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Calculation of the masses of the binary star HD 93205 by application of the theory of apsidal motion

O. G. Benvenuto,★† A. M. Serenelli,★‡ L. G. Althaus,§ R. H. Barbá§ and N. I. Morrell§
Facultad de Ciencias Astronómicas y Geofísicas, Universidad Nacional de La Plata, Paseo del Bosque S/N, (1900) La Plata, Argentina

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ABSTRACT

We present a method to calculate masses for components of both eclipsing and non-eclipsing binary systems as long as their apsidal motion rates are available. The method is based on the fact that the equation that gives the rate of apsidal motion is a supplementary equation that allows the computation of the masses of the components, if their radii and the internal structure constants can be obtained from theoretical models. For this reason the use of this equation makes the method presented here *model dependent*.

We apply this method to calculate the mass of the components of the *non-eclipsing* massive binary system HD 93205 (O3 V + O8 V), which is suspected to be a very young system. To this end, we have computed a grid of evolutionary models covering the mass range of interest, and taking the mass of the primary (M_1) as the only independent variable, we solve the equation of apsidal motion for M_1 as a function of the age of the system. The mass of the primary that we find ranges from $M_1 = 60 \pm 19\,\mathrm{M}_\odot$ for zero-age main-sequence models, which sets an upper limit for M_1, down to $M_1 = 40 \pm 9\,\mathrm{M}_\odot$ for an age of 2 Myr. Accordingly, the upper limit derived for the mass of the secondary ($M_2 = QM_1$) $M_2 = 25\,\mathrm{M}_\odot$ is in very good agreement with the masses derived for other O8 V stars occurring in eclipsing binaries.

Key words: binaries: eclipsing – stars: early-type – stars: evolution – stars: fundamental parameters – stars: individual: HD 93205 – stars: interiors.

1 INTRODUCTION

The motion of the apsis of a binary is mainly a direct consequence of the finite size of its components. If both stars were spherical objects and general relativity corrections were negligible, they would move on a Keplerian, fixed orbit. However, the presence of the companion object, and also its rotation, makes the structure of each star depart from a sphere. In such a situation, there appears a finite quadrupolar (and higher) momentum to the gravitational field of each object that forces the orbit to modify the position of the apsis. This is an effect well known from a long time ago (Cowling 1938; Sterne 1939). The rate of motion of the apsis is dependent on the internal structure of each component; thus, if we are able to determine the main characteristics of a binary, it provides an observational test of the theory of stellar structure and evolution (Schwarzschild 1958; Kopal 1959).

In spite of the age of the idea, apsidal motion of binary systems has been studied systematically only recently in a series of papers by Claret & Giménez (1993) and Claret (1995, 1997, 1998, 1999). Perhaps, one of the main reasons for such a situation is that the rate of motion of the apsis is very dependent on the stellar structure. Thus, the apsidal motion test has been useful only recently because of the availability of accurate stellar evolutionary models such as those of Claret (1995). These authors have performed detailed stellar models computing the coefficients that determine the rate of motion of the apsis and applied them to compare with observational data from eclipsing binaries.

The detection of apsidal motion in non-eclipsing binary systems is an elusive subject. It has to be determined through the time variation of the shape of the radial velocity curve caused by change in the longitude of periastron. Generally, the observed radial velocities in binary systems have large uncertainties that mask this effect in many cases. Moreover, the fact that times of light minima are usually determined with high precision for eclipsing binaries, acts as a selection effect in favour of detecting the motion of the line of apsides in such systems: 100 cycles is usually enough to note the change in relative position of the secondary minimum with respect to the primary minimum. Observations over much longer periods of time are needed to find evidence of apsidal motion in systems where eclipses are not seen and the only observable effect is the change in shape of the radial velocity orbit.

Empirical determinations of masses are scarce for early O-type

★E-mail: obenvenuto@fcaglp.fcaglp.unlp.edu.ar (OGB); serenell@fcaglp.fcaglp.unlp.edu.ar (AMS)
†Member of the Carrera del Investigador Científico, CICBA, Argentina.
‡Fellow of CONICET, Argentina.
§Member of Carrera del Investigador Científico, CONICET, Argentina.

stars (e.g. Burkholder et al. 1997; Schönberner & Harmanec 1995). Few early O-type stars are known to be members of double-lined binaries, and from them, those showing eclipses or some kind of light variations that enable the estimate of the orbital inclination (and then, absolute masses), are rare. The so-called 'mass discrepancy', first described by Herrero et al. (1992) and recently reviewed (Herrero, Puls & Villamariz 2000), relates to the difference between the masses derived via numerical evolutionary models and those obtained from spectral analysis (plus model atmospheres) or binary star studies. This discrepancy, amounting to 50 per cent in 1992, has been partially solved with the use of new evolutionary models that consider the effect of stellar rotation (Meynet & Maeder 2000) and new model atmospheres. However, large differences between 'predicted' and 'observed' masses are still present for the hottest and youngest (non-evolved) stars. A recent study of the massive double-lined O-type binary system HD 93205 (Morrell et al. 2001) yields minimum masses of 31.5 ± 1.1 and $13.3 \pm 1.1\,M_\odot$ for the O3 V primary and O8 V secondary components, respectively. This leads to a probable mass of $\sim$52–60 $M_\odot$ for the O3 V star, if a mass value according to those derived for other O8 V stars in eclipsing binaries is assumed for the secondary O8 V. This is much less than the 80–100 $M_\odot$ predicted from the position of this star on a theoretical Hertzsprung–Russell diagram (HRD) compared with stellar evolutionary tracks (see fig. 7 in Morrell et al. 2001).

