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OPTIMAL ECONOMIC GROWTH WITH RECURSIVE PREFERENCES: DECREASING RATE OF TIME PREFERENCE*

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1. Introduction

In the field of optimal growth theory, since Ramsey's time it is frequent to maximize a welfare function consisting of the discounted sum of instantaneous utilities, with a constant rate of time preference. Such an optimality criterion implies that preferences are independent over time.

Following in the tradition of Irwing Fisher (1930), Koopmans (1960) presented an alternative for the case of discrete time periods; he used an assumption of limited non-complementarity over time, and showed that there exist welfare functions for which the rate of time preference is variable. In a later study with Beals (Beals and Koopmans (1969); see also Iwai (1972)) he showed that the implications are that even in the simplest situations described by the neoclassical growth model initial conditions may affect the long run optimal path.

Equivalent results for the case of continuous time have been reached by the present author (Mantel 1966, 1967a, 1967b, 1970, 1993, 1995).

A similar approach by Uzawa (1968) reaches different results due to his particular assumptions; his optimal paths are, in the long run, independent of initial wealth. The same is true of the case of discrete time, as in the studies of Epstein (1987), Lucas and Stokey (1984), and others. Blanchard and Fischer (1991), referring to Uzawa's increasing rate of time preference, state that this "is not particularly attractive as a description of preferences and is not recommended for general use". Irving Fisher, the father of the creature, explains in his *Theory of Interest* (1930), pg 247 that "near the minimum of subsistence ... to give up one iota of this year's income in exchange for any amount promised for next year would mean too great a privation in the present. ...his rate of time preference will gradually decrease ... that is, the larger the income, other things remaining the same, the smaller the degree of impatience."

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The particular case -when time is continuous- in which the resulting welfare function can be explicitly represented as an integral as in Uzawa's essay has been analyzed elsewhere by the present author (Mantel 1967c), but is not covered by the other studies which assume that the welfare functional is quasi-concave. The results for growth theory obtained illustrate the use of such a welfare function taking into account Fisher's form for the pure rate of time preference; the qualitative behavior of optimal growth paths is there seen to be similar to that described previously, including the multiplicity of asymptotic growth paths, with long run situations depending on the initial endowments. Thus preferences may lead to a "poverty trap" even in the case of a well behaved neoclassical technology. In such cases the rich desire to become richer, whereas the poor prefer getting poorer; under high levels of initial capital stock society may wish to save and accumulate more, while the same preferences may lead to dissaving if initial wealth is below some critical level.

The translation of the previous results for continuous time to the present case of discrete time is not trivial, since the strong result due to Pontryagin and his associates valid in the first case —that the Hamiltonian is zero for all instants, result equivalent to the Keynes-Ramsey-Koopmans condition- has no equivalent in the second case. Only the weaker result due to the envelope theorem -i.e. that the current shadow price of the capital good measures the marginal increase in welfare due to an increase in the initial capital endowment- is true.

The present essay presents a parallel analysis for the case in which time is presented in discrete units.

2. Preference over time.

The present section presents briefly some of the results needed in the sequel. These results are in accord with those of Beals and Koopmans. Nevertheless some of the proofs are not appropriate for the present case, since here the welfare functional may not be quasi-concave.

A *time-path* or *program* is a real-valued sequence ${}_0z=(z_0, z_1,\ldots z_t,\ldots)$, where the non-negative integer argument t represents time. The present moment is $t=0$, and the planning horizon of the family or society extends to the infinite future. Admissible programs are bounded. The set of all admissible paths will be called Z.

A *consumption path* or *program* initiating at the present time 0, ${}_0c \in Z$, is an instance of an admissible path. The set of admissible consumption programs X consists of those admissible paths for which the consumption rate is never negative, so that $c_t \geq 0$ for all t. A *welfare function -prospective utility* in Koopmans' terminology- is a real valued function W defined on the set X of consumption programs. The *immediate* or *instantaneous utility* of a consumption at period t, c_t ,is the value of the real valued function $u(c_t)$. It should be noted that this definition is at variance with Koopmans' concept adhered to in Mantel (1993). In the present essay, the relation between the two concepts of immediate and prospective utility is given by the sum

$$W({}_0c) = \sum_{s=0}^{\infty} \left\{ \prod_{t=0}^{s-1} \alpha(c_t) \right\} u(c_s),$$

where the real-valued function $\alpha(c)$ is the (psychological) *factor of time preference*.

