



Inflationary gravitational waves in collapse scheme models



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ARTICLE INFO

Article history:

Received 22 December 2014
 Received in revised form 18 September 2015
 Accepted 29 November 2015
 Available online 2 December 2015
 Editor: S. Dodelson

Keywords:

Cosmology
 Inflation
 Collapse schemes
 Gravitational waves

ABSTRACT

The inflationary paradigm is an important cornerstone of the concordance cosmological model. However, standard inflation cannot fully address the transition from an early homogeneous and isotropic stage, to another one lacking such symmetries corresponding to our present universe. In previous works, a self-induced collapse of the wave function has been suggested as the missing ingredient of inflation. Most of the analysis regarding the collapse hypothesis has been solely focused on the characteristics of the spectrum associated to scalar perturbations, and within a semiclassical gravity framework. In this Letter, working in terms of a joint metric-matter quantization for inflation, we calculate, for the first time, the tensor power spectrum and the tensor-to-scalar ratio corresponding to the amplitude of primordial gravitational waves resulting from considering a generic self-induced collapse.

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1. Introduction

The vast majority of the cosmological community considers the inflationary paradigm on a stronger footing than ever given the agreement between its predictions and the latest observations (e.g. WMAP9 [1], Planck [2]). In particular, last year a claim by BICEP2 Collaboration regarding the detection of primordial tensor modes [3], in spite of the subsequent controversy of their results [4–6], has made some cosmologists think that this important prediction from the traditional inflation model will be confirmed in the foreseeable future, which in turn will reassert the standing of the model.

According to the traditional inflationary paradigm, the early universe undergoes an accelerated expansion (lasting at least some 70 e -folds or so), resulting in an essentially flat, homogeneous and isotropic space–time with an extreme dilution of all unwanted relics. Note that the dynamics of the space–time is governed by Einstein equations which are symmetry preserving, i.e. the symmetry being the homogeneity and isotropy (H&I). Another important aspect is that when considering the quantum features of the scalar field (the inflaton) driving the expansion. This field, is assumed to be in the vacuum state as a result of the same

exponential expansion, and one finds also that it contains “fluctuations” with the appropriate nearly-scale-invariant spectrum. These vacuum fluctuations are considered responsible for all the structures we observe in the actual universe, and in particular, the observed cosmic microwave background (CMB) anisotropies.

One cannot deny the favorable matching between the model predictions and observations; nevertheless, from the conceptual point of view something is missing. Even if the inflaton contains quantum uncertainties (or vacuum fluctuations), according to the Quantum Theory, the physical state of the system is encoded in the quantum state. The vacuum state of the quantum fields is H&I, i.e. it is an eigen-state of the operators generating spatial translations and rotations (see Appendix A of Ref. [7] for a proof). The fact that a system contains quantum uncertainties does not necessarily implies that it contains actual inhomogeneities and anisotropies, since the quantum state, which characterize the physical state of the system, can still be perfectly H&I. Additionally, the dynamics of the quantum state is governed by Schrödinger equation, which does not break translational and rotational invariance. Consequently, the initial quantum state cannot be evolved into a final state lacking such symmetries. Thus, there is an important issue, namely: what is the precise mechanism by which the primordial perturbations are born given that the equations governing the dynamics are symmetry preserving? In other words, it is not clear how from an initial condition that is H&I (both in the background

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space–time and in the quantum state that characterizes the quantum fields), and based on a dynamics that supposedly preserves those symmetries, one ends up with a non-homogeneous and non-isotropic state associated to the late observed universe.

The above described situation is sometimes related to the issue of the quantum-to-classical transition of the primordial quantum fluctuations. And, although decoherence provides a partial understanding of the issue [8,9], it does not fully address the problem; mainly because decoherence does not solve the quantum measurement problem, which appears in an exacerbated manner in the case of the inflationary universe. We invite the interested reader to consult, for instance, Refs. [10,11] where a more detailed analysis has been made regarding the issues with decoherence and other approaches to the problem at hand.

In order to account for the aforementioned problem, Sudarsky et al. [10] proposed a self-induced collapse of the wave function, i.e. a spontaneous change from the original quantum state associated to the inflaton field into a new quantum state lacking the symmetries of the initial state. Also, their approach relies on the semiclassical gravity framework, in which matter is described by a Quantum Field Theory and the space–time is always treated in a classical manner. The self-induced collapse is considered as being the responsible of generating the primordial perturbations. In particular, by relying on Einstein semiclassical equations, the expectation value in the post-collapse state of the quantum matter fields is related to the metric of the space–time which is always classical. The result of the evolution of the metric perturbations, born after the collapse, is related to the actual anisotropies and inhomogeneities observed in the CMB radiation. Thus, in this proposal, after the collapse, the universe is described by a space–time and a quantum state that are no longer H&I.

On the other hand, it is evident that the collapse mechanism should be a physical process independent of external entities, since in the early universe there is not a clear notion of observers, measurement devices, environment, etc. It is worthwhile to comment that models involving an objective dynamical reduction of the wave function (in different contexts from cosmology) have been proposed in past years [12–17]. These models attempt to provide a solution to the so-called measurement problem of Quantum Mechanics by eliminating from the theory the need of an external agent responsible for localizing the wave function. It is also interesting that these models give predictions that can be tested experimentally and that are different from the standard Quantum Theory [18]. We will not deal with all the conceptual framework concerning the self-induced collapse and instead we will refer the interested reader to Refs. [7,10,11,19] for a more in depth analysis.

Previous works, e.g. [10,20,21], have analyzed the characteristics of the spectrum associated with the scalar perturbations resulting from considering the self-induced collapse hypothesis in different inflationary scenarios, e.g. multiple collapses [22], correlation between the modes caused by the collapse [23], collapse occurring during the radiation dominated era [24], and also in a non-inflationary model [25]. Moreover, in Ref. [26] two quantum collapse schemes were tested with recent data from the CMB, including the 7 year release of WMAP [27] and the matter power spectrum measured using LRGs by the Sloan Digital Sky Survey [28]. However, as we have mentioned, most previous mentioned works have been based on the semiclassical gravity approximation, which enables a quantum treatment of the matter fields, while a classical description of gravitation is maintained. In particular, the amplitude of primordial tensor modes provided by the collapse hypothesis, within the semiclassical gravity approximation, is exactly zero at first-order in perturbation theory [10,21]. At second-order,

the model prediction for the amplitude is too low that is practically undetectable by any recent and future experiments [29].

On the other hand, last year an allegation concerning the detection of primordial B -modes polarization of the CMB by BICEP2 Collaboration [3] (notwithstanding the apparent tension with the results provided by Planck mission and a strong evidence of probable contamination by Galactic dust [30]), has made the revelation of primordial gravity waves a real possibility. In the plausible scenario of a confirmed detection of primordial B -mode polarization, the framework of semiclassical gravity applied to the inflationary universe faces several issues, nevertheless, one could still implement the self-induced collapse hypothesis. One possible option (and probably the simplest) is to apply the collapse proposal directly within the standard analysis, in terms of a quantum field jointly characterizing the inflaton and metric perturbations, the so-called Mukhanov–Sasaki variable. In Ref. [31] a first step, regarding the implications of considering the collapse of the wave function characterizing the state of the quantum field associated to the Mukhanov–Sasaki variable, was made. In particular, it was shown that the standard shape of the spectrum associated to the scalar perturbations becomes altered by introducing the collapse hypothesis. Furthermore, in Refs. [32,33] a particular objective collapse model, called Continuous Spontaneous Localization (CSL) collapse model [14–16], was implemented resulting in interesting modifications to the standard scalar power spectrum corresponding to the Mukhanov–Sasaki variable field.