Notably, HD 93205 is the first early O-type non-eclipsing[1] massive binary for which the rate of motion of the apsis has been determined with some accuracy. Morrell et al. (2001) derived an apsidal motion period of 185 ± 16 yr, which considering the orbital period of 6.0803 ± 0.0004 d, yields an apsidal motion rate $\dot{\varpi} = 0\overset{\circ}{.}0324 \pm 0\overset{\circ}{.}0031$ per orbital cycle. This is very interesting because, from a mathematical point of view, the rate of motion of the apsis provides another equation to be applied to the system apart from the standard ones. HD 93205 is an early-type massive short-period binary in a highly eccentric orbit ($e = 0.370 \pm 0.005$; Morrell et al. 2001), lying in the Carina Nebula, a galactic massive star-forming region. Thus, we can assume that its components are on, or very close to, the zero-age main sequence (ZAMS). Consequently, if we consider the evolutionary stage of HD 93205 to be known, the equation of apsidal motion can be written in a way that we can solve for the mass of the primary star. The aim of the present paper is to detail this method and to apply it to HD 93205.

The paper is organized as follows. In Section 2 we present the method we use to obtain the masses for components of non-eclipsing binary systems and we describe our evolutionary code and the calculations we have carried out. In Section 3 we present a test of our method by applying it to some eclipsing binary systems. Section 4 is devoted to showing the results obtained for the massive binary system HD 93205 and finally, in Section 5 we give some concluding remarks concerning the implications of our results.

2 COMPUTATIONAL DETAILS

Here we present an original method (as far as the authors are aware) of calculating the masses of the components of binary systems provided knowledge of the rate of motion of the apsis and the evolutionary status of the stars. In addition, we describe the main characteristics of our evolutionary code and the calculation of the internal structure constants (ISCs) on which the rate of the apsidal motion depends.

2.1 Equations of apsidal motion and description of the method

Sterne (1939) has shown that if the classical gravitational potential of each component of a binary system is expanded in a series of spherical harmonics, and terms up to the quadrupolar contribution are kept, then the rate of motion of the apsis is given by[2]

$$\frac{\dot{\varpi}_2}{\Omega} = k_{2,1}\left(\frac{a_1}{A}\right)^5\left[15\frac{M_2}{M_1}f_2(e) + \frac{\omega_1^2 A^3}{M_1 G}g_2(e)\right] + k_{2,2}\left(\frac{a_2}{A}\right)^5\left[15\frac{M_1}{M_2}f_2(e) + \frac{\omega_2^2 A^3}{M_2 G}g_2(e)\right], \quad (1)$$

where $\dot{\varpi}_2$ is the rate of secular motion of the apsis calculated considering only the quadrupolar contribution (the lowest order) of the gravitational potential; $k_{i,j}$ are the ISCs that depend on the internal mass distribution of the stars (see below, Subsection 2.2 for more details); i denotes the ith multipolar momentum considered (i.e. $i = 2$ throughout this paper); whereas j denotes the component of the binary system. G is the gravitational constant, Ω is the mean orbital angular velocity, A is the semi-axis of the relative orbit, a_1 and a_2 are the mean radii of the stars, ω_1 and ω_2 are their angular velocities of rotation, M_1 and M_2 are the stellar masses; and finally $f_2(e)$ and $g_2(e)$ are functions of the orbital eccentricity e given by

$$f_2(e) = \left(1 + \tfrac{3}{2}e^2 + \tfrac{1}{8}e^4\right)(1 - e^2)^{-5}, \quad (2)$$

$$g_2(e) = (1 - e^2)^{-2}. \quad (3)$$

In the following we shall reformulate equation (1) in order to write it to become an implicit equation for the mass of the primary star M_1. The semi-axis A is not directly known from observations, however, the projected semi-axis D given by

$$D = A \sin i \quad (4)$$

where $\sin i$ is the sine of the inclination of the orbit, can be assessed observationally. Let us define the mass ratio Q and the angular velocities ratio q_ω as

$$Q = \frac{M_2}{M_1}; \qquad q_\omega = \frac{\omega_2}{\omega_1}. \quad (5)$$

In order to eliminate $\sin i$ we can use the mass function, defined as

$$f = \frac{M_1^3 \sin^3 i}{(M_1 + M_2)^2} = \frac{M_1 \sin^3 i}{(1 + Q)^2}, \quad (6)$$

which can be determined from observations (Batten 1973). In the same trend, projected tangential velocities $v_1 = V_1 \sin i$ and $v_2 = V_2 \sin i$ are also observable quantities and their ratio $q = v_2/v_1$ can be used to eliminate q_ω.

One important point is that rotation modifies the internal structure of the stars. In a recent paper, Claret (1999) has shown that, within the quasi-spherical approximation, rotation can be taken into account in the apsidal motion analysis simply by reducing the ISC $k_{2,i}$ by

$$\log k_{2,i} = \log[k_{2,i}]_{\rm sph} - 0.87\lambda_i. \quad (7)$$

[1] Phase-dependent light variations with full amplitude of $\sim$0.02 mag in visual light were reported by Antokhina et al. (2000). These authors stated that the observed light variations are probably related to tidal distortions rather than eclipses.

[2] It is assumed that rotation of both components is perpendicular to the orbital plane.

Here, $[k_{2,i}]_{\rm sph}$ denotes the ISC obtained from spherical models and the parameter λ_i is defined by

$$\lambda_i = \frac{2V_i^2}{3g_i a_i} \tag{8}$$

where i denotes the component of the binary system and V_i, a_i and g_i are, respectively, the tangential velocity, the radius and the surface gravity of the component.

Up to this point, we have only considered the contributions to the motion of the apsis caused by Newtonian gravity. However, it is well known that general relativity predicts a secular motion of the apsis, which is independent of the classical contributions. The angular velocity of the apsis caused by general relativistic effects $\dot{\varpi}_{\rm GR}$ is given by (Levi-Civita 1937)

$$\frac{\dot{\varpi}_{\rm GR}}{\Omega} = 6.36 \times 10^{-6} \frac{M_1 + M_2}{A(1-e^2)}. \tag{9}$$

Defining $\mathcal{F}_2$ as

$$\mathcal{F}_2 = \frac{\dot{\varpi}}{\Omega} - \left(\frac{\dot{\varpi}_{\rm GR}}{\Omega} + \frac{\dot{\varpi}_2}{\Omega}\right) = 0 \tag{10}$$

and incorporating both, rotation and relativistic effects we obtain after some algebraic manipulation

$$\begin{aligned} \mathcal{F}_2 = \frac{\dot{\varpi}}{\Omega} - \Bigg\{ & 6.36 \times 10^{-6} \frac{[fM_1^2(1+Q)^5]^{1/3}}{D(1-e^2)} \\ & + \frac{15}{D^5} f_2(e) \left[\frac{f(1+Q)^2}{M_1}\right]^{5/3} \left(k_{2,1} a_1^5 Q + k_{2,2} a_2^5 \frac{1}{Q}\right) \\ & + \frac{v_1^2}{M_1 G D^2} g_2(e) \left(k_{2,1} a_1^3 + k_{2,2} a_2^3 \frac{q^2}{Q}\right) \Bigg\} = 0 \end{aligned} \tag{11}$$

where $k_{2,i}$ are the ISCs corrected by the effects of rotation. This is the fundamental equation for our purposes.