The welfare function satisfies the following postulates, originally stated by Koopmans for discrete time in a slightly different but equivalent way. For the continuous time case, see the author's essays already cited. Here only verbal statements will be provided.

P1.(Sensitivity). There exist two admissible programs which agree with each other from some time on with different welfare levels.
This postulate serves the purpose of excluding the uninteresting case in which all consumption programs are equivalent to each other, which then trivially would all be optimal.

P2.(Limited non-complementarity over time). The ordering of two programs with the same tail —i.e. which coincide after the first period— is not affected if their common tail is replaced by another one, as long as after the replacement both programs still have equal ending sections.
The limited non-complementarity postulate is the central assumption which allows writing the welfare function in recursive form.

P3.(Stationarity). The ordering of two programs which coincide in the first period is the same one obtains by discarding the common initial period and advancing these programs for a time duration equal to that period -thus only the ordering of the tails is relevant-. The purpose of this postulate is not its realism; one might argue that future generations have different tastes, so that the evaluation of a program from their perspective is not equal to the present generation's evaluation of the same program from today's perspective if it were to start today. The reason for requiring this postulate to be satisfied is to isolate the pure time preference effect from changes in tastes, in the belief that given sufficient freedom in the choice of preferences any development path may be justified. This would then provide no proof that development paths behave differently in the long run solely on the grounds of different initial endowments in response to a variable rate of time preference.

P4.(Extreme programs). There exist a best and a worst program, with finite welfare levels, $\underline{W}$, $\overline{W}$.

Thus the welfare of an admissible consumption program is bounded. Koopmans showed that under suitable continuity assumptions these postulates imply the existence of an *aggregator function*, V, whose arguments are the rate of consumption *c* and the welfare level Wwhich is strictly increasing in its arguments -if the representation of preferences is chosen appropriately-.

In the case of discrete time, the aggregator function to be considered in the present essay has the property that the welfare of a program can be evaluated by solving the following difference equation with bounded end condition for its initial value. The solution is given by a *welfare path Wt* such that.*Wt* is the prospective utility one would derive from implementing today the tail of the program intended to start at time t.
In the present case, this means that

$$W_t \equiv W({}_tc) = \sum_{s=t}^{\infty} \left\{ \prod_{v=t}^{s-1} \alpha(c_v) \right\} u(c_s),$$

It is easily checked that the sequence ${}_0W$ satisfies the difference equation

$$W_t \equiv u(c_t) + \alpha(c_t) W_{t+1} \qquad \textit{for all } t \geq 0 \qquad (1)$$

The interpretation of the difference equation (1) is as follows. The prospective utility of the consumption program starting at time t is W_t . The program offers a consumption c_t for that period. The aggregator function $V(c,W) \equiv u(c) + \alpha(c)\,W$ -the right hand side of the difference equation- uses this information to indicate that if those two quantities are known, advancing the program by discarding the consumption of the first period after the current time t achieves the prospective utility W_{t+1}..

For the purposes of the maximization of welfare to be carried out in the next section, it will be assumed that the following conditions hold for the instantaneous utility function $u(.)$ and the rate of time preference function $\rho(.) \equiv 1/\alpha(.) - 1$.

P5.(Utility aggregator). The *utility-aggregator* function $V(c,W) \equiv u(c) + \alpha(c) W$ satisfies

1. $u(c)$ and $\alpha(c)$ are continuous on R_+ and twice continuously differentiable for $c > 0$,
2. $u' > 0 > u''$; and $\lim_{c\to 0+} u'(c) = +\infty$, $u(0) \geq 0$
3. $\alpha' > 0 > \alpha''$; $\alpha(0) > 0$.
4. $0 < \alpha(c) \leq \bar{\alpha} < 1$ for some constant $\bar{\alpha}$, for all $c \geq 0$.

It is easily verified that such an aggregator function produces a welfare function which satisfies the postulates. Following Boyd (1990), one notes that $V(.,.)$ is continuous and increasing in its arguments, whereas

$$|V(c,W) - V(c,W')| = \alpha(c)|W - W'| \leq \bar{\alpha}|W - W'|$$

shows that it satisfies a Lipschitz condition of order one. Therefore both Boyd's conditions W2 are satisfied and a unique prospective utility or welfare function exists.

