A solution to the cosmological ^{7}Li problem?

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Abstract. In this work we report on the cosmological ^{7}Li problem, from a nuclear structure point of view, that is by including resonances in the calculation of the reactions which populate beryllium. It is found that the primordial abundance of lithium is reduced, as a consequence of the presence of resonant channels in the relevant cross sections. We establish constraints on the resonant energies, and make a comparison with the available observational data.

1. Introduction

BBN (for Big Bang Nucleosynthesis) is the framework which describes the formation of light nuclei in the early Universe. The theory relies upon nuclear reactions between primordial elements. It has only one parameter, the baryon density, which is related to the baryon-to-photon ratio. This parameter can be determined by the analysis of the data produced by the Wilkinson Microwave Anisotropy Probe (WMAP) satellite (1). The results are consistent with the observed abundances of deuterium and ^{4}He, but they disagree with the observed abundance of ^{7}Li. In the literature, this problem was analyzed using different theoretical approaches based on astrophysical considerations (2; 3; 4). Recently, the nuclear aspect of the problem concerning the abundance of ^{7}Li have been revisited (5; 6; 7), by considering nuclear-reaction and nuclear-structure mechanisms which may reduce the abundance of lithium due to a depletion of the production of beryllium.

In this work, we analyze the effect upon the production of ^{7}Li due to the inclusion of isolated resonances in the reactions involving ^{7}Be. To perform the calculations of the primordial abundances we use the formalism developed by (8), and to include the resonances we assume that the cross sections are described by the Breit-Wigner formula. We analyze the effects of the inclusion of these resonances separately, and, in particular, on the freeze-out temperature of beryllium, the ^{7}Be abundance and the lithium primordial abundance. We set constrains on the value of the resonant energy for each process using the available observable data.

2. Formalism

The nuclear reactions which are relevant for the present study are:

$$\begin{aligned} {}^7\mathrm{Be} + d &\rightarrow {}^4\mathrm{He} + {}^4\mathrm{He} + p \\ {}^7\mathrm{Be} + {}^4\mathrm{He} &\rightarrow \gamma + {}^{11}\mathrm{C} \\ {}^7\mathrm{Be} + n &\rightarrow p + {}^7\mathrm{Li}. \end{aligned} \quad (1)$$

The reaction rate $[ij;kl]$ for the process $i+j \rightarrow k+l$ [1]. is written (8)

$$[ij;kl] \;=\; \rho_B N_A \langle\sigma v\rangle = 0.93\times 10^{-3}\Omega_B h^2 T_9^3 N_A \langle\sigma v\rangle \ , \tag{2}$$

where σ is the cross section, v is the relative velocity, $\rho_B = 0.93\times 10^{-3}\Omega_B h^2 T_9^3$ is the density of baryonic matter, N_A is the Avogadro number per gram, T_9 is the temperature in units of $10^9 K$. The use of Maxwell-Boltzmann velocity-distribution leads to the estimate

$$\langle\sigma v\rangle \;=\; \left(\frac{\mu}{2\pi kT}\right)^{3/2} \int e^{-\frac{\mu v^2}{2kT}} v\sigma(E) d^3v \ , \tag{3}$$

where μ is the reduced mass.

In order to take into account the resonances in the reaction rates, we assume that the cross section for a generic process is describe by the Breit-Wigner formula (9)

$$\sigma(E) \;=\; \frac{\pi\hbar^2}{2\mu E}\frac{\omega_r\Gamma_1\Gamma_2}{(E-E_r)^2+\Gamma^2/4} \ , \tag{4}$$

where Γ_i is the partial width for the decay of the resonant state, Γ is the sum over all partial widths, $\omega_r = \frac{(1+\delta_{ab})g_r}{g_a g_b}$ and $g_r = 2J_r+1$, J_r being the spin of the resonant state and E_r is the resonance energy in the center of momentum system. The average cross section $\langle\sigma v\rangle$ is