In this work, we will make a step further and obtain the spectrum associated to the tensor modes within the framework of quantizing both the matter and metric perturbations. We will show that, as in the scalar case, the tensor power spectrum becomes modified by introducing the collapse hypothesis. Additionally, we will obtain the tensor-to-scalar ratio r and show that it is of the same order of magnitude as the one predicted by standard single-field slow-roll inflation. Nevertheless, an interesting result is that r is independent of the collapse parameters. Thus, the precise measurement of r sets the energy scale of inflation (the same as in the standard case), but cannot yield any significant information concerning the collapse. Moreover, we will not consider a specific collapse mechanism, but we will parameterize the collapse generically through the expectation values of the field and its conjugated momentum evaluated in the post-collapse state. It is worthwhile to mention that, in Ref. [34], the CSL collapse model was used to analyze the tensor modes in the same context as the present work, in relation to the quantum treatment of the fields. The authors conclude that accurate measurements of r and the tensor spectral index n_T can help to constraint such model parameters. However, their point of view regarding the physical implications of the collapse is different from ours. Specifically, in our picture if there is no quantum collapse the quantum state of the field is homogeneous and isotropic and there are no perturbations of the space–time, thus $r = 0$. On the other hand, within the model analyzed in [34], in the absence of a quantum collapse one recovers the standard inflationary predictions concerning the tensor and scalar power spectra. This is an important distinction with further implications regarding the observational quantities, as it will be shown in future work, but more importantly it constitutes a difference in the physical implication of the self-induced collapse.

The present Letter is organized as follows: in Section 2 we review some basics about previous results regarding the power spectrum of scalar perturbations, in the framework of collapse scheme models and working in terms of a joint metric-matter quantization for inflation; in Section 3 we show our results for the power spectrum of tensor modes and the tensor-to-scalar ratio; and finally, in Section 4 we summarize our conclusions.

2. Brief review of previous results

In this Section, we will present a brief review of the results obtained in Ref. [31], where the self-induced collapse hypothesis was added to the standard quantum treatment characterizing the primordial perturbations, namely to the scalar field associated to the Mukhanov–Sasaki variable. Specifically, we will mention the problem with the standard picture, and then we will motivate the addition of the self-induced collapse. Later, we will focus on the power spectrum corresponding to the scalar perturbations within our model. There is no original work in this Section, and detailed analyses can be found in Refs. [7,10,11,21].

We start characterizing the inflationary universe by Einstein theory $G_{ab} = 8\pi G T_{ab}$ ($c = 1$) along with the dynamics of the matter fields corresponding to the inflaton. Also, we shall work with the standard single-field slow-roll inflaton ϕ . Specifically, the background space-time is described by an approximately de Sitter expansion. Thus, the scale factor, in conformal time η , is given by $a(\eta) \simeq -1/H\eta$ with H the Hubble parameter, approximately constant. On the other hand, the matter sector is dominated by the inflaton, which is “rolling slowly” down the potential V ; consequently, the slow-roll parameter is defined $\epsilon \equiv 1 - \mathcal{H}'/\mathcal{H}^2$. Here, a prime denotes partial derivative with respect to conformal time η , and $\mathcal{H} \equiv a'/a$ is the conformal expansion rate. Also, during slow-roll inflation $\epsilon \simeq M_p^2/2(\partial_\phi V/V)^2$ where $M_p^2 \equiv (8\pi G)^{-1}$ is the reduced Planck mass; additionally, we will work with the assumption that $\epsilon = \text{constant}$.¹

We choose to work in the longitudinal gauge, and we assume no anisotropic stress. So, the scalar perturbations of the metric are represented, in comoving coordinates, by the following line element:

$$ds^2 = a^2(\eta)[-(1 + 2\Psi)d\eta^2 + (1 - 2\Psi)\delta_{ij}dx^i dx^j] \quad (1)$$

with $\Psi(\eta, \vec{x}) \ll 1$. Decomposing the scalar field into an homogeneous and isotropic part plus small perturbations $\phi(\vec{x}, \eta) = \phi_0(\eta) + \delta\phi(\vec{x}, \eta)$, one can construct the Mukhanov–Sasaki variable

$$v \equiv a \left(\delta\phi + \frac{\phi'_0}{\mathcal{H}} \Psi \right). \quad (2)$$

Einstein perturbed equations at first-order $\delta G_{ab} = 8\pi G \delta T_{ab}$, imply

$$\nabla^2 \Psi = -\sqrt{\frac{\epsilon}{2}} \frac{H}{M_p} \left(v' - \frac{z'}{z} v \right) \quad (3)$$

where $z \equiv a\phi'_0/\mathcal{H}$. Moreover, since we are assuming an approximately de Sitter expansion, i.e. assuming $\epsilon' = 0$ and slow-roll type of inflation, then $z'/z = a'/a$. It is important to mention that in the longitudinal gauge, the field Ψ represents the curvature perturbation of the background and is related to the Mukhanov–Sasaki variable v as in Eq. (3).

As it is well known, one of the advantages of working with the variable v is that the quantum theory of primordial perturbations is reduced to an action describing a free scalar field with a time-dependent mass term. The question then is: which are the appropriate observables that emerge from the quantum theory encoded in the quantum field \hat{v} ?

The standard answer is the power spectrum, which is normally associated with the quantum two-point correlation function of the quantum field \hat{v} . That is, the quantum theory of the variable v , simultaneously sets the quantum theory of $\delta\phi$ and $\hat{\Psi}$ [see Eq. (2)]. Afterwards, one calculates the Fourier transform of $\langle 0|\hat{\Psi}(\vec{x}, \eta)\hat{\Psi}(\vec{y}, \eta)|0\rangle$ and relates it with the scalar power spectrum of the curvature perturbation. In other words, in the standard approach, one identifies the Fourier transform of the quantum two-point correlation function with an average over an ensemble of classical anisotropic universes of the same correlation function:

$$\langle 0|\hat{\Psi}_{\vec{k}}\hat{\Psi}_{\vec{k}'}|0\rangle = \overline{\Psi_{\vec{k}}\Psi_{\vec{k}'}} \equiv 2\pi^2\delta(\vec{k} + \vec{k}')P_\Psi(k). \quad (4)$$

As mentioned in Sec. 1, one usually encounters in the literature that decoherence helps to understand the identification made in Eq. (4), e.g. [8,9]. The line of reasoning is as follows: the dynamics of the inflationary universe leads the vacuum state of the field \hat{v} to a highly squeezed state, and in this limit, all the quantum predictions can be reproduced if one assumes that the system always followed classical laws but had random initial conditions with a given probability density function. Although we do not subscribe to such posture (for a detailed analysis see Refs. [7,11]), that argument alone does not say anything concerning the physical mechanism leading to a particular realization of the field Ψ corresponding to our universe. One cannot apply the usual postulates of Quantum Mechanics based on the Copenhagen interpretation, since entities such as observers, measurements or measurement devices are not well defined in the early universe. Moreover, even if in principle there exist many universes, the fact is that we only have observational access to one – our own – universe. Therefore, the situation is completely different than the ordinary laboratory setup, where one would check that the predictions provided by the Quantum Theory can be verified by repeating the experiment many times.

