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Vector mesons within nonlocal chiral quark models for strong interactions

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Abstract. We analyze meson properties within an SU(2) chiral quark model that includes nonlocal four fermion couplings with wave function renormalization. Model parameters are determined from meson phenomenology, considering different nonlocal form factor shapes. We concentrate in the description of basic features of nonstrange vector and axial vector mesons, considering nonlocal form factors that are based in lattice QCD results for effective quark propagators.

1. Introduction

Given the nonperturbative character of QCD in the low energy regime, the analysis of hadron phenomenology starting from first principles still represents a challenge for theoretical physics. Although a significant progress has been achieved in this sense through lattice calculations, this approach is not free of difficulties, e.g. when dealing with small quark masses or nonzero chemical potentials. Alternatively, low energy hadron phenomenology can be analyzed in the framework of effective models based on QCD symmetry properties. For two light flavors QCD supports an approximate SU(2) chiral symmetry which is dynamically broken at low energies, and pions play the role of the corresponding Goldstone bosons. A simple scheme including these properties is the well known Nambu−Jona-Lasinio (NJL) model [1, 2], in which quarks interact through a local, chiral invariant four fermion coupling. Due to the local nature of this interaction, the corresponding Schwinger-Dyson and Bethe-Salpeter equations become relatively simple. However, the main drawbacks of the model are direct consequences of the locality: loop integrals are divergent (and therefore have to be somehow regulated), and the model is nonconfining. As a way to improve upon the NJL model, extensions which include nonlocal interactions have been proposed [3]. In fact, nonlocality arises naturally in quantum field theory, in particular, in several well established approaches to low energy quark dynamics, as e.g. the instanton liquid model and the Schwinger-Dyson resummation techniques [4, 5]. Effective nonlocal interactions are also supported by lattice QCD calculations, which lead to a given momentum dependence of both the mass and the wave function renormalization (WFR) in the effective quark propagators [6, 7]. In addition, it has been argued that nonlocal extensions of the NJL model do not show some of the problems of the local theory. For example, nonlocal form factors regularize the model in such a way that the effective interaction is finite to all orders in the loop expansion and there is no need to introduce sharp cutoffs.

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Previous works on nonlocal NJL-like models can be found in the literature, focusing on different aspects of strong interaction physics [8, 9, 10, 11, 12, 13, 14, 15]. In particular, some works consider models that lead to momentum-dependent mass and WFR in the quark propagators, thus it is possible to choose nonlocal form factors so as to reproduce the behavior obtained through lattice QCD calculations [13, 15]. Here we consider a model of this type, concentrating in particular in the description of the vector meson sector. Therefore we include couplings between vector and axial vector nonlocal quark currents that satisfy the proper symmetry requirements, and analyze possible form factor choices and parametrizations in order to describe the basic properties of low energy vector meson phenomenology.

The article is organized as follows. In Sect. 2 we introduce the model, and in Sect. 3 we describe how to obtain the physical vector and axial vector meson states in the mean field approximation. In Sect. 4 and 5 we briefly sketch how to calculate some basic phenomenological quantities, namely the f_π and f_v decay constants and the $\rho \to \pi\pi$ decay width. In Sect. 6 we describe the parametrizations considered and quote the corresponding results for meson observables. Finally in Sect. 7 we present our conclusions.

2. Model

We consider a nonlocal chiral quark model than includes two light flavors. The corresponding Euclidean effective action is given by

$$\begin{aligned} S_E &= \int d^4x \left\{ \bar\psi(x)\left(-i\partial\!\!\!/ + \hat m\right)\psi(x) - \frac{G_S}{2}\Big[j_S(x) j_S(x) + j_P^a(x) j_P^a(x) + j_M(x) j_M(x)\Big] \right. \\ &\quad \left. - \frac{G_V}{2}\Big[j_V^{\mu\,a}(x) j_{V\mu}^a(x) + j_A^{\mu\,a}(x) j_{A\mu}^a(x)\Big]\right\} , \qquad (1) \end{aligned}$$

where $\psi(x)$ is the $N_f = 2$ quark doublet $\psi = (u\ d)^T$, and $\hat m = \mathrm{diag}(m_u, m_d)$ is the current quark mass matrix. We will work in the isospin symmetry limit, assuming $m_u = m_d$, which will be called from now on m_c.