$$\langle\sigma v\rangle \;=\; \left(\frac{2\pi\hbar^2}{\mu kT}\right)^{3/2} \frac{(\omega\gamma)_r}{\hbar} e^{E_r/kT} \ , \tag{5}$$

where $\gamma_r = \left(\frac{\Gamma_1\Gamma_2}{\Gamma}\right)_r$ The reaction rate is written

$$[ij;kl] = 9.69\times 10^{-33}\Omega_B h^2 \left(\frac{T_9}{\mu}\right)^{3/2} (\omega\gamma)_r e^{-11.605 E_r/T_9} \ , \tag{6}$$

where T_9 is the temperature in units of 10^9 K. Empirical values of the widths may be obtained from nuclear reaction data. This is a sensitive aspect of the calculations, because the actual value of the resonances and their widths, extracted from experimental data, may restrict the range of values where the effect upon the abundance of ^{7}Li become noticeable (10). Having these limitations into account we shall explore the mechanism (that is the excitation and decay of resonances) for which the population of states leading to ^{7}Li may be depleted.

2.1. Calculation of primordial abundances

The equation that governs the primordial abundance Y_i of a given element i, is

$$\dot{Y}_i \;=\; J(t) - \Gamma(t) Y_i \tag{7}$$

The static solution of this equation is the quasi-static equilibrium (QSE) solution (8)

$$f_i \;=\; \frac{J(t)}{\Gamma(t)}. \tag{8}$$

In order to compute the primordial abundances, one must solve the system of differential equations, considering only the dominant reaction rates (8). The semi-analytical approach

[1] Hereafter, we denote the reactions rates as $[\mathrm{Be}\,d;\,\alpha\,\alpha\,p]$, $[\mathrm{Be}\,\alpha;\,\gamma\,\mathrm{C}]$ and $[\mathrm{Be}\,n;\,p\,7]$, respectively, where p refers to proton, d to deuterium, α to ^{4}He, Be to ^{7}Be, γ to the photon, C to ^{11}C, n is the neutron and 7 stands for ^{7}Li.

consists in calculating the abundances between fixed points or stages. One solves the equations only for one element at each step and, for the other elements, one considers the quasi static equilibrium solution. The freeze-out of the production of each element takes places when the dominant destruction reaction rate equals the expansion rate of the Universe (11).

The differential equation that governs the primordial abundance of beryllium is

$$\begin{aligned}\dot{Y_{\rm Be}} &= Y_pY_6[6\,p;\,\gamma\,{\rm Be}] + Y_3Y_\alpha[3\,\alpha;\,\gamma\,{\rm Be}] - Y_\gamma Y_{\rm Be}[{\rm Be}\,\gamma;\,3\,\alpha] \\ &\quad -Y_nY_{\rm Be}[{\rm Be}\,n;\,p\,7] - Y_pY_{\rm Be}[{\rm Be}\,p;\,\gamma\,8] - Y_dY_{\rm Be}[{\rm Be}\,d;\,\alpha\,\alpha\,p] \\ &\quad -Y_\alpha Y_{\rm Be}[{\rm Be}\,\alpha;\,\gamma\,{\rm C}]. \end{aligned} \tag{9}$$

According to (8), the predominant destruction reaction rate is $[{\rm Be}\,n;\,p\,7]$ and the predominant source term is $Y_3Y_\alpha[3\,\alpha;\,\gamma\,{\rm Be}]$. The freeze-out temperature of ^{7}Be $\left(T_9^{\rm Be}\right)$ is calculated by equating the expansion rate of the Universe $\left(H = \frac{1}{356}T_9^2\right)$ with the dominant reaction (destruction) rate

$$H = Y_n[{\rm Be}\,n;\,p\,7] + f\,(E_r, T_9) \quad , \tag{10}$$

where $f\,(E_r, T_9)$ stands for $Y_d[{\rm Be}\,d;\,\alpha\,\alpha\,p]$, and $Y_\alpha[{\rm Be}\,\alpha;\,\gamma\,{\rm C}]$. The neutron abundance can be determined by solving the quasi-static equation $\dot{Y_n} = 0$

$$Y_n = Y_d\frac{Y_d[d\,d;\,n\,3] + Y_t[d\,t;\,n\,\alpha]}{Y_p[n\,p;\,\gamma\,d] + Y_3[3\,n;\,p\,t] + [n]} \quad . \tag{11}$$