The previous described problem can be addressed by invoking a self-induced collapse of the wave function [10,11]. In particular, we assume that the vacuum state associated to each mode of the field $\hat{v}_{\vec{k}}$ spontaneously changes at a certain time $\eta_{\vec{k}}^c$, called the time of collapse, into a new state, i.e. $|0_{\vec{k}}\rangle \rightarrow |\Xi_{\vec{k}}\rangle$. The state $|\Xi_{\vec{k}}\rangle$ is no longer invariant under rotations and spatial translations. Thus, the post-collapse state characterizing the field is no longer homogeneous and isotropic. These collapses for each mode will be assumed to occur according certain rules called *collapse schemes*, and we will detail them in the next Section.

At this point, we must focus on the connection between the classical and quantum prescriptions. In particular, here we will focus on the scalar perturbation Ψ , representing the curvature perturbation, which is intrinsically related to the temperature anisotropies of the CMB. As precisely explained in Ref. [31], the relation between $\hat{\Psi}$ and Ψ is made by taking the view that the classical description, encoded in Ψ , is only relevant for those particular states for which the quantity in question is sharply peaked and that the classical description corresponds to the expectation value of said quantity. For example, one can take the wave packet characterizing a free particle, where clearly the wave function is sharply peaked around some value of the position. In that context, one could claim that the particle position is well defined and corresponds to the expectation value of the position operator in that state described by the wave packet. Given the previous discussion, we identify

$$\Psi(\vec{x}, \eta) = \langle \Xi|\hat{\Psi}(\vec{x}, \eta)|\Xi\rangle, \quad (5)$$

with $|\Xi\rangle$ a state of the quantum field $\hat{v}(x)$ characterizing jointly the metric and the field perturbation, which only acquires a physical meaning as long as the state corresponds to a sharply peaked

¹ As it is well known, assuming ϵ to be exactly constant leads to a perfect scale-invariant power spectrum (scalar and tensor). It is only by considering $\epsilon' \neq 0$, also known as a quasi-de Sitter expansion, that one obtains a small dependence on k in the traditional power spectrum of the form k^{n_s-1} with $n_s \neq 1$. On the other hand, in this work we are mainly interested in the amplitude and not in the shape of the tensor spectrum. Therefore, we can perform all the analysis in an the approximation that ϵ is exactly a constant, without loss of generality, and finally obtain the amplitude of the corresponding spectra.

one associated to the quantum field $\hat{\Psi}(x)$. In other words, after establishing the quantum theory of \hat{v} , Eqs. (3) and (5) imply

$$\nabla^2 \Psi = \nabla^2 \langle \hat{\Psi} \rangle = -\sqrt{\frac{\epsilon}{2}} \frac{H}{M_P} \left(\langle \hat{v}' \rangle - \frac{z'}{z} \langle \hat{v} \rangle \right) \quad (6)$$

It is worthwhile to mention that if we consider the vacuum state, as it is in the standard approach, we would have $\langle 0 | \hat{\Psi}(\vec{x}, \eta) | 0 \rangle = \Psi(\vec{x}, \eta) = 0$. Consequently, the space-time would be perfect homogeneous and isotropic. It is only after the collapse that generically $\langle \Xi | \hat{\Psi}(\vec{x}, \eta) | \Xi \rangle = \Psi(\vec{x}, \eta) \neq 0$. This illustrates how the metric perturbations are born from the self-induced collapse.

After establishing how the primordial curvature perturbation is generated within our approach, we can make contact with the observational quantities. This is, we can extract the scalar power spectrum from

$$\overline{\Psi_{\vec{k}} \Psi_{\vec{k}'}} = \overline{\langle \Xi_{\vec{k}} | \hat{\Psi}_{\vec{k}} | \Xi_{\vec{k}} \rangle \langle \Xi_{\vec{k}'} | \hat{\Psi}_{\vec{k}'} | \Xi_{\vec{k}'} \rangle} \quad (7)$$

The bar appearing in $\overline{\Psi_{\vec{k}}(\eta) \Psi_{\vec{k}'}(\eta)}$ denotes an average over possible realizations of $\Psi_{\vec{k}}$, which is a random field and its randomness is inherited by the stochastic nature of the collapse. In other words, the average is over possible outcomes of the field $\Psi_{\vec{k}}$. The set of all modes of the field $\{\Psi_{\vec{k}_1}, \Psi_{\vec{k}_2}, \Psi_{\vec{k}_3}, \dots\}$ characterizes a particular universe \mathcal{U} . Thus, the average is over possible realizations characterizing different universes $\mathcal{U}_1, \mathcal{U}_2, \dots$. Our universe, is just one particular materialization \mathcal{U}^* . Note that this is different from the standard inflationary account, in which the power spectrum is obtained from $\langle 0 | \hat{\Psi}_{\vec{k}}(\eta) \hat{\Psi}_{\vec{k}'}(\eta) | 0 \rangle$, with all the mentioned shortcomings. Meanwhile, in our picture, the power spectrum is obtained from the expression $\langle \Xi_{\vec{k}} | \hat{\Psi}_{\vec{k}} | \Xi_{\vec{k}} \rangle \langle \Xi_{\vec{k}'} | \hat{\Psi}_{\vec{k}'} | \Xi_{\vec{k}'} \rangle$ where every element can be clearly justified.

Finally, the scalar power spectrum, within the collapse proposal, is [31]:

$$P_{\Psi}(k) \propto \frac{H^2}{\epsilon M_P^2} C(z_k) \quad (8)$$

with

$$C(z_k) \equiv \lambda_{\pi}^2 \left(1 - \frac{1}{z_k^2} + \frac{1}{z_k^4} \right) \left[\cos z_k - \frac{\sin z_k}{z_k} \right]^2 + \lambda_v^2 \left(1 + \frac{1}{z_k^2} \right) \left[\frac{\cos z_k}{z_k} - \left(\frac{1}{z_k^2} - 1 \right) \sin z_k \right]^2 \quad (9)$$

The parameters λ_{π} and λ_v can only take the values 0 or 1 depending on which variable is affected by the collapse, e.g. if only the momentum is affected by the collapse then $\lambda_{\pi} = 1$ and $\lambda_v = 0$. The parameter z_k is defined as $z_k \equiv k \eta_k^c$, so it is directly related to the time of collapse η_k^c . Therefore, the time of collapse substantially modifies the scalar power spectrum in a very particular manner that, in principle, can be used to distinguish it from the traditional prediction. The specific technical details regarding the implementation of the self-induced collapse hypothesis that guided to result (8) can be consulted in Refs. [19,31]; nevertheless, the steps are quite similar to the ones that will be presented in the next section concerning the tensor modes.

Another important aspect concerning the collapse scalar power spectrum (8), is that it is of the form $P_{\Psi}(k) = \mathcal{A}C(z_k)$, which is different from the traditional prediction $P_{\Psi}(k) = \mathcal{A}k^{n_s-1}$. The reason for this apparent difference is because the collapse spectrum was obtained using the approximation that ϵ is exactly constant, thus, leading to $n_s = 1$. We could have worked with a better approximation in which $\epsilon' \neq 0 = \text{constant}$, known as quasi-de Sitter inflation and the final result would have been of the form $P_{\Psi}(k) = \mathcal{A}Q(z_k)k^{n_s-1}$. However, it can be shown [35] that $Q(z_k) \simeq C(z_k)$ if

the time of collapse occurs during the earlier stages of the inflationary regime; furthermore, since in this article we are primarily interested in the amplitude of the tensor modes, rather than the exact shape of their spectrum, we can continue working in the approximation $\epsilon' = 0$ and, thus, use the result (8).

3. Tensor modes and the tensor-to-scalar ratio

In order to proceed to find our results, in this Section we will study the incorporation of the self-collapse hypothesis to the description of primordial tensor perturbations.