The fermion currents in Eq. (1) are given by

$$\begin{aligned} j_S(x) &= \int d^4z\, g(z)\, \bar\psi\left(x + \frac{z}{2}\right)\, \psi\left(x - \frac{z}{2}\right), \\ j_P^a(x) &= \int d^4z\, g(z)\, \bar\psi\left(x + \frac{z}{2}\right)\, i\gamma_5\tau^a\, \psi\left(x - \frac{z}{2}\right), \\ j_M(x) &= \int d^4z\, f(z)\, \bar\psi\left(x + \frac{z}{2}\right) \frac{i\,\overleftrightarrow{\partial\!\!\!/}}{2\kappa_P}\, \psi\left(x - \frac{z}{2}\right), \\ j_{V\,\mu}^a(x) &= \int d^4z\, h(z)\, \bar\psi\left(x + \frac{z}{2}\right) \gamma_\mu\tau^a\, \psi\left(x - \frac{z}{2}\right), \\ j_{A\,\mu}^a(x) &= \int d^4z\, h(z)\, \bar\psi\left(x + \frac{z}{2}\right) \gamma_\mu\gamma_5\tau^a\, \psi\left(x - \frac{z}{2}\right) . \qquad (2) \end{aligned}$$

where τ_a, $a = 1, 2, 3$, are the Pauli matrices, while the functions $f(z)$, $g(z)$ and $h(z)$ are covariant form factors responsible for the nonlocal character of the interactions. The coupling of the "momentum" current $j_M(x)$ will be responsible for a momentum dependent WFR of the quark propagators.

We proceed by performing a standard bosonization of the theory. This is done by introducing scalar (σ_1, σ_2), pseudoscalar ($\vec\pi$), vector ($\vec v^\mu$) and axial vector ($\vec a^\mu$) meson fields, and integrating out the quark fields. Then we consider the mean field approximation (MFA), in which we expand the boson fields around their vacuum expectation values. From QCD symmetry considerations

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only nonzero VEVs for the scalar fields, say $\bar{\sigma}_1$ and $\bar{\sigma}_2$, are allowed. Thus the bosonized effective action can be rewritten through an expansion in powers of meson fluctuations. Details of this procedure (for a nonlocal model that does not include the vector meson sector) can be found e.g. in Ref. [13].

3. Quadratic fluctuations

The meson masses can be obtained from the terms in the Euclidean action that are quadratic in the bosonic fields. In particular, in the vector meson sector the quadratic piece has a form

$$S_E^{\rm quad}(\vec{v}_\mu) \;=\; \frac{1}{2}\int \frac{d^4p}{(2\pi)^4}\; G_v^{\mu\nu}(p^2)\;\delta\vec{v}_\mu(p)\cdot\delta\vec{v}_\nu(-p)\;. \tag{3}$$

Here the tensor $G_v^{\mu\nu}(p^2)$ can be written as

$$G_v^{\mu\nu}(p^2) = G_\rho(p^2)\left(g^{\mu\nu} - \frac{p^\mu p^\nu}{p^2}\right) + L_+(p^2)\frac{p^\mu p^\nu}{p^2}\;, \tag{4}$$

where $G_\rho(p^2)$ and $L_+(p^2)$ are one loop integrals arising from the fermion determinant in the bosonized action. We obtain

$$\begin{aligned} G_\rho(p^2) &= \frac{1}{G_V} + 8N_C\int\frac{d^4q}{(2\pi)^4}h^2(q)\frac{z(q^+)z(q^-)}{D(q^+)D(q^-)}\left[\frac{-q^2}{3} - \frac{2(p\cdot q)^2}{3p^2} + \frac{p^2}{4} - m(q^-)m(q^+)\right], \\ L_+(p^2) &= \frac{1}{G_V} + 8N_C\int\frac{d^4q}{(2\pi)^4}h^2(q)\frac{z(q^+)z(q^-)}{D(q^+)D(q^-)}\left[-q^2 + \frac{2(p\cdot q)^2}{p^2} - \frac{p^2}{4} + m(q^-)m(q^+)\right], \end{aligned} \tag{5}$$

where N_C is the number of quark colors, and we have used the definitions $z(p) = [1-\bar{\sigma}_2 f(p)]^{-1}$, $m(p) = z(p)\,[m_c + \bar{\sigma}_1\, g(p)]$, $D(p) = p^2 + m^2(p)$ and $q^\pm = q \pm p/2$. Here the functions $z(p)$, $g(p)$ and $h(p)$ are Fourier transforms of the nonlocal form factors in Eqs. (2).

