ON THE MEASUREMENT OF THE SOCIAL BENEFITS
OF A CUSTOM UNION

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Introduction

Since Viner's (1950) time it is customary to measure the welfare effects of economic integration by using his concept of trade creation and trade diversion. This reflect the welfare of the world economy as a whole. It is therefore difficult to imagine that countries wishing to form a custom union should take such measurements into account. In fact a custom union will be formed not only to facilitate trade among its members, but also to exploit as much as possible its trade possibilities with the rest of the world.

It is furthermore usual to compare the final post integration situation with the actual pre-integration position. In this the additional error is incurred of assigning to the creation of the union benefits which really stem from a more rational commercial policy, benefits which could have been obtained without having to form the union.

It is the purpose of the present paper to present a procedure to estimate the benefits of economic integration from the point of view of the members of the union, separating these benefits from those obtained from a more rational commercial policy, in the spirit of a previous essay (Mantel and Martirena-Mantel, 1973).

The paper can be logically divided between two parts differing markedly in the degree of generality. The first part corresponds to the first five sections. In it the arguments are presented geometrically for a very simple two commodity - three country world. Section one presents some introductory comments on the type of benefits we do not wish to measure. Section two analyzes the formation of a union in such a way that it has no effect on the rest of the world. Section three determines the individually rational pre-union tariff equilibrium and section four the collectively rational postunion optimal tariff. Section five summarizes the gains from integration in the simple model.

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The second part is expounded in section six and the appendix. In the text the main results are summarized following in the main aspects the more expository first part while more technical aspects are treated in the Appendix.

1. The standard story and the true benefits from integration

The present section will give an example showing that the usual analysis measures benefits which cannot be truly attributed to the process of integration.

Figure 1 is taken from Johnson (1962), and represents a world economy consisting of three countries. Items refering to the home country will be identified with $D$; similarly $S$ and $W$ are reserved for the partner country and the rest of the world respectively. These three countries trade in two commodities $m$ and $e$, which correspond to the home country imports and exports and to the other two countries exports and imports.

The Figure shows the quantity of the home country’s imports on the horizontal axis and its price in terms of exports in the international market on the vertical axis. The curve labeled $D_D$ represents the demand schedule of the home country for its imports, given its income in terms of exports, whereas $S_D$ is the corresponding supply curve. The horizontal lines $S_W$ and $S_S$ represent the supply curves of the other two countries assumed to be perfectly elastic with the partner producing at higher costs than the rest of the world. The horizontal line $S'_W$ lies above $S_W$ by the amount of the tariff initially set by the home country on its imports. Obviously, pre-integration equilibrium requires that the home country obtains all its imports from the lower cost producer, in this case the rest of the world. This equilibrium is given by the intersection of the two domestic curves with the horizontal supply curve cum tariff $S'_W$.

On the other hand the post integration equilibrium will be given by the intersection of the domestic supply and demand curves with the supply curve of the partner $S_S$, since the discriminatory removal of the tariff on imports from the partner will give domestic importers access to this cheaper source.
The net benefits to the home country are obtained by subtracting from the sum of the gain in consumer surplus $B_1$ and the savings of domestic resources $B_2$, the additional cost $C$ due to replacing the imports by a more expensive foreign source. Thus in the present example one could conclude that economic integration will be beneficial to the home country.

Nevertheless this conclusion is false. The theory of the optimal tariff so well presented by H. Johnson (1958) himself shows that under the present assumptions, there should be no tariffs in the pre-integration situation. Therefore the benefits measured by the two triangles $B_1$ and $B_2$ are really not attributable to the process of integration. In fact, the cost of integration will be measured by the trapeze between the two supply curves $S_S$ and $S_W$ and the domestic supply and demanded curves $S$ and $D_D$. Any benefit must be attributed to a more rational commercial policy, which could be followed by the home country without having to set up an agreement with its partner.
2. Gains from an arbitrary pre-integration position

In the present section it will be shown how an agreement to eliminate tariffs between partners and setting a common tariff with the rest of the world will bring about a benefit to the union as a whole whatever the initial tariff levels are. The situation can be analyzed with the help of Figure 2 which is a simplification of the graph presented by Johnson (1962) for a different question.

