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Chern–Simons–Higgs theory with visible and hidden sectors and its $\mathcal{N}=2$ SUSY extension

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ABSTRACT

We study vortex solutions in Abelian Chern–Simons–Higgs theories with visible and hidden sectors. We first consider the case in which the two sectors are connected through a BF-like gauge mixing term with no explicit interaction between the two scalars. Since first order Bogomolny equations do not exist in this case, we derive the second order field equations. We then proceed to an $\mathcal{N}=2$ supersymmetric extension including a Higgs portal mixing among the visible and hidden charged scalars. As expected, Bogomolny equations do exist in this case and we study their string-like solutions numerically.

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The possible existence of unobserved particles in a hidden sector coupled to the standard model particles (in the visible sector) or its supersymmetric extensions has recently received much attention in connection with dark matter, supersymmetry breaking and also in the context of phenomenological superstring studies (see [1–3] and references therein).

Concerning gauge theories coupled to Higgs scalars there are two possible renormalizable and gauge invariant couplings between the visible and hidden sectors: the gauge kinetic mixing (GKM) and the Higgs portal (HP), originally introduced in Refs. [4–9]. Concerning the GKM coupling, in the case of an Abelian gauge symmetry $U(1)$ and visible and hidden coupled Maxwell–Higgs models, the mixing term L_{GKM} in the Lagrangian reads

$$L_{GKM} = \frac{1}{2}\xi F_{\mu\nu}G^{\mu\nu} \tag{1}$$

where

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$
$$G_{\mu\nu} = \partial_\mu G_\nu - \partial_\nu G_\mu$$

with A_μ and G_μ the visible and hidden gauge fields respectively and $\xi < 10^{-3}$ since larger values are experimentally ruled out [1,3].

In the case of the Higgs portal coupling, when the visible ϕ and hidden η scalars are both taken as complex charged fields, the mixing term takes the form

$$L_{HP} = -\frac{1}{2}\lambda_{\phi\eta}|\phi|^2|\eta|^2 \tag{2}$$

Concerning the portal coupling $\lambda_{\phi\eta}$, for the case in which ϕ is the standard model's scalar doublet it is often taken as $\mathcal{O}(1)$ when it is motivated by baryogenesis or naturalness [10] while in the proposal in which η introduces a massless Goldstone boson in connection with the fractional cosmic neutrinos' problem $\lambda_{\phi\eta} \sim 0.005$ [11].

Interestingly enough, if one wishes to construct a supersymmetric extension in models with visible and hidden Maxwell–Higgs sectors, one should necessarily have to include both kinetic gauge mixing and Higgs portal couplings. Indeed, the mixing of the two auxiliary fields D, D' belonging to the gauge superfield multiplets forces a mixing of the scalars in the chiral superfields. One also needs to add Fayet–Iliopoulos terms in such theories, to set up, via the Higgs mechanism, the required spontaneous breaking of the gauge groups. As a result a relation between the mixing parameters ξ and $\Lambda_{\phi\eta}$ is imposed. A discussion of new physics arising from such mixing can be found in [12] and references therein.

Now, we know that there is tight connection between supersymmetry and self-dual (Bogomolny or BPS) equations whose solutions also solve the classical field equations of gauge field theories (see [13] and references there). In the case of supersymmetric models with visible and hidden Maxwell–Higgs sectors this con-

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nection was exploited to find first order BPS equations which exhibit topologically stable string-like solutions [14,15].

Working in $2+1$ dimensions the Chern–Simons–Higgs action also exhibits vortex-like solutions which one can find from self-dual equations when the symmetry breaking is achieved with sixth-order symmetry breaking potential [16,17]. As in the Maxwell action case, such a specific potential is precisely the one required for the supersymmetric extension of the model [18].

It is the purpose of the present work to study $2+1$ dimensional models with visible and hidden sectors in which the gauge field dynamics is dictated by Chern–Simons (CS) actions. The natural gauge-field mixing in this case takes the form of a *BF*-term [19]

$$L_{GM} = \frac{1}{2}\xi\varepsilon^{\alpha\beta\gamma}A_\alpha G_{\beta\gamma} \tag{3}$$

It should be noticed that apart from the interest *per se* of the *BF* theories, this kind of term has played a relevant role in the study of self-duality in topologically massive gauge theories [20,21] and $2+1$ bosonization [22,23] with interesting applications in condensed matter problems [24,25].

