

# An upper limit to the secular variation of the gravitational constant from white dwarf stars

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**Abstract.** A variation of the gravitational constant over cosmological ages modifies the main sequence lifetimes and white dwarf cooling ages. Using an state-of-the-art stellar evolutionary code we compute the effects of a secularly varying  $G$  on the main sequence ages and, employing white dwarf cooling ages computed taking into account the effects of a running  $G$ , we place constraints on the rate of variation of Newton’s constant. This is done using the white dwarf luminosity function and the distance of the well studied open Galactic cluster NGC 6791. We derive an upper bound  $\dot{G}/G \sim -1.8 \times 10^{-12} \text{ yr}^{-1}$ . This upper limit for the secular variation of the gravitational constant compares favorably with those obtained using other stellar evolutionary properties, and can be easily improved if deep images of the cluster allow to obtain an improved white dwarf luminosity function.

**Keywords:** Stars, white dwarfs, gravity

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## 1 Introduction

General Relativity is currently the preferred theory of gravitation, and relies on the equivalence principle. Consequently, it also relies on the assumption that the gravitational constant,  $G$ , does not vary with time or space location. However, the usual assumption that  $G$  is indeed constant is just a hypothesis, though quite an important one, which deserves to be explored. In fact, several modern grand-unification theories predict that the gravitational constant is a slowly varying function of low-mass dynamical scalar fields — see, for instance, Refs. [1–3] and references therein, for recent descriptions of the theoretical approaches which can be used to formally describe the variation of fundamental constants. If these theories are correct, we expect that the gravitational constant should experience slow changes over cosmological timescales.

In recent years, several constraints have been placed on the variation of the fine structure constant [2, 3]. This is a controversial issue, since there have been recent claims that for a range of redshifts ( $0.5 < z < 3.5$ ) the results are consistent with a time-varying fine structure constant [4–6], whereas other authors have challenged these results [7–12], or, at least, have put forth doubts on such a possible detection of a time-varying  $\alpha$  [13]. In sharp contrast with the vivid debate about whether (or not) there is evidence for a varying fine structure constant, relatively few works have been devoted to study a hypothetical variation of the gravitational constant. Probably, one of the reasons is the intrinsic difficulty of measuring the present value of this constant [14]. Actually, the gravitational constant is the fundamental constant for which we have the less accurate determination, and the several measures of  $G$  differ considerably among them. However, there are other reasons, being the weakness of the gravitational interaction another probable reason. Therefore, it is not surprising that many methods aimed to bound any hypothetical variation of  $G$  have been devised. At present, the most tight constraints are those obtained using Lunar Laser Ranging and Big Bang nucleosynthesis. The most recent analysis of the Lunar Laser Ranging experiments provides an upper bound  $\dot{G}/G = (0.2 \pm 0.7) \times 10^{-12} \text{ yr}^{-1}$  [15], whereas the bounds obtained from Big Bang nucleosynthesis bounds are of the same order of magnitude  $-0.3 \times 10^{-12} \text{ yr}^{-1} \lesssim \dot{G}/G \lesssim 0.4 \times 10^{-12} \text{ yr}^{-1}$  [16, 17]. However, Lunar Laser Ranging provides only local limits to the secular rate of variation of  $G$ , whereas Big Bang nucleosynthetic arguments are model-dependent. At intermediate cosmological ages the Hubble diagram of Type Ia supernovae can also be used to constrain the rate of variation of the gravitational constant, but the constraints are somewhat weaker  $\dot{G}/G \lesssim 1 \times 10^{-11} \text{ yr}^{-1}$  at  $z \sim 0.5$  [18, 19].

White dwarf stars allow to constrain in an independent way any hypothetical variation of  $G$ . The primary reason for this is that the evolutionary timescales of white dwarfs are very long. Thus, even small secular variations of  $G$  become prominent in the course of their lives. However, this is not the only reason whatsoever. In fact, because the inner cores of these stars are almost completely degenerate the mechanical structure of white dwarfs is very sensitive to the precise value of  $G$ , and any, otherwise small, secular change of its value should be quite apparent for long enough times. Moreover, white dwarfs do not have nuclear energy sources, and their evolution can be well described as a slow cooling process in which the gravothermal energy of their cores is released through a partially degenerate, insulating convective or radiative envelope. This cooling process is now very well understood for sufficiently large luminosities, say  $\log(L/L_{\odot}) \gtrsim -4.5$  [20]. Additionally, it has been recently shown [21] that the specific rate at which white dwarfs cool is not only sensitive to its secular rate of variation,  $\dot{G}/G$ , confirming previous theoretical evidence [22], but also to

the specific value of  $G$  at which the white dwarf was born.