The description of the world economy is the same as in the previous section except that in the present more general model increasing costs are allowed.

FIGURE 2
As before $D_D$ denotes the excess demand for imports of the home country and $S_S$, now rising the excess supply of the home country's imports by the partner. In the initial situation - preintegration equilibrium - the international price is $OP$, $QR$ is the tariff set by the home country and $UV$ is the export tax levied by the partner. The partner exports $PV$ to the home country, who imports the quantity $PQ$; the segment $VQ$ represents the quantity imported from the rest of the world.

The joint gain from trade of the two candidate partners can be represented by the area enclosed by the irregular polygon $QRSUTV$. This area is composed by the benefits accruing to the home country, the trapeze $PQRS$ and those accruing to the partner, the trapeze $PTV$.

The benefits of the home country come from consumer surplus, the area under the demand curve above a horizontal through point $R$ plus the proceeds of the tariff, the area of the rectangle with base $PQ$ and height $QR$. Benefits of the partner country stem from producer surplus, the area above its supply curve and below a horizontal through point $U$ plus the proceeds from its export tax given by the area of the rectangle with base $PV$ and height $VU$.

Consider now the effect of liberalizing trade among partners. In order to facilitate graphical analysis, assume that the union sets a common tariff for its imports from the rest of the world is not affected.

The new equilibrium point will be $R'$ on the home country's demand curve obtained in the following way. Shift the horizontal segment $VQ$ vertically downwards to the position $UW$ and then shift it upwards sliding it along the partner's supply curve $S_S$ keeping the point $U$ on that curve until the other extreme touches the domestic demand curve at $R'$. Due to this construction $V'R'$ represents the same imports from the rest of the world as previously did $VQ$. The segment $P'V'$ now represents the imports from the partner at a new common domestic price $OP'$, which also induces a domestic demand of $P'R'$.

The new tariff is given by the segment $PP'$. The gains from trade to the union at the new situation are given by the area of polygon $TU'Q'R'S$ and can be decomposed as follows. The domestic consumer surplus is the triangle $P'R'S$ under the demand curve. The home country's tariff proceeds is given by its imports from the rest of the world $V'R'$ multiplied by the tariff $PP'$ and can be represented by the area of the parallelogram $V'Q'R'Y'$. Finally the
partner's producer surplus which is given by the triangle $T V' P'$ above its supply curve.

Since the area of the triangle $U U' V$ equals that of the triangle $W Q' Q$ it is easily seen that the new situation represents a gain with respect to the initial situation equal to the area of the triangle $W R' R$. This area is one half the increase in the home country's imports times the segment $R W$ which by construction is equal to the sum of the tariffs set by the partner before the union is formed. This segment therefore represents the pre-integration discrepancy between the internal prices of the home country's imports in the two candidate partner countries.

The gains obtained in this analysis represent a lower bound to the possible gains. A common external tariff set optimally cannot provide lower benefits. Note that the final position always implies a gain in the benefits accruing to the partner, but not necessarily to those of the home country, so that some kind of transfer from the partner to the home country may be needed in certain cases.

3. **The individually rational or Cournot tariff equilibrium**

As argued in a previous essay (Mantel and Martirena-Mantel 1973) the true welfare effects of integration should not include the benefits resulting from a more rational post-integration behavior. Thus the natural starting point for the measurement of benefits should be the equilibrium solution to a tariff war, formally a non-competitive game.

In the present simple model, such a Cournot equilibrium - the Nash solution to the associated game of strategy - can be easily determined as is done in Figure 3, which represents the same international economy as Figure 2. For simplicity in the graphical presentation it will be assumed that tariffs are specific, a fixed amount per unit imported.

The individual maximum for each country is obtained by maximizing its utility subject to the net supply of the rest of the world, given their tariffs. In other words, the excess demand curve of each country should intersect the sum of the marginal excess supply of the non-partner and that of the candidate partner, the latter shifted by the amount of the tariff.