As mentioned above, in equation (11) some quantities are determined observationally $(\Omega, \dot{\varpi}, e, D, Q, v_1, f, q)$. On the other hand, $k_{2,1}$ and $k_{2,2}$ must be computed from evolutionary models and, if we are dealing with non-eclipsing systems, as is the case for HD 93205, then the radii a_1 and a_2 must be obtained from theoretical models as well. Now, if we assume that both components of the binary system have the same age, and we use the observational constraint $Q = M_2/M_1$, then $k_{2,1}, k_{2,2}, a_1$ and a_2 can be derived from evolutionary calculations as a function of M_1 and the age of the system.

The method presented here to calculate M_1 is as follows: we compute a grid of evolutionary models covering the range of masses of interest with a small mass step. Using this grid, we construct isochrones starting at the ZAMS with a given time-step and for each isochrone we seek the solution of equation (11). In this procedure, the only independent quantity is M_1, so when the solution of equation (11) is found, the corresponding value of M_1 is the mass of the primary star that corresponds to the age of the isochrone.

Thus, for a given age of the system we have one solution for the mass of the primary star M_1, and using $Q = M_2/M_1$ we can derive the mass of the secondary. However, the age of the system must be constrained by other means. In addition, the masses of the components that can be found with the present method are *model dependent*. Note especially the sensitivity of the tidal and rotational terms in equation (11) to the value of the stellar radius. It is clear that we need accurate stellar models in order to obtain a physically reliable value of M_1.

Finally, it is worth mentioning that the solution of equation (11) is subject to some constraints. Two of them were already mentioned: the age of both components in a binary system must be the same, and $M_2 = QM_1$. In addition, the value of the mass function imposes a minimum value for M_1,

$$[M_1]_{\min} = f(1+Q)^2 \tag{12}$$

and also the condition for the system to be detached

$$a_1 + a_2 < D, \tag{13}$$

must be fulfilled.

2.2 Evolutionary models and calculation of the ISCs

As stated above, in order to solve equation (11), the ISCs and radii of both components must be obtained from evolutionary calculations. This leads us to the necessity of having a set of evolutionary tracks of objects covering the range of masses expected for the components of the system. In the case of HD 93205, the primary O3 V star is a candidate to be one of the most massive stars known. Thus, we have carried out calculations up to quite large stellar masses such as $106\,{\rm M}_\odot$ because, as far as we are aware, there is no computation of the ISCs for such massive stars available in the literature.

The calculations have been carried out with the stellar evolution code developed at La Plata Observatory. It is essentially the same code employed for studying white dwarf stars (see, for example, Benvenuto & Althaus 1998) and intermediate mass stars (Brunini & Benvenuto 1997) and has been adapted for properly handling the case of massive stars.

Let us briefly describe the main ingredients of the code. The equation of state employed is that of OPAL (Rogers, Swenson & Iglesias 1996). Radiative opacities are the latest version of OPAL (Iglesias & Rogers 1996), while for low temperatures they are complemented with the molecular opacities Alexander & Ferguson (1994). Conductive opacities and neutrino emission rates are the same as in Benvenuto & Althaus (1998). Nuclear reaction rates are taken from Caughlan & Fowler (1988) and weak electron screening is taken from Graboske et al. (1973).

As we are dealing with massive stars, it is important to mention that we have accounted for the occurrence of overshooting by employing the formalism described in Maeder & Meynet (1989). We have adopted the distance of overshooting $d_{\rm ov}$ to be a fraction of the pressure scaleheight H_P at the canonical border of the convective zone: $d_{\rm ov} = \alpha_{\rm ov} H_p$. Also, we allowed for mass loss following De Jager, Neiuwenhuijzen & van der Hutch (1986).

Using the evolutionary code just described, we have calculated a set of evolutionary sequences covering the mass range of $4-106\,{\rm M}_\odot$ with a mass step of $\approx$5 per cent. We followed the evolution starting at the ZAMS until the depletion of hydrogen at the centre of the star. The initial helium content of our models is $Y = 0.275$ and the adopted value for metallicity is $Z = 0.02$, while two values for overshooting, $\alpha_{\rm ov} = 0.25$ and 0.40, were considered.

After convergence of each model was reached, we solved the Clairaut–Radau differential equation (Sterne 1939) that accounts for the apsidal motion to the lowest (second) order

$$a\frac{{\rm d}\eta_2}{{\rm d}a} + \eta_2^2 - \eta_2 - 6 + 6\frac{\rho}{\bar{\rho}}(\eta_2 + 1) = 0 \tag{14}$$

Table 1. Astrophysical parameters for selected test systems.

