Selection rules in the $\beta\beta$ decay of deformed nuclei

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Abstract

The 2ν $\beta\beta$ decay half-lives of six nuclei, whose decays were previously reported as theoretically forbidden, are calculated by including the pairing interaction, which mixes different occupations and opens up the possibility of the decay. All allowed channels for the 0ν $\beta\beta$ decay are also computed. The estimated 2ν $\beta\beta$ half-lives suggest that measurements in ^{244}Pu may find positive signals, and that planned experiments would succeed in detecting the $\beta\beta_{2\nu}$ decay in ^{160}Gd. Limits for the zero neutrino mode, in the analyzed deformed emitters, are predicted.

Neutrinoless double beta decay ($\beta\beta_{0\nu}$), if detected, would offer definitive evidence that the neutrino is a Majorana particle, i.e., that it is its own antiparticle [1,2]. It would also provide the information needed to determine neutrino masses, complementary to the one obtained from solar and atmospheric neutrino experiments [3,4].

Theoretical nuclear matrix elements are needed to convert experimental half-life limits, which are available for many $\beta\beta$-unstable isotopes [2,5], into constrains for the effective Majorana mass of the neutrino and the contribution of right-handed currents to the weak interactions. Thus, these matrix elements are essential to understand the underlying physics [6–8]. The two neutrino mode of the double beta decay ($\beta\beta_{2\nu}$) is allowed as a second-order process in the standard model. It has been detected in ten nuclei [5] and it has served to test a variety of nuclear models [2].

The pseudo-SU(3) approach has been used to describe many low-lying rotational bands, as well as B(E2) and B(M1) intensities, in rare earth and actinide nuclei, both with even–even and odd-mass numbers. The theoretical results show, in general, a very good agreement with the data [9]. The $\beta\beta$ half-lives of some heavy deformed nuclei, which may decay to the ground and excited states of the daughters, were evaluated for the two and zero neutrino emitting modes [10–14] using the pseudo-SU(3) scheme. The predictions were in good agreement with the available experimental data for ^{150}Nd and ^{238}U. The double electron capture decay channel was studied for the decay of other three nuclei [15].

The simplest pseudo-SU(3) model predicts the complete suppression of the $\beta\beta_{2\nu}$ decay for the following five nuclei: ^{154}Sm, ^{160}Gd, ^{176}Yb, ^{232}Th and ^{244}Pu [10,14]. Recently, it was argued that the cancellation

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Table 1
Deformations, proton and neutron occupation numbers, pairing mixing h_{pair} and excitation ΔE energies (both in MeV) and mixing coefficients x_i

Nucleus	ϵ_2	n_p^N	n_p^A	n_n^N	n_n^A	ΔE	h_{pair}	x_i
^{154}Sm (1)	0.250	8	4	6	4			0.933
^{154}Sm (2)		8	4	8	2	1.27	0.575	0.359
^{154}Gd	0.225	8	6	6	2			1.000
^{160}Gd	0.258	8	6	8	6			1.000
^{160}Dy (1)	0.250	10	6	6	6			0.923
^{160}Dy (2)		8	8	6	6	1.71	0.865	0.385
^{170}Er (1)	0.267	10	8	12	8			0.934
^{170}Er (2)		12	6	12	8	1.24	0.554	0.356
^{170}Yb	0.267	12	8	10	8			1.000
^{176}Yb	0.250	12	8	16	8			1.000
^{176}Hf (1)	0.250	14	8	14	8			0.943
^{176}Hf (2)		12	10	14	8	1.23	0.493	0.332
^{232}Th (1)	0.192	4	4	10	6			0.829
^{232}Th (2)		6	2	10	6	0.318	0.391	0.559
^{232}U	0.192	6	4	8	6			1.000
^{244}Pu (1)	0.208	6	6	16	8			0.740
^{244}Pu (2)		8	4	16	8	0.080	0.419	0.673
^{244}Cm	0.217	8	6	14	8			1.000

of the $\beta\beta_{2\nu}$ decay in ^{160}Gd would suppress the background for the detection of the 0ν mode [16].

