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A new combined quasigeoid model in Tierra del Fuego

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ABSTRACT

This work focuses on the development of a combined quasigeoid model for Tierra del Fuego province. The Equivalent Source Technique (EST) is applied together with the remove-compute-restore technique in order to combine gravity and GPS/levelling observations and to obtain a quasigeoid model. This model features an improved accuracy in relation to previous models. A discussion about the geodetic reference system is also presented. Geodetic coordinates of all stations used were transformed to TDF08 to be in accordance with the new geodetic reference frame of Argentina. After a cross validation procedure it is determined that a 5cm (r.m.s.) quasigeoid model has been achieved for the major part of the province, fulfilling the requirements for its practical use. New Global Geopotential Models (GGM) are introduced in the discussion, particularly the EGM2008 which is used for evaluation purposes. It shows a 9cm agreement after its evaluation on the levelling lines.

KEYWORDS **Tierra del Fuego. Quasigeoid. GPS/levelling. Gravity. Equivalent source technique.**

INTRODUCTION

Tierra del Fuego province is located in the southernmost part of South America, specifically in the eastern side of Isla Grande de Tierra del Fuego and belongs to the Argentine Republic. The terrain goes from flat lands in the north to mountains of 2600m height in the south. The latter are mainly located in the Chilean part and belong to the Andes range. From 1998 to 2008, several gravimetric and GPS/levelling campaigns were carried out in the province of Tierra del Fuego. In spite of the measurements performed, there is still lack of information about the centre-west part of the province mainly due to the rough terrain which makes the access very difficult. Previous estimations of the geoid in that region were made by Del Cogliano *et al*. (2001) and Tocho (2006, ARG05 model). They were totally geometric (GPS/levelling) or just gravimetric geoid models, respectively. The estimated accuracy for the geometric model, after comparing it with observed geoid undulations, was a few centimetres within a narrow region not so far from the levelling lines. The accuracy determined for the gravimetric one was 15cm (Tocho, 2006). The latter value was obtained from the comparison of the gravimetric model ARG05 with observed geoidal undulations in GPS/levelling points located in Tierra del Fuego province. ARG05 (Tocho, 2006) was based on the remove-compute-restore technique and the geoidal component and terrain effects were both estimated by applying Fast Fourier Transform (FFT). Part of the GRAVSOFT package and many routines developed in the University of Calgary were used.

This work presents a combined quasigeoid (gravimetric/geometric) model by means of the Equivalent Source Technique (EST) for almost the whole province of Tierra del Fuego. The new model combines GPS/levelling data with gravity measurements in order to augment the area where a high precision quasigeoid can be achieved. Geodetic coordinates of all data were transformed to TDF08 (Mendoza, 2008), representing an improved geodetic reference frame for the ellipsoidal heights. TDF08 is in accordance to the present Argentine reference frame which makes possible this quasigeoid model to be used as a height reference for many activities in the province.

OBSERVATIONAL DATA INCLUDED

GPS/levelling data

The included data consisted in GPS/levelling and gravity observations. GPS coordinates of levelling marks were determined along several field campaigns involving different geodetic reference frames. Most of the GPS coordinates were expressed in the local geodetic network TDF95 (Hormaechea, personal communication) and a few other coordinates were referred to ITRF00 and ITRF97. TDF95 was the first geodetic network for that province and it was a local densification of the national geodetic network POSGAR94 (Posiciones Geodésicas Argentinas 94). In order to adopt a common frame, transformation parameters were calculated to change coordinates from their original reference frames to TDF08 which is nowadays the best materialization of a Terrestrial Reference Frame (TRF) in the island. TDF08 is aligned to IGS05 (Ferland, 2006), epoch 2000.0.