We shall first consider the case in which visible and hidden sectors are coupled solely by the gauge field mixing (3) and then discuss the supersymmetric extension. In this last case a Higgs portal is necessarily present and consists of two sixth order terms of the form $(|\phi|^2)^2|\eta|^2$ and $|\phi|^2(|\eta|^2)^2$ with mixing coupling constants related to the gauge field parameters in such a way as to ensure supersymmetry and, a fortiori, the existence of BPS equations.

A Chern–Simons–Higgs model with visible and hidden sectors

We start from the Lagrangian for $2+1$ dimensional with visible and hidden $U(1)$ gauge fields and dynamics governed by Chern–Simons terms coupled to two charged scalars and a gauge mixing term of the form (1),

$$\begin{aligned}\mathcal{L} = {} & \frac{1}{4}\kappa\varepsilon^{\alpha\beta\gamma}A_\alpha F_{\beta\gamma} + \frac{1}{4}\kappa_h\varepsilon^{\alpha\beta\gamma}G_\alpha G_{\beta\gamma} + \frac{1}{2}\xi\varepsilon^{\alpha\beta\gamma}A_\alpha G_{\beta\gamma} \\ & + (D_\mu\phi)^*(D^A_\mu\phi) + (D_\mu\eta)^*(D^G_\mu\eta) - V[\phi] - V[\eta]\end{aligned} \tag{4}$$

where the covariant derivatives are defined as

$$D^A_\mu\phi = \partial_\mu\phi - ieA_\mu\phi\,, \qquad D^G_\mu\eta = \partial_\mu\eta - igG_\mu\eta \tag{5}$$

The symmetry breaking potentials $V[\phi]$ and $V[\eta]$ will be fixed below. The Chern–Simons couplings κ, κ_h and ξ are real constants with dimensions of a mass and Minkowski metric is taken as $g_{\mu\nu} = \mathrm{diag}(1,-1,-1)$. Clearly, in the limit of $\xi \to 0$ we have two decoupled Chern–Simons–Higgs theories which, for a particular choice of symmetry breaking potential have shown to be self-dual [16,17], this being connected with the possibility of an $\mathcal{N}=2$ supersymmetric extension [18].

The gauge field equations read

$$\begin{aligned}&\frac{\kappa}{2}\varepsilon^{\mu\alpha\beta}F_{\alpha\beta} + \frac{\xi}{2}\varepsilon^{\mu\alpha\beta}G_{\alpha\beta} = J^\mu[\phi]\,, \\ &\frac{\kappa_h}{2}\varepsilon^{\mu\alpha\beta}G_{\alpha\beta} + \frac{\xi}{2}\varepsilon^{\mu\alpha\beta}F_{\alpha\beta} = J^\mu[\eta],\end{aligned} \tag{6}$$

with the conserved matter currents $J^\mu(\phi)$, $J^\mu(\phi)$ given by

$$\begin{aligned}&J_\mu[\phi] = ie(\phi^* D^A_\mu\phi - \phi(D^A_\mu\phi)^*)\,, \\ &J_\mu[\eta] = ig(\eta^* D^G_\mu\eta - \eta(D^G_\mu\eta)^*)\end{aligned} \tag{7}$$

The time components of Eqs. (21)–(22) read

$$-\kappa B_A - \xi B_G = J^0[\phi]\,, \qquad -\kappa_h B_G - \xi B_A = J^0[\eta] \tag{8}$$

Magnetic and electric fields are defined as

$$B_A = -F^{12}\,, \qquad B_A = -G^{12}\,, \qquad E^i_A = F^{0i}\,, \qquad E^i_B = G^{0i} \tag{9}$$

Concerning the field equations for the visible and hidden charged scalars, they read

$$D_\mu(A)D^\mu(A)\phi = -\frac{\partial V(\phi)}{\partial\phi^*}\,, \qquad D_\mu(G)D^\mu(G)\eta = -\frac{\partial V(\eta)}{\partial\eta^*} \tag{10}$$

As already mentioned, when there is just one sector and the Higgs potential is sixth order the CS–Higgs model becomes self-dual and one finds first order field equations whose static rotationally symmetric solutions also solve the Euler–Lagrange equations. This can be done à la Bogomolny [26], by writing the energy as a sum of squares plus a topological term [16,17]. In the present case, because of the gauge field mixing the energy cannot be written in this way so that first order Bogomolny equations do not exist for any choice of Higgs potentials in which the two scalars do not interact. Of course, one can solve the second order field equations (4) with vortex like solutions. We shall now discuss this issue choosing sixth-order potentials for both scalars, which is the standard choice for the system with one sector only leading to BPS equations [16,17],

$$V[\phi] = \frac{e^4}{4\kappa^2}|\phi|^2(|\phi|^2-\phi_0^2)^2\,, \qquad V[\phi] = \frac{g^4}{4\kappa_h^2}|\eta|^2(|\eta|^2-\eta_0^2)^2. \tag{11}$$