For example for the home country, whatever the export tax set by the candidate partner, its optimal tariff will be the vertical distance between the
lines \( \tilde{D} \) and \( D \) corresponding to its imports in Figure 3. Line \( \tilde{D} \) goes through the intersection \( B \) of the vertical through the free trade point \( F \) and the horizontal through \( A \), where \( S + W \) hits the vertical axis. That this is so can be seen if one notes that changes in the partner's export tax shift the supply line \( S + W \) parallel to itself. One of these shifted supply lines goes through \( B \). By construction, this line must intersect the vertical axis at a distance below \( A \) exactly equal to \( F B \), so that the corresponding marginal aggregate supply must go through the free trade point \( F \), and \( F B \) would then give the home country's optimal tariff. It is easily verified that any point on \( \tilde{D} \) has the property that if one of the family of \( S + W \) curves goes through it, then the corresponding marginal curve intersects \( D \) above it. Therefore the equilibrium price-quantity combination must lie on \( \tilde{D} \).

A similar argument provides the other equilibrium locus \( \tilde{S} + W \) drawn through point \( D \). Here \( C' \) and \( D \) play the roles of \( A \) and \( B \), respectively, \( E \) that of \( R \). Point \( C \) is the intersection of the home demand and the excess supply of the rest of the world; \( D \) is on the horizontal through \( C \) and the parallel to \( W \) through \( F \), and \( E \) is the intersection of the two supply lines.

The intersection of the equilibrium lines \( \tilde{D} \) and \( \tilde{S} + W \) determines the Cournot equilibrium point \( I \), giving the home country's imports and their international price in terms of exports. The home country's optimal tariff is, by construction the segment joining \( I \) vertically to its demand line \( D \). Net world exports are given by the segment joining \( I \) horizontally to the equilibrium line \( \tilde{S} \) net of world supply, whereas the vertical segment joining \( S \) with \( \tilde{S} \) represents the optimal export tax for the partner, given the home country's tariff. The resulting picture is as in Figure 2, giving a complete description of the pre-union Cournot or Nash tariff equilibrium.
4. **The collectively rational, optimal tariff union**

Our post-integration reference point will be the tariff optimizing union. We cannot find empirical significance to the benevolent union, carefully designed so as not to hurt any non-union member. Instead we assume that the union behaves aggressively exploiting its foreign trade possibilities as much as possible.

The optimal tariff union has been depicted in Figure 4. As before, $D$, $S$, $W$ represent the excess demand of the home country and the excess supplies of the partner and of the rest of the world. The curve $W Ma$ is the marginal curve corresponding to $W$, and $S + W Ma$ is the partner excess supply added to the
world marginal supply. The intersection of this curve at $T$ gives the equilibrium imports of the home country and the common domestic price $p + t$. The horizontal through $T$ intersects curve $S$ at point $U$, given the equilibrium quantity of the partner's exports and curve $WMa$ at giving the equilibrium quantity of the exports of the rest of the world. The corresponding international price is found by dropping a vertical from the letter intersection to curve $W$, thus providing point $Y$.

It is important to note that point $Z$, the intersection of the horizontal through $Y$ with the supply curve $S$, defines with point $T$ a segment - not shown in the figure - which is parallel to the aggregate supply curve $S + W$. This follows from the definition of the marginal supply curve. The line $WMa$ bisects the angle formed by the line $W$ and the vertical axis. Since aggregation is a linear operation, the same property holds for corresponding aggregates, so that the line $S + Ma$ bisects the angle formed by the line $S + W$ and $S$ - the latter being of course the sum of itself with the vertical axis.

FIGURE 4
Thus the point at which these three lines meet is on $S$, to the left of $Z$ and at a distance equal to $\overline{ZU}$ from it. Therefore a line parallel to $S + W$ through $T$ must bisect the segment defined by this triple intersection and point $U$, so that it goes through $Z$.