Three and 7-parameter Helmert transformation were tested to transform coordinates from TDF95 to TDF08. These coordinates were not part of the GPS/levelling data. Both transformations gave satisfactory results: around 3cm r.m.s. for the first adjustment and 4cm for the last one. Taking into account that the province of Tierra del Fuego is a small region, the authors of this work do not consider that a 7 parameters transformation could be justified between two networks of enough quality. This reason and the result of the statistics justify a 3 parameters transformation. The transformation applied to those TDF95 coordinates consisted in the following transformation:

$$\Delta x = 0{,}667\text{m} \pm 0.011\text{m}$$
$$\Delta y = -1{,}057\text{m} \pm 0.035\text{m}$$
$$\Delta z = -0{,}311\text{m} \pm 0.034\text{m}$$

This implied a correction of almost 1m in the original ellipsoidal heights as it was expected for TDF95 coordinates. In the case of ITRF00 and ITRF97 coordinates, the transformation parameters published by Mc Carthy and Petit (2003) and Ferland (2006) were applied. The impact produced on ellipsoidal heights was almost negligible. After the mentioned transformations, a coherent set of coordinates referred to a unique reference frame was obtained.

Regarding the height system, all data located on Tierra del Fuego province was referred to Ushuaia tide gauge and the accuracy of levelling information was about 1mm. Since there was not any information in the west side of the province, three chilenian GPS/levelling points were included in the present quasigeoid model and their ITRF00 coordinates were transformed to TDF08. Although it has not been yet determined, the difference between the origin of these levelling heights and the Ushuaia height datum was estimated in 10cm. This value was inferred from the published works of Sanchez (2005) and Del Cogliano (2004).

Gravimetric data

The gravimetric data was completely referred to the International Gravity Standardization Net 1971 (IGSN71) and the error of an isolated gravity observation was estimated in 0.01mgal. The data distribution is shown in Figure 1. The model is valid in the area located between 66.5ºW, 69ºW, 55ºS and 52.5ºS. This region is bigger than the area considered in the models previously mentioned but there are still wide areas without any data.

GRAVIMETRIC REDUCTION

One of the most critical topics in gravimetric reduction is the handling of the shortest wavelengths and the best Digital Elevation Model (DEM) is required. At the computation time, the SRTM3 (Farr *et al.*, 2007) was available and the corresponding topography map is shown in Figure 2. Terrain corrections (TC) used in the gravimetric reduction process were estimated with the TC program (Forsberg, 1984). This routine computes terrain corrections in a classic way. It uses prisms to simulate different topographic structures. Fast Fourier methods used to compute terrain corrections were not applied because of the rough topography. It was demonstrated by Kirby and Featherstone (2001) that they overestimate terrain correction values due to instabilities of the FFT when there are slopes higher than 45°, like in Tierra del Fuego. The FFT series development cannot converge under these circumstances. In Figure 3 a map of terrain corrections is presented. Although there are some terrain correction values that exceed the expected ones, more than 85% are bellow 35mgal.

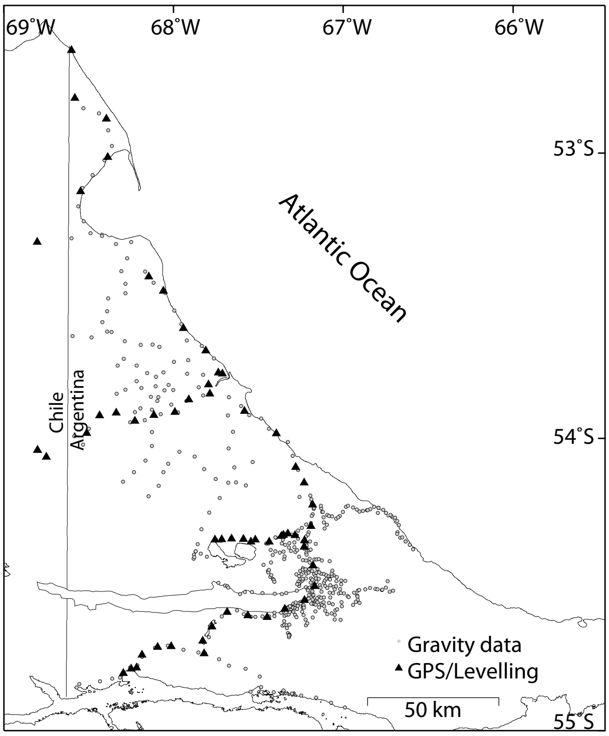

FIGURE 1. Distribution map of the 534 gravity stations (gray circles) and 58 GPS/levelling marks (black triangles) on Tierra del Fuego.

Bouguer anomalies were also computed as part of the study. In Figure 4 there is a map of their spatial distribution. Their values range from -63mgal, in the South, to 30mgal in the North of Tierra del Fuego. There are two negative minimums in the area of Fagnano Lake in correspondence with the results published by Lodolo *et al.* (2007).