It is convenient to work with dimensional variables $x \to x/(e\phi_0^2)$, $\phi \to \phi\phi_0$, $\eta \to \eta\phi_0$, $A_\mu \to A_\mu\phi_0^2$, $G_\mu \to G_\mu\phi_0^2$ and redefine coupling constants $h \equiv g/e$ and $k = \kappa/e$, $k_h = \kappa_h/g$, $\bar\xi = \xi/g$, $\bar\eta_0 = \eta_0/\phi_0$.

Then, after proposing the usual Nielsen–Olesen [27] ansatz, which in polar coordinates (t,θ) reads

$$\begin{aligned}&\phi = f(r)e^{in_1\theta}, \quad A_\theta = -\frac{a(r)}{r}, \quad A_r = 0 \\ &\eta = q(r)e^{in_2\theta}, \quad G_\theta = -\frac{b(r)}{r}, \quad G_r = 0\end{aligned} \tag{12}$$

the resulting field equations read

$$2A_0 f^2 = k\frac{a'}{r} + h\bar\xi\,\frac{b'}{r}, \tag{13}$$

$$2hG_0 q^2 = k_h\frac{b'}{r} + \bar\xi\,\frac{a'}{r}, \tag{14}$$

$$\frac{2}{r}(-n_1+a)\,f^2 = kA_0' + h\bar\xi G_0', \tag{15}$$

$$\frac{2}{r}(-n_2+hb)\,q^2 = k_h G_0' + \bar\xi A_0', \tag{16}$$

$$f'' + \frac{f'}{r} - \frac{1}{r^2}(-n_1+a)^2 f + A_0^2 f = \frac{1}{4k^2}f\left(4f^2 - 3f^4 - 1\right), \tag{17}$$

$$q'' + \frac{q'}{r} - \frac{1}{r^2}(-n_2+hb)^2 q + h^2G_0^2 q = \frac{h^2}{4k_h^2}q\left(4q^2 - 3q^4 - \tilde\eta_0^4\right). \tag{18}$$

The vortex-like solutions of these equations, in which both scalar fields condense in the vacuum breaking their respective gauge symmetries, are qualitatively similar to those found in an analogous supersymmetric model which we investigate below. In particular, the results of variations of the gauge mixing parameter on the field profiles are identical in both systems. Therefore, in order

not to present similar plots repeatedly, we refer the reader to the supersymmetric section for numerical solutions.

At this point it is interesting to note that when just one of the $U(1)$ symmetries is spontaneously broken the theory can be cast in a very simple way. Let us assume that the hidden sector has an unbroken $U(1)$ symmetry, while the visible one has a broken one (we could have chosen the other way around as well). The simplest way to achieve this is by eliminating the hidden scalar sector so that all η dependent terms in Lagrangian (4) are absent. In that case the kinetic mixing term can be eliminated by rotating the hidden field through the term

$$\tilde{G}_\mu = G_\mu + \frac{\xi}{\kappa_h} A_\mu, \tag{19}$$

so that the Lagrangian reduces to one with no gauge fields mixing,

$$L = \frac{1}{4}\kappa_h \varepsilon^{\alpha\beta\gamma}\tilde{G}_\alpha \tilde{G}_{\beta\gamma} + \frac{1}{4}\tilde{\kappa}\varepsilon^{\alpha\beta\gamma}A_\alpha F_{\beta\gamma} + (D_\mu[A]\phi)^*(D_\mu[A]\phi) - V[\phi], \tag{20}$$

where $\tilde{\kappa} \equiv \kappa - \xi^2/\kappa_h$. The equations of motion for A_μ then become the ones of the ordinary CS–Higgs model,

$$\frac{\tilde{\kappa}}{2}\varepsilon^{\mu\alpha\beta}F_{\alpha\beta} = ie(\phi^* D^\mu\phi - \phi(D^\mu\phi)^*) \tag{21}$$

while that for $\tilde{G}_\mu$ corresponds to the pure CS theory,

$$\frac{\kappa_h}{2}\varepsilon^{\mu\alpha\beta}\tilde{G}_{\alpha\beta} = 0 \;\Rightarrow \frac{\kappa_h}{2}\varepsilon^{\mu\alpha\beta}\tilde{G}_{\alpha\beta} = -\xi\varepsilon^{\mu\alpha\beta}\tilde{F}_{\alpha\beta} \tag{22}$$