	EM Car		GL Car		QX Car		Y Cyg		V478 Cyg	
P (d)	3.415		2.422		4.478		2.996		2.881	
$\dot{\varpi}$ (deg d^{-1})	0.0237		0.039 10		0.0027		0.0206		0.013 01	
	0.0029		0.000 05		0.000 05		0.000 08		0.001 34	
e	0.0120		0.1457		0.278		0.142		0.019	
	0.0005		0.0010		0.003		0.002		0.002	
A (R$_\odot$)	33.70		22.64		29.79		28.44		27.32	
	0.15		0.62		1.04		0.2		0.64	
i (deg)	81.5		86.4		85.7		85.5		78.0	
	0.2		0.2		0.2		0.5		0.6	
	Prim.	Sec.	Prim.	Sec.	Prim.	Sec.	Prim.	Sec.	Prim.	Sec.
Sp	O8V	O8V	B0.5	B1	B2V	B2V	O9.3	O9.4	O9.5V	O9.5V
$\log T_{\rm eff}$	4.531	4.531	4.476	4.468	4.377	4.354	4.491	4.499	4.485	4.485
	0.026	0.026	0.007	0.007	0.009	0.010	0.029	0.029	0.015	0.015
$\log g$	3.926	3.856	4.17	4.2	4.140	4.151	4.12	4.17	3.916	3.908
	0.17	0.17	–	–	0.014	0.015	0.04	0.04	0.027	0.027
M (M$_\odot$)	22.89	21.42	13.5	13.0	9.27	8.48	17.57	17.04	16.6	16.3
	0.32	0.33	1.4	1.4	0.122	0.122	0.27	0.26	0.9	0.9
a (R$_\odot$)	9.35	8.34	4.99	4.74	4.29	4.05	5.93	5.78	7.43	7.43
	0.17	0.16	–	–	0.06	0.06	0.07	0.07	0.12	0.12
V (km s^{-1})	150	130	141	134	120	110	147	138	135	135
	20	15	SR	SR	10	10	10	10	SR	SR
Refs.	1		2		3,4		5.6		7	

SR: synchronous rotation is assumed.
References: (1) Andersen & Clausen (1989); (2) Giménez & Clausen (1986); (3) Giménez, Clausen & Jensen (1986); (4) Andersen et al. (1983); (5) Simon, Sturm & Fiedler (1994); (6) Hill & Holmgren (1995); (7) Petrova & Orlov (1999) and references therein.

subject to the boundary condition $\eta_2 = 0$ at $a = 0$. In this expression a is the mean radius of a given equipotential, $\rho(a)$ is the density at a and $\bar{\rho}(a)$ is the mean density interior to a, η_i is given by

$$\eta_i \equiv \frac{a}{Y_i}\frac{\mathrm{d}Y_i}{\mathrm{d}a} \tag{15}$$

and the radius r of the distorted configuration and a are related by

$$r = a\left(1 + \sum_{i=0}^{n} Y_i(a, \theta)\right), \tag{16}$$

where $Y_i(a,\theta)$ describes the amplitude of the distortions. In order to integrate equation (14), we recall that $\rho(a)$ and $\bar{\rho}(a)$ are provided by the structure of the evolving model. Close to the centre $\rho(a)$, and consequently $\bar{\rho}(a)$ and $\eta_2(a)$, are expanded following an analogous treatment to that presented in Brooker & Olle (1955). The integration of equation (14) is started at the mesh point adjacent to the centre. Numerical integration is carried out with a standard Runge–Kutta routine (Press et al. 1986) up to the surface of the stellar model in order to obtain $\eta_2(a_i)$. Then, the ISC $k_{2,i}$ is finally given by

$$k_{2,i} = \left[\frac{3 - \eta_2(a)}{4 + 2\eta_2(a)}\right]_{a=a_i}. \tag{17}$$

3 A TEST OF THE METHOD EMPLOYING ECLIPSING BINARY SYSTEMS

The method we are presenting here is, to the best of our knowledge, original. In view of this fact, we have applied this method to some previously studied massive eclipsing binary systems with the aim of testing the method before applying it to HD 93205. We have focused our attention on detached systems in which both components are massive stars in the main sequence (MS) and have a rather well-measured apsidal motion. We have finally selected the following systems: EM Car, QX Car, GL Car, Y Cyg and V478 Cyg. Observational parameters for these systems are summarized in Table 1. In order to test the method, we compare the masses it yields with the *observed* ones, i.e. those obtained from the simultaneous analysis of light and radial velocity curves of the systems.

Components of a binary system must have the same age, so the first test we apply to the evolutionary code is that for each system considered there must be a single isochrone fitting the mass and radius of both components on the $M-R$ plane. The results of our calculations for the choice $\alpha_{\rm ov} = 0.25$ can be appreciated in Fig. 1, in which we show the mass and radius of the components of the selected binary systems. Note that for each system there is one isochrone that fits both components well, so the constraint that the ages of the components impose on evolutionary calculations is clearly satisfied by our models. In Fig. 2 we show the effective temperatures derived from our models for each star as a function of the observed effective temperature. Again, a good agreement between our evolutionary calculations and observations is found. As we have already stated, we have also considered a larger amount of overshooting, by fixing $\alpha_{\rm ov}$ to 0.40. We find no significant differences with the case of a smaller amount of overshooting, so we adopt the lower value ($\alpha_{\rm ov} = 0.25$) as the standard one in our calculations.

Based on these preliminary results, we are confident that our models are appropriate for studying the evolution of stars in detached binary systems. Thus, we apply our models to the study of apsidal motion through the calculation of the ISCs. Let us consider equation (1) again. We can rewrite it in order to define a *mean* observational ISC $\bar{k}_{2,\rm obs}$

$$\frac{\dot{\varpi}_2}{\Omega} = k_{2,1}c_{2,1} + k_{2,2}c_{2,2} = \bar{k}_{2,\rm obs}(c_{2,1} + c_{2,2}), \tag{18}$$

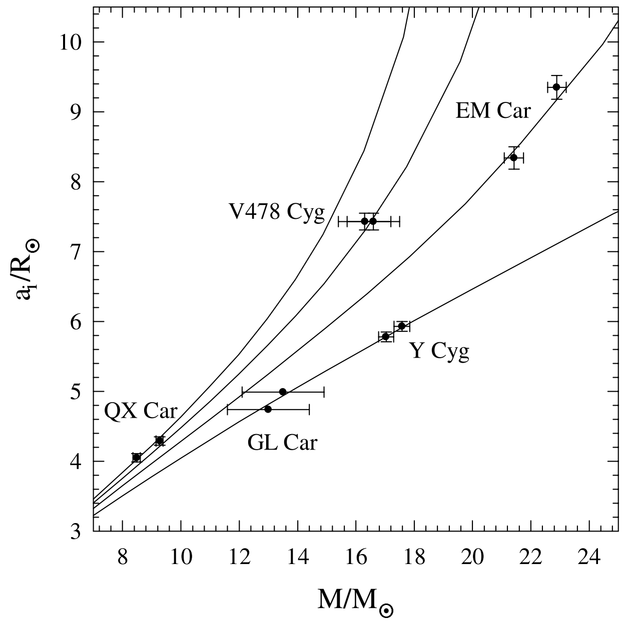

Figure 1. The radius versus mass relationship for the components of binary systems EM Car, QX Car, Gl Car, Y Cyg and V478 Cyg together with their corresponding error bars. Solid lines represent our theoretical isochrones for 7.8, 6.3, 4.3 and 1.8 Myr (from right to left). For each system considered there is one isochrone that fits both components.

