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ON SEARCH THEORY AND CUSTOMERS*

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1. Introduction

The development of customer relationships is a very common phenomenon from the every day life. One possible explanation for this is that consumers may have to *search* for an appropriate provider of the good. If these consumers have some searching cost, once they have found a "good" store they will stick to it. This can be thought of as the consequence of a "perceived" switching cost.

The formation of customers is associated in the literature with a number of factors: (i) switching costs on the side of the consumers ("brand loyalty", see Klemperer (1995)); (ii) experience goods, where issues of quality and imperfect information play an important role; and (iii) repeat purchases and search over a price distribution (see McMillan and Morgan (1988), and Benabou (1993)).[1] In our model, we will have repeat purchases of a certain good but the search phenomenon that generates the long-term relationship will be over types of stores and not over prices. In many respects, our model will be a special case of Jovanovic's Matching Model where the quality of a match for the consumer is either of two things: a good match if she gets paired with a "sophisticated" store; or a bad match if the store is a standard one.[2]

These customer-seller relationships may cause a switching lag among the different activities that the sellers perform. For example, even if after some change in the environment it is no longer profitable for the seller to develop a customer relationship, she will still keep the customers that she already has. The general idea behind this is that it is economically cheaper for a seller to maintain a current customer than to acquire a new one. But, then, the existence of customer relationships will impose some stickiness on the type of activities that are performed in the economy (especially in those associated with customers). This effect will resemble (but is not) a switching cost on the side of the firm, and is caused by the fact that firms will be reluctant to change their production structures after they have already invested in the formation of a customer relationship.[3]

In this paper, we study a simple model of search where a buyer have to search for a seller who provides the services that she wants. Since not all sellers will get matched with a consumer, if the seller wants to become a provider of the "so-

[1] See Tirole (1988) for an excellent survey.

[2] See Jovanovic (1979) or Prescott and Townsend (1980) for details of the standard quality-of-the-match models.

[3] This idea should be reminiscent of the irreversible investment literature. For a good discussion of the hysteresis phenomenon induced by irreversibilities in the investment process see for example Pindyck (1993).

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phisticated" good, she will face only a partial probability that an actual consumer finds her. We will study the conditions under which this turns out to be profitable. Once a buyer finds a "sophisticated" seller, a customer relationship between the two develops. The buyer wants to avoid the searching cost and the seller wants to sell the "sophisticated" good, assumed to be a more profitable activity.

The idea is that the sophisticated good has to be produced in advance and the probability of actually selling it is less than one. But, once the seller has been found by a consumer, the latter becomes a customer of the former, to avoid searching costs. We assume the relationship lasts a certain period of time. During that time the "good" store produces the sophisticated good with the knowledge that it will be able to sell it with certainty. This in turn is assumed to be a profitable activity.

The search costs for the buyer are two-fold; on the one hand, there will be an explicit cost of searching s that we can associate with the time employed in it, the disutility of doing it, etc. On the other hand, since the buyer only finds a "good" seller with certain probability, there always exist the possibility that the buyer is not able to find that seller and end up consuming a standard good. Since we assume no bargaining power on the side of the consumer in any stage of the customer's relationship, this last factor gets washed out in equilibrium.

An interesting consequence of our setup is the emergence of a certain sluggishness effect in the response of firms' activities to changes in the environment. Suppose the economy is in an equilibrium where customer relationships have been developed. Assume now that an unexpected change in cost or in consumers' tastes (or even in the market conditions) makes the formation of this kind of relationship no longer profitable. At first sight, we would expect a complete halt in the production of "customers-type" services in the economy; however, if the change in cost is moderate enough, we may have firms already associated with customers by the time of the shock that will still provide those services to their commercial partners. Then, the fading of those activities in the economy will be gradual and spread over time.

The paper is organized as follows. The next section presents the environment and analyses in detail the choice problems that consumers and firms face. Section 3 defines a steady state equilibrium with customers and establishes sufficient conditions under which this equilibrium exists. Section 4 shows how these customer relationships can impose a certain sluggishness on the economy's response to changes in the cost of production of goods or in the tastes of the consumers. Section 5 concludes.