For uniformly bounded admissible consumption programs ${}_0c$ one has

$$W({}_0c) = \lim_{T\to\infty} W(0;T,W(T))$$

where $W(t;T,W(T))$ is a solution of the difference equation (1) with any end condition satisfying

$$W(T;T,W(T)) = W(T)$$

with

$$0 \leq \underline{W} \leq W(T) \leq \bar{W}$$

The solution can be given in closed form. For example, if we take W(T)=0 for all T, this solution is

$$W(0;T,0) = \sum_{s=t}^{T-1} \left\{ \prod_{v=t}^{s-1} \alpha(c_v) \right\} u(c_s),$$

We shall give $1/\alpha\,() - 1 \equiv \rho\,()$ the name of *instantaneous rate of time preference*. As will be seen it acts as a discount rate. Note that it is independent of the representation of preferences only for constant programs; in the general case its value depends on the (welfare) utility scale. In the present situation, it coincides with the concept of a *pure rate of time preference* used elsewhere; in more general situations the two concepts are equal only in the case of stationary programs (see Koopmans 1960).

3. The technology and feasibility

The technology will be described by a simple neoclassical aggregate production function with the following properties.

P6.(Technology). The real-valued production function $f(k)$-where the non-negative real number k denotes capital per capita- is

1. continuous, twice continuously differentiable for $k > 0$,
2. $f(0) = 0; f' > 0; f'' < 0$, $lim_{k \to 0+} f(k) = +\infty$.[1]
3. There exists a $k_m > 0$ such that $f(k_m) = k_m$.

Here it is assumed that there exists only one good, used both for consumption and for accumulation. The symbol k stands for the *capital-labor ratio*, $f()$ for the *gross output-labor ratio* —the latter net of maintenance and other costs, including the investment necessary for keeping the capital-labor ratio constant and the capital stock of the previous period—. The second assumption is standard —see e.g. Stokey and Lucas (1989)—, and states that capital is an indispensable input and that output plus capital per capita is an increasing and concave function of capital per capita. The last line can be justified in an economy with a growing labor force, where it is conceivable that as labor becomes scarce it will be impossible to produce enough to sustain the capital-labor ratio. In the sequel no reference will be made to the rate of growth of labor, which will be assumed to be constant. All relevant variables will be expressed equivalently in absolute or in *per capita* terms.

Denote the highest sustainable -"golden rule"- consumption rate by

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[1] Nota del editor: se reproduce exactamente como el artículo original pero se estima que el límite cuando k tiende a 0^+ hace referencia a la derivada, es decir, $f'(k)$.

$$\bar{c} \equiv arg\ max \{f(k) - k \mid 0 \le k \le k_m\},$$

the corresponding level of capital by $\bar{k}$, so that both quantities are positive and $f'(\bar{k}) = 1; c = f(\bar{k}) - \bar{k}$

A *capital path* is an admissible path ${}_0k \in X$; it is *feasible for an initial capital stock k* if $k_0 = k$ and $0 \le t$ imply

$$0 \le k_{t+1} \le f(k_t)$$

The associated consumption path ${}_0c$ satisfies

$$c_t = f(k_t) - k_{t+1} \qquad (2)$$

so that $0 \le c_t \le f(k_t)$

To simplify the exposition, the analysis will be restricted without loss to those situations in which the initial capital stock is productive, i.e.$0 < k_0 < k_m$. In that case feasibility implies $0 < k_t < k_m$ for all t. Consequently the capital path is uniformly bounded, and so is the consumption path, with $0 < c_t < \bar{c}$. The problem to be solved now consists in determining the optimal feasible capital, consumption and welfare programs.

Define the (psychological) *discount factor*, β, *associated* with a feasible consumption path, ${}_0c$, as follows. The discount factor is

$$\beta_t \equiv \prod_{s=0}^{t-1} \alpha(c_s) \qquad (3)$$

and satisfies the inequalities

$$\alpha(0)^t \le \beta_t \le \bar{\alpha}^t \le \bar{\alpha} < 1$$

for all $t \ge 0$, due to **P5**.

This expression uses the instantaneous rate of time preference $\rho \equiv 1/\alpha - 1$ as a discount rate to evaluate the relative merit of events at time t as seen from the perspective of the present time 0.

These definitions allow the following problems to be stated, in the fashion of Stokey and Lucas (1989).

4. Optimality

Consider the sequential maximization problem
(SP)

$$v(k_0) \equiv max \sum_0^{\infty} \beta_t u(c_t)$$
$$s.t. \quad \beta_{t+1} \le \alpha(c_t)\beta_t \quad \beta_0 = 1$$
$$k_{t+1} \le f(k_t) - c_t \quad k_0 > 0$$
$$\beta, c, k \ge 0$$

and the associated functional equation
(FE)

$$v(k) = \max_c \{u(c) + \alpha(c) v(y) | c + y \le f(k); c, y \ge 0\}$$

Given the assumptions in the present essay, both problems have the same, non-empty, set of solutions —see Stokey and Lucas (1989)—. The results in section 5 of Boyd (1990) guarantee the existence of an optimal path starting from k_0 with a continuous, strictly increasing value function $v(.)$.