Consequently if one multiplies the tariff - the length $\overline{AY}$ - by the slope of the world supply curve $W$ one obtains the length of the segment $UA$. This is of course the well known formula which states that the optimal rate of the tariff equals the reciprocal of the foreign supply elasticity. It also follows that if $\overline{TV}$ is parallel to $S$ then $ZV$ equals $\overline{UT}$.

5. The gains from integration

According to our previous analyses it is possible to decompose the gains from integration accruing to the members of the union into two parts. These parts will be measured in monetary terms using the traditional consumer surplus analysis and correspond to the gains from the formation of a benevolent union and those obtained from exploiting the rest of the world respectively.

According to our analysis in Sections 2 and 3 the gains from the formation of a benevolent union are given by one half of the intraunion trade creation times the difference between the domestic prices of the home country's importables in the two partner countries in terms of the domestic country's exportables.

In other words, these benefits are one of the quantity by which the home country's importables increase when going from the Cournot point to the benevolent union equilibrium times the sum of the home country's tariff plus the partner export tax at the Cournot equilibrium. This benefit can be visualized in Figure 2 as the area of the triangle $RWR'$. The gains from exploiting the rest of the world measured in monetary terms, are given by one half of the increase of the rest of the world exports to the union multiplied by the sum of the increase in the international price of the home country importables in terms of exportables plus the reduction in the common tariff on the same goods.
The analysis leading to this formula can be followed in Figure 5 which essentially reproduces Figure 4 with the addition of the representation of the benevolent union. As in Figure 4 point T in Figure 5 represents the final optimal tariff equilibrium point on the home country's demand curve. The points U, V and Z retain their position. The equilibrium of the benevolent union is at C on the demand curve of the home country. Drawing horizontals and parallel to S through C one obtains E, F and G, similar in meaning as the points corresponding to the optimal tariff. The points M, N and P are common to both equilibria.

Consider the surpluses in both situations for the union. They are given in each case by consumer surplus - area under the home demand curve above the domestic price-, producer surplus - area above the partner supply curve below the domestic and international price lines, to the right of S and to the left of the parallel to S through the equilibrium point.

It is then easily checked that the benefit from more aggressive behavior of the union can be measured by the differences between two surfaces: the trapeze enclosed by the polygonal C T V M minus the parallelogram M Z G F.

From the analysis in section 4, the main diagonal of the parallelogram T V Z U is parallel to that of V P F M, so that these two are similar. And their heights are in the same relation than their sides. Thus one finds

\[ \frac{LK}{ZV} = \frac{KJ}{FP} \]

or equivalently

\[ \frac{LK}{FP} = \frac{KJ}{ZV} \]

showing that the parallelograms T V M N and V P G Z have equal area. The gain for the union then reduces to the sum of the areas of the triangle C T N and the parallelogram V P F M.

Denoting by \( \Delta w, \Delta t \) and \( \Delta p \) the increases in imports from the rest of the world, tariffs and international prices, respectively, one obtains
\[ \text{Area of } CTN = \frac{1}{2} \Delta w \left[ -\Delta(p + t) \right] \]

\[ \text{Area of } VPFM = \Delta w \Delta p \]

\[ \text{Benefits} = \frac{1}{2} \Delta w (\Delta p - \Delta t) \]

Thus the optimal tariff provides benefits of one half the creation of trade with the rest of the world multiplied by the sum of the increase in the international price of world exports and the tariff reduction. These three increments will be all positive if the current tariff is too high, negative in the opposite situation.

**FIGURE 5**
6. The benefits from Integration with many countries and commodities

In the general case we have to start from an initial Nash or Cournot equilibrium. The existence of such equilibrium has been shown for the general three country, two commodity, many consumer's case in a previous paper (Mantel and Martirena-Mantel, 1973) and the 1-commodity, n-country case in Otani (1978).

The welfare effects of benevolent union has been analyzed by Kemp and Wan (1976) who showed that such a union can be designed so as to improve the welfare of the union members without hurting the rest of the world.