2. The Model

There are two types of entrepreneurs and one type of consumers in the economy. Also, there are two types of goods. Good 1 is a "standard" good and good 2 is a "sophisticated" good. Both entrepreneurs and consumers have a taste for consumption of good 1, but only consumers like good 2. They buy one unit of good 2 per period if at all. All goods are perishable; i.e., they last only one period. Each firm is owned by an entrepreneur who starts every period with an endowment of R units of type 1 consumption goods. Before the matching process takes place, the firm can use some of those goods to produce one unit of good 2 with the intention of selling it later to incoming customers. However, we will assume that only a limited number of the entrepreneurs have a cost advantage in the production of good 2 that makes them be the ones producing it in equilibrium. Consumers also have an endowment of R goods of type 1 per period. Only consumers search for firms. If they decide not to search or if they can not find a "good" store, they will consume their endowment. When a "sophisticated" store is reached by a consumer, the former sell good 2 to the latter at a price p in terms of good 1. Note that consumers can not transform good 1 into good 2, and they have to find an appropriate entrepreneur to be able to buy and consume the "sophisticated" good.

Let good 1 be the numeraire, i.e., $p_1 = 1$ and c_2 be the unit cost of production of good 2. Since firms meet one and only one consumer per period, they will produce at most one unit of good 2 per period.[4]

Consider the problem for the consumers. Assume they have a utility function with the form

$$U_2(x_1, x_2, \chi_s) = u(x_1) + \nu x_2 - s\chi_s, \tag{2.1}$$

where $u(\cdot)$ is strictly increasing and concave, $x_2 \in \{0, 1\}$ is the consumption of good 2, and χ_s is an indicator function that takes value one if the agent has searched in the current period and zero otherwise. We have assumed here that the consumer pays a cost of search in terms of utility but we could have assumed that the firms pay that cost (say in advertising that they are a "good" store).

[4] We are using a version of the random matching hypothesis. All consumers get match with a firm (we will assume one to one matching) after paying a search cost. However, they are randomly match with any one of the firms. It might then turn out that a consumer gets match with a standard firm; i.e., we allow for bad matches. In this case, no trade takes place and each one consumes their endowment. This setup is similar to Diamond (1971), but here consumers search not for "good" prices but for "good" stores (see also Benabou (1993)).

The results would be equivalent (see Klemperer (1995)).

Let W be the value function for a consumer looking for a "good" store. Then, we have

$$\begin{aligned} W &= \psi\left[u(R-p)+\nu-s+\delta\max\{u(R-p')+\nu+\delta W, W, u(R)+\delta W\}\right]+ \\ &\quad +(1-\psi)\left[u(R)-s+\delta\max\left\{\frac{u(R)}{1-\delta}, W\right\}\right], \end{aligned} \tag{2.2}$$

where ψ is the probability that a consumer finds an appropriate store and primes denote the next period values.[5] δ is the discount factor. Note that we are assuming zero returning costs; once the consumer has found a "good" store she will be able to go back to it next period without paying any cost. The search cost we are assuming is, therefore, more of an "inspection" cost rather than expenses per purchase (like trips or orders) (see Benabou (1993)). Additionally, we are assuming that after two periods the customer relationship is exogenously terminated.

We will be looking for an equilibrium where customer relationships get developed. For this to be compatible with consumers' behavior, it is necessary that the following two conditions hold. First, we need them wanting to search for "good" stores. This requires that

$$W \geq \frac{u(R)}{1-\delta} \tag{2.3}$$

Second, consumers must want to become customers; i.e. once they have found a "good" store they are willing to stick to it for a second period. If inequality (2.3) holds, then this second requirement is satisfied when the following inequality obtains

$$u(R-p')+\nu+\delta W \geq W. \tag{2.4}$$

Inequality (2.4) implies that at price p' the consumer prefers to go back to the good store in the second period over searching again. Note however that in combination with (2.3) this also implies that the consumer would rather become a customer than to consume his endowment and wait for the next period to search again. We can summarize these inequalities as follows

$$u(R-p')+\nu \geq (1-\delta)W \geq u(R) \tag{2.5}$$

[5] We are using here the Single Deviation Principle of dynamic programming theory. As a consequence, for example, when we consider that the agent doesn't want to search in the next period, we know that she will not search in any of the following ones. See Mas Colell et. al (1995).

Assume that entrepreneurs consume only the standard good 1 and their preferences are monotonic. Therefore, entrepreneurs will maximize

$$E\left\{\sum_{t}^{\infty}\delta^{t}\left[R+(p\chi_{2}-c_{2})\chi_{3}\right]\right\} \tag{2.6}$$

where χ_2 is a random variable that take value 1 if the firms get to sell good 2 and zero otherwise. And, χ_3 is an indicator function that takes value 1 if the firm has decided to produce good 2 and zero otherwise.