Using this equation and eq. (19) one finds a relation between the original hidden and visible magnetic fields

$$B_G = -\frac{\xi}{\kappa_h}B_A. \tag{23}$$

Hence, although the hidden gauge symmetry is unbroken the existence of vortices with magnetic flux in the sector where the symmetry is broken implies a non-trivial magnetic flux Φ_G for the hidden sector, due to gauge mixing of the sectors. In particular, the vortex like solution for the magnetic field profile in the broken symmetry sector implies through this relation a vortex like magnetic field profile for the unbroken sector.

The time component of eq. (21) allows us to write the usual Gauss-like relations of CS theories between A_0 and the magnetic field, namely

$$A_0 = \tilde{\kappa}\frac{B_A}{2e^2|\phi|^2} \tag{24}$$

Inserting this relation in the T_{00} component of the energy–momentum tensor we find for the energy

$$E = \int d^2x \left(|\nabla\phi - ieA_i\phi|^2 + \frac{\kappa^2 B_A^2}{4e^2|\phi|^2}\left(1 - \frac{\xi^2}{\kappa\kappa_h}\right)^2 + V[\phi]\right) \tag{25}$$

which is just the expression for the energy of the ordinary self-dual Chern–Simons vortices discussed in [16,17] if the Chern–Simons κ parameter is taken as

$$\kappa = \kappa\left(1 - \frac{\xi^2}{\kappa\kappa_h}\right) \tag{26}$$

so that the usual magnetic flux/matter charge can be seen to be, in the visible sector

$$Q_{vis} = \int d^2x J^0 = -\kappa\Phi_A \tag{27}$$

One can also see the visible sector acts as a source of electric charge Q_{hid} for the hidden one, since taking the zeroth component of eq. (22) and using (27) one finds that

$$Q_{\text{hid}} = -\frac{\xi}{\kappa_h}Q \tag{28}$$

Since in the present case the energy of the model with visible and hidden sector has been reduced to an effective model with just one sector, one can find a Bogomolny bound for the energy (25) that is saturated satisfying the usual CS–Higgs self-dual equations first obtained in Refs. [16,17]

In this case, where the system diagonalizes to a single sector, the numerical solutions have been described in detail in [16,17] and we shall solely mention here that in contrast to the case of the Nielsen–Olesen vortex, in which the magnetic field has its maximum at its center, the magnetic field for the Chern–Simons vortex is concentrated in a ring surrounding the zero of the Higgs field and a similar behavior holds for the electric field.

The $\mathcal{N} = 2$ supersymmetric extension

Within the $\mathcal{N} = 2$ supersymmetric extension of Chern–Simons theories coupled to charged fields one introduces (in a language analogous to 4D $\mathcal{N} = 1$ supersymmetry) a vector superfield V, composed (in the Wess–Zumino gauge) of a gauge field A_μ, a 2-component complex spinor χ, a real scalar field M and the auxiliary scalar field D [18,28–30]

$$V = 2i\theta\bar{\theta}M + 2\theta\gamma^\mu\bar{\theta}A_\mu + \sqrt{2}i\theta^2\bar{\theta}\bar{\chi} - \sqrt{2}i\bar{\theta}^2\theta\chi + \theta^2\bar{\theta}^2 D \tag{29}$$

with the γ-matrices taken as the Pauli matrices with $\gamma^0 = \sigma^3$, $\gamma^i = \sigma^i$.

Concerning matter content, one introduces a chiral superfield Φ a complex scalar ϕ, a complex fermion σ and an auxiliary field F components.

The Chern–Simons-matter action composed of these superfields reads

$$S = \int d^3x \int d^2\theta d^2\bar{\theta}\left(\kappa V\Sigma + \Phi^\dagger \exp^{-ieV}\Phi\right) \tag{30}$$

with Σ (the linear multiplet) defined as

$$\Sigma = \bar{D}^\alpha D_\alpha V \tag{31}$$

The component fields Lagrangian derived from the superaction (30) takes the form

$$L = \kappa(\epsilon^{\mu\nu\rho}A_\mu\partial_\nu A_\rho - \bar{\chi}\chi + 2DM) + |D_\mu\phi|^2 + i\bar{\sigma}\gamma^\mu D_\mu\sigma + |F|^2 - (e^2M^2 - eD)|\phi|^2 - e\phi_0^2 D - e\bar{\sigma}\sigma M + ie\bar{\chi}\sigma\phi - ie\bar{\sigma}\chi\phi. \tag{32}$$

