μ. Then, the contact transformation $\psi^\mu \to \psi'^\mu = R(a)^{\mu\nu}\psi_\nu \equiv (g^{\mu\nu} + a\gamma^\mu\gamma^\nu)\psi_\nu$, only affects $\psi_{1/2\mu}$ components of ψ_μ and let $\mathcal{L}_{free}$ invariant we having a whole family of valid one parameter Lagrangians ($a = 1/2(1 + 3A)$)

$$\mathcal{L}_{free}(A) \;=\; \overline{\psi}(x)R(A)\left\{i\partial_\alpha\Gamma^\alpha - mB\right\}R(A)\psi(x), \tag{3}$$

where for $A = -1/3$ we recuperate the RS one and where some indexes are omitted by simplicity. Finally the spin-$3/2$ propagator $G(p, A)$ can be obtained as usually giving

$$G(p, A) \;=\; R^{-1}(A)G\left(p, -\frac{1}{3}\right)R^{-1}(A),$$

$$G\left(p, -\frac{1}{3}\right)_{\mu\nu} \;=\; -\left[\frac{\not{p} + m}{p^2 - m^2}\hat{P}^{3/2}_{\mu\nu} + \frac{2}{m^2}(\not{p} + m)(\hat{P}^{1/2}_{11})_{\mu\nu} + \frac{\sqrt{3}}{m}(\hat{P}^{1/2}_{12} + \hat{P}^{1/2}_{21})_{\mu\nu}\right], \tag{4}$$

this last being the RS propagator and P^k_{ij} the projectors on the $k = 3/2, 1/2$ sector of the representation space and with $i, j = 1, 2$ indicating sub-sectors of the $1/2$ subspace [8]. At difference of the on-shell case where the subsidiary conditions select only the $3/2$ states, when Δ is off-shell ($p^2 \neq m^2$) the $1/2$ ones appear. One can wonder if this situation is exclusive of the RS field. And the answer is no, for example the W propagator appearing in $\pi \to W \to \overline{\nu}\mu$ or pion-pole terms in $\nu N \to \mu N'\pi$ reactions (see Figure 2)

$$\Delta^{\mu\nu}(p) = -\left[\frac{P^{\mu\nu}_1(p)}{p^2 - m^2} + \frac{P^{\mu\nu}_0(p)}{m^2}\right], \tag{5}$$

has P_0 and P_1 projectors on the $S = 0$ and $S = 1$ sectors, respectively. Again here we have an off-shell lower spin contribution as for RS case.
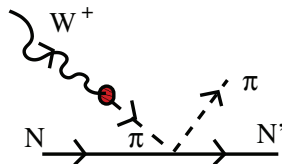

Figure 2. *W* meson propagation between the lepton weak vertex and a conversion to a pion in a pion production reaction.

3. Interactions

$\mathcal{L}_{int}$ must be invariant under contact transformations and lead to A-independent amplitudes. We analyze the strong $\pi(\phi)N(\Psi)\Delta(\psi_\mu)$ vertex, as appears in the amplitude of Figure 3, but the analysis can be extended to another type of interactions. We will analyze two different models for the $\pi N\Delta$ vertex, the "conventional" (C) and the "3/2-gauge invariant" (G) one. The C vertex is inspired in the nonlinear realization of the chiral symmetry[9], which leads to a derivative of the pion field

$$\mathcal{L}_{int_C} = \frac{f_{\pi N\Delta}}{m_\pi}\bar{\Psi}\partial^\mu\phi R\left(\frac{1}{2}(1 + 3A)\right)_{\mu\nu}\psi^\nu + \text{h.c.}, \tag{6}$$

and as this vertex couples to the spin-1/2 off-shell Δ sector and one must pursue the constraints analysis to generate new subsidiary conditions, this fact is viewed as a dof counting inconsistence.
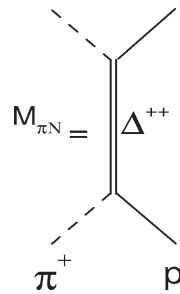
Figure 3. Δ-pole contribution to $\pi^+ p$ elastic scattering.