In the last equation, t stands for tritium. Considering $\dot{Y_t} = 0$

$$Y_n = Y_d^2\frac{[d\,d;\,n\,3] + [d\,d;\,p\,t]}{Y_p[n\,p;\,\gamma\,d] + Y_3[3\,n;\,p\,t] + [n]} \quad . \tag{12}$$

Then, the quasi-static equation for ^{7}Be, we can compute the beryllium primordial abundance at the freeze-out temperature (8)

$$Y_{\rm Be} = \frac{Y_3Y_\alpha[3\,\alpha;\,\gamma\,{\rm Be}]}{Y_n[{\rm Be}\,n;\,p\,7] + f\,(E_r, T_9^{\rm Be})} \quad . \tag{13}$$

The differential equation which governs the primordial abundance of lithium is

$$\dot{Y_7} = Y_nY_6[6\,n;\,\gamma\,7] + Y_nY_{\rm Be}[{\rm Be}\,n;\,p\,7] + Y_tY_\alpha[t\,\alpha;\,\gamma\,7] - Y_pY_7[7\,p;\,\alpha\,\alpha] \quad .$$

The quasi-static equation $\dot{Y_7} = 0$ is solved by considering the dominant production and destruction terms

$$Y_7 = \frac{Y_nY_{\rm Be}[{\rm Be}\,n;\,p\,7] + Y_tY_\alpha[t\,\alpha;\,\gamma\,7]}{Y_p[t\,p;\,\alpha\,\alpha]} \quad , \tag{14}$$

where $Y_{\rm Be}$ is the value obtained by evaluating Eq.(13) at the freeze-out temperature of beryllium. The abundance of tritium is determined by solving the equation $\dot{Y_t} = 0$, and the expression for Y_n is given in Eq. (12). In order to calculate the final lithium abundance, we evaluate this expression at the freeze-out temperature of lithium $\left(T_9^{\rm Li}\right)$ which is the solution of

$$Y_p[7\,p;\,\alpha\,\alpha] = H \quad . \tag{15}$$

As one can see, the inclusion of an isolated resonance in either $[{\rm Be}\,d;\,\alpha\,\alpha\,p]$, $[{\rm Be}\,\alpha;\,\gamma\,{\rm C}]$ or $[{\rm Be}\,n;\,p\,7]$ does not affect the freeze-out temperature of lithium. However, its abundance is reduced due to the change in the beryllium primordial abundance resulting from the change in the freeze-out temperature of it.

3. Results

To compute the light nuclei abundances we have adopted the WMAP value of the baryon density $(\Omega_B h^2)_{\rm WMAP} = 0.0224 \pm 0.0008$ (1; 12; 13), the primordial abundances of proton, deuterium, ^{3}He and ^{4}He are

$$\begin{aligned} Y_p &= 0.749 \ , \\ Y_d &= 2.36 \times 10^{-5} \ , \\ Y_3 &= 6.76 \times 10^{-6} \ , \\ Y_\alpha &= 0.24915 \ , \end{aligned} \tag{16}$$

respectively. Thus, the freeze-out temperature of lithium (see Eq. (15)) is estimated at the value

$$T_9^{\rm Li} = 0.1849 \ . \tag{17}$$

As an example, we show in Figure 1 the normalized freeze-out temperature of beryllium $\left(\left(T_9^{\rm Be}\right)_{res}/T_9^{\rm Be}\right)$ as a function of the resonant energy, for different values of the parameter a. The freeze-out temperatures have been calculated with, $\left(T_9^{\rm Be}\right)_{res}$, and without, $T_9^{\rm Be}$, including a resonance in the spectrum of ^{7}Be. The temperature is lower than the standard one for resonant energies lower than 0.25 MeV, for all the values of the parameter a. The effect of the variation of a is noticeable. The freeze-out temperature decreases if a increases. It means that a resonant state with a large width, in the entrance channel of the reaction, will cause a significant reduction of the freeze-out temperature and the effect will be dominant at lower energies. Similarly, the