In this work, Faye anomalies were obtained instead of free air anomalies. This type of gravimetric anomaly was used because we were seeking for the middle frequency of the gravity field signal which is appreciated in geoid undulations. In mountainous areas like the one studied, most of observations are usually made on lower lands and this situation may cause aliasing effects. In order to avoid them, the gravimetric reduction followed the steps proposed by Featherstone and Kirby (2000). The gravimetric remove-compute-restore procedure applied is described below:

i) Surface free air anomalies were built and the Bouguer Plate (BP) was removed from them.

A'= *gobs*- γ(H)-BP(H)

gobs being observed gravity and γ represents normal gravity. H is the height involved which is described below.

As the gravity measured points did not have associated normal or orthometric heights, those derived from GPS and a geometric height transformation model were used in this step. Owing to the density needed in the DEM and the area to be covered, SRTM3 was applied for the rest of the process.

ii) In order to avoid possible errors in terrain TC owing to DEM deficiencies, it was decided not to interpolate refined Bouguer anomalies. A' anomalies were used instead. The interpolation was made on a grid of 1km resolution. Then:

A'grid= A' interpolated on a 1km DEM resolution grid

iii) Terrain corrections (cp) and the geopotential model were removed from A'grid to avoid aliasing effects. The atmospheric corrections (atm) were applied afterwards. The residual part obtained after this removal was called Ares.

Ares=A'grid+cp+atm-GGM

EIGEN-GL04c up to degree and order 200 was the GGM applied to remove the long wavelength part of the spectrum, after testing other models like EIGEN-CG01c (Reigber, 2006) available at that time. EGM2008 (Pavlis *et al.* 2012) has a similar behaviour to the same degree and order but it was decided to be used for evaluation.

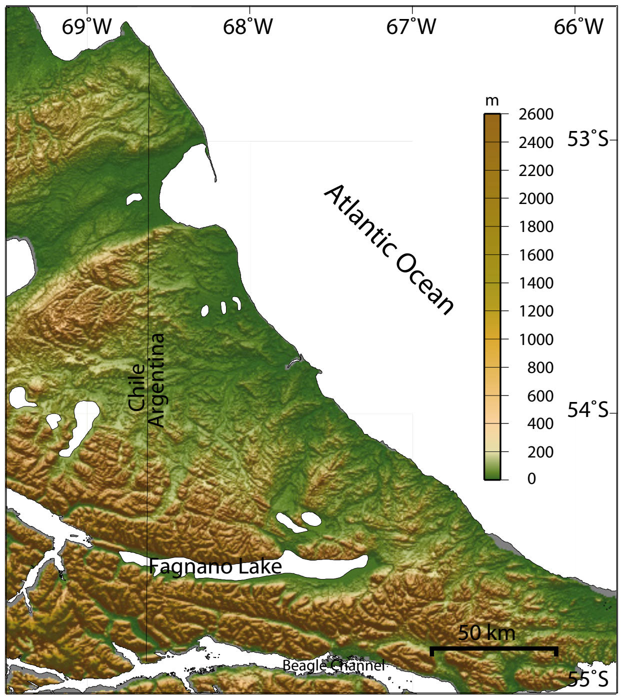

FIGURE 2. Topography map of Tierra del Fuego using the SRTM3 dataset. Although the limit in the scale is 2600m for the entire island, the maximum height over the Argentine portion does not exceed 1060m.

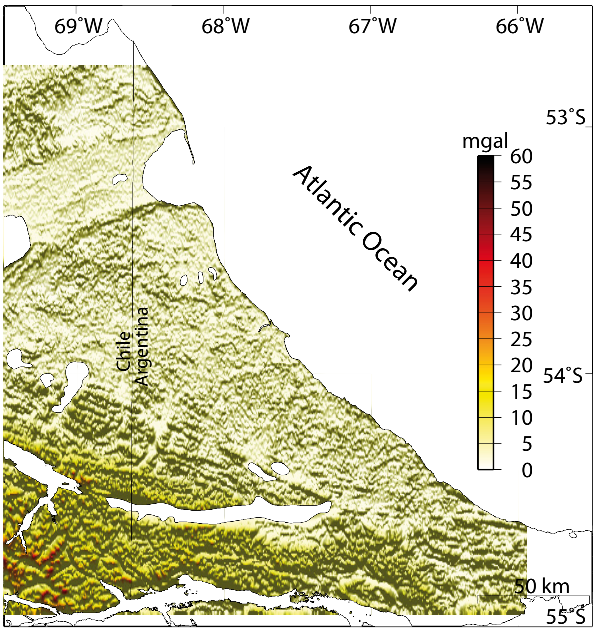

FIGURE 3 Terrain corrections estimated on the island.