As it is known, these perturbations represent gravitational waves, and they are characterized by a symmetric, transverse and traceless tensor field. These properties lead to the existence of only two degrees of freedom. Therefore, the tensor h_{ij} , representing the gravitational waves, is usually decomposed as [36]:

$$h_{ij} = h_+ e_{ij}^+ + h_{\times} e_{ij}^{\times} \quad (10)$$

where e_{ij}^{α} , $\alpha = +, \times$ is a time-independent polarization tensor. We will work with only one polarization $\alpha = +, \times$. As each polarization term is independent, and as each polarization leads to the same result, we will just multiply by a factor of two the spectrum associated to an individual case, at the end of our calculations, to obtain the final result.

The action for the gravitational waves can be obtained by expanding the Einstein action up to the second order in transverse, traceless metric perturbations $h_{ij}(\vec{x}, \eta)$. The result is [37,38],

$$S = \frac{1}{64\pi G} \int d^3x d\eta a^2 \left(h_{ij}' h_i^{j'} - h_{ij}^i h_j^i \right) \quad (11)$$

where the spatial indices are raised and lowered with the help of the unit tensor δ_{ik} .

Then, we expand h_{ij} in Fourier modes,

$$h_{ij}(\vec{x}, \eta) = \int \frac{d^3k}{(2\pi)^{3/2}} h_{\vec{k}}(\eta) e_{ij}(\vec{k}) e^{i\vec{k}\cdot\vec{x}}, \quad (12)$$

and substituting (12) into the action (11) it is obtained,

$$S = \frac{1}{64\pi G} \int d^3k d\eta a^2 e^i_j e_i^j \left(h_{\vec{k}}' h_{-\vec{k}}' - k^2 h_{\vec{k}} h_{-\vec{k}} \right) \quad (13)$$

Next, we perform the change of variable:

$$v_{\vec{k}} = \sqrt{\frac{e^i_j e_i^j}{32\pi G}} a h_{\vec{k}} \quad (14)$$

and then, the action (13) can be rewritten as:

$$S = \frac{1}{2} \int d^3k d\eta \left[v_{\vec{k}}' v_{-\vec{k}}' - \left(k^2 - \frac{a''}{a} \right) v_{\vec{k}} v_{-\vec{k}} \right]. \quad (15)$$

This action describes a real scalar field in terms of its Fourier transform,

$$v(\vec{x}, \eta) = \int \frac{d^3k}{(2\pi)^{3/2}} v_{\vec{k}}(\eta) e^{i\vec{k}\cdot\vec{x}} \quad (16)$$

Thus, the action for the variable $v(\vec{x}, \eta)$ results:

$$S = \frac{1}{2} \int d^3x d\eta \left[(v')^2 - (v_{,i})^2 + \frac{a''}{a} v^2 \right]. \quad (17)$$

Note that the momentum canonical to $v(\vec{x}, \eta)$ is $\pi(\vec{x}, \eta) \equiv \frac{\partial(\sqrt{-g}\mathcal{L})}{\partial v'} = v'(\vec{x}, \eta)$.

In the quantization process, the field $v(\vec{x}, \eta)$ and its conjugate momentum $\pi(\vec{x}, \eta)$ are promoted to operators acting on a Hilbert

space \mathcal{H} . These satisfy the standard equal time commutation relations:

$$[\hat{v}(\vec{x}, \eta), \hat{v}(\vec{x}', \eta)] = [\hat{\pi}(\vec{x}, \eta), \hat{\pi}(\vec{x}', \eta)] = 0$$

$$[\hat{v}(\vec{x}, \eta), \hat{\pi}(\vec{x}', \eta)] = i\delta(\vec{x} - \vec{x}')$$

The standard procedure is to decompose \hat{v} and $\hat{\pi}$ in terms of the time-independent creation and annihilation operators. For practical reasons, we will work with periodic boundary conditions over a box of size L , where $k_i L = 2\pi n_i$ for $i = 1, 2, 3$. So we write,

$$\hat{v}(\vec{x}, \eta) = \frac{1}{L^{3/2}} \sum_{\vec{k}} \hat{v}_{\vec{k}}(\eta) e^{i\vec{k}\cdot\vec{x}}$$

$$\hat{\pi}(\vec{x}, \eta) = \frac{1}{L^{3/2}} \sum_{\vec{k}} \hat{\pi}_{\vec{k}}(\eta) e^{i\vec{k}\cdot\vec{x}}$$

where $\hat{v}_{\vec{k}}(\eta) = v_k(\eta)\hat{\beta}_{\vec{k}} + v_k^*(\eta)\hat{\beta}_{-\vec{k}}^\dagger$ and $\hat{\pi}_{\vec{k}}(\eta) = v'_k(\eta)\hat{\beta}_{\vec{k}} + v_k^{*\prime}(\eta)\hat{\beta}_{-\vec{k}}^\dagger$.

The mode functions $v_k(\eta)$ are normalized such that

$$v_k^* v'_k - v_k v_k^{*\prime} = -i, \quad (18)$$

and then, the creation and annihilation operators $\hat{\beta}_{\vec{k}}$ y $\hat{\beta}_{\vec{k}}^\dagger$ satisfy the commutation relations:

$$[\hat{\beta}_{\vec{k}}, \hat{\beta}_{\vec{k}'}^\dagger] = [\hat{\beta}_{\vec{k}}^\dagger, \hat{\beta}_{\vec{k}'}] = 0$$

$$[\hat{\beta}_{\vec{k}}, \hat{\beta}_{\vec{k}'}] = \delta(\vec{k} - \vec{k}')$$

From (17), the equation of motion for $v_{\vec{k}}$ results:

$$v_k'' + \left(k^2 - \frac{a''}{a}\right) v_k = 0 \quad (19)$$

As it was mentioned earlier, we are working in an approximately *de Sitter* inflation where $H \simeq \text{const.}$, and hence $a(\eta) \simeq -1/(H\eta)$. Using this approximation, the last equation takes the form:

$$v_k'' + \left(k^2 - \frac{2}{\eta^2}\right) v_k = 0 \quad (20)$$

whose solution, choosing the Bunch–Davies vacuum as the initial state, is:

$$v_k = \frac{1}{\sqrt{2k}} \left(1 - \frac{i}{k\eta}\right) e^{-ik\eta}. \quad (21)$$

At this point, we introduce the self-induced collapse proposal: we suppose that at time (dependent on the mode k), η_k^c , called the *time of collapse*, the vacuum state associated to each mode of the field $\hat{v}_{\vec{k}}$ spontaneously changes into a new state, i.e. $|0_{\vec{k}}\rangle \rightarrow |\Xi_{\vec{k}}\rangle$. The state $|\Xi_{\vec{k}}\rangle$ is no longer invariant under rotations and spatial translations. Thus, the post-collapse state characterizing the field is no longer homogeneous and isotropic. We will not consider a specific collapse mechanism, but we will parameterize the collapse through the expectation values of the field and its conjugated momentum evaluated in the post-collapse state, as it will be shown below.