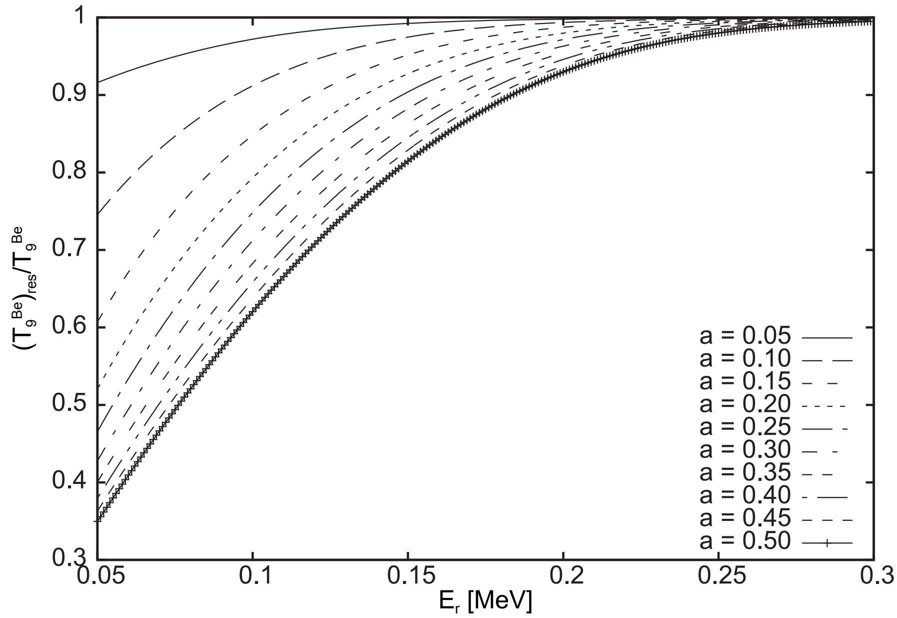

Figure 1. Ratio $\left(T_9^{\rm Be}\right)_{res}/T_9^{\rm Be}$ of the freeze-out temperature of beryllium as a function of the resonant energy E_r, for different values of the parameter a, considering an isolated resonance in the reaction $^7{\rm Be} + d \rightarrow ^4{\rm He} + ^4{\rm He} + p$

results for the primordial abundance of lithium with, $(Y_7)_{res}$, and without, Y_7, including a resonance in the spectrum of ^{7}Be are shown in Figure 2, as a function of the resonant energy, and for different values of the parameter a. The effect is quite noticeable. It is seen that the value of Y_7 can be reduced almost to one half of the standard value depending on the values of E_r and a. These results are in agreement with the results obtained by (5) and by (7).