The functions $G_\rho(p^2)$ and $L_+(p^2)$ correspond to the transverse and longitudinal parts of the vector fields, which describe meson states with spin 1 and 0, respectively. Thus the masses of the physical ρ^0 and $\rho^\pm$ vector mesons (which are degenerate in the isospin limit) can be obtained by solving the equation

$$G_\rho(-m_\rho^2) \;=\; 0\;, \tag{6}$$

where the minus sign is due to the fact that the action is given in Euclidean space. In addition, physical states $\tilde{\vec{v}}_\mu$ have to be normalized through

$$\tilde{\vec{v}}_\mu(p) = Z_\rho^{-1/2}\;\vec{v}_\mu(p)\;, \tag{7}$$

where

$$Z_\rho^{-1} = g_{\rho qq}^{-2} = \left.\frac{dG_\rho(p)}{dp^2}\right|_{p^2=-m_\rho^2}\;. \tag{8}$$

For the axial vector meson sector one has to take into account the mixing between the $\vec{\pi}$ and $\vec{a}^\mu$ fields. The corresponding quadratic action reads

$$\begin{aligned} S_E^{\rm quad}(\vec{\pi},\vec{a}_\mu) \;=\; \frac{1}{2}\int\frac{d^4p}{(2\pi)^4}\Bigg\{ & G_\pi(p^2)\,\delta\pi(p)\delta\pi(-p) + G_a^{\mu\nu}(p^2)\,\delta\vec{a}_\mu(-p)\delta\vec{a}_\nu(p) \\ & + i\,G_{\pi a}(p^2)\left[\delta\vec{a}_\mu(p)\,p^\mu\delta\vec{\pi}(-p) - \delta\vec{a}_\mu(-p)\,p^\mu\delta\vec{\pi}(p)\right]\Bigg\}\;, \end{aligned} \tag{9}$$

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where $G_\pi(p^2)$ and $G_{\pi a}(p^2)$ are one loop integrals [16]. The tensor $G_a^{\mu\nu}(p^2)$ is similar to $G_v^{\mu\nu}(p^2)$, the only difference being the signs of the mass terms $m(q^+)m(q^-)$ in both the spin 1 and spin 0 pieces. The physical states $\tilde{\vec{a}}^\mu$ and $\tilde{\vec{\pi}}$ can be obtained by the relations

$$\begin{aligned} \vec{\pi}(p) &= Z_\pi^{1/2}\, \tilde{\vec{\pi}}(p) \ , \\ \vec{a}^\mu(p) &= Z_a^{1/2}\, \tilde{\vec{a}}^\mu + i\,\lambda(p)\, p^\mu\, Z_\pi^{1/2}\, \tilde{\vec{\pi}}(p) \ , \end{aligned} \tag{10}$$

where Z_π and Z_a are wave function renormalization factors, and $\lambda(p)$ is determined by requiring that the cross terms in the quadratic expansion vanish.

4. f_π and f_v decay constants

The pion weak decay constant f_π is given by the matrix elements of the axial currents $J^a_{A\mu}(x)$ between the vacuum and the physical one-pion state at the pion pole:

$$\langle 0|J^a_{A\mu}(0)|\delta\tilde{\pi}^b(p)\rangle = i\,\delta_{ab}\, f_\pi(p^2)\; p_\mu \ , \tag{11}$$

with $p^2 = -m_\pi^2$. On the other hand, the matrix elements of the electromagnetic current $J_{\mathrm{em}\,\mu}(0)$ between the neutral vector meson state $\delta\tilde{v}^3_\mu \equiv \rho^0_\mu$ and the vacuum determine the vector decay constant f_v:

$$\langle 0|J_{\mathrm{em}\,\mu}(0)|\delta\tilde{v}^3_\nu(p)\rangle \ = \ e\, f_v(p^2)\,(g_{\mu\nu}\, p^2 - p_\mu p_\nu) \ , \tag{12}$$

with $p^2 = -m_\rho^2$, where e is the electron charge.

In order to obtain these matrix elements within our model, we have to "gauge" the effective action through the introduction of gauge fields, and then we have to calculate the functional derivatives of the bosonized action with respect to the electromagnetic and axial gauge fields and the renormalized meson fields. Notice that if the action is written in terms of the original states $\delta\vec{\pi}$, in order to calculate the matrix element in Eq. (11) one has to take into account the mixing with the $\vec{a}^\mu$ fields. In addition, due to the nonlocality of the interaction, the gauging procedure requires the introduction of gauge fields not only through the usual covariant derivative in the Euclidean action but also through a transport function that comes with the fermion fields in the nonlocal currents (see e.g. Refs. [3, 8, 12]), giving rise to tadpole-like contributions to the decay constants. The various diagrams contributing to f_π are schematized in Fig. (1). Analytical expressions for the functions $f_\pi(p^2)$ and $f_v(p^2)$ will be given in a forthcoming article [16].