Bellman's recursive relation is, for the optimal programs $({}_0W, {}_0c, {}_0k)$ with $W_t \equiv v(k_t)$ for all $t \ge 0$,

$$W_t \equiv v_t = u_t + \alpha_t v_{t+1} \quad (4)$$

where

$$v_t \equiv v(k_t), u_t \equiv u(c(k_t)), \alpha_t \equiv \alpha(c(k_t)), c(k_t) \equiv f(k_t) - g(k_t), v_{t+1} \equiv v(g(k_t))$$

and the corresponding policy function is given by

$$g(k) = f(k) - \arg\max_c \{u(c) + \alpha(c) v(y) | c + y \le f(k); c, y > 0\} \qquad (5)$$

Since v is maximum with respect to consumption c, one has the necessary first order

$$u'_t + \alpha'_t v_{t+1} = \alpha_t v'_{t+1} \quad (6)$$

and second order conditions.

$$u''_t + \alpha''_t v_{t+1} - 2\alpha'_t v'_{t+1} + \alpha_t v''_{t+1} \le 0 \qquad (7)$$

Differentiating 6 one has

$$(u''_t+\alpha''_t v_{t+1})(f'_t-g'_t)+\alpha'_t v'_{t+1} g'_t-(\alpha'_t (f'_t-g'_t)v'_{t+1}+\alpha_t v''_{t+1} g'_t)=$$
$$(u''_t+\alpha''_t v_{t-1})f'_t-(u''_t+\alpha''_t v_{t+1})g'_t+2\alpha'_t v'_{t+1} g'_t-(\alpha'_t f'_t v'_{t+1}+\alpha_t v''_{t+1} g'_t)=0$$

Solving for g'.

$$g'_t=\frac{u''_t+\alpha''_t v_{t+1}-\alpha'_t v'_{t+1}}{u''_t+\alpha''_t v_{t+1}-2\alpha'_t v'_{t+1}+\alpha_t v''_{t+1}} f'_t \qquad (8)$$

The numerator of the fraction is negative since u'', α'' are so, whereas α', v, v' are positive. The denominator is negative because of the second order condition for a maximum (7). Consequently, $g(.)$ is increasing.

For c' one has

$$c'_t=f'_t-g'_t=\frac{\alpha_t v''_{t+1}-\alpha'_t v'_{t+1}}{u''_t+\alpha''_t v_{t+1}-2\alpha'_t v'_{t+1}+\alpha_t v''_{t+1}} f'_t$$

so $c(.)$ is increasing if the numerator is negative; for this it suffices that $v(.)$ be concave —which it need not be—.

From Bellman's relation (4) above, the envelope theorem implies that

$$v'_t=\alpha_t v'_{t+1} f'_t \qquad (9)$$

From equation (8) one has that $g'>0$, so that

$$k_1>k_0 \Rightarrow g(k_1)>g(k_0) \qquad so \qquad {}_1k>{}_0k$$

•**Definition**. A capital path is *strictly monotone* if it is constant or either always strictly increasing or else always strictly decreasing.

One has the following results.

1.The value function $v(.)$ is strictly increasing in k. For if $\varepsilon>0$, since $(k_0+\varepsilon, {}_1k)$ is feasible,

$$v(k_0+\varepsilon)\geq u(f(k_0+\varepsilon)-k_1)+\alpha(f(k_0+\varepsilon)-k_1)v(k_1)$$
$$>u(f(k_0)-k_1)+\alpha(f(k_0)-k_1)v(k_1)=v(k_0)$$

where the first inequality follows from the definition of a maximum, and the second strict inequality is due to the fact that $u(.)$ and $\alpha(.)$ are strictly increasing, whereas as can be seen from (SP), $v(.)$ is nonnegative.

2. The necessary condition (9), due to the envelope theorem, implies that

$$f'>0, \quad \text{so that} \quad k_t<\bar{k}, \quad \forall t$$

3. $v_t \geq 0$ implies that

$$u_t \leq v_t \qquad \forall t \geq 0$$

because of Bellman's equation (4), since α, $v \geq 0$.