Let V be the value function for the producer deciding whether to produce good 2 in advance or not. Define π to be the probability that a consumer arrives to a particular store (firm). Then

$$\begin{aligned} V &= \max\{\pi\left[(R+p-c_2)+\delta\left(R+p'-c_2+\delta V\right)\right]+ \\ &\quad +(1-\pi)\left(R-c_2+\delta V\right), R+\delta V\}. \end{aligned} \tag{2.7}$$

The firm will be deciding between producing the sophisticated good or being a standard firm and have only good 1 available for sale. If a seller decide to produce good 2 in advance, with probability π, she gets to sell it.[6] Besides, she develops a customer relationship. The buyer will come back to the store next period to buy again good 2.

We assume that the firm fix p and p' at will[7] but they will satisfied conditions (2.3) and (2.4) if customer relationships are to be developed in equilibrium. We assume that the buyer does not have any bargaining power when she meets a seller. Her only choice is between accepting or not the deal the seller proposes to her. Her reservation utility is given by $u(R)$, which she can obtain by rejecting the deal and consuming her endowment of type 1 good. Then, the seller will fix a vector of prices $(p,p')=(\widehat{p},\widetilde{p})$ such that[8]

$$u(R-\widetilde{p})+\nu=(1-\delta)W=u(R). \tag{2.5'}$$

[6] We will assume that there is more firms than consumers searching. This is equivalent to assuming that only a fraction of the population have a preference for good 2. Note that under our assumption some of the "sophisticated" entrepreneurs will not get a match in equilibrium and they will consume $R-c_2$.

[7] We deal with the case where $p=p'$ or "flat price" system at the end of this section.

[8] This is an application of the usual result in the literature on both search (Diamond, 1971) and switching costs (Basu, 1993): the second stage Nash Equilibrium for price-setting firms is the monopoly outcome.

Given that the above conditions hold, using (2.2) we can find an explicit expression for the value function of the searcher

$$W = \frac{1}{1-\delta}\left[u(R-\widehat{p})+\nu\right] - \frac{s}{(1-\delta)\psi}. \tag{2.8}$$

Using this expression and (2.5') we have that if the firm wants to develop a customer relationship, it has to fix a price $\widehat{p}$ for the first period that is implicitly given by the solution to

$$u(R-p)+\nu = u(R)+\frac{s}{\psi}. \tag{2.9}$$

Using (2.9) we can see that the payoff for the consumer of terminating the relationship after the first period and search again in the second is the same as the one of just consuming her endowment of good 1 in the second period.

$$\begin{aligned}\psi[u(R-\widehat{p})+\nu-s]+(1-\psi)[u(R)-s] &= \\ = \psi[u(R)+\tfrac{s}{\psi}-s]+(1-\psi)[u(R)-s] &= u(R).\end{aligned}$$

In other words, if the consumer is indifferent between buying good 2 in the same store for the second time and consuming her own endowment of good 1, then she is also indifferent between those two alternatives and searching once and again every period.

Note that once the consumer gets to find a "good" store, since she cannot search again in this period, the store have some incentive to set $p=\widetilde{p}$ even in the first period. But since the only possible commitment price for the second period is $\widetilde{p}$, which makes the consumer indifferent between buying good 2 or not, we assume that she will buy it only if the seller honor the announced price for the first period $\widehat{p}$. Then, the seller will chose to commit to the announced prices if

$$\begin{aligned}R-c_2+\widetilde{p}+\delta\left[\pi\left(R-c_2+\widetilde{p}\right)+(1-\pi)\left(R-c_2\right)\right] \\ \leq R-c_2+\widehat{p}+\delta\left[R-c_2+\widetilde{p}\right],\end{aligned} \tag{2.10}$$

which reduces to

$$\widetilde{p} \leq \frac{1}{1-\delta(1-\pi)}\widehat{p}. \tag{2.11}$$

Therefore, we assume that prices $(\widehat{p},\widetilde{p})$ are being announced before search takes place and maintained along the relationship.

From (2.5') and (2.9) we have that

$$u(R - \widetilde{p}) + \nu = u(R) < u(R) + \frac{s}{\psi} = u(R - \widehat{p}) + \nu, \quad (2.12)$$

which implies that $\widetilde{p} > \widehat{p}$. We can think of this rise-in-price effect as a consequence of the interaction between the firm's desire to attract consumers and the monopoly power that the firm can exert over the consumer once the customer relationship has been set. Note that this result strongly depends on the existence of a positive s. That is to say, the cost of search is what stops the firm from exploiting the entire monopoly power in the first stage of the match. In a way, the firm is paying the search cost incurred by consumers. The other part of the cost of search that we mentioned in the introduction gets washed out by the assumption that consumers have no bargaining power on the price negotiations. If the consumer were to obtain a positive surplus from finding a good store (something that may easily be the case) then search frictions will be even more prejudicial to consumers. Even in the early stages of the relationship, our consumer does not obtain any surplus from the formation of the match. Hence, in the second stage she has nothing to lose in that respect that she has not lost already.