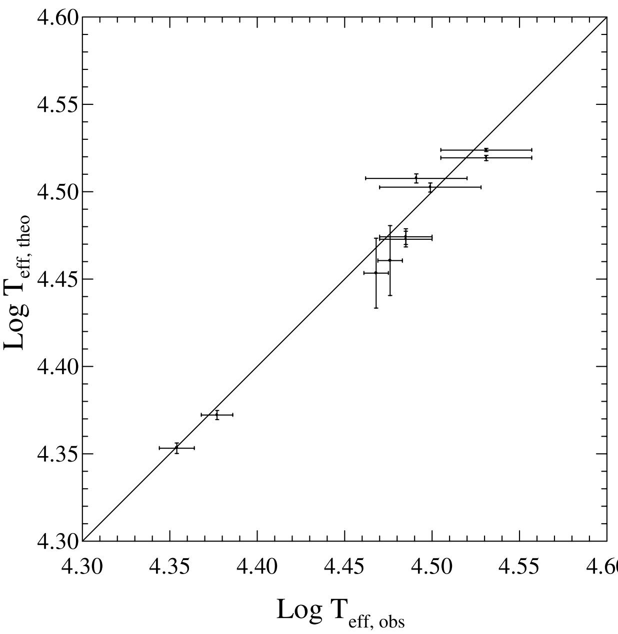

Figure 2. Comparison between theoretical and observed effective temperatures for the components of the same binary systems shown in Fig. 1.

and a *mean* theoretical ISC by

$$\bar{k}_{2,\rm theo} = \frac{k_{2,1}c_{2,1} + k_{2,2}c_{2,2}}{c_{2,1} + c_{2,2}} \tag{19}$$

where $c_{2,i}$ is given by

$$c_{2,i} = \left(\frac{a_i}{A}\right)^5 \left[15\frac{M_{3-i}}{M_i} f_2(e) + \frac{\omega_i^2 A^3}{M_i G} g_2(e)\right]. \tag{20}$$

Even when the quotient $\dot{\varpi}_2/\Omega$ can be assessed from observation, it

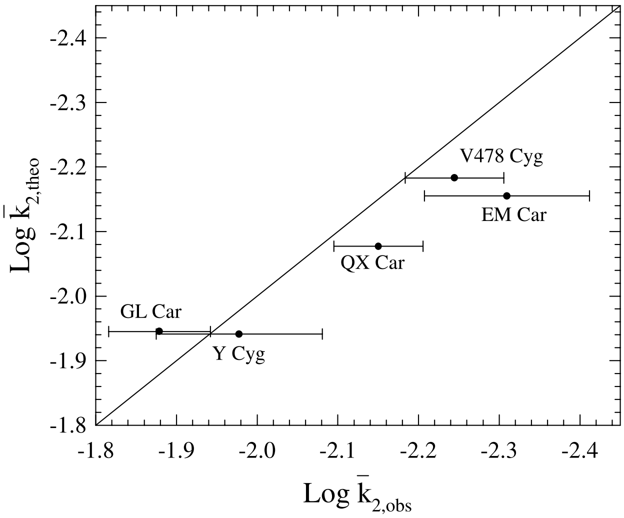

Figure 3. Comparison between theoretical and observed $\bar{k}_2$ for the components of the same binary systems included in Fig. 1. 1σ error bars for $\bar{k}_{2,\rm obs}$ are also shown. Note that theoretical models are slightly less concentrated than indicated by observations.

is not possible to separate the contribution of each component to the rate of apsidal motion. Instead, we can determine $\bar{k}_{2,\rm obs}$ and it is this value that is currently used to contrast evolutionary models with observation. In Fig. 3 we show the theoretical values for $\bar{k}_2$ derived from our models against the observed ones. We find that our models predict mean ISCs that are in reasonably good agreement with the observed ones for the less concentrated models (those with a higher value of $\bar{k}_2$). The most discrepant case we find is that of EM Car, which is the most evolved system considered by us, as we find that its primary star has spent $\approx$60 per cent of its lifetime on the MS. In this case, the result of our model is less concentrated than what we should expect from the observed value $\bar{k}_{2,\rm obs}$. However, as stated by Andersen & Clausen (1989), the apsidal motion rate for this system is based on observations covering only $\sim$1/6 of the apsidal motion period, thus the accuracy of apsidal motion parameters is still limited. In addition, information on its chemical composition is also missing. The theoretical values $\bar{k}_{2,\rm theo}$ we find for the other systems are in good agreement with the observed values and, for QX Car and Y Cyg, also with those derived theoretically by Claret (1997).