i) BP was restored

Ares"=Ares+BP(SRTM3)

ii) Ares" values were averaged on a grid of 7x7km to improve the distribution and to minimize the errors in the DEM. This is in accordance with the real separation of gravimetric data, which is close to 7km.

Alrec= <Ares">7km

The anomalies obtained in this way resulted in 654 Faye gravity anomalies distributed in most of the Argentine province of Tierra del Fuego. More details regarding this reduction scheme can be found in Featherstone (2000), Bajracharya (2003) and Sanchez (2003). As it was written in this manuscript, gravity anomalies were reduced by the short and long wavelengths of the Earth gravity field spectrum. These effects were restored after the quasigeoid model was estimated, as part of the restore step.

THE EQUIVALENT SOURCE TECHNIQUE

In order to start the computation part of the process, the EST was chosen. The EST routines were written in FORTRAN 77/90, considering previous works published by Dampney (1969), Del Cogliano (2006) and Guspí *et al.* (2004). Only the main principles of this method will be exposed because it is not new and it is beyond the scope of this article. The routines included little modifications from those published works, related to precise formulae for gravity anomalies estimation or the possibility to generate a combined model as well as a geometric or just a gravimetric model. The EST is based on the premise of the inverse gravimetric problem: the external gravitational potential can be uniquely determined by its values on an equipotential surface. However, the distribution of masses below this surface, and responsible for such gravitational potential, is non-unique. Point masses or equivalent sources are located below each observation site at a depth proportional to the distance of the nearest station of the same kind (Fig. 5). This is done for each observation type. Depth factors are required as an input in the EST routine and they are empirically determined. The masses are estimated by means of a dump least squares process, which adjusts a set of masses capable of reproducing observed height anomalies, geoid undulations, gravity anomalies or any quantity of the gravity field, which can be measured on the topographic surface. EST can work just with one observation type (*i.e.* only gravity data, to obtain a gravimetric geoid) or more than one type. In this case, gravity anomalies were combined with height anomalies in order to extend the model as much as possible in Tierra del Fuego.

QUASIGEOID GENERATED MODEL

The reduced 654 gravity anomalies were introduced as part of the input data in the EST software. At the same

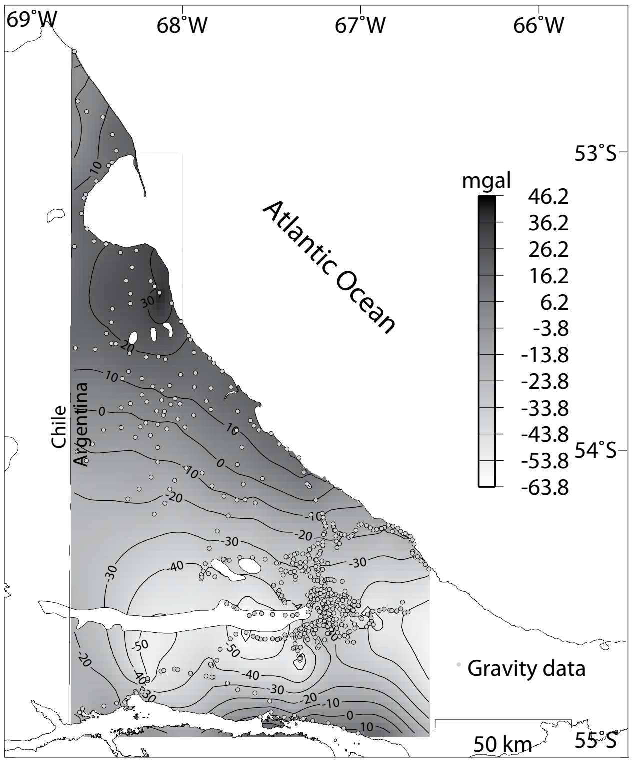

FIGURE 4 Map of Bouguer anomalies. The contours are valid only in the area covered with gravity data.