Concerning SUSY variations, one has

$$\begin{aligned} -2i\delta A_\mu &= \bar{\epsilon}\gamma_\mu\chi - \bar{\chi}\gamma_\mu\epsilon \\ 2\delta M &= \bar{\epsilon}\chi - \bar{\chi}\epsilon \\ \delta\phi &= \bar{\epsilon}\sigma \\ 2\delta D &= \bar{\epsilon}\gamma^\mu\partial_\mu\chi - \partial_\mu\bar{\chi}\gamma^\mu\epsilon \qquad\qquad i\delta F = \bar{\epsilon}\gamma^\mu D_\mu\sigma. \\ \delta\chi &= [i\epsilon^{\mu\nu\rho}\gamma_\rho F_{\mu\nu} - iD - \gamma^\mu\partial_\mu M]\epsilon \\ \delta\sigma &= (-i\gamma^\mu D_\mu\phi - M\phi + F)\epsilon \end{aligned}$$

The F term vanishes, since we impose no superpotential, and one can eliminate the non-dynamical fields M and D from their algebraic field equations:

$$-2\kappa^2 D = e^3|\phi|^2(|\phi|^2 - \phi_0^2), \quad -2\kappa M = e(|\phi|^2 - \phi_0^2). \tag{33}$$

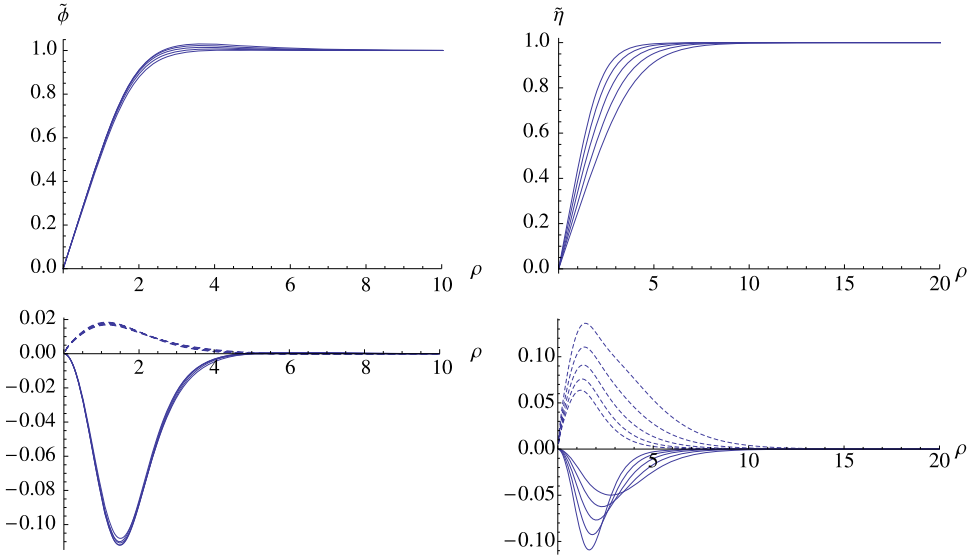

Fig. 1. Field profiles at $\kappa = 2$, $\kappa_h = 3$, $\phi_0/\eta_0 = 1$, $e = 3$ and $\xi = 1$, changing g between 1.4 and 2.2 in steps of 0.2, with highest g corresponding to tighter vortex core. Plots on the second line include electric (dashed line) and magnetic fields (solid line) for both the A (left column) and G (right column) fields.

When reintroduced back into the action, this generates the sixth-order potential originally proposed in [16,17] in the search of BPS equations and derived in the $\mathcal{N} = 2$ supersymmetric context in [18]

$$V_{eff} = e^4 \frac{|\phi|^2}{4\kappa^2}(|\phi|^2 - \phi_0^2)^2 \tag{34}$$

Indeed, within the context of supersymmetry the first order BPS equations can be obtained by imposing that all fermions and their supersymmetric variations vanish everywhere [13], as first shown for the CS theory in [18].