Nevertheless, the $W \leftrightarrow \pi$ vertex also couples to the spin-0 sector, more it is impossible for a pion to decay without coupling to the off-shell spin-0 piece of the W propagator. The G vertex is inspired by the fact that in the massless case $\mathcal{L}_{free}$ is invariant under the spin-3/2 gauge transformation[10] $\psi_\mu(x) \to \psi_\mu(x) + R^{-1}(\frac{1}{2}(1+3A))_{\mu\nu} R(-1)^{\nu\mu'} \partial_{\mu'} \chi(x)$, and one looks for a Lagrangian respecting the same symmetry[1]

$$\mathcal{L}_{int_G}(A) = \frac{f_{\pi N\Delta}}{m_\pi m} \bar{\Psi} \partial_\mu \phi^\dagger \epsilon^{\mu\nu\alpha\beta} \gamma_\beta \gamma_5 R(-\frac{1}{2}(A+1))_{\nu\sigma} \partial_\alpha \psi^\sigma + \text{h.c.} . \tag{7}$$

This vertex contracted with the Δ momentum is zero, which in turn nullifies the contraction with $P_{ij}^{\frac{1}{2}\mu\nu}$ avoiding the coupling with the spin-1/2 sector in (4)(this is easy to check for $A = -1$). Nevertheless, when we couple a photon making the minimal substitution $\partial_\mu \to \partial_\mu - iq_\Delta A_\mu$, the obtained electromagnetic Lagrangians are no more spin-3/2 gauge invariant appearing terms $q_\Delta \partial_\mu \bar{\chi} \epsilon^{\mu\nu\alpha\beta} A_\alpha \gamma_\beta \gamma_5 \psi_\nu + ...$, this indicating that both gauge symmetries cannot coexist.

4. $\pi^+ p$ cross section calculation

Viewing the evident complexity of the problem and since the C vertex constraints problem is not present in perturbative calculations, leaving for a moment the G vertex 3/2-electromagnetic gauge problem, we simply compare C and G couplings in the case of πN elastic scattering within an isobar model. The different contributions considered to built the amplitude are depicted in Figure 4. We put attention in the Δ-pole amplitude contribution (the Δ-cross one can be
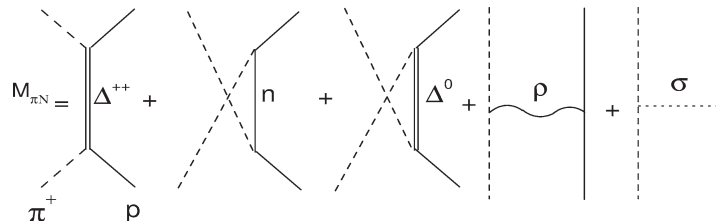

Figure 4. Feynman graphs corresponding to different contributions to the elastic $\pi^+ p$ scattering amplitude.

obtained on the same footing) since the other ones can be constructed as usually from the

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[1] Now the corresponding propagator should be $G^{\mu\nu}(p,A) = R^{-1}\left(-\frac{1}{2}(A+1)\right)_\alpha^\mu G^{\alpha\beta}(p,-1) R^{-1}\left(-\frac{1}{2}(A+1)\right)_\beta^\nu$.

corresponding hadronic Lagrangians [11], we have

$$
\mathcal{M}^C_{\Delta^{++}} = \frac{f^2_{\pi N\Delta}}{m^2_\pi}\overline{u}(p'_p,m'_s)p^\mu_\pi G_{\mu\nu}(p,-\frac{1}{3})p^\nu_\pi u(p_p,m_s),
$$

$$
\mathcal{M}^G_{\Delta^{++}} = \frac{f^2_{\pi N\Delta}p^2}{m^2_\pi m^2}\overline{u}(p'_p,m'_s)p^\mu_\pi (-)\frac{\not{p}+m}{p^2-m^2}\hat{P}^{\frac{3}{2}}_{\mu\nu}p^\nu_\pi u(p_p,m_s).
$$

(8)

To overcome the singular behavior of the resonant amplitude at $p^2 = m^2$ we use the complex mass scheme (see Ref.[11]) making the replacement $m^2 \to m^2 - im\Gamma$. The Δ mass, width and coupling constants $g_\sigma = g_{\sigma\pi\pi}g_{\sigma NN}$ and $f_{\Delta N\pi}$ are left as the only free parameters to be determined from the fitting of the total cross section of $\pi^+ p$ scattering to the data [12]. The
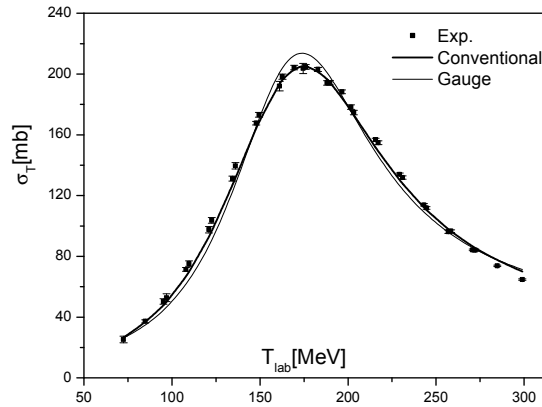