We now turn to the determination of conditions over the price-costs structure compatible with the existence of "sophisticated" firms. Firms set $(p, p') = (\widehat{p}, \widetilde{p})$ so that the potential customer wants to buy good 2 in the two periods the relationship lasts. Let V_2 be the value function if the seller chooses to produce good 2 in advance, and V_1 if she does not produce. Then

$$V_2 = \frac{R - c_2 + \pi p + \delta\pi\,(R - c_2 + p')}{1 - \pi\delta^2 - (1 - \pi)\delta} \quad (2.13)$$

and

$$V_1 = \frac{R}{1 - \delta}. \quad (2.14)$$

If V_2 is greater than (smaller than) V_1, the firm will choose (not choose) to produce the sophisticated good and attempt to enter a customer relationship. Note that as long as $p > c_2$, firms will always prefer to produce good 2 when they know ex-ante that they will be able to sell it.[9] This means that firms will want to keep

[9] This condition follows from the expression

$$R < R + p - c_2$$

which says that the profits in term of good 1 from selling one unit of good 2 are positive.

their already acquired customers if $\widetilde{p} \geq c_2$.

The condition $V_2 > V_1$ reduces to

$$\widetilde{p} > \frac{1+\delta\pi}{\delta\pi}c_2 - \frac{1}{\delta}\widehat{p}. \tag{2.15}$$

We will assume that only a proportion q_1 of the total number of firms face a value of c_2 such that (2.15) and $\widetilde{p} \geq c_2$ are satisfied in equilibrium.[10] In conclusion, there will be two types of sellers that a consumer can meet in equilibrium. On one side, there will be stores that have available only good 1. On the other side, there will be "good" stores that sell the sophisticated good 2. This composition of the population of existing stores is what motivates the search theoretic structure for the problem of the consumers introduced before.

An alternative approach to the price setting behavior studied above is to think that stores have to charge the same price to new consumers as to returning customers. Even though at first sight it might sound more appealing to the reader this type of arrangement, this is not so in the present setup. In the specific environment of our model, different prices at each of the stages of the customer relationship does not mean that the stores are selling the same good at different prices at the same time. The matches are one store-one customer, and in the second stage, stores sell good 2 only to customers. In any case, we will consider this alternative strategy for the sake of comparisons.

Firms-stores will choose a uniform p to solve the following problem

$$\begin{aligned} &\max_p \{R - c_2 + p + \delta[R - c_2 + p + \delta V]\} \\ &\text{s. to} \\ &\quad W \geq \tfrac{u(R)}{1-\delta} \end{aligned} \tag{2.16}$$

where the constraint in (2.16) is a participation constraint and W, according with (2.2) for $p = p'$, is given by[11]

$$W = \frac{\psi(1+\delta)[u(R-p)+\nu] - s + (1-\psi)u(R)}{1 - \psi\delta^2 - (1-\psi)\delta}. \tag{2.17}$$

[10] Clearly, firms with a lower value of c_2 are the ones that will develop a customer relationship in equilibrium, because they have larger benefits from doing so.

[11] Note that we are assuming that the seller announces the prices before the searching process and that maintains them after that. This will be more consistent with stores trying to attract customers *every* period. In the model as it stands, there is incentives for the firm to rise prices once the consumer has already incur the costs of search.

Then, the participation constraint reduces to

$$u(R-p)+\nu \geq u(R)+\frac{s}{\psi(1+\delta)} \tag{2.18}$$

which the firm will set to equality with $p = p^*$.

It is interesting to note that

$$\begin{aligned} u(R-\widehat{p})+\nu = u(R)+\frac{s}{\psi} &> u(R)+\frac{s}{\psi(1+\delta)} \\ &= u(R-p^*)+\nu > u(R) = u(R-\widetilde{p})+\nu \end{aligned} \tag{2.19}$$

and therefore $\widehat{p} < p^* < \widetilde{p}$.

This trade-off that firms face in setting the price in a customer-type environment is a well-known result in the switching cost literature (see Klemperer, 1995). Firms have to set a low enough price so that they would attract customers; but they also want to exploit as much as they can the monopoly power they acquire in the customer relationship. Note that the discount factor plays an important role in the determination of the value of p^*. A flat-price scheme implies that the buyer experience a net loss in the first period (given by $p^* - p$) and a net surplus in the second (given by $p' - p^*$). The relative size of the loss-surplus portion depends on δ because the surplus is available one period in the future.