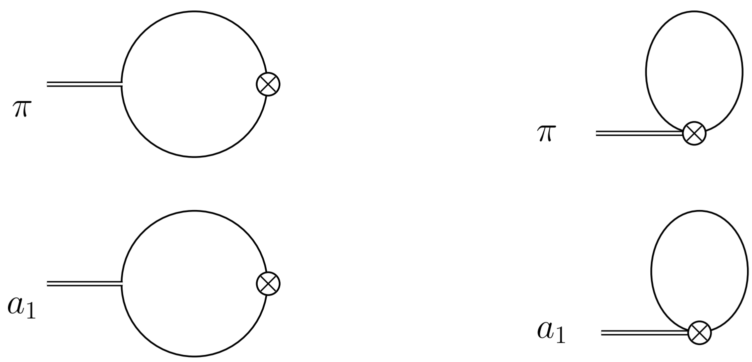

Figure 1: Diagrams contributing to the calculation of the pion decay constant.

5. $\rho \to \pi\pi$ decay

In general, different transition amplitudes can be calculated by expanding the bosonized action to higher orders in the meson fluctuations. In order to obtain the decay amplitude of the ρ meson into two pions we have to calculate the corresponding functional derivative of the effective action, namely

$$\left.\frac{\delta^3 S_E}{\delta\tilde{\pi}^a(q_1)\delta\tilde{\pi}^b(q_2)\delta\tilde{v}^c_\mu(p)}\right|_{\delta v_\mu=\delta\pi=0} = (2\pi)^4\,\delta^{(4)}(p+q_1+q_2)\;i\,\epsilon_{abc}\Big[\tilde{F}_{\rho\pi\pi}(p^2,q_1^2,q_2^2)\;(q_{1\mu}+q_{2\mu}) + \tilde{G}_{\rho\pi\pi}(p^2,q_1^2,q_2^2)\;(q_{1\mu}-q_{2\mu})\Big]\;. \quad (13)$$

The $\rho \to \pi\pi$ decay width is then given by

$$\Gamma_{\rho\to\pi\pi} = \frac{1}{12\pi}\,m_\rho\,g^2_{\rho\pi\pi}\left(1-\frac{4m_\pi^2}{m_\rho^2}\right)^{3/2}\;, \quad (14)$$

where $g_{\rho\pi\pi} \equiv \tilde{G}_{\rho\pi\pi}(-m_\rho^2,-m_\pi^2,-m_\pi^2)$.

The meson fields in Eq. (13) are physical states. In terms of the unrenormalized fields, taking into account the $\pi - a$ mixing we have

$$\tilde{G}_{\rho\pi\pi}(p^2,q_1^2,q_2^2) = Z_\rho^{1/2}\,Z_\pi\Big[G_{\rho\pi\pi}(p^2,q_1^2,q_2^2) + \lambda(p)\;G_{\rho\pi a}(p^2,q_1^2,q_2^2) + \lambda(p)^2\;G_{\rho aa}(p^2,q_1^2,q_2^2)\Big]\;. \quad (15)$$

The one-loop functions $G_{\rho\pi\pi}(p^2,q_1^2,q_2^2)$, $G_{\rho\pi a}(p^2,q_1^2,q_2^2)$ and $G_{\rho aa}(p^2,q_1^2,q_2^2)$, which can be obtained after a rather lengthy calculation [16], can be associated with the diagrams of Fig. (2).

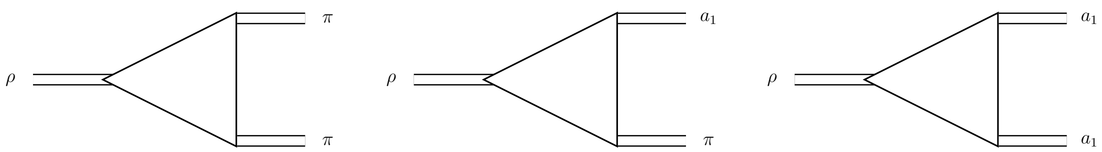

Figure 2: Diagrams contributing to the $\rho \to \pi\pi$ decay amplitude.