In the present contribution we extend the previous research [10–14,17] and evaluate the $\beta\beta$ half-lives of ^{154}Sm, ^{160}Gd, ^{170}Er, ^{176}Yb, ^{232}Th and ^{244}Pu using the pseudo-SU(3) model. In these nuclei the $2\nu\beta\beta$ mode is forbidden when the most probable occupations are considered. To be able to evaluate finite halflives, we were forced to include in the calculations states with different occupation numbers which can be mixed through the pairing interaction. The amount of this mixing is evaluated, and the possibility of observing the $\beta\beta$ decay is discussed for both the 2ν and 0ν modes.

Pseudo-spin symmetry [18] describes the quasidegeneracy of the single-particle orbitals with $j = l - 1/2$ and $j = (l - 2) + 1/2$ in the η shell. It allows these orbitals to be classified as pseudo-spin partners with quantum numbers $\tilde{j} = j$, $\tilde{\eta} = \eta - 1$ and $\tilde{l} = l - 1$. The first step in the pseudo-SU(3) description of any nuclei is to find the occupation numbers for protons (p) and neutrons (n) in the normal and abnormal parity states n_p^N, n_n^N, n_p^A, n_n^A. These numbers are determined by filling the Nilsson levels from below, as discussed in [10]. The deformations [19] and occupancies for the

12 isotopes studied in the present work are shown in Table 1.

In the $\beta\beta_{2\nu}$ decay each Gamow–Teller operator annihilates a proton and creates a neutron in the same oscillator shell and with the same orbital angular momentum. As a consequence, the $\beta\beta_{2\nu}$ decay is allowed only if the occupation numbers fulfill the following relationships

$$n_{p,f}^A = n_{p,i}^A + 2, \qquad n_{n,f}^A = n_{n,i}^A,$$
$$n_{p,f}^N = n_{p,i}^N, \qquad n_{n,f}^N = n_{n,i}^N - 2. \qquad (1)$$

This selection rule [10] forbids the $\beta\beta_{2\nu}$ decay between the nuclei marked with (1) or without comments in Table 1. However, the pairing interaction allows the mixing between states in the same nuclei with pairs of nucleons transferred between different configurations. These excited configurations are indicated by (2) in Table 1. An energy difference ΔE is required to promote a pair of nucleons from the last occupied normal parity orbital to the next intruder orbital (or vice versa), in the deformed single particle Nilsson scheme. It is listed in the seventh column of Table 1.

The "leading SU(3) irreps", those which are the most bounded under the quadrupole–quadrupole inter-

Table 2
The proton, neutron, and total irreps assigned to each nucleus

Nucleus	(λ_p, μ_p)	(λ_n, μ_n)	(λ, μ)
^{154}Sm (1)	(10, 4)	(18, 0)	(28, 4)
^{154}Sm (2)	(10, 4)	(18, 4)	(28, 8)
^{154}Gd	(10, 4)	(18, 0)	(28, 4)
^{160}Gd	(10, 4)	(18, 4)	(28, 8)
^{160}Dy (1)	(10, 4)	(18, 0)	(28, 4)
^{160}Dy (2)	(10, 4)	(18, 0)	(28, 4)
^{170}Er (1)	(10, 4)	(24, 0)	(34, 4)
^{170}Er (2)	(4, 10)	(24, 0)	(28, 10)
^{170}Yb	(4, 10)	(20, 4)	(24, 14)
^{176}Yb	(4, 10)	(18, 8)	(22, 18)
^{176}Hf (1)	(0, 12)	(20, 6)	(20, 18)
^{176}Hf (2)	(4, 10)	(20, 6)	(24, 16)
^{232}Th (1)	(12, 2)	(30, 4)	(42, 6)
^{232}Th (2)	(18, 0)	(30, 4)	(48, 4)
^{232}U	(18, 0)	(26, 4)	(44, 4)
^{244}Pu (1)	(18, 0)	(34, 8)	(52, 8)
^{244}Pu (2)	(18, 4)	(34, 8)	(52, 12)
^{244}Cm	(18, 4)	(34, 6)	(52, 10)

action, are the dominant component of the ground state wave function in these heavy deformed nuclei, representing in most cases more than 60% of the total wave function [9]. For the eight rare earth isotopes listed in Table 1, this dominant wave function component can be written as

$$|\text{Nucleus}, 0^+\rangle$$
$$= \left|(h_{11/2})^{n_p^A}, J_p^A = 0; (i_{13/2})^{n_n^A}, J_n^A = 0\right\rangle_A$$
$$+ \left|\{2^{n_p^N/2}\}(\lambda_p, \mu_p); \{2^{n_n^N/2}\}(\lambda_n, \mu_n);\right.$$
$$\left.1(\lambda, \mu)K = 1, J = 0\right\rangle_N. \quad (2)$$

For the actinide isotopes, the intruder sector is $|(i_{13/2})^{n_p^A}, J_p^A = 0; (j_{15/2})^{n_n^A}, J_n^A = 0\rangle$.