time, 58 residual height anomalies were also included. They were called "residuals" because the GGM was removed from them at the same degree and order as it was used for the gravimetric reduction. This data generated a model with 712 equivalent sources. The distribution of gravimetric information was almost equal to that of GPS/leveling data, thus the depth factors were similar for both kinds of observables. The error assumed for a GPS/levelling observation was 2cm and the corresponding for the reduced gravity anomalies was 1mgal. The present model included all the observed height anomalies. Therefore, a cross validation procedure (Fotopoulos, 2003) over the height anomalies was prepared to evaluate its quality on quasigeoid prediction. Cross validation consists of a set of data (sample) and the estimation leaves one observation out from the computation of the model. Then, the model is evaluated in the observation which is left aside. Once the evaluation is done, the observation goes back to the sample and the next observation is used as test. This repeated procedure was done with the 58 heights anomalies, always keeping the same set of gravity anomalies. The described process resulted in 58 error values with an r.m.s. of 6cm and a mean value of 1cm. In Figure 6 the error distribution after the cross validation procedure is shown. In most of the study region, the errors range from 0 to 5cm. As expected, the largest errors occur in the border zone reaching 19cm, especially in the West side where three GPS/levelling points that belong to Chile were included in the statistic (see Fig. 1.). As it was explained in the corresponding section, these points were included as part of the model because they contributed with significant contour information. When these points, located in the Chilean side, are excluded from the statistic, the obtained model presents a 5cm r.m.s. The residuals shown in Figure 6 were determined on the levelling lines and the 5cm are representative of those populated places where there is either gravimetric or GPS/levelling information.

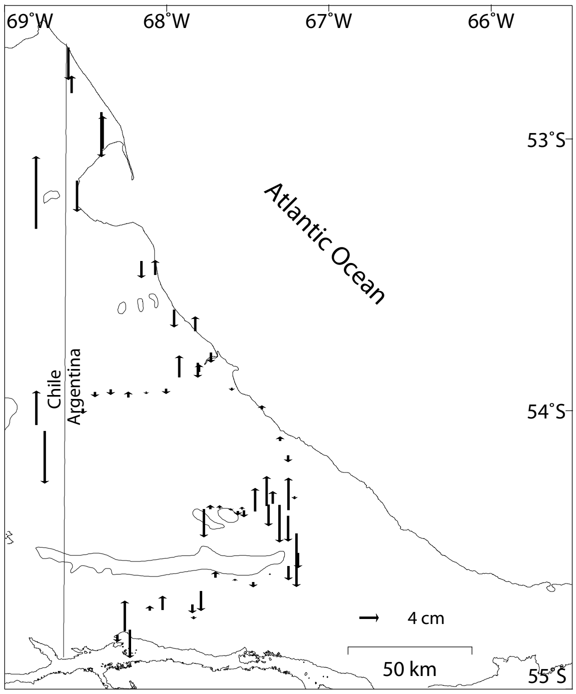

FIGURE 6 Distribution of errors obtained after the cross validation process. As it can be seen, the largest residuals take place near the data limits or close to the highest places.

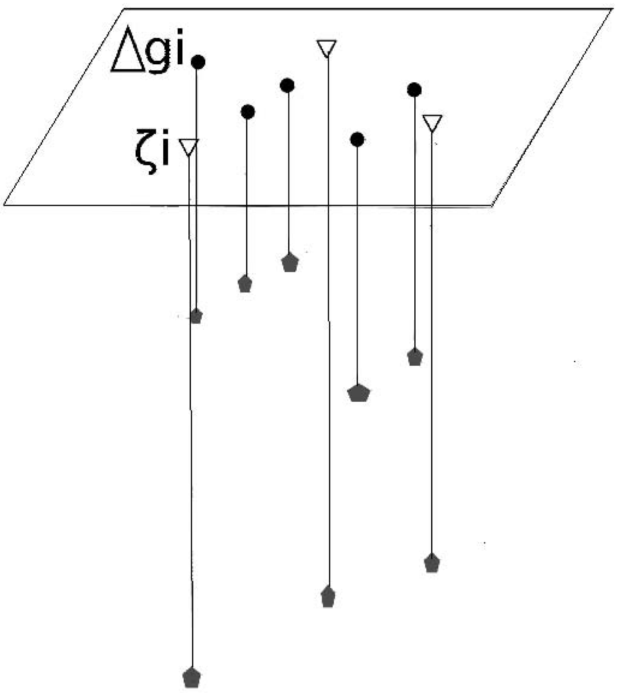

FIGURE 5 Equivalent sources distribution scheme. Dots represent gravity stations, while triangles, GPS/levelling marks. Pentagons represent point masses below each station.