6. Numerical results

6.1. Model parameters and form factors

In order to properly define the model it is necessary to give the model parameters and to specify the form factors $f(z)$, $g(z)$ and $h(z)$ in the fermion currents. Here we consider two parametrizations, corresponding to different functional dependences for the Fourier transforms $f(p)$ and $g(p)$ that guarantee the ultraviolet convergence of loop integrals. The first one, which we call P1, is given by simple Gaussian functions [8, 9, 10, 11, 12]

$$g(q) = e^{-q^2/\Lambda_0^2}\;, \qquad f(q) = e^{-q^2/\Lambda_1^2}\;. \quad (16)$$

The second, that we call P2, is chosen so as to reproduce lattice QCD results [6] for the momentum dependence of effective quark propagators. In this case the form factor functions are taken as [13]

$$g(q) = \frac{1+\alpha_z}{1+\alpha_z f_z(q)} \, \frac{\alpha_m f_m(q) - m\alpha_z f_z(q)}{\alpha_m - m\alpha_z} \ , \qquad f(q) = \frac{1+\alpha_z}{1+\alpha_z f_z(q)} \, f_z(q) \ , \tag{17}$$

where

$$f_m(q) = \left[1 + \left(\frac{q^2}{\Lambda_0^2}\right)^{3/2}\right]^{-1} \ , \qquad f_z(q) = \left[1 + \left(\frac{q^2}{\Lambda_1^2}\right)\right]^{-5/2} \ . \tag{18}$$

Notice that the form factors introduce two additional parameters, Λ_0 and Λ_1, which play the role of effective momentum cutoffs. Both types of form factors have been previously considered in the context of nonlocal chiral quark models in Refs. [13, 15].

In our model there is also another form factor $h(p)$ coming from the vector and axial vector currents. For definiteness and simplicity we assume the effective behavior of quark interactions to be similar in the $J = 0$ and $J = 1$ channels, therefore we choose this form factor to have the same form as the function $g(p)$.

Now, given the form factor functions, one can fix the model parameters so as to reproduce the observed meson phenomenology. We recall that in our model the parameters in the effective action are the quark mass m_c and the coupling constants G_S, G_V, and κ, to which we have to add the scale parameters Λ_0 and Λ_1.

The parameters are determined as follows. The effective cutoffs Λ_0 and Λ_1 are fixed in such a way that they lead to momentum dependences of the effective mass and WFR in the quark propagator in agreement with lattice QCD results. The remaining four parameters are determined by fixing the value of the quark WFR at momentum zero, $Z(0) \simeq 0.7$ (as dictated by lattice QCD estimations), and by requiring that the model reproduces the empirical values of three physical quantities. Here we have chosen the masses of the π and ρ mesons and the pion weak decay constant f_π. The numerical results for the model parameters corresponding to the above described form factor functions are quoted in Table 1.

	P1	P2
m_c [MeV]	3.36	1.48
$G_S\Lambda_0^2$	29.7	21.84
Λ_0 [GeV]	1.03	0.95
Λ_1 [GeV]	1.10	1.50
κ_P [GeV]	3.41	6.35
$G_V\Lambda_0^2$	19.31	15.56

Table 1: Parameter values for parametrizations P1 and P2.

The momentum dependence of quark propagators is illustrated in Fig. 3, where we show the curves for the functions $f_m(p)$ and $Z(p)$ that correspond to our parametrizations P1 and P2, together with lattice QCD results.

6.2. Meson properties

Once the parameters have been determined, we can calculate the values of the above mentioned meson observables for the pseudoscalar, vector and axial-vector sectors. Our numerical results for parametrizations P1 and P2 are summarized in Table 2, together with the corresponding phenomenological values.

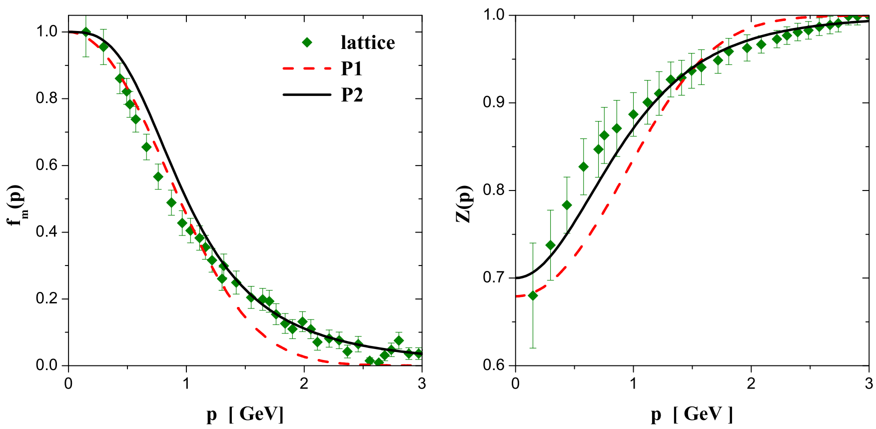

Figure 3: Momentum dependence of the functions $f_m(p)$ and $Z(p)$ for parametrizations P1 and P2, in comparison with lattice results.