In order to proceed, we decompose the operators $\hat{v}_{\vec{k}}(\eta)$ and $\hat{\pi}_{\vec{k}}(\eta)$ in their real and imaginary parts,

$$\hat{v}_{\vec{k}}(\eta) = \hat{v}_{\vec{k}}^R(\eta) + i\hat{v}_{\vec{k}}^I(\eta) \quad (22)$$

$$\hat{\pi}_{\vec{k}}(\eta) = \hat{\pi}_{\vec{k}}^R(\eta) + i\hat{\pi}_{\vec{k}}^I(\eta) \quad (23)$$

where

$$\hat{v}_{\vec{k}}^{R,I}(\eta) = \frac{1}{\sqrt{2}} \left(v_k(\eta)\hat{\beta}_{\vec{k}}^{R,I} + v_k^*(\eta)\hat{\beta}_{\vec{k}}^{\dagger R,I} \right), \quad (24)$$

and

$$\hat{\pi}_{\vec{k}}^{R,I}(\eta) = \frac{1}{\sqrt{2}} \left(v'_k(\eta)\hat{\beta}_{\vec{k}}^{R,I} + v_k^{*\prime}(\eta)\hat{\beta}_{\vec{k}}^{\dagger R,I} \right) \quad (25)$$

with $\hat{\beta}_{\vec{k}}^R = \frac{1}{\sqrt{2}}(\hat{\beta}_{\vec{k}} + \hat{\beta}_{-\vec{k}})$ and $\hat{\beta}_{\vec{k}}^I = \frac{-i}{\sqrt{2}}(\hat{\beta}_{\vec{k}} - \hat{\beta}_{-\vec{k}})$. In this manner, $\hat{v}_{\vec{k}}^{R,I}(\eta)$ and $\hat{\pi}_{\vec{k}}^{R,I}(\eta)$ are Hermitian operators, which we know from standard Quantum Mechanics that these kind of operators can be subjected to a “measurement” type of process.

The commutation relations for these operators read,

$$\begin{aligned} [\hat{\beta}_{\vec{k}}^R, \hat{\beta}_{\vec{k}'}^{\dagger R}] &= \delta_{\vec{k},\vec{k}'} + \delta_{\vec{k},-\vec{k}'} \\ [\hat{\beta}_{\vec{k}}^I, \hat{\beta}_{\vec{k}'}^{\dagger I}] &= \delta_{\vec{k},\vec{k}'} - \delta_{\vec{k},-\vec{k}'} \end{aligned} \quad (26)$$

with all the other commutators vanishing. Note that, in the last equation, \vec{k} and $-\vec{k}$ are not independent.

Being $\hat{v}_{\vec{k}}^{R,I}(\eta)$ and $\hat{\pi}_{\vec{k}}^{R,I}(\eta)$ Hermitian operators, we will evaluate their expectation values:

$$\begin{aligned} \langle \hat{v}_{\vec{k}}^{R,I}(\eta_k^c) \rangle_{\Xi} &= \lambda_v x_{k,1}^{R,I} \sqrt{[\Delta \hat{v}(\eta_k^c)]_0^2} \\ &= \lambda_v x_{k,1}^{R,I} \frac{1}{\sqrt{2}} |v_k(\eta_k^c)| \end{aligned} \quad (27)$$

$$\begin{aligned} \langle \hat{\pi}_{\vec{k}}^{R,I}(\eta_k^c) \rangle_{\Xi} &= \lambda_\pi x_{k,2}^{R,I} \sqrt{[\Delta \hat{\pi}(\eta_k^c)]_0^2} \\ &= \lambda_\pi x_{k,2}^{R,I} \frac{1}{\sqrt{2}} |v'_k(\eta_k^c)| \end{aligned} \quad (28)$$

where, the numbers $x_{k,1}^{R,I}$ and $x_{k,2}^{R,I}$ are a collection of independent random quantities (selected from a Gaussian distribution centered at zero with unit-spread), and $[\Delta \hat{v}(\eta_k^c)]_0^2$ and $[\Delta \hat{\pi}(\eta_k^c)]_0^2$ are the quantum uncertainties of the operators $\hat{v}_{\vec{k}}^{R,I}$ and $\hat{\pi}_{\vec{k}}^{R,I}$ in the vacuum state $|0\rangle$ at time η_k^c .

The parameters λ_v and λ_π are viewed as “switch-off/on” parameters. This is, they can only take the values 0 or 1 depending on which variable $\hat{v}_{\vec{k}}^{R,I}$, $\hat{\pi}_{\vec{k}}^{R,I}$ or both is affected by the collapse. For instance, in past works [10,22], the name *independent scheme* was coined for the case $\lambda_v = \lambda_\pi = 1$, i.e. $\hat{v}_{\vec{k}}^{R,I}$ and $\hat{\pi}_{\vec{k}}^{R,I}$ are both affected independently by the collapse. Nevertheless, there are other options, e.g. $\lambda_v = 0$ and $\lambda_\pi = 1$. In the rest of the present Letter, we will keep the λ_v and λ_π parameters without referring to a particular collapse scheme.

Given a post-collapse state $|\Xi\rangle$, next to equations (24) and (25), it can be seen that,

$$\langle \hat{v}_{\vec{k}}^{R,I}(\eta) \rangle_{\Xi} = \sqrt{2} \Re \left[v_k(\eta) \langle \hat{\beta}_{\vec{k}}^{R,I} \rangle_{\Xi} \right] \quad (29)$$

$$\langle \hat{\pi}_{\vec{k}}^{R,I}(\eta) \rangle_{\Xi} = \sqrt{2} \Re \left[v'_k(\eta) \langle \hat{\beta}_{\vec{k}}^{R,I} \rangle_{\Xi} \right] \quad (30)$$

Now, we evaluate (29) and (30) at time of collapse η_k^c . This allows us to obtain an expression for $\langle \hat{\beta}_{\vec{k}}^{R,I} \rangle_{\Xi}$ in terms of the quantities $\langle \hat{v}_{\vec{k}}^{R,I}(\eta_k^c) \rangle_{\Xi}$ and $\langle \hat{\pi}_{\vec{k}}^{R,I}(\eta_k^c) \rangle_{\Xi}$. Once this is done, we can rewrite (29) which now reads:

$$\begin{aligned} \langle \hat{v}_{\vec{k}}^{R,I}(\eta) \rangle_{\Xi} &= \langle \hat{v}_{\vec{k}}^{R,I}(\eta_k^c) \rangle_{\Xi} \left\{ \left(1 + \frac{1}{k\eta z_k} - \frac{1}{z_k^2} \right) \times \right. \\ &\quad \left. \times \cos(k\eta - z_k) + \left[\frac{1}{k\eta} \left(\frac{1}{z_k} - 1 \right) + \frac{1}{z_k} \right] \sin(k\eta - z_k) \right\} + \end{aligned}$$

$$\begin{aligned}
& + \frac{\langle \hat{\pi}_{\vec{k}}^{R,I}(\eta_k^c) \rangle_{\Xi}}{k} \left\{ \left(\frac{1}{k\eta} - \frac{1}{z_k} \right) \cos(k\eta - z_k) + \right. \\
& \left. + \left(1 + \frac{1}{k\eta z_k} \right) \sin(k\eta - z_k) \right\} \quad (31)
\end{aligned}$$

where $z_k \equiv k\eta_k^c$.