4. The necessary first order condition for a maximum, (6), implies that

$$v_{t+1} \leq \frac{\alpha_t}{\alpha'_t} v'_{t+1}$$

hence, another way of seeing that $v(.)$ is increasing can be obtained from here;

$$v'_{t+1} \geq \alpha'_t v_{t+1} / \alpha_t > 0 \quad (10)$$

so that v (.) is strictly increasing if $v > 0$, since $\alpha' > 0$.

5. Capital paths are strictly monotone. For let $k_1 > k_0$; then, because of 10, $v(k_1) > v(k_0)$, and

$$g' > 0 \Rightarrow g(k_1) > g(k_0) \quad \text{so} \quad k_2 > k_1$$

Thus if $k_1 > k_0$, by induction k_t is always increasing over time; similarly, if $k_1 < k_0$ then k_t is always decreasing . Hence optimal capital paths are strictly monotone in the sense of Beals and Koopmans, i.e. they are either constant, or strictly increasing, or strictly decreasing over time.

6. Stationary solutions imply

Bellman's equation	$v = u + \alpha v$
Maximum f.o.g	$u' + \alpha' v = \alpha v'$
Envelope theorem	$v' = \alpha v' f'$

The last relation, for positive v', implies the equilibrium condition $\alpha f' = 1$

stating that the marginal productivity of capital f'- 1 should equal the rate of time preference $1/\alpha - 1$. Both are positive since $0< \alpha \leq \alpha$. Define the set of stationary capital levels

$$K \equiv \{k \,\varepsilon\, N ; \phi(k) \equiv \alpha[f(k) - k] f'(k) = 1\}$$

Assume nondegeneracy, that is to say, if $k \in K$ is a stationary capital level then

$$\alpha f'' + f'\alpha'(f' - 1) \neq 0$$

so that it is locally unique. Thus the number of capital levels is finite.

But the assumptions on the functions imply that $\phi(0) = \alpha(0) f'(0) > 1$ and $\phi(\infty) = \alpha f'(\infty) = 0 < 1$ so that one has

$$\#K = odd, \qquad hence\ K \neq \emptyset$$

Order the elements in K according to magnitude, so that $K = (k^1, \ldots, k^n)$, $n = \#K$ odd, with $0 \equiv k^0 < k^1 < k^2 < \ldots < k^n < +\infty \equiv k^{n+1}$. Define the attraction intervals

$$K^i \equiv \{k \,\varepsilon\, R \mid k^{i-1} < k < k^{i+1}\}$$

for i odd. Then if $k_0 = k^i$ for i even one has $k_t = k_0$ for all t. For other values of k_0,.

$k_0 \,\varepsilon\, K^i$, for some i odd, and
if $k_0 < k^i$ then k_t increases toward k^i
if $k_0 > k^i$ then k_t decreases toward k^i

Thus one has the following result.

•**Proposition**. Optimal capital paths are strictly increasing (decreasing, constant) if the marginal product of the initial capital stock exceeds (is less than, equals) the pure rate of time preference corresponding to a constant capital path equal to that initial capital stock, that is, if

$$f'(k) - 1 > (<,=)\ \rho[f(k) - k] \equiv 1/\alpha[f(k) - k] - 1 \qquad (11)$$

This result is also true if the rate of time preference is constant or increasing. The difference resides in that if the pure rate of time preference is constant or increasing then there is only one capital-labor ratio with a marginal product equal to it, whereas if it is decreasing there may be several solutions to the equality in relations (11). This central result of the present investigation is summarized in the next proposition.

•**Proposition**. If the initial capital stock is very large, the optimal path will be strictly decreasing. If $\rho(0) < f'(0) - 1$ and the initial capital stock is very low the path will be strictly increasing, else it will decrease toward zero. For intermediate initial capital stocks, there may be several intervals for which the path rises or for which it falls, separated by constant paths along which the pure rate of time preference equals the marginal product of capital.

Some remarks on the relation between continuous and discrete time follow.

In continuous time, the main result of Pontryagin's maximum principle is that for autonomous systems the Hamiltonian is identically zero. This means that, in our notation, the current price of capital is given by

$$\pi = \frac{dW}{dt} \Big/ \frac{dk}{dt}$$

On the other hand, in discrete time the envelope theorem says that

$$\pi = v'$$

Relating both results, approximately,

$$v' \equiv \frac{\partial v}{\partial k} = \pi \approx \Delta W / \Delta K.$$

This shows the consistency of both approaches; it is clear that the continuous time case is easier to handle, since then the constancy of the Hamiltonian is exact, providing a first integral for the system of differential equations which allows the elimination of one of the state variables, W, in terms of the other, k, and the control, c.