We are now ready to consider Chern–Simons–Higgs actions both for the visible and for the hidden sectors coupled through a CS like mixing term. We then consider as a starting point the supersymmetric action

$$S_{\mathcal{N}=2} = \kappa S_1[V, \Phi] + \kappa_h S_2[U, \Omega] - \xi S_{int}[V, U]. \tag{35}$$

Here V, Φ and U, Ω are the vector and scalar superfields of the visible and hidden sectors respectively. The components of V and Φ are those introduced in above. Concerning the hidden vector and chiral scalar superfields, we denote their components as $U = (N, G_\mu, \tau, d)$ and $\Omega = (\eta, \rho, f)$.

The interaction S_{int} takes the form

$$\begin{aligned}
&\int d^3x d^4\theta\, U\Sigma \\
&= \int d^3x d^4\,\theta V \Upsilon \\
&= \int d^3x \left(\epsilon^{\mu\nu\rho} A_\mu G_{\nu\rho} - \frac{1}{2}(\bar{\chi}\tau + \bar{\tau}\chi) + 2DN + 2dM\right)
\end{aligned} \tag{36}$$

From the field equations of auxiliary fields D, d, M, N one gets:

$$\begin{aligned}
&\tfrac{\delta}{\delta D}: \ \kappa M = -\tfrac{e}{2}(|\phi|^2 - \phi_0^2) + \xi N \\
&\tfrac{\delta}{\delta d}: \ \kappa_h N = -\tfrac{g}{2}(|\eta|^2 - \eta_0^2) + \xi M \\
&\tfrac{\delta}{\delta M}: \ \kappa D = e^2 M|\phi|^2 + \xi d \\
&\tfrac{\delta}{\delta N}: \ \kappa_h d = g^2 N|\eta|^2 + \xi D
\end{aligned} \tag{37}$$

These are two 2x2 linear systems of equations, the solution of which is

$$\begin{aligned}
M &= -\tfrac{1}{2(\kappa\kappa_h - \xi^2)}\left(\kappa_h e(|\phi|^2 - \phi_0^2) + \xi g(|\eta|^2 - \eta_0^2)\right) \\
N &= -\tfrac{1}{2(\kappa\kappa_h - \xi^2)}\left(g\kappa(|\eta|^2 - \eta_0^2) + e\xi(|\phi|^2 - \phi_0^2)\right) \\
D &= -\tfrac{e^2|\phi|^2}{2(\kappa\kappa_h - \xi^2)(\kappa - \xi)}\left(e\kappa_h(|\phi|^2 - \phi_0^2) + \xi g(|\eta|^2 - \eta_0^2)\right) \\
d &= -\tfrac{g^2|\eta|^2}{2(\kappa\kappa_h - \xi^2)(\kappa_h - \xi)}\left(e\xi(|\phi|^2 - \phi_0^2) + g\kappa(|\eta|^2 - \eta_0^2)\right)
\end{aligned} \tag{38}$$

Replacing these values in the Lagrangian the effective potential becomes simply

$$\begin{aligned}
V_{\text{eff}} = \frac{1}{4}\frac{1}{(\kappa\kappa_h - \xi^2)}\Big(&\left(e\kappa_h(|\phi|^2 - \phi_0^2) + \xi g(|\eta|^2 - \eta_0^2)\right)^2 e^2|\phi|^2 \\
&+ \left(g\kappa(|\eta|^2 - \eta_0^2) + e\xi(|\phi|^2 - \phi_0^2)\right)^2 g^2|\eta|^2\Big)
\end{aligned} \tag{39}$$

and we have the symmetric BPS equations

$$\begin{aligned}
&F_{12} = \pm\tfrac{e^2|\phi|^2}{4(\kappa\kappa_h - \xi^2)(\kappa - \xi)}\left(e\kappa_h(|\phi|^2 - \phi_0^2) + g\xi(|\eta|^2 - \eta_0^2)\right) \\
&D_1[A]\phi \pm D_2[A]\phi = 0 \\
&G_{12} = \pm\tfrac{g^2|\eta|^2}{4(\kappa\kappa_h - \xi^2)(\kappa_h - \xi)}\left(g\kappa(|\eta|^2 - \eta_0^2) + e\xi(|\phi|^2 - \phi_0^2)\right) \\
&D_1[G]\eta \pm D_2[G]\eta = 0
\end{aligned} \tag{40}$$

These are the Bogomolny equations of the supersymmetric Lagrangian (32) with all fermionic fields put to zero and auxiliary fields replaced using their algebraic field equations

$$\begin{aligned}
L = {}&\kappa\epsilon^{\mu\nu\rho} A_\mu \partial_\nu A_\rho + \kappa_h \epsilon^{\mu\nu\rho} G_\mu \partial_\nu G_\rho + 2\xi\epsilon^{\mu\nu\rho} A_\mu \partial_\nu G_\rho \\
&+ |D_\mu[A]\phi|^2 + |D_\mu[G]\eta|^2 - V_{\text{eff}}[\phi, \eta]
\end{aligned} \tag{41}$$