Working with equations (21), (27), (28) and (31), we arrive to the expression:

$$\langle \hat{\nu}_{\vec{k}}^{R,I}(\eta) \rangle_{\Xi} = \frac{1}{2k^{1/2}} \left[\lambda_{\nu} F(k\eta, z_k) x_{\vec{k},1}^{R,I} + \lambda_{\pi} G(k\eta, z_k) x_{\vec{k},2}^{R,I} \right] \quad (32)$$

where,

$$\begin{aligned}
F(k\eta, z_k) & \equiv \left(1 + \frac{1}{z_k^2} \right)^{1/2} \left\{ \left(1 + \frac{1}{k\eta z_k} - \frac{1}{z_k^2} \right) \times \right. \\
& \left. \times \cos(k\eta - z_k) + \left[\frac{1}{k\eta} \left(\frac{1}{z_k^2} - 1 \right) + \frac{1}{z_k} \right] \sin(k\eta - z_k) \right\} \\
G(k\eta, z_k) & \equiv \left[\frac{1}{z_k^2} + \left(1 - \frac{1}{z_k^2} \right)^2 \right]^{1/2} \left\{ \left(\frac{1}{k\eta} - \frac{1}{z_k} \right) \times \right. \\
& \left. \times \cos(k\eta - z_k) + \left(1 + \frac{1}{k\eta z_k} \right) \sin(k\eta - z_k) \right\}
\end{aligned}$$

On the other hand, from equation (22), we will evaluate the expectation value of $\hat{\nu}_{\vec{k}}(\eta)$ in the post-collapse state,

$$\langle \hat{\nu}_{\vec{k}}(\eta) \rangle_{\Xi} = \langle \hat{\nu}_{\vec{k}}^R(\eta) \rangle_{\Xi} + i \langle \hat{\nu}_{\vec{k}}^I(\eta) \rangle_{\Xi} \quad (33)$$

By using (32), we obtain:

$$\langle \hat{\nu}_{\vec{k}}(\eta) \rangle_{\Xi} = \frac{1}{2k^{1/2}} \left[\lambda_{\nu} F(k\eta, z_k) x_{\vec{k},1} + \lambda_{\pi} G(k\eta, z_k) x_{\vec{k},2} \right] \quad (34)$$

where $x_{\vec{k},j} = x_{\vec{k},j}^R + i x_{\vec{k},j}^I$ with $j = 1, 2$.

Since we have quantized $\hat{\nu}_{\vec{k}}(\eta)$, we can return to the original variable $h_{ij}(\vec{x}, \eta)$, describing the metric tensor perturbations. Therefore, we find that,

$$\hat{h}_{ij}(\vec{x}, \eta) = \frac{1}{L^{3/2}} \sum_{\vec{k}} \hat{h}_{\vec{k}}(\eta) e_{ij}(\vec{k}) e^{i\vec{k}\cdot\vec{x}} \quad (35)$$

where

$$\hat{h}_{\vec{k}}(\eta) = \sqrt{\frac{32\pi G}{e^i_j e^j_i} \frac{1}{a(\eta)}} \hat{\nu}_{\vec{k}}(\eta) \quad (36)$$

Evaluating the expectation value of the last quantity, in the post-collapse state, we obtain:

$$\langle \hat{h}_{\vec{k}}(\eta) \rangle_{\Xi} = \sqrt{\frac{32\pi G}{e^i_j e^j_i} \frac{1}{a(\eta)}} \langle \hat{\nu}_{\vec{k}}(\eta) \rangle_{\Xi} \quad (37)$$

Similarly to what was said to the equation (5), here we will identify

$$\langle \hat{h}_{\vec{k}}(\eta) \rangle_{\Xi} \simeq h_{\vec{k}}(\eta) \quad (38)$$

This means that the expectation value of $\hat{h}_{\vec{k}}$ coincides approximately with the amplitude value of the classical gravitational wave $h_{\vec{k}}$. After this identification is made, we can evaluate the classical amplitude during the inflationary phase. Since we are

considering *slow-roll* inflation, and because we are working in the approximation $a(\eta) \simeq -1/H\eta$, the classical amplitude results:

$$\begin{aligned}
h_{\vec{k}}(\eta) & = \frac{2H}{M_P} \frac{(-\eta)}{\sqrt{e^i_j e^j_i}} \frac{1}{k^{1/2}} \left[\lambda_{\nu} F(k\eta, z_k) x_{\vec{k},1}^* \right. \\
& \left. + \lambda_{\pi} G(k\eta, z_k) x_{\vec{k},2}^* \right] \quad (39)
\end{aligned}$$

As it is usual in the literature, if the Hubble radius is representative of the horizon, the observational relevant modes are those satisfying the condition $k \ll \mathcal{H}$. Since during inflation $\mathcal{H} \simeq -1/\eta$, the condition for modes that are outside the horizon becomes $-k\eta \rightarrow 0$. In this limit, it can be shown that:

$$\lim_{-k\eta \rightarrow 0} F(k\eta, z_k) = \left(\frac{1}{-k\eta} \right) f(z_k)$$

$$\lim_{-k\eta \rightarrow 0} G(k\eta, z_k) = \left(\frac{1}{-k\eta} \right) g(z_k)$$

where,

$$f(z_k) \equiv \left(1 + \frac{1}{z_k^2} \right)^{1/2} \left[-\frac{1}{z_k} \cos(z_k) + \left(\frac{1}{z_k^2} - 1 \right) \sin(z_k) \right] \quad (40)$$

$$g(z_k) \equiv \left(1 - \frac{1}{z_k^2} + \frac{1}{z_k^4} \right)^{1/2} \left(-\cos(z_k) + \frac{1}{z_k} \sin(z_k) \right) \quad (41)$$

Therefore, for modes outside the horizon we obtain:

$$h_{\vec{k}}(\eta) = \frac{2H}{M_P} \frac{1}{\sqrt{e^i_j e^j_i}} \frac{1}{k^{3/2}} \left[\lambda_{\nu} f(z_k) x_{\vec{k},1} + \lambda_{\pi} g(z_k) x_{\vec{k},2} \right] \quad (42)$$

This quantity is approximately constant (since $H \simeq \text{const.}$). Additionally, it depends on the random numbers $x_{\vec{k},1}$ and $x_{\vec{k},2}$, and also on the time of collapse through the variable $z_k \equiv k\eta_k^c$. Note that this expression is only possible by considering the self-induced collapse, and every element has a clear physical origin. It has no counterpart in the traditional approach, where $h_k(\eta)$ is only assumed to acquire a classical meaning somehow (e.g. decoherence, squeezing of the vacuum state, many-world interpretation of Quantum Mechanics, etc.) only after the proper wavelength associated to the mode k becomes bigger than the Hubble radius H^{-1} .

Now, considering that $x_{\vec{k},1}^{R,I}$ and $x_{\vec{k},2}^{R,I}$ are independent random numbers, and since \vec{k} and $-\vec{k}$ are not independent quantities, we have:

$$\overline{x_{\vec{k},i}^R x_{\vec{k}',i}^R} = \delta_{\vec{k},\vec{k}'} + \delta_{\vec{k},-\vec{k}'} \quad (43)$$

$$\overline{x_{\vec{k},i}^I x_{\vec{k}',i}^I} = \delta_{\vec{k},\vec{k}'} - \delta_{\vec{k},-\vec{k}'} \quad (44)$$

where $i = 1, 2$. This leads to:

$$\overline{x_{\vec{k},1}^* x_{\vec{k}',1}^*} = \overline{x_{\vec{k},2}^* x_{\vec{k}',2}^*} = 2\delta_{\vec{k},\vec{k}'} \quad (45)$$

and because $x_{\vec{k},1}$ and $x_{\vec{k},2}$ are not correlated,

$$\overline{x_{\vec{k},1}^* x_{\vec{k}',2}^*} = 0 \quad (46)$$

Thus, from equation (42) we arrive to:

$$\overline{h_{\vec{k}}(\eta) h_{\vec{k}'}^*(\eta)} = \frac{8H^2}{M_P} \frac{1}{e^i_j e^j_i} \frac{1}{k^3} \left[\lambda_{\nu}^2 f^2(z_k) + \lambda_{\pi}^2 g^2(z_k) \right] \delta_{\vec{k},\vec{k}'} \quad (47)$$

As discussed previously for (7), from (47) the power spectrum for the primordial gravitational wave amplitudes can be extracted. We obtain:

$$P_h(\eta) = \frac{H^2}{\pi^5 M_p^2} C(z_k) \quad (48)$$

where $C(z_k) \equiv \lambda_v^2 f^2(z_k) + \lambda_\pi^2 g^2(z_k)$ and it coincides exactly with (9).