Below we present some of the results of applying our method to the solution of equation (11). First of all, let us emphasize that for an assumed age of the system, the solution of equation (11) is very well determined, i.e. only one solution is found as $\mathcal{F}_2$ is a very well-behaved, monotonically decreasing function of the independent quantity M_1. We illustrate this general behaviour with one example: in Fig. 4 we show $\mathcal{F}_2$ as a function of M_1 for the case of V478 Cyg, assuming an age of 6 Myr for both components. In view of these results, we find the method to be very reliable, from a mathematical point of view, in yielding a well-determined value of M_1. In Fig. 5 we show the mass M_1 of the primary component of EM Car obtained as a function of the age of the system. The figure corresponds to the choice $\alpha_{\rm ov} = 0.25$ for overshooting. We find a good agreement between our theoretical prediction for M_1 and its *observed* value for the whole range of ages considered, within a $\pm 1\sigma$ error. The upper limit for the age considered is given by the fact that, for larger ages, M_1 falls below the minimum mass derived for this system from its mass function *f*. Results are very similar if an overshooting amount of $\alpha_{\rm ov} = 0.40$ is considered, the only main difference being that the solution curve is slightly shifted to larger

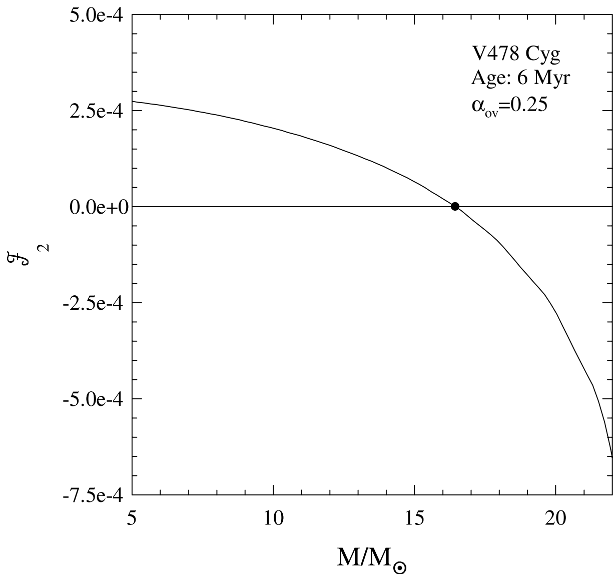

Figure 4. $\mathcal{F}_2$ as a function of M_1 for the case of V478 Cyg, assuming an age of 6.0 Myr for the system and setting $\alpha_{ov} = 0.25$. Note that the solution value M_1 for which $\mathcal{F}_2 = 0$ is very well determined, since $\mathcal{F}_2$ is a monotonically decreasing function of M_1.

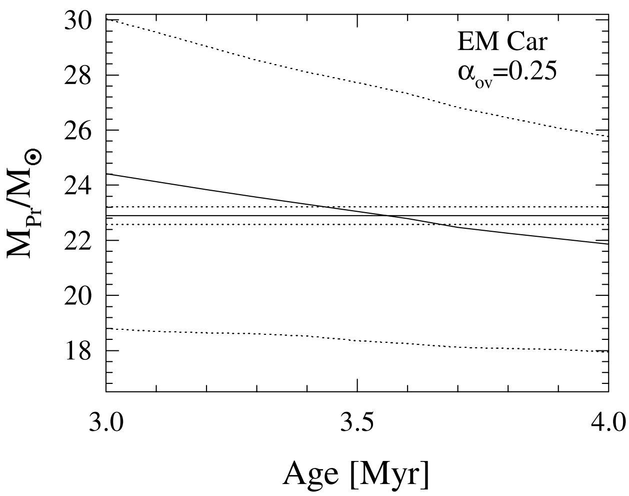

Figure 5. The mass of the primary star of the EM Car system assuming $\alpha_{ov} = 0.25$ as a function of its age. The horizontal solid line corresponds to the preferred value deduced from radial velocity and light curves. Short-dashed horizontal lines represent the uncertainty in this *observational value*. The other solid and short-dashed lines represent the preferred value and its 1σ uncertainty, respectively, deduced from the apsidal motion rate.

ages ($\approx$10 per cent). Fig. 6 shows the results obtained for V478 Cyg again for $\alpha_{ov} = 0.25$. For this binary system, an excellent agreement is achieved between our method and the *observed* mass. The same trend as before is found, i.e. the larger overshooting shifts the solution toward ages $\sim$10 per cent larger. Note also that the larger the age considered, the smaller the mass of the primary (and also the mass of the secondary) that can account for the observed rate of apsidal motion.

4 CALCULATION OF THE MASSES OF HD 93205 AND RELATED PARAMETERS

From the results of the previous sections, we judge our method to be good enough to be employed in the mass estimation of the

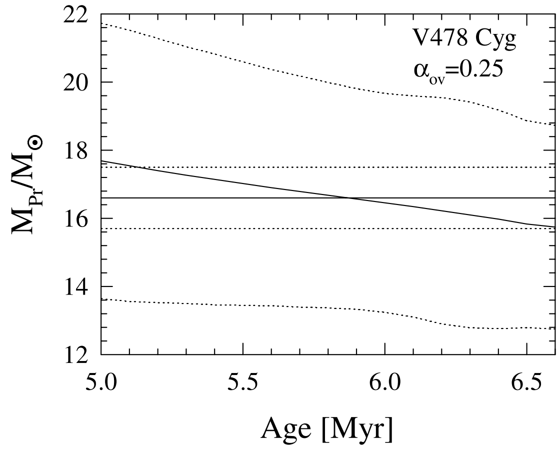

Figure 6. Same as in Fig. 5 but for V478 Cyg.

components of HD 93205. As stated before in the Introduction, HD 93205 is a highly eccentric system, which strongly suggests that it must be very young. However, the age estimates for such early O-type stars are very uncertain, either one tries to derive them considering the region in which the star is located, or comparing the position of the star on the theoretical HRD with isochrones calculated from evolutionary stellar models. HD 93205 belongs to the open cluster Trumpler 16, the most massive stars of which have an age spread of between 1 and 2 Myr (DeGioia-Eastwood et al. 2001). Besides, there is evidence of ongoing star formation in the molecular cloud complex associated with the Carina Nebula (Megeath et al. 1996). Consequently, a lower limit to the age of the members of Tr 16 cannot be established. On the other hand, de Koter, Heap & Hubeny (1998) showed that if we increase by $\sim$10 per cent the effective temperature of O3-type stars, the age would decrease from 2 to 1 Myr. Regarding the interpretation of theoretical isochrones for the most massive stars, these authors stated: ‘The derived T_{eff} values are so similar because the isochrone for $\sim$2 Myr runs almost vertical and because the distance in temperature between the isochrones of 1 and 3 Myr is very small.’