There is another eventual time inconsistency problem in the flat price argument when applied to the model as it stands (see previous footnote). Firms that have attracted consumers in the second period with the flat price offer might try to set the second stage price at $\widetilde{p}$ (the monopolistic price) once the consumer has arrived to their location. The consumer will have nothing to lose by accepting this different price if she has already lost the ability to search in the current period.[12]

In what follows we will assume that the firms set a price schedule that satisfies equations (2.5') and (2.9). The main reasons are the ones just mention and the fact that the analysis does not change in a substantial way by introducing the flat scheme. For most of the arguments flat prices are almost a special case and we will mention them when the variations are not straightforward. Besides, the increasing price scheme seems to have some empirical foundation. Probably a good example of this type of practice is the sales strategy often used by US telephone companies to attract new customers.[13] See Klemperer (1995) for other examples.[14]

[12] The flat price policy would not be subgame perfect.

[13] In 1996, while this paper was in preparation, one of them was offering a very low rate for the first six month, and the standard rate thereafter.

[14] Klemperer (1995) mention that "unregulated TV stations show fewer advertisements at the

3. Equilibrium

To simplify matters, we will be assuming that there is a measure γ of consumers with $0 < \gamma < 1$ and a measure one of entrepreneur-firms. A proportion $0 < q_1 < 1$ of these firms have a cost advantage in the production of good 2, i.e. $c_2^{q_1} < c_2^{q_2}$, where $q_2 = 1 - q_1$. Consumers, as we said before, have an endowment R per period of the perishable good 1. Only consumers can search after paying a cost s in terms of utility.

Assume for the moment that parameters in the model are such that an equilibrium with customer relationships exists.[15] Let n be the fraction of firms (and consumers) that have a customer relationship at the beginning of a period.[16] We will analyze steady state equilibria, where n is constant over time. Suppose $c_2^{q_2}$ is *not* small enough as to make every firm a potential "good" store. Each period, then, only $q_1 - n$ of the potentially "good" stores are available to form a match. Clearly, ψ, the probability that a consumer gets a "good" match in equilibrium is given by the following expression

$$\psi = \frac{q_1 - n}{1 - n}. \tag{3.1}$$

(This is the probability for a match of being good).

As we said before, there are γ consumers of which only $\gamma - n$ are actually looking for a good store. Then, the probability that a good store gets matched with a consumer is

$$\pi = \frac{\gamma - n}{1 - n}. \tag{3.2}$$

Let n' be the value of n next period. Then, the law of motion for n will be

$$n' = \pi(q_1 - n), \tag{3.3}$$

and

$$n' = \psi(\gamma - n). \tag{3.4}$$

Checking for consistency using (3.1) and (3.2) we have,

$$n' = \psi(\gamma - n) = \frac{q_1 - n}{1 - n}(\gamma - n) = \pi(q_1 - n). \tag{3.5}$$

beginning of a film than they show later on when viewers are 'hooked'."

[15]We will carefully specify this later in the section when we introduce the definition of a Customers Equilibrium.

[16]A match in our model is a pair of one firm-one consumer.

In steady state we require $n' = n$. Then n^*, the steady state value of the number of customers at the beginning of the period solves

$$n^* = \frac{(q_1 - n^*)(\gamma - n^*)}{1 - n^*} \tag{3.6}$$

or equivalently

$$2\,(n^*)^2 - (1 + \gamma + q_1)\, n^* + q_1\gamma = 0. \tag{3.7}$$

Assuming that $(1 + \gamma + q_1)^2 > 8q_1\gamma$ there will exist at least one (but eventually two) $n^* \in [0, 1]$ that solve (3.7).[17]

We are ready now to state the definition of equilibrium on which we are interested in this paper.

Definition 3.1. *A* Customers Steady State Equilibrium *is a vector of prices* $(p_1, p_2) \in \mathbb{R}^2_+$ *and a steady state number of customers* n^* *such that*

1) "good" stores produce type 2 commodities (see equation (2.15))

$$c_2^{q_1} \le \varphi(p_1, p_2, n^*) \equiv \frac{\pi}{1 + \delta\pi} p_1 + \frac{\delta\pi}{1 + \delta\pi} p_2 < c_2^{q_2}, \tag{3.8}$$

2) "good" stores want to keep their customers

$$c_2^{q_1} < p_2, \tag{3.9}$$

3) "good" stores honor the committed prices

$$p_2 \le \frac{1}{1 - \delta(1 - \pi)} p_1, \tag{3.10}$$

4) prices are feasible for consumers

$$p_1 < R, \qquad p_2 < R, \tag{3.11}$$

[17] Equation (3.7) has solution

$$n_i = \frac{(1 + \gamma + q_1) \pm \sqrt{(1 + \gamma + q_1)^2 - 8q_1\gamma}}{4}$$

It is easy to show that only $n^* = \frac{(1+\gamma+q_1) - \sqrt{(1+\gamma+q_1)^2 - 8q_1\gamma}}{4} < \min\,[\gamma,\ q_1]$ (so that π and ψ are well defined) and therefore it is the unique possible steady state equilibrium value of n.