Since any dependence on k is in the function $C(z_k)$ through $z_k \equiv k\eta_k^c$, if the time of collapse scales as $\eta_k^c \propto 1/k$, then z_k is independent of k . In this manner, the power spectrum (48) [as in the scalar case (8)] becomes a scale free spectrum. Also, small variations in the relation $\eta_k^c \propto 1/k$ would yield deviations in the spectrum shape with respect to the standard prediction, which could be observationally distinguished.

Finally, from equations (48) and (8), we can evaluate the tensor-to-scalar ratio r . This quantity results to be:

$$r \equiv \frac{P_h}{P_\Psi} \propto \frac{(H^2/M_p^2) C(z_k)}{(H^2/M_p^2 \epsilon) C(z_k)} \quad (49)$$

Hence, this means:

$$r \propto \epsilon \quad (50)$$

A few remarks are in order. According to latest observations from Planck mission, the scalar power spectrum is practically scale invariant [2]; on the other hand, even if the tensor power spectrum is also expected to be close to scale invariant, the fact is that detection of primordial gravity waves is still waiting for confirmation [3,6].

As is clear from expression (50), the prediction for the tensor-to-scalar ratio r is independent of our model parameters. In particular, it does not depend on the time of collapse. This can be seen from Eqs. (9) and (48) where the modification to both power spectra (scalar and tensor) is given by exactly the same function $C(z_k)$. Therefore, a possible confirmation regarding the detection of primordial gravitational waves will not help to constraint the collapse parameters, but only will set, as in the standard case, an energy scale for inflation. The constriction of the collapse parameters can be made by focusing on the scalar power spectrum and also the primordial bispectrum [40].

The fact that r is independent of the collapse parameters can be understood as follows. The quantum theory of the scalar perturbations, using the Mukhanov–Sasaki variable, can be considered as a theory representing a collection of parametric oscillators (i.e. one oscillator per mode), whose time-dependent frequency can be expressed as $\omega_s^2(\eta, k) = k^2 - z''/z$. Furthermore, the quantum theory of the tensor perturbations can also be viewed as a theory representing a collection of parametric oscillators [see Eq. (15)]. In this case, the time-dependent frequency is given by $\omega_t^2(\eta, k) = k^2 - a''/a$. Additionally, $z''/z = a''/a$ up to first-order in the slow-roll parameters. Therefore, the physical mechanism behind what we effectively describe as a self-induced collapse, should not in principle distinguish between the quantum theory of the scalar and tensor perturbations because they are essentially the same, i.e. a collection of harmonic oscillators with a time-dependent frequency that happens to be practically the same in both cases. As matter of fact, the parameters λ_v and λ_π that control which variable is affected by the collapse (recall that the values of these parameters can only be 0 or 1 depending on which field \hat{v}_k or $\hat{\pi}_k$ or both is affected by the collapse) should be the same for the scalar and tensor modes because there is no difference in the quantum theory characterizing the scalar and tensor perturbations.

We think this is the main reason behind the fact that the modification to both power spectra is given by the same function $C(z_k)$ and consequently r is independent of the collapse parameters.

Moreover, as it was mentioned in Sec. 2, in order for our model prediction for the scalar power spectrum to be consistent with CMB data, the time of collapse must satisfy $\eta_k^c \propto 1/k$. That is, if the tensor power spectrum is also expected to be close to scale invariant, then the time of collapse must also be of the form $\eta_k^c \propto 1/k$. Thus, the dependence on the wave number k of the time of collapse is exactly the same for the scalar and tensor modes. This result is consistent with our previous discussion in the sense that the self-induced collapse somehow affects all kind of perturbations (scalar and/or tensor) in the same way. On the contrary, the situation in which the self-induced collapse proposal is based on the semiclassical gravity framework, is different from the one based on the Mukhanov–Sasaki variable. That is, in the semiclassical gravity approximation the source terms that generate the curvature perturbations does not affect equally the scalar and tensor modes, consequently in that approach the prediction for r is different as in the present work [29].

It is worthwhile to mention that in the expression for the scalar power spectrum, Eq. (8), the slow-roll parameter ϵ appears explicitly, while in the expression for the tensor power spectrum Eq. (48) it does not. The reason for this difference can be traced back in the way we have linked the scalar and tensor curvature perturbations to the quantum variables affected by the collapse. The scalar curvature perturbation Ψ_k^- is generated by evaluating the field variables \hat{v}_k^- and \hat{v}_k^+ (which is essentially $\hat{\pi}_k^-$) at the post-collapse state, Eq. (6). In this expression, ϵ appears explicitly and it was obtained using Einstein equations. On the other hand, the tensor curvature perturbation is generated by the expectation value of \hat{v}_k^- only, Eq. (37), which is independent of the slow-roll parameter.²

Furthermore, the fact that $r \propto \epsilon$, within the framework of the present manuscript, makes this prediction indistinguishable from the standard case. However, this only applies to the amplitude of the tensor modes. The scalar power spectrum is substantially different from the traditional inflationary paradigm. The difference is encoded in the function $C(z_k)$, and one can perform an analysis using the observational data, as the one done in e.g. [26]. Additionally, a possible improvement in future experiments, regarding the detection of the shape and amplitude of the primordial bispectrum, can also help to discriminate between our proposal and the standard prediction [40]. Moreover, the main consequence of the result obtained in this Letter is that a confirmed detection of a non-vanishing value for r can differentiate between the two frameworks of the self-induced collapse proposal, namely, the semiclassical gravity approach and the joint matter-metric quantization, as reflected in the quantization of the Mukhanov–Sasaki variable. In the former case, the predicted value for r is suppressed by a factor of $10^{-9}\epsilon^2$ [29]; thus, practically undetectable. While in the latter, r is of the same order of magnitude as the slow-roll parameter ϵ and, hence, from the observational point of view, in the same footing as the standard picture.

² Note that in Eq. (6) the slow-roll parameter ϵ appears in the numerator, while in the expression for the scalar power spectrum (8) appears in the denominator. The reason for this difference is that, in the longitudinal gauge, the scalar curvature perturbation Ψ becomes amplified by a factor of $1/\epsilon$ during the transition from inflation to the radiation dominated stage [21,39], in which the CMB is originated. Consequently, in order to obtain a consistent prediction to be compared with the observations, we must multiply by a factor of $1/\epsilon^2$ the scalar power spectrum obtained during inflation associated to $\overline{\Psi_k^- \Psi_k^-}$.

4. Conclusions

As it has been mentioned in previous works e.g. [10,21], working in the framework of semiclassical gravity, the collapse hypothesis, which serves to address the transition from an homogeneous and isotropic state to another one which is not, leads to a practically undetectable amplitude for the primordial gravitational waves. For this reason, and motivated by the implications of a possible detection of primordial B polarization modes, we have calculated the amplitude of tensor modes in the joint metric-matter quantization of the primordial perturbations, but taking into account the self-induced collapse hypothesis. We have accomplished this task by assuming a slow-roll type of inflation and characterizing the collapse by the expectation values of the field and its conjugated momentum; in this sense, we have considered a generic type of collapse.