The proton, neutron, and total SU(3) irreps associated to each set of occupation numbers are listed in Table 2.

As a first approximation, we will describe the ground state of each nucleus as a linear combination of these two states:

$$|\text{Nucleus}, 0^+\rangle = x_1|\text{Nucleus}(1), 0^+\rangle$$
$$+ x_2|\text{Nucleus}(2), 0^+\rangle,$$

with $x_1^2 + x_2^2 = 1$.

Many multipole–multipole pairing type interactions can remove a pair of nucleons from an unique parity orbital and create another pair in a normal parity one. In the present approach we are restricting the pairs of nucleons in intruder orbits to be coupled to $J = 0$, i.e., to have seniority zero. Under this approximation the only term in the Hamiltonian which can connect states with different occupation numbers in the normal and unique parity sectors is pairing. In the present case, the Hamiltonian matrix has the simple form

$$H = \begin{pmatrix} 0 & h_{\text{pair}} \\ h_{\text{pair}} & \Delta E \end{pmatrix}, \quad (3)$$

with $h_{\text{pair}} = \langle\text{Nucleus}(2), 0^+|H_{\text{pair}}|\text{Nucleus}(1), 0^+\rangle$, whose explicit expression is given in [17]. The values of h_{pair}, x_1 and x_2 are presented in the last two columns of Table 1. In the case of ^{244}Pu we are using a small deformation [19], for which the two configurations listed in Table 1 are nearly degenerate, and have maximal mixing. It has important consequences upon its predicted $\beta\beta$ half-life.

The inverse half-life of the two neutrino mode of the $\beta\beta$ decay, $\beta\beta_{2\nu}$, can be evaluated as [20]

$$\left[\tau_{2\nu}^{1/2}(0^+ \to 0^+)\right]^{-1} = G_{2\nu}|M_{2\nu}|^2, \quad (4)$$

where $G_{2\nu}$ is a kinematical factor which depends on $Q_{\beta\beta}$, the total kinetic energy available in the decay.

The nuclear matrix element is

$$M_{2\nu} \approx M_{2\nu}^{\text{GT}} = \sum_N \frac{\langle 0_f^+\|\Gamma\|1_N^+\rangle\langle 1_N^+\|\Gamma\|0_i^+\rangle}{E_f + E_N - E_i}, \quad (5)$$

being Γ the Gamow–Teller operator. The energy denominator contains the intermediate E_N, initial E_i and final E_f energies. The kets $|1_N^+\rangle$ denote intermediate states.

The mathematical expressions needed to evaluate the nuclear matrix elements of the allowed g.s. → g.s. $\beta\beta$ decay in the pseudo-SU(3) model were developed in [10]. Using the summation method described in [10], exploiting the fact that the two body terms of the $\widetilde{SU}(3)$ Hamiltonian commutes with the Gamow–Teller operator [11], resuming the infinite series and recoupling the Gamow–Teller operators, the following

expression was found:

$$M_{2\nu}^{GT} = \frac{\sigma(p,n)^2}{\mathcal{E}}$$
$$\times \langle 0_f^+ | [[a_p^\dagger \otimes \tilde{a}_n]^1 \otimes [a_p^\dagger \otimes \tilde{a}_n]^1]^{J=0} | 0_i^+ \rangle, (6)$$

where $\sigma(p,n)$ are the 1-body Gamow–Teller matrix elements and the energy denominator $\mathcal{E}$ is determined by demanding that the excitation energy of the Isobaric Analog State equals the difference in Coulomb energies [10,17]. The index p, n refer to the orbitals $i_{11/2}^p$, $i_{13/2}^n$ for ^{154}Sm, ^{160}Gd, ^{170}Er, and ^{176}Yb, and to the orbitals $j_{13/2}^p$, $j_{15/2}^n$ for ^{232}Th and ^{244}Pu.