5) sophisticated consumers search for "good" stores (see equation (2.3))

$$u(R-p_1)+\nu=u(R)+\frac{s}{\psi}, \tag{3.12}$$

6) sophisticated consumers wants to become customers (see equation (2.4))

$$u(R-p_2)+\nu=u(R), \tag{3.13}$$

7) the steady state number of customers n^* solves equation (3.7).
Where $\pi=\frac{\gamma-n^*}{1-n^*}$ and $\psi=\frac{q_1-n^*}{1-n^*}$.

Consider the following system of equations

$$\left\{\begin{array}{l} u(R-p_1)+\nu=u(R)+\frac{s}{\psi} \\ u(R-p_2)+\nu=u(R) \end{array}\right\} \tag{3.14}$$

and call its solution $(p_1,p_2)=(\widehat{p},\widetilde{p})\equiv \mathbf{f}\left(R,\nu,\frac{s}{\psi},\xi\right)$, where ξ represents the set of parameters in the functional form of the utility function.

Proposition 3.2. *Assume the following four conditions* over the parameters *of the economy hold.*

(A)

$$(1+\gamma+q_1)^2>8q_1\gamma, \tag{3.15}$$

(B)

$$u(R)>\nu, \tag{3.16}$$

(C)

$$\widetilde{p}\leq\frac{1}{1-\delta(1-\pi)}\widehat{p}, \tag{3.17}$$

(D)

$$\widetilde{p}\geq\frac{1+\delta\pi}{\delta\pi}c_2-\frac{1}{\delta}\widehat{p}, \tag{3.18}$$

where $\pi=\frac{\gamma-n^*}{1-n^*}$, $\psi=\frac{q_1-n^*}{1-n^*}$, and $n^*\in[0,1]$ is a solution to (3.7).

Then, there exist a Customers Steady State Equilibrium for this economy with n^* customers, and $(\widehat{p},\widetilde{p})$ the equilibrium prices given by (3.14).

Proof. Conditions (B) imply that $(\widehat{p},\widetilde{p})$ satisfies (3.11). Condition (C) implies (3.10). By definition, $(\widehat{p},\widetilde{p})$ satisfies conditions (3.12) and (3.13). Condition (D) implies (3.8), where $c_2^{q_1} = c_2$ and $c_2^{q_2}$ is consider big enough to be irrelevant. Note that from (3.12) and (3.13) we have that $\widehat{p} < \widetilde{p}$, then together with (3.18) this implies that $\widetilde{p} > c_2$ by the following inequality

$$\delta\pi\widetilde{p} \geq (1+\delta\pi)c_2 - \pi\widehat{p} > (1+\delta\pi)c_2 - \widetilde{p}. \tag{3.19}$$

Finally, we know by condition (A) that there exist $n^* \in [0,1]$ satisfying (3.7). This shows that $\{n^*, (\widehat{p},\widetilde{p})\}$ is a SS equilibrium. ■

Note that $(\widehat{p},\widetilde{p})$ is the outside corner of the feasible set for equilibrium prices (see **Figure 1**).[18]

(insert Figure 1 here)

Example 3.3. *To obtain an example with $u(c) = c$ one can proceed in the following way. First, set values for q_1 and γ so that (3.15) is satisfied. Then, obtain the value of $n^* \in [0,1]$ and of π and ψ according with the proposition. Given those, set the values of ν, s and δ so that*

$$\nu > \frac{1}{\delta(1-\pi)}\frac{s}{\psi}. \tag{3.20}$$

Finally, set the values of R and c_2 such that

$$R > \nu > \frac{1+\delta\pi}{(1+\delta)\pi}c_2 + \frac{1}{(1+\delta)}\frac{s}{\psi}. \tag{3.21}$$

This values for the parameters will provide an example of a Customers Equilibrium.

The following set of parameters $\{\gamma = .6,\ q_1 = .3,\ R = 2,\ s = .1,\ \nu = 1.3,\ \delta = .9,\ c_2 = .6\}$ derive in an equilibrium with $\widehat{p} = 0.84$, $\widetilde{p} = 1.3$, $n^* = .11$, $\pi = .55$, $\psi = .22$.

[18] In **Figure 1**, line **D** is the boundary of the set defined by expression (3.8), **C** the boundary defined by (3.10) and **e** is the solution to the system of equations (3.14).