A heuristic argument for the characterization of an optimal program, the Keynes-Ramsey-Koopmans argument, is valid for continuous time. First presented by Ramsey (1928), who attributes it to Keynes for the case of a zero rate of time preference, and later by Koopmans (1965) for a constant rate of time preference, it runs as follows. At any time t, increasing consumption by a fraction ε of the investment rate, dk / dt, during a sufficiently short time period means an increase in consumption of $\delta c = \varepsilon \, dk / dt$. This produces a gain in welfare equal to

$$\begin{aligned}\delta W &= \delta\big(W({}_0c) - W({}_1c)\big) \\ &\approx -\delta\frac{dW}{dt} = -\delta[\rho W - u] \\ &\approx -[\rho' W - u']\delta c = \varepsilon[u' - \rho' W]\frac{dk}{dt}\end{aligned}$$

and a loss -due to postponement of capital accumulation by a fraction ε - equal to $\varepsilon\, dW / dt$. The net gain is therefore

$$-\left([\rho' W - u']\frac{dk}{dt} + \frac{dW}{dt}\right)\varepsilon$$

and should not be positive if the capital path is to be optimal. Since ε can have any sign, it follows that

$$\frac{dW}{dt} + [\rho' W - u']\frac{dk}{dt} = 0 \qquad (12)$$

Replacing the time derivatives of W, k by the corresponding differences, one has approximately $\partial v/\partial k = u' + W\alpha' \approx \Delta W/\Delta k$.

This says that the undiscounted price of the consumption good measures the welfare effect of a.marginal addition to the capital stock.

5 Conclusion.

The present investigation started with setting out a welfare function for a family or a social planner wishing to design an optimal growth program in a neoclassical setting. "The proof of the cake is in the eating", which in the case of an economist in the position to advise the planner means that it is desirable to try out several criteria for optimal growth so as to ascertain the effects these have on the shape of the resulting optimal programs. It is difficult to ask the planners for their preferences, so it will be simpler to deduce them from their choice among optimal paths obtained from different optimality criteria.

A welfare function has been presented which is not so simple as to reduce to one with a constant pure rate of time preference, but still simple enough to be amenable to analysis.

The results that have been obtained show that on the one hand there are similarities with the case of a constant rate of time preference, in that the capital paths are one of three types,

1. constant for all time, in case that initially the pure rate of time preference coincides with the marginal productivity of capital;

2. strictly increasing, accumulating capital by consuming less than is produced, approaching a long run capital-labor ratio asymptotically in case the pure rate of time preference falls initially short of the marginal productivity of capital;

3. strictly decreasing, decumulating capital by consuming more than is produced, again approaching a long run capital-labor ratio asymptotically, in case the pure rate of time preference exceeds initially the marginal productivity of capital.

On the other hand there are important differences.

1. In the case of a constant -or increasing, as proposed by Uzawa- rate of time preference there exists a unique capital-labor ratio to which all capital programs tend in the long run independently of the initial endowment of the economy. In other words, poor societies will restrict their consumption to accumulate capital until the long run capital- labor ratio is reached, whereas rich societies will eat up their capital until that same long run capital-labor ratio is attained.

2. In the case of a falling rate of time preference -as proposed by Irving Fisher- there may exist a multiplicity of long run relative endowments. This means that the development path of an economy depends on its initial endowments; society is not willing to disregard its past.

Observing the development paths of different countries, it seems quite reasonable to expect to find situations in which there are at least two different capital-labor ratios at which the pure rate of time preference equals the marginal product of capital. In such a case, a very poor society may decide that the effort to accumulate capital is too high, that the benefits will take too long to be reaped, and thus embark in a high consumption program leading to a low -perhaps zero- long run capital-labor ratio. On the other hand, a somewhat richer society with an initial capital endowment exceeding some critical amount, may have sufficient incentives to decide to undertake the effort, to tighten their belts by consuming less, to accumulate and reach a long run capital-labor ratio that is higher than the present one.

When the rate of time preference falls with increasing consumption rates, a country may decide not to undertake the effort of economic development when its initial capital endowment is below some critical level, whereas if it were above that level it would be willing to sacrifice its present generation for the well-being of the future ones. It is impossible to obtain such a result with a constant or increasing rate of time preference in the case of a simple neoclassical technology.

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