As it was discussed in [10] Eq. (6) has no free parameters, being the denominator $\mathcal{E}$ a well defined quantity. The reduction to only one term comes as a consequence of the restricted proton and neutron spaces of the model. The initial and final ground states are strongly correlated with a very rich structure in terms of their shell model components. The evaluation of the matrix elements in the normal space of Eq. (6) is performed by using SU(3) Racah calculus to decouple the proton and neutron normal irreps, and expanding the annihilation operators in their SU(3) tensorial components.

For the six potential $\beta\beta$ emitters listed in Tables 1 and 2, the $\beta\beta_{2\nu}$ decay can only proceed through the second component of the ground state wave function, and for this reason it is proportional to the amplitude x_2. Its explicit expression is given in [17].

For massive Majorana neutrinos one can perform the integration over the four-momentum of the exchanged particle and obtain a neutrino potential, which for a light neutrino ($m_\nu < 10$ MeV) has the form

$$H(r, \overline{E}) = \frac{2R}{pr} \int_0^\infty dq \frac{\sin(qr)}{q + \overline{E}}, (7)$$

where $\overline{E}$ is the average excitation energy of the intermediate odd–odd nucleus and the nuclear radius R has been added to make the neutrino potential dimensionless. In the zero neutrino case this closure approximation is well justified [21]. The final formula, restricted to the term proportional to the neutrino mass, is [20,22]

$$\left(\tau_{0\nu}^{1/2}\right)^{-1} = \left(\frac{\langle m_\nu \rangle}{m_e}\right)^2 G_{0\nu} M_{0\nu}^2, (8)$$

where $G_{0\nu}$ is the phase space integral associated with the emission of the two electrons. The nuclear matrix elements $M_{0\nu}$ are [20]

$$M_{0\nu} \equiv \left| M_{0\nu}^{GT} - \frac{g_V^2}{g_A^2} M_{0\nu}^F \right|,$$
$$\text{with} \quad M_{0\nu}^\alpha = \langle 0_f^+ \| O^\alpha \| 0_i^+ \rangle, (9)$$

where the kets $|0_i^+\rangle$ and $|0_f^+\rangle$ denote the corresponding initial and final nuclear states, the quantities g_V and g_A are the dimensionless coupling constants of the vector and axial vector nuclear currents, and

$$O^{GT} = \sum_{m,n} \vec{\sigma}_m t_m^- \cdot \vec{\sigma}_n t_n^- H(|\vec{r}_m - \vec{r}_n|, \overline{E}),$$
$$O^F = \sum_{m,n} t_m^- t_n^- H(|\vec{r}_m - \vec{r}_n|, \overline{E}), (10)$$

being $\vec{\sigma}$ the Pauli matrices related with the spin operator and t^- the isospin lowering operator, which satisfies $t^- |n\rangle = |p\rangle$. The superscript GT denotes the Gamow–Teller spin–isospin transfer channel, while F indicates the Fermi isospin one. In the present work we use the effective value $(g_A/g_V)^2 = 1.0$ [6].

Transforming this operator to the pseudo SU(3) space, we arrive to the expression

$$O^\alpha = O_{N_p N_n}^\alpha + O_{N_p A_n}^\alpha + O_{A_p N_n}^\alpha + O_{A_p A_n}^\alpha, (11)$$

where the subscript index NN, NA, ... are indicating the normal or abnormal spaces of the fermion creation and annihilation operators, respectively.

In previous works [10,11] we restricted our analysis to six potential double beta emitters which, within the approximations of the simplest pseudo SU(3) scheme, were also decaying via the 2ν mode. They included the observed ^{150}Nd $\rightarrow$ ^{150}Sm and ^{238}U $\rightarrow$ ^{238}Pu decays. In these cases two neutrons belonging to a normal parity orbital decay in two protons belonging to an abnormal parity one. The transition is mediated by the operator $O_{A_p N_n}^\alpha$.

In Table 3 the $\beta\beta_{2\nu}$ decays of the six nuclei, previously reported as forbidden, are presented. Those with the larger $Q_{\beta\beta}$ values have the larger phase-space integrals $G_{2\nu}$. The $\beta\beta_{2\nu}$-decay matrix elements $M_{2\nu}^{GT}$ are suppressed by a factor $x_2 \approx 1/3$ compared with the "allowed" ones (see [10]), which reflects in $\beta\beta_{2\nu}$ half-lives a factor 10 larger than in other nuclei with similar Q-values. The exception is ^{244}Pu, which

Table 3

The Q-values, phase-space integrals, matrix elements and predicted half-lives for the $\beta\beta_{2\nu}$ beta decay