4. Customers and Sluggishness

The development of customer relationships like the ones studied above will imply that the sophisticated firms respond with some lag to changes in the cost structure for the production of goods or in other property of the economy that originally made the production of good 2 efficient for those firms. Once firms have "invested" in the formation of customers, they will try to maintain them even if the conditions of the economy have changed to make the development of customer relationships not profitable any more. However, we will see that the changes in the environment have to be moderate because otherwise the firm will terminate the relationship with its customers immediately. Clearly, the study of this sluggishness-in-response is a relevant issue when we analyze policy changes in economies where these long-lasting relationships are important.

Suppose we have an economy in a Customers Steady State Equilibrium $\{n^*, (\widehat{p}, \widetilde{p})\}$. Now, suppose that at the beginning of period t, the current period, an unexpected exogenous factor increase the cost of producing good 2, $c_2^{q_1}$. Let the change in $c_2^{q_1}$ be such that (3.8) does not hold any more and the value of producing good 2 in advance, V_1, is smaller than the value of producing only good 1, V_2. Call the new value of the cost c_2' (see **Figure 2**). Then, in the current period it will not be profitable any more to form a long-term relationship. However, as we saw before, there will be n^* firms and consumers already engaged in a customer relationship. These firms can charge a maximum of $p_M = \widetilde{p}$ for good 2 if they decide to produce it.[19,20] If they do produce the good, their payoff will be

$$\Omega_1 = R + p_M - c_2' + \delta\frac{R}{1-\delta}. \tag{4.1}$$

And if they do not produce it

$$\Omega_2 = R + \delta\frac{R}{1-\delta}. \tag{4.2}$$

[19] They know that the consumer will come to the store. Then, these consumers will buy good 2 only if

$$u(R - p_M) + \nu \geq u(R)$$

This expression is equivalent to (2.4). And the solution when it holds with equality is $p_M = \widetilde{p}$.

[20] Under the flat rate case $p = p' = p^*$, there will be an increase in price from p^* (defined in equation (2.18)) to $\widetilde{p}$ in the period of the change in cost (this seems to be a potentially testable fact).

Thus, the n^* firms that begin period t with a customer will still produce good 2 in that period if $\Omega_1 \geq \Omega_2$. This reduces to the condition

$$p_M \geq c_2'. \tag{4.3}$$

(insert Figure 2)

Then, if the increase in c_2 is small enough so that (4.3) holds, we will have that the economy stops to produce good 2 only gradually. If, on the other hand, c_2' is big enough so that $p_M < c_2'$, the halt in the production of good 2 will be immediate. Note that, in any case, sooner or later good 2 will disappear from the economy.

A similar effect can take place as a consequence of a decrease in the equilibrium value of the probability that a "good" store receives a consumer, π. However, in this case firms will *always* maintain their current customers. One possible shock that might eventually reduce the value of π is an exogenous decrease in γ, the number of consumers in the economy.[21] In this case the line **D** in **Figure 2** rotates upwards and might leave point **e** outside the feasible set of equilibrium prices (actually, the feasible set will be empty if this happens).

It is clear from the arguments above that economies with a well developed network of customer relationships will tend to adjust gradually to certain changes in the structure of production and demand. This "customers effect" is a manifestation of the irreversible investment phenomenon involved in the acquisition of those customers.

5. Conclusion

When a bakery opens up in some place, the grocery stores in the neighborhood will continue to have bread on their shelves. Even though the bakery may be a better place to buy bread, consumers will need to find out this by themselves and that can be a costly process. As a consequence, for a reasonable period of time, surrounding grocery stores will be able to sell bread to their customers. Of course, after some time, if it is truly more convenient to buy bread in the bakery, almost all the people in the neighborhood will switch and the stores will not be able to sell bread anymore.

[21] Note that one should be careful with this example because changes in γ also derive in changes in the equilibrium number of matches, n^* (and therefore in ψ and $\widehat{p}$).

We think that the ideas in this paper can help in the study and understanding of these type of phenomena.[22] We set up a model where consumers search for "good" stores and then become customers for a certain period of time. Stores produce specific goods in advance, called "sophisticated" goods. Since under normal conditions firms do not get a consumer with certainty each period to whom they could sell the good, they are eager to form long-lasting relationships. We defined a *Customers Steady State Equilibrium* and found conditions over the parameters of the economy under which this type of equilibrium exists. A high probability of finding a "good" store, a high probability for the store to be found by a potential customer and a high valuation of the "sophisticated" good in terms of utility, all of them contribute to the possibility that a Customers Equilibrium would arise.