Switching to the dimensionless units $\rho \to \phi_0^2 r$, $\tilde{\phi} = \phi/\phi_0$, $\tilde{\eta} = \eta/\eta_0$ and using a rotational ansatz like that in (12) the BPS equations reduce to

$$\begin{aligned}
&\tfrac{1}{\rho}\partial_\rho A_\theta = -\tfrac{e^2|\tilde{\phi}|^2}{4(\kappa\kappa_h - \xi^2)(\kappa - \xi)}\left(e\kappa_h(|\tilde{\phi}|^2 - 1) + \xi g(|\tilde{\eta}|^2 - \tfrac{\eta_0^2}{\phi_0^2})\right) \\
&\rho\partial_\rho\tilde{\phi} = \tilde{\phi}(1 - eA_\theta) \\
&\tfrac{1}{\rho}\partial_\rho G_\theta = -\tfrac{|\tilde{\eta}|^2}{4(\kappa\kappa_h - \xi^2)(\kappa_h - \xi)}\left(\xi e(|\tilde{\phi}|^2 - 1) + \kappa_h g(|\tilde{\eta}|^2 - \tfrac{\eta_0^2}{\phi_0^2})\right) \\
&\rho\partial_\rho\tilde{\eta} = \tilde{\eta}(1 - gG_\theta)
\end{aligned} \tag{42}$$

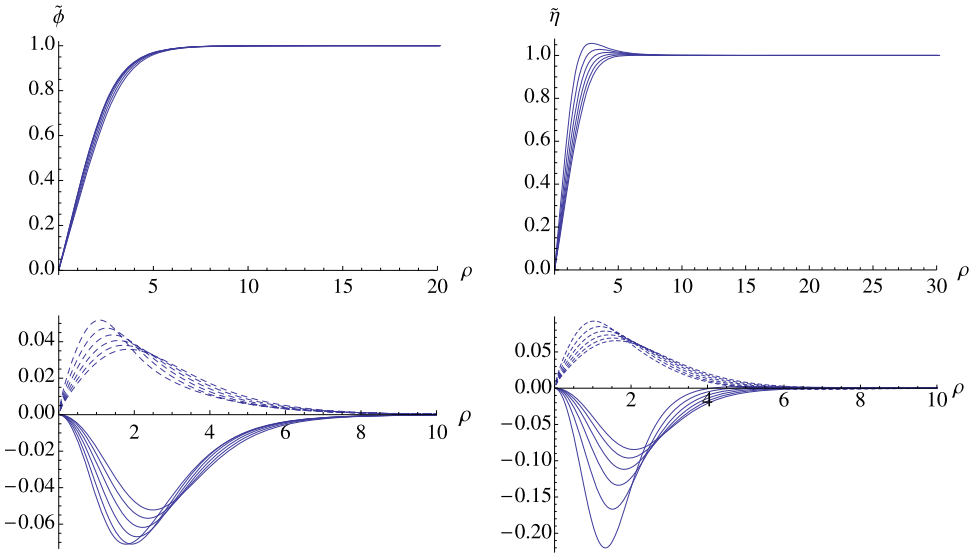

Fig. 2. Field profiles at $\kappa = 2$, $\kappa_h = 3$, $e = 2$, $g = 2$ and $\eta_0/\phi_0 = 1$ changing ξ between 0.6 and 1.6 in steps of 0.2, with highest ξ corresponding to larger magnitude of magnetic fields. Plots on the second line include electric (dashed line) and magnetic fields (solid line) for both the A (left column) and G (right column) fields.