It is also worthwhile to mention that our approach differs drastically from the one considered in Ref. [34]. As mentioned in the Introduction, our point of view is that the quantum collapse is directly related to the generation of the primordial perturbations. Therefore, if there is no quantum collapse, then $\Psi_k = 0 = h_k$. In turn, the authors in Ref. [34] consider a particular collapse mechanism, known as CSL, and apply it directly to the Mukhanov–Sasaki variable obtaining a prediction for r (as well as for the scalar and tensor power spectra) that depends on the CSL model parameters. However, in their work, if there is no quantum collapse, then P_ψ , P_h and r are exactly the same as in the standard approach, thus, changing drastically the physical implication of assuming a self-induced collapse, as well as, the theoretical prediction for r .

Our results indicate that it is possible to obtain a detectable amplitude associated to the primordial gravitational waves even by adding the self-induced collapse hypothesis. The predicted amplitude is quite similar to the one provided by standard inflation, i.e. $r \propto \epsilon$. This result implies that our model prediction is consistent with the latest findings from the joint BICEP/Planck collaboration [30]. Also, as a consequence of our result $r \propto \epsilon$, the amplitude is independent of the collapse mechanism; particularly, is independent of the time of collapse η_k^c . Therefore, even if the power spectra (scalar and tensor) do depend on η_k^c each one, they do in the exactly same way making r independent of the time of collapse. On the other hand, a detection of primordial gravity waves cannot help to distinguish between the collapse proposal *à la* Mukhanov–Sasaki and the standard inflation case. In order to discriminate between the two approaches, one must focus on the scalar power spectrum and the primordial bispectrum.

Finally, if a detection of primordial gravitational waves is confirmed, and consequently, r turns out to be non-vanishing, the collapse hypothesis applied to the inflationary universe, within the framework of the semiclassical gravity approximation, would face serious issues; in consequence, the most viable option would be to consider the self-induced collapse applied to the Mukhanov–Sasaki variable. Therefore, the result obtained in this work, along with future observational data, can help to improve our overall knowledge of the collapse mechanism behind the primordial perturbations;

in particular, the relation between the collapse and the gravitational aspects in the early universe.

Acknowledgements

G.R.B. is supported by CONICET (Argentina). G.R.B. acknowledge support from the PIP 112-2012-0100540 of CONICET (Argentina). G.L.'s research funded by Consejo Nacional de Investigaciones Científicas y Técnicas, CONICET (Argentina) and by Consejo Nacional de Ciencia y Tecnología, CONACYT (Mexico).

References

- [1] G. Hinshaw, et al., *Astrophys. J. Suppl. Ser.* 208 (2013) 19.
- [2] Planck Collaboration, P.A.R. Ade, et al., *Paper XVI, Astron. Astrophys.* 571 (2014) A16.
- [3] BICEP2 Collaboration, *Phys. Rev. Lett.* 112 (2014) 241101.
- [4] R. Flauger, J. Colin Hill, D.N. Spergel, *J. Cosmol. Astropart. Phys.* 08 (2014) 039.
- [5] M.J. Mortonson, U. Seljak, *J. Cosmol. Astropart. Phys.* 10 (2014) 035.
- [6] Planck Collaboration, *Paper XXX, Astron. Astrophys.*, accepted for publication, arXiv:1409.5738.
- [7] G. Leon, S.J. Landau, D. Sudarsky, *Phys. Rev. D* 88 (2013) 023526.
- [8] C. Kiefer, D. Polarsky, *Adv. Sci. Lett.* 2 (2009) 164; J.J. Halliwell, *Phys. Rev. D* 39 (1989) 2912; C. Kiefer, *Nucl. Phys. B, Proc. Suppl.* 88 (2000) 255; D. Polarsky, A.A. Starobinsky, *Class. Quantum Gravity* 13 (1996) 377.
- [9] J. Martin, *Lect. Notes Phys.* 738 (2008) 193.
- [10] A. Perez, H. Sahlmann, D. Sudarsky, *Class. Quantum Gravity* 23 (2006) 2317.
- [11] D. Sudarsky, *Int. J. Mod. Phys. D* 20 (2011) 509.
- [12] L. Diosi, *Phys. Lett. A* 120 (1987) 377.
- [13] R. Penrose, *Gen. Relativ. Gravit.* 28 (1996) 581.
- [14] G.C. Ghirardi, A. Rimini, T. Weber, *Phys. Rev. D* 34 (1986) 470.
- [15] P.M. Pearle, *Phys. Rev. A* 39 (1989) 2277.
- [16] G.C. Ghirardi, P.M. Pearle, A. Rimini, *Phys. Rev. A* 42 (1990) 78.
- [17] A. Bassi, G.C. Ghirardi, *Phys. Rep.* 379 (2003) 257.
- [18] M. Bahrani, M. Paternostro, A. Bassi, H. Ulbricht, *Phys. Rev. Lett.* 112 (2014) 210404.
- [19] A. Diez-Tejedor, D. Sudarsky, *J. Cosmol. Astropart. Phys.* 045 (2012) 1207.
- [20] A. De Unanue, D. Sudarsky, *Phys. Rev. D* 78 (2008) 043510.
- [21] G. Leon, D. Sudarsky, *Class. Quantum Gravity* 27 (2010) 225017.
- [22] G. Leon, A. De Unanue, D. Sudarsky, *Class. Quantum Gravity* 28 (2011) 155010.
- [23] G. Leon, D. Sudarsky, *SIGMA* 8 (2012) 024.
- [24] G. Leon, S.J. Landau, M.P. Piccirilli, *Phys. Rev. D* 90 (2014) 083525.
- [25] G.R. Bengochea, P. Cañate, D. Sudarsky, *Phys. Lett. B* 743 (2015) 484.
- [26] S.J. Landau, C.G. Scoccola, D. Sudarsky, *Phys. Rev. D* 85 (2012) 123001.
- [27] D. Larson, et al., *Astrophys. J. Suppl. Ser.* 192 (2011) 16.
- [28] B.A. Reid, et al., *Mon. Not. R. Astron. Soc.* 404 (2010) 60.
- [29] G. León, L. Kraiselburd, S.J. Landau, *Phys. Rev. D* 92 (2015) 083516.
- [30] P.A.R. Ade, et al., BICEP2 and Planck Collaborations, *Phys. Rev. Lett.* 114 (2015) 101301.
- [31] A. Diez-Tejedor, G. Leon, D. Sudarsky, *Gen. Relativ. Gravit.* 44 (2012) 2965.
- [32] J. Martin, V. Vennin, P. Peter, *Phys. Rev. D* 86 (2012) 103524.
- [33] S. Das, K. Lochan, S. Sahu, T.P. Singh, *Phys. Rev. D* 88 (8) (2013) 085020; S. Das, K. Lochan, S. Sahu, T.P. Singh, *Phys. Rev. D* 89 (10) (2014) 109902 (Erratum).
- [34] S. Das, et al., *Phys. Rev. D* 90 (2014) 043503.
- [35] G. León, S.J. Landau, M.P. Piccirilli, *Eur. Phys. J. C* 75 (8) (2015) 393.
- [36] S. Weinberg, *Cosmology*, Oxford University Press, USA, 2008.
- [37] V.F. Mukhanov, et al., *Phys. Rep.* 215 (1992) 203.
- [38] V.F. Mukhanov, *Physical Foundations of Cosmology*, Cambridge University Press, UK, 2005, pp. 340–348, section 8.3.3.
- [39] N. Deruelle, V.F. Mukhanov, *Phys. Rev. D* 52 (1995) 5549.
- [40] G. León, D. Sudarsky, *J. Cosmol. Astropart. Phys.* 06 (2015) 020.