Finally, we studied how this economy would transit from a Customers Equilibrium to a No-Customers Equilibrium after an unexpected change in the cost of production of the sophisticated good. The main result is that the economy will show some degree of sluggishness for changes in the structure of production due to the existence of these customer relationships.

Even though the correspondence between the model and the example at the beginning of this section is not exactly one-to-one, we think that the model captures fairly well the essence of the sluggishness produced by the formation of long-lasting relationships in the economy.

A potentially interesting extension to the present work would be to formulate a fully stochastic model where the cost of production of sophisticated goods follows a random process and to consider the response of the different economic agents to these dynamic stochastic changes.

To conclude, we also think that several elements in the model that was presented here may show to be specially relevant for the new developments on the theory of market organization (see Howitt and Clower (1999) for example), decentralized trading mechanisms and even monetary exchange. In particular, we conjecture that the determinants of currency substitution and the well documented hysteresis associated to this process (see, for example, Guidotti and Rodriguez (1992)) can probably be better understood in an environment where a version of the main insights of the present paper are somehow included. This is of course a major item in the author's future research agenda.

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[22] One can think that the appearance of a bakery is an exogenous shock modifying, for example, the number of potential consumers γ on the market of the groceries.

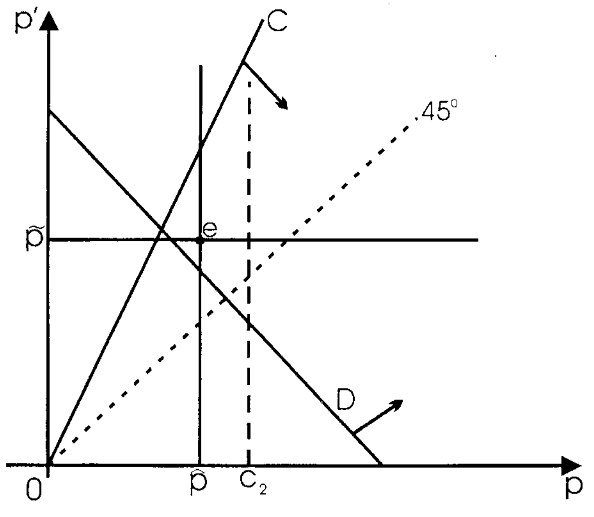

Figure 1

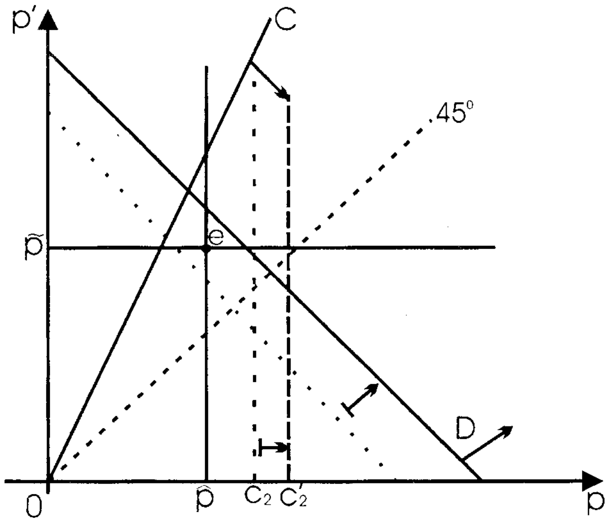

Figure 2

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SOBRE LA TEORIA DE BUSQUEDA Y LA FORMACION DE CLIENTES

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RESUMEN

Este artículo estudia un modelo simple de intercambio descentralizado donde los consumidores deben buscar por un vendedor que ofrezca ciertos servicios que se desea adquirir. Una vez que los compradores encuentran uno de estos "buenos" negocios, se desarrollauna relación de clientela entre ambos. Estudiamos los factores que determinan la formación de estos vículos y sus propiedades y características. Se define un Equilibrio de Estado Estacionario con Clientes y se establecen condiciones bajo las cuales tal equilibrio existe. Finalmente, se muestra como estas relaciones de clientela imponen cierto rezago en la respuesta de la estructura existente a cambios en los costos de producción (u otros parámetros de la economía).

ON SEARCH THEORY AND CUSTOMERS

HUBERTO M. ENNIS

SUMMARY

This paper studies a simple model of decentralized exchange where consumers have to search for a seller that provides the services they want. Once a buyer finds one of these "good" stores, a customer relationship between the two develops. We study the factors on which the formation of this link depends and some of their properties and characteristics. We define a Customers Steady State Equilibrium and give conditions under which one of these exists. Finally, we show how these customer relationships impose a certain sluggishness on the economy as it responds to changes in the cost of production.