Figure 5. Elastic $\pi^+ p$ total cross section calculated with the amplitude with $\mathcal{M}^C_{\Delta^{++}}$ and $\mathcal{M}^G_{\Delta^{++}}$.

mass of the hypothetical σ meson that runs on the range $400 - 1000$ MeV region [13] is strongly correlated with the g_σ value (see Ref.[11]). We have moved m_σ on the full mentioned range and have looked for the value of g_σ to get the best fitting for each adopted m_σ, comparing at the same time fittings done with different m_σ. We find that for $m_\sigma = 450$ MeV or $m_\sigma = 650$ MeV, depending on the model for $\mathcal{M}_{\Delta^{++}}$, we get in each case the best fitting and we are sure that for these values we have the better description with each coupling. For the C couplings the best fitting corresponds to $m_\sigma = 650$ MeV, and we get : $f^2_{\Delta N\pi}/4\pi = 0.317\pm0.003$, $m_\Delta = 1211.2\pm0.4$ MeV, $\Gamma = 88.2 \pm 0.4$ MeV, $g_\sigma/4\pi = 1.50 \pm 0.12$, and $\chi^2/dof = 4.5$, while for the G coupling we use $m_\sigma = 450$ MeV, and get 0.278 ± 0.002, 1211.6 ± 0.3 MeV, 76.62 ± 0.25 MeV, 1.00 ± 0.05 and 13.5, respectively. Results are shown in Figure 5. With these obtained parameters we also show the predicted differential cross section at two fixed T_{lab} energy values. Our results are compared with available data for both C and G couplings in the Figure 6. Note that following the philosophy of effective Lagrangian models we have not introduced *ad-hoc* form factors for the vertices, since the description of low-energy hadron interactions must incorporate only the (structureless) relevant degrees of freedom. In addition, our model does not consider the final state rescattering between the pion and the nucleon. These last two effects surely improve the fitting, but more free parameters should be considered increasing the model-dependency of the calculation and obscuring the C-G comparison.
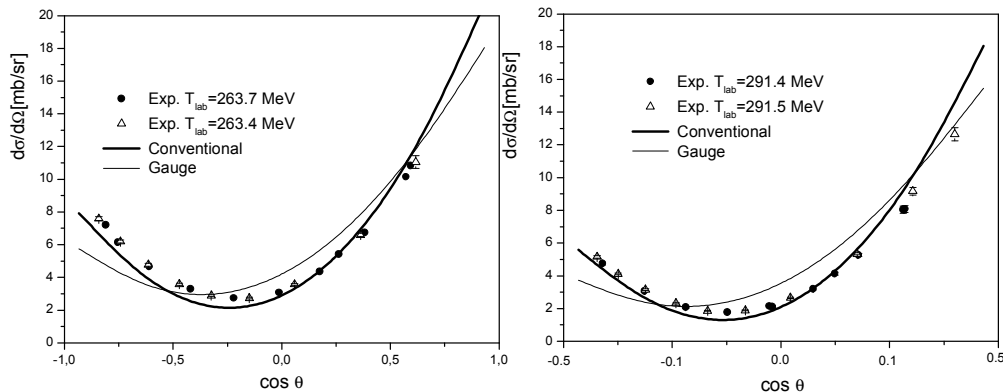
Figure 6. Differential $\pi^+ p$ cross section calculated with the conventional (C) and gauge (G) amplitudes for $T_{lab} = 263.7, 291.4$ MeV. Circles and triangles indicate experimental data from Refs. [14] and [15], respectively.

5. Conclusions

The so called "inconsistent" off-shell propagation of the 1/2 lower spin components of the Δ field is clearly present in other cases, as it is for example the W boson induced reactions. Within a simple isobar model including resonant and background terms, the fitting achieved with the C couplings are clearly better than those obtained with G ones. Seems not possible accommodate the parameters of the σ meson (those of ρ are fixed in both approaches by low energy phenomenology) to get identical results with both types of couplings. Giving the problems with the constraint analysis within the C couplings and those of coexistence between the spin-3/2 and electromagnetic gauge invariance within the G ones, one can appreciate the complexity of the problem and that is not closed.

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