Taking into account the problem in the age determination described in the previous paragraph we choose to solve equation (11) for a whole set of isochrones ranging from the ZAMS up to 2 Myr. We consider as a zero-age isochrone that corresponding to the time when the stellar radius reaches its minimum value. In our models, this happens for ages of a few $\times 10^4$ yr.

In Fig. 7, we present the mass M_1 of the primary component of HD 93205 as a function of the age of the system. Two curves are shown, each of them corresponding to a particular choice of the overshooting parameter ($\alpha_{ov} = 0.25, 0.40$). Let us emphasize that the amount of overshooting that actually occurs is a rather uncertain quantity, so we consider it as a free parameter and study its influence on the solution of equation (11). As can be seen from Fig. 7, for a given age, M_1 is almost insensitive to our different choices of α_{ov}. Both curves are almost overlapped over the whole range of ages, though differences tend to increase with age. This is not surprising because for a given mass the initial model (a ZAMS model) is the same in both cases so no initial discrepancy exists between them. As models evolve both sequences depart from one another and different internal mass concentrations slowly arise. In view of this insensitivity, we shall concentrate ourselves on the case $\alpha_{ov} = 0.25$, but there is no particular reason to prefer this value instead of the higher one.

Let us consider again Fig. 7. The mass of the primary is a

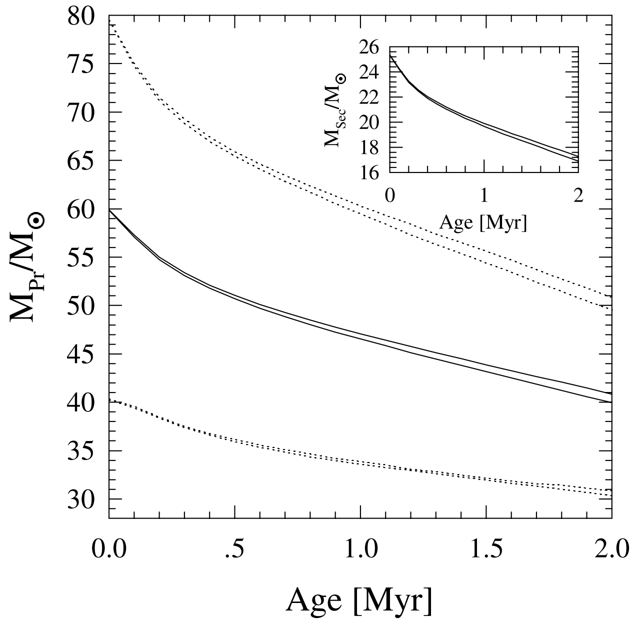

Figure 7. The mass of the components of HD 93205 deduced from its apsidal motion rate as a function of its assumed age. In each figure, the solid lower (upper) line corresponds to the preferred value assuming $\alpha_{ov} = 0.25$ ($\alpha_{ov} = 0.4$). Short-dashed lines represent its 1σ uncertainty. For more details, see the text.

decreasing function of the age of the system. We find that its maximum value, corresponding to ZAMS models, is $M_1 = 60 \pm 19\,M_\odot$. This is the upper limit for the mass M_1 of the O3 V component of HD 93205. At increasing ages, it decreases rapidly and reaches $53\,M_\odot$ at just 0.3 Myr and $46.5\,M_\odot$ at approximately 1 Myr and finally $M_1 = 40 \pm 9\,M_\odot$ at 2 Myr. Within observable quantities, the main source of uncertainty in determining M_1 is the apsidal motion rate (known up to a 9 per cent accuracy) and to a smaller extent the projected semi-axis and the projected rotational velocities, so better determinations of these quantities (especially the apsidal motion rate) are needed in order to decrease the error in the determination of M_1. We recall here that the apsidal motion rate is a critical parameter because the necessity of a very long time baseline (decades) for high-quality observations. With the mass M_1 determined, it is straightforward to calculate the mass M_2 of the secondary if we recall (Table 2, see Morrell et al. 2001 for further details) that the mass ratio $Q = M_2/M_1$ for HD 93205 is 0.423 ± 0.009. We find that M_2 ranges from $25.3 \pm 8\,M_\odot$ at the ZAMS down to $17 \pm 4\,M_\odot$ if a rather large value of 2 Myr is adopted for the age of the system. These mass values are in good agreement with those expected for an O8 V star such as this one (consider, particularly, the well-known short-period eclipsing binary EM Car, the primary component of which is an O8 V and its mass is $22.89 \pm 0.32\,M_\odot$, Andersen & Clausen 1989). Once M_1 is determined it is easy to obtain the inclination i of the orbit from equation (6). In Fig. 8 it is shown the resulting inclination from the set of calculations corresponding to $\alpha_{ov} = 0.25, 0.40$. It can be seen that the inclination of the system increases with age. This is a direct consequence of the behaviour of M_1 (which decreases with age) but it is worth noting that within the whole range of ages considered, the resulting inclination does not allow eclipses to occur. Indeed, if we assume that HD 93205 is not older than 2 Myr we find that $54° \leq i \leq 68°$, in coincidence with Antokhina et al. (2000) who found a most probable value of $i = 60°$.

Table 2. Observed parameters for HD 93205.