Nuclei	$Q_{\beta\beta}$ [MeV]	$G_{2\nu}$ [MeV^2 yr^{-1}]	$M_{2\nu}^{GT}$ [MeV^{-1}]	$\tau_{2\nu}^{1/2}$ [yr]
^{154}Sm	1.251	4.872×10^{-21}	0.0445	1.04×10^{23}
^{160}Gd	1.730	8.028×10^{-20}	0.0455	6.02×10^{21}
^{170}Er	0.654	6.496×10^{-23}	0.0374	1.10×10^{25}
^{176}Yb	1.086	3.866×10^{-21}	0.0306	2.77×10^{23}
^{232}Th	0.858	7.410×10^{-21}	0.0504	5.30×10^{22}
^{244}Pu	1.352	4.081×10^{-19}	0.0617	6.43×10^{20}

Table 4

The phase-space integrals, matrix elements and predicted half-lives for the 0ν double beta decay

Nucleus	$G_{0\nu}$ [yr^{-1}]	$M_{0\nu}$	$\tau_{0\nu}^{1/2} \langle m_\nu \rangle^2$ [yr eV^2]
^{154}Sm	4.898×10^{-15}	2.384	9.38×10^{24}
^{160}Gd	1.480×10^{-14}	0.919	2.09×10^{25}
^{170}Er	1.673×10^{-15}	0.731	2.92×10^{26}
^{176}Yb	6.817×10^{-15}	0.772	6.43×10^{25}
^{232}Th	3.160×10^{-14}	1.232	5.44×10^{24}
^{244}Pu	1.463×10^{-13}	1.171	1.30×10^{24}

for the deformation used has a large mixing, and the shorter $\beta\beta_{2\nu}$ half-life, which is not far from the limits reported in the Livermore experiment [23]. The decay of ^{160}Gd is suppressed but it is still not far from the present limits [24], and large enough to be seen in the proposed experiments [16].

Those configurations in which the $\beta\beta_{2\nu}$ transitions are forbidden can still be connected through the zero neutrino mode, due to presence of the neutrino potential. In this way there are two terms in the $\beta\beta_{0\nu}$ decay: one connecting to the basis state which has allowed $\beta\beta_{2\nu}$ decay, and one to the state with forbidden $\beta\beta_{2\nu}$ decay. The equations needed in the first case are the same employed in the study of allowed decays [10,11]. The second one involves the annihilation of two neutrons in normal parity orbitals, and the creation of two protons in normal parity orbitals (intruder–intruder in ^{154}Sm). The transition is mediated by the operator $O_{N_p N_n}^{\alpha}$ ($O_{A_p A_n}^{\alpha}$). A detailed description of the calculations involved is presented in [17]. $\beta\beta_{0\nu}$ phase-space integrals, nuclear matrix elements and half-lives are shown in Table 4.

As a consequence of the explicit inclusion of deformation in the present model, the $\beta\beta_{0\nu}$ half-lives are larger than those reported in [25]. In ^{160}Gd the $\beta\beta_{0\nu}$-decay half-life is at least three orders of

magnitude larger than the $\beta\beta_{2\nu}$-decay half-life. It implies that the background suppression due to a large $\beta\beta_{2\nu}$ half-life would be effective, although not as noticeably as was optimistically envisioned in [16]. In any case, the results presented strongly suggest that the planned experiments using GSO crystals [16] would be able to detect the $\beta\beta_{2\nu}$ decay of ^{160}Gd, and to establish competitive limits to the $\beta\beta_{0\nu}$ decay.

The present results consider only the dominant pseudo SU(3) irrep for each configuration. We have learned from realistic calculations, where the single particle term and pairing interactions induce the mixing of different irreps, that the leading irreps represent in most even–even heavy deformed nuclei at least 60% of the total wave function [9]. The inclusion of spin dependent terms in the Hamiltonian, relevant to the description of the Gamow–Teller resonance, is not expected to strongly modify the ground state wave function of the even–even initial and final nuclei. This dominance lead us to expect that future calculations, which will take into account contributions from various irreps, would slightly affect the present predictions. Given the leading role play by the quadrupole–quadrupole interaction in heavy deformed nucleus, we are confident that the order of magnitude of the predicted $\beta\beta$ half-lives, when various irreps are included in the calculations, will remain unchanged, as compared to the results reported above.

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