Grinols (1981) later showed how the compensations necessary for the Kemp and Wan results can be implemented basing his analysis on the Grandmont and McFadden theorem on classical gains from Trade (1972).

It should be emphasized that all these authors analyze the formation of what we may call a "benevolent union" starting from an initially irrational position. This is so because they are careful to design a commercial policy which will not hurt the residents of non union countries.

In contrast with this our aim is to measure the true benefit from integration, starting from an individually rational situation to a collectively rational final solution. Rationality being defined in terms of applying commercial policies which make the best of the countries or union foreign trade opportunities.

Thus the third step in our analysis has not as far as we know been taken before and consists in obtaining further gains for the union by imposing the common tariff structure in an optimal way. The first difficulty that has to be taken care of in defining an optimal tariff in the general Arrow and Debreu model of competitive equilibrium - in which no interpersonal utility comparisons are possible - is to define what is meant by optimal.

For this purpose we will adopt Debreu's coefficient of resource utilization, which is adapted to comparing a given state of the economy with an optimal one in which individual preference levels are kept unchanged, to our presents needs which refer to a case in which individual welfare has to be improved.

The technique we shall employ is to determine for every individual residing in the union member country a direction of preference over the status quo. By this we mean a bundle of commodities for each consumer which when
added to the statu quo or Cournot allocation of that consumer will make him better off, and the more so the larger the scale of this bundle. It is then easy to define optimality, in terms of the scale associated with this bundle. We shall consider a tariff structure optimal if this scale common to all individuals in the union, is maximal. As is shown in the Appendix such a maximum exists and represents a Pareto improvement for the individuals residing in the union.

Three questions can be raised about this solution
1. Are there always strict Pareto improvements?
2. How do we measure the benefits?
3. Have we implemented the union?

The first question can be answered by considering in which case there will be no improvements. The most obvious case is that in which the members of the union and the union itself are small countries. We know that in that case no tariff can be optimal so that both the Cournot solution and the belligerent union solution coincide with free trade.

A somewhat less extreme case is that in which the union even though the countries and the union are small with respect to the rest of the world there are some local commodities traded among the members of the union for which supply and demand elasticities are finite. In this case there will be tariffs among the candidate partners at the Cournot solution. Integration will then lead to free trade.

In order to answer the second question the total pure benefits of integration will be broken up into two components as was done in section 5. The first component measures the benefits from moving from the Cournot equilibrium to a benevolent union whereas the second component measures the benefits of converting the benevolent union into a belligerent one.

Since consumer surplus has no meaning in the present context we propose to measure the benefits by the percentage increase of the value of the aggregate consumption over the union measured at final domestic prices.

Thus we have for the relative benefits in the first step the expression $(\overline{q} \overline{x} / \overline{q}, \overline{x}) - 1$ where $\overline{q}$ denotes the common domestic price vector in the benevolent union, $\overline{x}$ the corresponding aggregate consumption of the members countries and $x$ the same aggregate consumption at the Cournot equilibrium.

Similarly for the second step the relative benefits are given by $(\overline{q} \overline{x} / \overline{q} \overline{x}) - 1$ where $\overline{q}$ and $\overline{x}$ refer to domestic prices and aggregate
consumption of the union after an optimal tariff structure has been set by the union.

A measure of the total increase in benefits is given along similar lines by \( \frac{q_x}{\bar{q}_x} - 1 \) corresponding to a direct move from the Cournot equilibrium to the optimal tariff.

The third question can be answered following the lead of Grinols by computing the values of the transfers needed. With some modifications this analysis applies to our case of forming a belligerent union starting from Cournot equilibrium.

Let \( z^i \) denote the ith union member net import vector at the Cournot point traded at international prices \( p \). Similarly let \( \bar{z}^i \) and \( \bar{p} \) be the corresponding net trade and price vector of the belligerent union. Then \( \bar{p} \bar{z}^i \) is the value of the ith country post union trade at international prices; since it represents the difference between the value of its consumption at the net trade of the union, which must equal the value of the net exports of the rest of the world which by Walras law is zero. In other words, total net subsidies valued at international prices are zero.