$a_1 \sin i$ (km)	$(1.03 \pm 0.02) \times 10^7$	Morrell et al. (2001)
$a_2 \sin i$ (km)	$(2.44 \pm 0.02) \times 10^7$	"
K_1 (km s^{-1})	132.6 ± 2.0	"
K_2 (km s^{-1})	313.6 ± 1.8	"
P (d)	6.0803 ± 0.0004	"
e	0.370 ± 0.005	"
$M_1 \sin^3 i$(M$_\odot$)	31.5 ± 1.1	"
$M_2 \sin^3 i$ (M$_\odot$)	13.3 ± 1.1	"
$Q(M_2/M_1)$	0.423 ± 0.009	"
$\dot{\varpi}$ (deg d^{-1})	$0.005\,33 \pm 0.000\,51$	"
$V_1 \sin i$ (km s^{-1})	135	Howarth et al. (1997)
$V_2 \sin i$ (km s^{-1})	145:	"

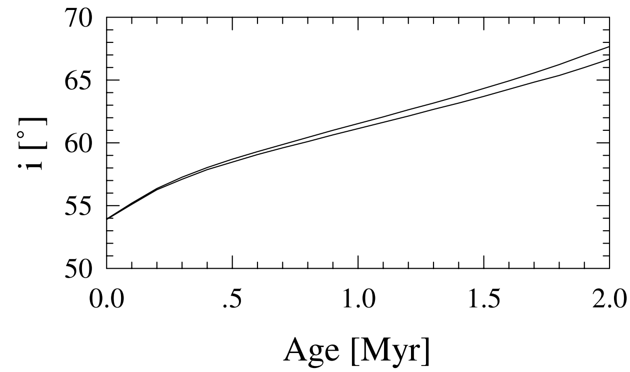

Figure 8. Inclination of HD 93205 as a function of its age. For more details, see the text.

However, a problem arises when we try to compare the luminosity derived from the corresponding models with the observed value for HD 93205. Let us explain this with an example: if we consider the 60-M$_\odot$ model, it predicts, for zero age, a radius $R_1 = 10.7\,R_\odot$ and $\log T_{eff} = 4.68$, resulting in a luminosity, $\log L = 5.72\,L_\odot$. This corresponds to a bolometric magnitude, $M_{bol} = -9.55$, which is almost 1 mag fainter than $M_{bol} = -10.41$, derived by Morrell et al. (2001) from the visual magnitude of the O3 V component of HD 93205, the distance modulus of 12.55 obtained by Massey & Johnson (1993) for Tr 16, and the bolometric correction (BC) for an O3 V star taken from the calibration by Vacca, Garmany & Shull (1996). This large disagreement between the expected and observed bolometric magnitudes, points to a large error in some (or any) of the involved assumptions. If the distance modulus is right, then we can suspect that the BC must be wrong by approximately 1 mag. On the other hand, the distance modulus of the Carina Nebula is still a matter of discussion. A distance modulus of the order of that derived by Massey & Johnson (1993) arises from the consideration of colour–magnitude diagrams for the stellar component of the clusters. Some other independent determinations, such as those obtained recently by Davidson et al. (2001) from a kinematic study of the Homunculus nebula surrounding Eta Car, give a distance modulus as low as 11.76, which would decrease the referred discrepancy significantly. However, if we suppose this last distance modulus to be correct, then all of the stars in Tr 16 will have $M_V \sim 0.8$ mag fainter than the values accepted to date. Here we arrive at a point the importance of which is obvious for many astrophysical issues, and deserves to be studied carefully. The referred discrepancies might also arise in a combination of different sources of error (BCs, distances and the adopted absolute magnitude scale for ZAMS stars). A detailed discussion of these issues will be presented in a forthcoming paper.

Finally, let us comment briefly that the tidal contribution to the apsis motion of HD 93205 is the most important one, ranging from ~60 per cent at the ZAMS to 70 per cent at 2 Myr. The rotational contribution ranges from 30 per cent to 20 per cent and the relativistic one is almost constant and approximately 10 per cent of the apsidal motion rate is caused by this effect. In this sense, HD 93205 could be classified as a relativistic binary system (Claret 1997).

5 CONCLUSIONS

We present a method of calculating masses for components of non-eclipsing binary systems if their apsidal motion rate is provided. The method consists in solving equation (11) if the radius and the internal structure constant of each component can be obtained from a grid of stellar evolution calculations. In order to test this method, we have selected some eclipsing binary systems and have derived the masses of their components. A very good agreement was achieved between masses obtained with our method and those derived from the analysis of their radial velocity and light curves.

The main goal of this article, besides presenting the method, is to calculate the masses of the components of HD 93205. This is an O3 V + O8 V system. Its O3 V component has the earliest known spectral type of a normal star found in a double-lined close binary system, and thus is potentially a very massive star. Although HD 93205 is not an eclipsing binary, Morrell et al. (2001) have measured its apsidal motion rate and found it to be $\dot{\varpi} = 0\overset{\circ}{.}0324 \pm 0\overset{\circ}{.}0031$ per orbital cycle so we have been able to apply the method presented here to this system. The resulting mass of the primary star (M_1) is obtained as a function of the assumed age of the system. HD 93205 is a highly eccentric system ($e = 0.370 \pm 0.005$) which suggests a very low age. However, we do not adopt a particular value for the age as its determination is quite uncertain, and prefer to consider a range of ages starting at the ZAMS. We find that for zero-age models the resulting mass is $M_1 = 60 \pm 19\,\mathrm{M}_\odot$ and that it decreases monotonically as the age is increased (Fig. 7), reaching $M_1 = 40 \pm 9\,\mathrm{M}_\odot$ at 2 Myr. Now, if we take into account the mass ratio $Q = 0.423$ for HD 93025, the mass of the secondary lies in the range $M_2 = 25.3-17\,\mathrm{M}_\odot$ for this range of ages. It is worth mentioning again that these M_2 values are in good agreement with the masses derived for other O8 V stars in eclipsing binaries such as the well-studied system EM Car (Andersen & Clausen 1989). The mass value derived for M_1 is also in the range ($52-60\,\mathrm{M}_\odot$) obtained from the observed Q assuming a 'normal' mass for the O8 V secondary component (i.e. $22-25\,\mathrm{M}_\odot$). In addition, we have estimated the inclination of the system through equation (6) and the results obtained (Fig. 8) are consistent with the non-eclipsing condition of HD 93205.

Our results corresponding to zero age give an upper limit to the mass of the O3 V component of HD 93205, a result that places a strong constraint to the masses of theoretical stellar models for the most massive stars. Also, the luminosity derived from the stellar models for the O3 V component gives rise to a problem when compared with the observed value, the theoretical value of M_{bol} being almost 1 mag fainter than the value derived from the observations. This discrepancy raises the need to review both the distance and BC scales for the earliest-type ZAMS stars, a subject that will be addressed in the near future.

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