	P1	P2	Empirical
$\bar{\sigma}_1$ [MeV]	560	560	-
$\bar{\sigma}_2$ [MeV]	-0.470	-0.430	-
$-\langle\bar{q}q\rangle^{-1/3}$ [MeV]	295	380	-
m_π [MeV]	146	139	139
f_π [MeV]	91.2	91.8	92.4
m_ρ [MeV]	784	779	775.5
f_v	0.16	0.21	0.20
$\Gamma_{\rho\to\pi\pi}$ [MeV]	122	148	149.1

Table 2: Numerical results for various phenomenological quantities.

7. Summary

We analyze the description of light pseudoscalar and vector mesons within an SU(2) chiral quark model that includes nonlocal four fermion couplings with wave function renormalization. In particular we calculate in this framework the masses and decay constants for the ρ and π mesons, as well as the decay width of the ρ meson into two pions. The model parameters are determined considering two different nonlocal form factor shapes, namely simple Gaussian functions and lattice QCD inspired form factors.

From our numerical results it is seen that in both cases it is possible to find sets of model parameters that lead to reasonable phenomenological values for the meson masses and decay constants. The agreement is particularly good for our parametrization P2, which is also able to reproduce adequately the momentum dependence of mass and WFR in the quark propagators found in lattice QCD calculations.

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References

[1] Nambu Y and Jona-Lasinio G 1961 *Phys. Rev.* **122** 345; **124** 246
[2] Vogl U and Weise W 1991 *Prog. Part. Nucl. Phys.* **27** 195; Klevansky S P 1992 *Rev. Mod. Phys.* **64** 649; Hatsuda T and Kunihiro T 1994 *Phys. Rept.* **247** 221
[3] Ripka G 1997 *Quarks bound by chiral Fields* (Oxford: Oxford University Press)
[4] Schafer T and Shuryak E V 1998 *Rev. Mod. Phys.* **70** 323
[5] Roberts C D and Williams A G 1994 *Prog. Part. Nucl. Phys.* **33** 477; Roberts C D and Schmidt S M 2000 *Prog. Part. Nucl. Phys.* **45** S1
[6] Parappilly M B, Bowman P O, Heller U M, Leinweber D B, Williams A G and Zhang J B 2006 *Phys. Rev. D* **73** 054504
[7] Furui S and Nakajima H 2006 *Phys. Rev. D* **73** 074503
[8] Bowler R D and Birse M C 1995 *Nucl. Phys. A* **582** 655; Plant R S and Birse M C 1998 *Nucl. Phys. A* **628** 60
[9] Schmidt S M, Blaschke D and Kalinovsky Y L 1994 *Phys. Rev. C* **50** 435
[10] Gomez Dumm D and Scoccola N N 2002 *Phys. Rev. D* **65** 074021; Gomez Dumm D and Scoccola N N 2005 *Phys. Rev. C* **72** 014909
[11] Scarpettini A, Gomez Dumm D and Scoccola N N 2004 *Phys. Rev. D* **69** 114018; Contrera G A, Gomez Dumm D and Scoccola N N 2010 *Phys. Rev. D* **81** 054005
[12] Gomez Dumm D, Grunfeld A G and Scoccola N N 2006 *Phys. Rev. D* **74** 054026
[13] Noguera S and Scoccola N N 2008 *Phys. Rev. D* **78** 114002
[14] Hell T, Roessner S, Cristoforetti M and Weise W 2010 *Phys. Rev. D* **81** 074034
[15] Contrera G A, Orsaria M and Scoccola N N 2010 *Phys. Rev. D* **82** 054026; Gomez Dumm D, Noguera S and Scoccola N N 2011 *Phys. Lett. B* **698** 236; Pagura V, Gomez Dumm D and Scoccola N N 2012 *Phys. Lett. B* **707** 76; Carlomagno J P, Gomez Dumm D and Scoccola N N 2013 *Phys. Rev. D* **88** 074034
[16] Gomez Dumm D and Izzo Villafañe M F, work in preparation