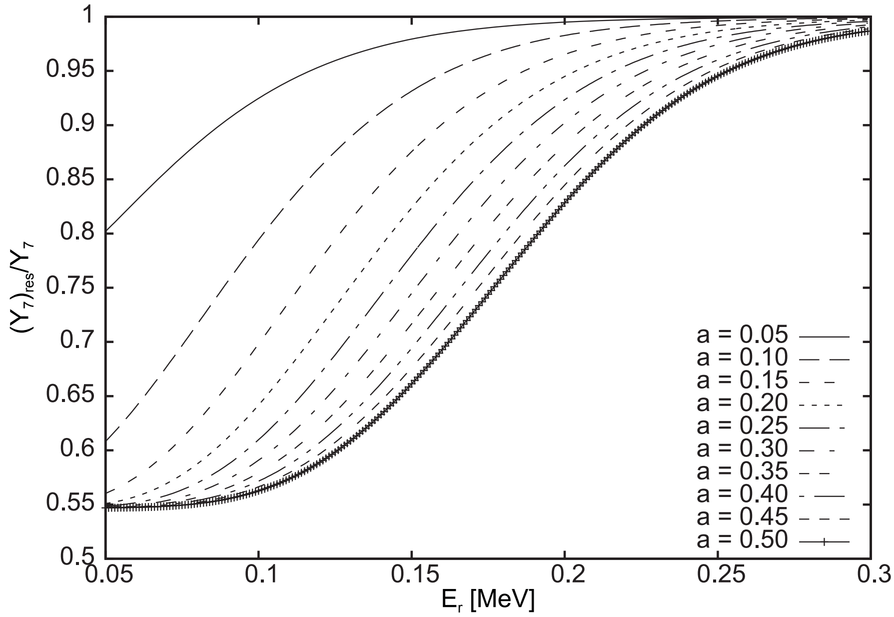

Figure 2. Ratio between the abundance of lithium with ($(Y_7)_{res}$) and without including a resonant state (Y_7) in the beryllium spectrum, as a function of the resonant energy E_r, for different values of the parameter a, in the reaction $^7\mathrm{Be} + d \rightarrow^4 \mathrm{He} +^4 \mathrm{He} + p$

The available data of lithium may be used to to obtain the best-value of the resonant energy for different values of the parameter a, by implementing a χ^2-test. The results of the analysis are presented in Table 1. There is a good fit for all values of the parameter a considered,

Table 1. Best-fit and 1σ errors of the resonant energy of an isolated resonance in reaction $^7\mathrm{Be} + d \rightarrow^4 \mathrm{He} +^4 \mathrm{He} + p$, for different values of the parameter a (the parameter remains fixed in the statistical test).

a	$E_r \pm \sigma$ [MeV]	$\chi^2/(N-1)$
0.15	$0.066^{+0.014}_{-0.040}$	1.00
0.20	$0.080^{+0.016}_{-0.040}$	1.00
0.25	$0.091^{+0.016}_{-0.040}$	1.00
0.30	$0.099^{+0.017}_{-0.043}$	1.00
0.35	$0.105^{+0.018}_{-0.044}$	1.00
0.40	$0.111^{+0.018}_{-0.046}$	1.00
0.45	$0.116^{+0.018}_{-0.048}$	1.00
0.50	$0.120^{+0.019}_{-0.049}$	1.00

since $\chi^2/(N-1) = 1$ (N is the number of observational data, $N = 11$). The best-fit value of the resonant energy increases its value for larger values of the parameter a. These results for the resonant energy are dependent on the observable data set used in the statistical analysis, however, the favored value of the resonant energy is lower than 0.2 MeV.

4. Conclusion

In this work we have computed the primordial abundances of light elements using a semi-analytical approach and considering the inclusion of isolated resonances in the reactions leading to beryllium. We have found that there exists a reduction of the primordial abundance of lithium due to a depletion of the beryllium abundance, produced by resonances in the reactions, particularly in $^7\mathrm{Be} + d \rightarrow ^4\mathrm{He} + ^4\mathrm{He} + p$ and $^7\mathrm{Be} + ^4\mathrm{He} \rightarrow \gamma + ^{11}\mathrm{C}$. However, if the resonance is present in the reaction $^7\mathrm{Be} + n \rightarrow p + ^7\mathrm{Li}$, the amount of primordial lithium is not modified substantially, a result which is also in agreement with the results of Refs. (5; 7). We have performed a statistical analysis in order to obtain the best-fit value of the resonant energy, and found that, for the reaction $^7\mathrm{Be} + d \rightarrow ^4\mathrm{He} + ^4\mathrm{He} + p$, the best-fit value of E_r lies between 50 to 150 keV (given a partial width in the range of 10 to 60 keV). The resonant energy for $^7\mathrm{Be} + ^4\mathrm{He} \rightarrow \gamma + ^{11}\mathrm{C}$ is higher, between 250 to 350 keV (given partial width in the range of 30 to 164 keV).

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