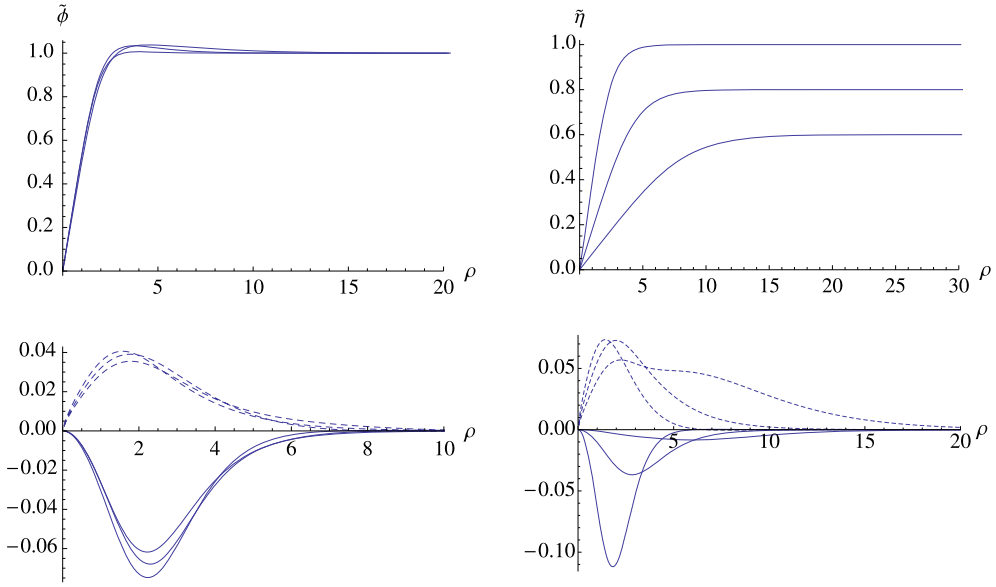

Fig. 3. Field profiles at $\kappa = 2$, $\kappa_h = 3$, $e = 2$, $g = 2$ and $\xi = 1$, changing η_0/ϕ_0 between 0.6 and 1 in steps of 0.2, with highest η_0/ϕ_0 corresponding to tighter vortex core. Plots on the second line include electric (dashed line) and magnetic fields (solid line) for both the A (left column) and G (right column) fields.

The appropriate boundary conditions for string-like solutions are

$$\tilde{\phi}(0) = 0, \quad \tilde{\eta}(0) = 0 \qquad A(0) = 0, \quad G(0) = 0$$
$$\tilde{\phi}(R) = 1, \quad \tilde{\eta}(R) = \eta_0/\phi_0 \qquad A(R) = 1/e, \quad G(R) = 1/g, \qquad (43)$$

where in the second line the limit of the radial variable $R \to \infty$ should be taken. These solutions are topologically stable.

In order to solve (42) we used a finite difference numerical procedure taking a value $R = 40$ for the large radial distance in (43). We show in Fig. 1 the resulting hidden and visible charged scalars, magnetic and electric fields for different values of the hidden gauge coupling g keeping all other parameters fixed. Although the value chosen for the mixing parameter is not very small, we see that the visible fields are not much affected by the changes in g. In contrast, as it was to be expected, as the coupling between the hidden gauge field and charge scalar grows the vortex core becomes tighter and tighter and the magnitude of the magnetic and electric fields maxima get higher. Due to the symmetry of the BPS equations under interchange of the visible and hidden fields an identical result with their role interchanged for the visible sector is obtained when instead of g one considers different values of the visible coupling constant e.

We study in Fig. 2 the behavior of scalar, magnetic and electric fields under changes in the mixing parameter ξ. We see that both the magnitude of magnetic and electric fields become larger as ξ grows. Fig. 3 shows the dependence of the scalar and magnetic fields under changes in the scalar vevs relative value. Tighter vortex cores and larger magnetic field maximums correspond to larger ratios η_0/ϕ_0. We see that as the scalar vev is lowered, there is a smearing of the electric field towards larger distances.

Summary and conclusions

We have studied visible and hidden sectors in theories in which gauge dynamics is dictated by Chern–Simons actions including the $\mathcal{N} = 2$ supersymmetric extension. Instead of the usual kinetic

gauge mixing introduced in $3+1$ space–time dimensions, in the planar case the natural gauge mixing is a BF term. As for the portal term arising in the supersymmetric extension it corresponds to sixth-order polynomials mixing the two charged scalars.

When the portal term is not included, no Bogomolny first order equations can be found. A particularly interesting phenomenon takes place in this case when one of the two charged scalars is absent. Hence there is no gauge symmetry breaking in the corresponding sector but, nevertheless, vortex-like solutions exist with both quantized magnetic flux and electric charge.

Concerning the supersymmetric extension, we found the associated BPS equations and studied them numerically, determining in particular how the magnetic, electric and Higgs field profiles depend on the relevant parameters constructed from coupling constants and masses of the model.

As stated in the introduction, BF terms associated to CS theories have played a relevant role concerning self-duality in topologically massive theories and bosonization of Thirring like theories in $2+1$ dimensions [22–25]. We expect to study this issue in the case of models with hidden sectors in a forthcoming publication.

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