If on the other hand one measures this subsidy at domestic prices one obtains \( \bar{q} \bar{z}^i \). The difference \( \bar{q} - \bar{p} = \bar{t} \) is the tariff so that the aggregate subsidy at domestic prices equals the proceeds of the tariff \( \bar{t} \bar{z} \), since as has been said the value of the union's net import is zero.
APPENDIX

A.0. In order to obtain more general results then those in the text it will be assumed that the world economy consists of a certain number of countries, of which those labeled $i = 1, \ldots, m$ are candidate partners of the custom union. The rest of the world is seen only through its excess supply function $f(p)$, where $p$ represents the $n$-coordinate vector of international prices, whereas the coordinates of $f$ represents the corresponding net exports. This function satisfies the usual assumption of zero degree homogeneity in prices and also satisfies Walras' Law, so that $pf(p) = 0$ for all $p$. Except for being differentiable as need no additional restrictions will be imposed on it.

Country $i$ is inhabited by $m_i$ consumers and production is carried out within its frontiers by $n_i$ firms. Consumer $j$ of country $i$ consumes a vector $x^{ij}$ of commodities and owns initial resources $w^{ij}$. His consumption to be feasible must be in his consumption set $X^{ij}$. Each firm produces a vector of net outputs $y^{ik}$ which is in its production possibility set $Y^{ik}$.

For each vector of net imports to the country $x^i$ the Government redistributes income optimally according to a welfare function $b^i$ depending on the individual utility functions $u^{ij}$. From this information it is possible to deduce the ith's country Meade (1952) / Rader (1972) induced preferences on its net imports, defined by the formula

$$ u^i(x^i) = \max \left\{ b^i(u^{i1}(x^{i1}), \ldots, u^{in}(x^{in}) / \sum_j (x^{ij} - w^{ij}) \leq x^i + \sum_k y^{ik}, x^{ij}, X^{ij} ; y^{ik} \in Y^{ik} \right\} $$

Given the usual assumptions in general equilibrium theory for example as presented by Debreu (1959) such utility function are continuous and quasi concave and well defined on a convex closed trade possibility set $X^i$.

In the sequel no further reference will be made to individual consumers and firms. It is always possible to retrace one's steps to obtain individual allocations from the net trades of each country, but this information is irrelevant for the purposes of this essay.
A.1. Measurement of the benefits of the beligerant union over the benevolent union.

The authorities of the union are assumed to set the tariffs optimally using some welfare indicator \( B(u) \) which depends on the levels of satisfaction \( u^i \) of the countries. The optimization problem can be set up as the maximization of this welfare indicator \( B(u) \) subject to \( u_i = u^i(x^i) \) and the usual feasibility condition that the aggregate consumption should not exceed the net imports from the rest of the world. Substituting the excess supply functions of the rest of the world \( f(p) \) for the net imports, one can apply the theorem of the dependence of the objective function on parameters - the international prices in our case - to obtain the first order differential of the function maximized with respect to all variables except \( p \) in the following form:

\[
dB = qf^\circ_p(p)\,dp
\]

where \( q \) denotes the vector of Lagrange multipliers associated with the market balance equations and will therefore be taken as domestic prices. If welfare is also to be maximized with respect to international prices, one obtains immediately the first order conditions

\[
q^\circ f^\circ_p(p^\circ) = 0
\]

Note that because of Walras' Law \( pf(p) = 0 \) one has the identity \( f^\circ(p) + pf^\circ_p(p) = 0 \). Since the tariff is \( t = q - p \), this necessary condition for optimality can be written as

\[
t^\circ f^\circ_p(p^\circ) = f(p^\circ)
\]

This says that, given the optimal tariff structure, the marginal effect on tariff revenue due to a price increase equals the net quantity imported of the corresponding commodity. This formula defines a whole family of optimal tariff structures. In the two commodity world, with a zero tariff on exports, it reduces immediately to the familiar reciprocal relation between the tariff as a fraction of the price of imports and the foreign price elasticity of supply.