QUASIGEOID OR GEOID MODEL

In the present work, the quasigeoid is computed. The geoid can be approximated after computing Bouguer anomalies and using the formula published by Heiskanen and Moritz (1967):

$$(N - \varsigma)[m] = \overline{AB}[gal].H[km]$$

AB being a mean Bouguer anomaly and *H* being an orthometric height. *N* represents geoid undulations and ζ, height anomalies.

Considering the formula above, the existent gravimetric information and the digital elevation model SRTM3, it is possible to compute the geoid–quasigeoid differences. These differences show the distribution and magnitude exposed in Figure 7. According to Figure 7, we may consider that the quasigeoid approximates the geoid at the 2cm level over the area where the model is valid.

EGM2008 EVALUATION ON GPS/LEVELLING LINES

When EGM2008 is evaluated in the existent GPS/levelling points, it presents the geographic distribution of errors shown in Figure 8. The statistic of this evaluation indicates a standard deviation of 9cm and a mean value of 35cm, being the latter the result of datum differences. That means that the adjustment of EGM2008 and observations is in accordance with its published accuracy. It is important to remark that the gravimetric data included in such geopotential model is not representative of the entire region but of the areas located not so far from the levelling lines. The mentioned GGM has no real information included in the southwestern part of Tierra del Fuego, which causes bigger discrepancies when the geopotential model is externally validated. This has been part of another study (Gomez *et al.*, 2013).

Comparison between EGM2008 and the new quasigeoid model

Figure 9 presents the new quasigeoid model compared to the corresponding one derived from EGM2008. The

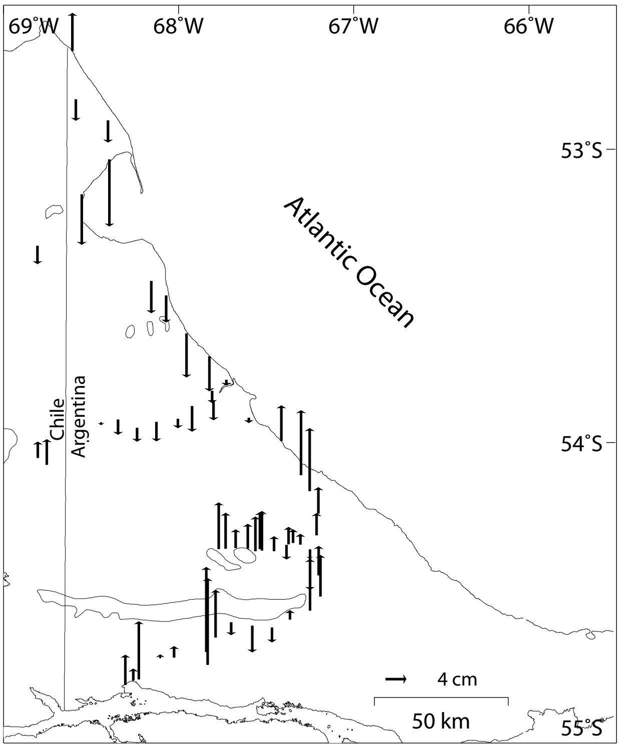

FIGURE 8. Differences obtained after the evaluation of EGM2008 in 58 GPS/levelling marks. The mean value of the differences was subtracted.