The second differential is then of course negative so that the loss of setting a non optimal tariff structure can be measured approximately by \( -d^2\lambda / 2 \). One has, ignoring cubic and higher terms,
\[ B_\circ - B = -\Delta B \equiv -\left( B_p + \frac{1}{2} \Delta B_p \right) \Delta p \]

\[ = -\frac{1}{2} \left[ qf_p(p) - q_{o} f_p(p_o) \right] (p - p_o) \]

But the second term in the square brackets is zero because of the optimality condition, whereas zero homogeneity in international prices of the foreign excess supply function implies by Euler's theorem that \( f_p(p), p = 0 \). Hence

\[ B_\circ - B = \frac{1}{2} qf_p(p)p_o \]

\[ = \frac{1}{2} \left[ f_p(p) - f(p) \right] p_0 \]

where the last relation is obtained as in the similar transformation of the necessary condition for optimality.

This equation gives as a measure of the benefits of setting the common tariff at an optimal level as one half of the value of the differences between the marginal effects of international prices and the corresponding net import to the union, evaluated at the international prices valid at the final equilibrium point.

A.2. Benefits of the benevolent union over the Cournot solution

The benevolent union redistributes the net imports from the rest of the world attained at the Cournot solution in an optimal way using a welfare function \( B(u) \) where \( u \) is the vector of utilities.

Thus the allocation of trades to countries in a benevolent union corresponds to the maximum of \( B_u \) subject to

A.2.1. \[ u_i = u^i(x^i) \]

A.2.2. \[ \sum x^i = f(p) \]

For an interior maximum there exists a vector of Lagrange multipliers \( q \) such that
A.2.3. \[ B_i u^i_x = q \]

indicating the equality of the marginal rates of substitution of the different countries. It will be assumed that a welfare function has been chosen so that the individual countries trade balances measures at the international prices are zero. That this can be done has been shown by Grinols (1981), so that one has A.2.4. In general the competitive equilibrium conditions for distorted trade differ from the relations listed before only in that the common tariff structure is replaced by a different tariff structure for each country. Therefore equation A.2.3. has to be replaced in the case of a tariff ridden initial point by

A.2.5. \[ B_i u^i_x = q^i \]

where \( q^i = p + t^i \), the sum of international prices plus the tariffs set by country \( i \).

Consider the effect of small changes moving the economy away from a benevolent union.

One has

A.2.6. \[ db = \sum B_i u^i_x dx^i = \sum q^i dx^i \]

Which at the initial benevolent union point becomes

A.2.7. \[ dB_v = \sum q^i dx^i = q \sum dx^i = 0 \]

where use has been made of the fact that initially all \( q^i \) are equal to \( q \) and that the differential of aggregate trade of the union is zero due to equation A.2.2. since the international prices and therefore the net imports from the rest of the world remain unchanged.

In order to determine the welfare effect it is necessary therefore to compute the second differential

A.2.8. \[
\begin{align*}
    d^2 B_v &= \sum \left( dq^i dx^i + q^i d^2 x^i \right) \\
    &= \sum dq^i dx^i + q \sum d^2 x^i
\end{align*}
\]
where again use has been made of the equality of the initial domestic price vectors. Again the market balance equation A.2.2 guarantees that the second term in the above formula is zero.

On the other hand the constancy of the international prices also implies that changes in domestic prices are equal to changes in the tariff so that finally one obtains as an approximate measure of benefits of moving to a benevolent union

A.2.9.\[ \frac{1}{2} d^2 B_o \approx \frac{1}{2} \sum \Delta t^i \Delta x^i \]

This formula can be interpreted by describing it as one half the value of the increase in net imports multiplied by the reduction in the tariff.

Note that in the special case analyzed in the text the same result was obtained. The corresponding formula was much simpler because in the 2 commodity world only distortions on one of them need to be considered so that in the summation in formula A.2.9, only two terms remain.
REFERENCES