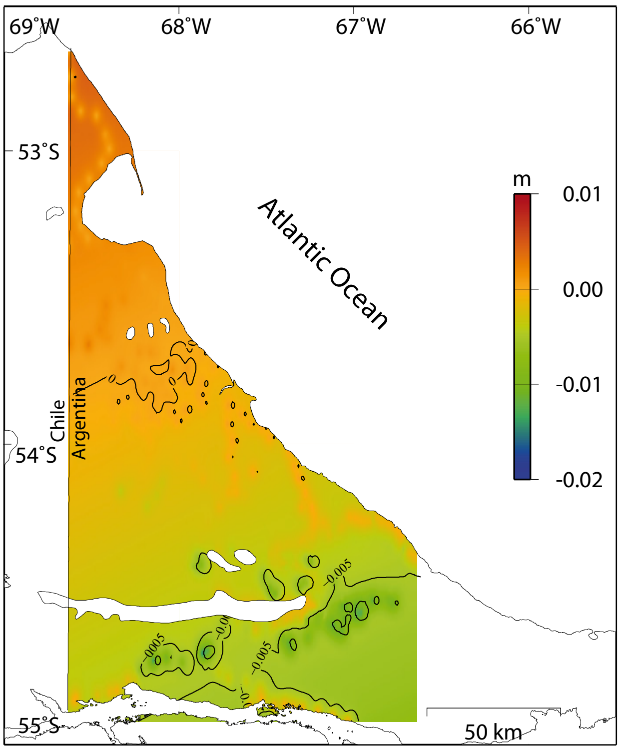

FIGURE 7. Distribution map of the differences between geoid undulations (*N*) and height anomalies (ζ) in Tierra del Fuego. The maximum, in absolute value, is 1.4cm.

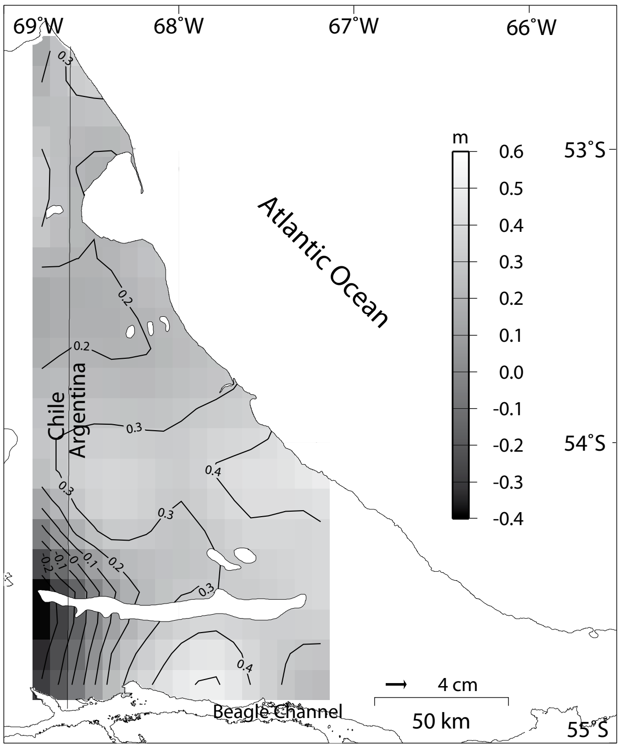

FIGURE 9. Differences obtained after the comparison of the new quasigeoid model and EGM2008 on a grid of 20km resolution.

comparison was done on a grid of 20km resolution that covered the region of study. The agreement between both models can be appreciated in those places where EGM2008 has included observed data, but not in the southwest part of the province where there is a lack of information. The standard deviation of the differences is 15cm for the whole grid.

SUMMARY, CONCLUSIONS AND REMARKS

The information of GPS/levelling in combination with gravity data allowed us to obtain a 5cm quasigeoid model for most of the Argentine part of the island. This has been made through a basic concept applied by the EST. It is a classic method but not so much used in geoid/quasigeoid modelling. It was also shown that the formally obtained quasigeoid can be considered a geoid model for the most of practical purposes at the level of a few centimetres. Regarding the method applied, the EST is capable enough of reproducing the quasigeoid by means of gravimetric and geometric observations. It is worth saying that the standard deviation reaches 9cm when EGM2008 geoid undulations are compared to those implied by GPS/levelling data. This agreement is also seen between the presented quasigeoid model and EGM2008, except in the South–West areas where none of these models have enough observed gravimetric information on land. Tierra del Fuego is located in a tectonically complex region related to the South America and Scotia plates transform boundary. The knowledge of a geoid with this precision and resolution could be useful to validate cortical models considering the interesting geological and geophysical characteristics of the region.

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