In this paper we use white dwarfs to place an upper limit to the secular rate of change of the gravitational constant. To do this we use the number counts of white dwarfs of the very old, well-populated, metal-rich Galactic open cluster NGC 6791. This cluster is so close to us that very deep images can be obtained [23]. This has allowed to measure with unprecedented accuracy the white dwarf cooling sequence of the cluster, and to obtain reliable number counts which ultimately have allowed to derive a reliable white dwarf luminosity function [24, 25] — that is, the number of white dwarfs as a function of the magnitude. As it will be shown below the shape of the white dwarf luminosity function depends on  $\dot{G}/G$ , and this will allow us to derive an upper bound to the secular rate of change of  $G$ .

Our paper is organized as follows. In Sect. 2 we describe the evolution of white dwarf progenitors, while in Sect. 3 we overview the white dwarf cooling tracks employed in this work. We emphasize that these cooling sequences are state-of-the-art, while the progenitor evolutionary sequences with a varying  $G$  have been specifically computed for this work. Sect. 4 is devoted to describe how the white dwarf luminosity function of NGC 6791 was built. It follows Sect. 5, in which we show how  $\dot{G}/G$  can be constrained using the available data. Finally, in Sect. 6 we summarize our major findings, we discuss its significance and present our concluding remarks.

## 2 Evolution of white dwarf progenitors

As previously mentioned, in the present study white dwarf cooling tracks that were derived following in a self-consistent way the evolution of white dwarfs in the case of a varying  $G$  will be used. However, a slowly varying  $G$  also affects the evolutionary properties of white dwarf progenitors, and particularly their ages [26]. This is an important issue and has to be taken into account when evaluating the properties of Galactic globular or open clusters. To assess the effects of a varying  $G$  on the evolutionary times of progenitor stars, we have computed the main sequence evolution of two model stars of  $1.0$  and  $2.0 M_{\odot}$  — the progenitors of our  $0.525$  and  $0.609 M_{\odot}$  white dwarfs, see table 1 of Ref. [21] — considering three values for the rate of change of  $G$ , namely  $\dot{G}/G = -5 \times 10^{-11} \text{ yr}^{-1}$ ,  $\dot{G}/G = -1 \times 10^{-11} \text{ yr}^{-1}$ , and  $\dot{G}/G = -1 \times 10^{-12} \text{ yr}^{-1}$ . All the evolutionary calculations were done using the LPCODE stellar evolutionary code [27, 28], appropriately modified to take into account the effect of a varying  $G$ . In particular, we assume that the rate of change of  $G$  is so small that the evolution can be considered as adiabatic and we simply allow  $G$  to vary in the equation of hydrostatic equilibrium. This approach is fair and has been adopted in previous studies of this kind [26]. Additionally, it is important to realize that the main sequence evolution of progenitor stars in the case of a varying  $G$  is strongly dependent on the initial value of  $G$  at the Zero Age Main Sequence (ZAMS), when hydrogen starts to be burned in the center of the star. Accordingly, for each value of  $\dot{G}/G$  we have computed several sequences with different values of  $G_i/G_0$ , where  $G_i$  stands for the value of  $G$  at the ZAMS, and  $G_0$  corresponds to the present value of  $G$ . It is important to realize at this point of our discussion that in previous works [26] the important fact that the initial value of  $G$  should be larger (or smaller, depending on the adopted value of  $\dot{G}/G$ ) than its present-day value was not taken into account. Thus, our results clearly improve the only existing calculations. All in all, we have computed 19 evolutionary sequences that are listed in Table 1, which cover the most important stellar evolutionary phases, namely the hydrogen and helium burning phases, and the formation of

**Table 1.** Pre-white dwarf evolutionary sequences computed in this work. We list the stellar mass at the ZAMS (in solar units) and, for each value of  $\dot{G}/G$  (in units of  $\text{yr}^{-1}$ ), the initial value of  $G$  at the ZAMS,  $G_i/G_0$ . The numbers in brackets give the factor by which the main sequence age is reduced when a varying  $G$  is considered.

$M_{\text{ZAMS}}/M_{\odot}$	$G_i/G_0$		
	$G/G=-5 \times 10^{-11}$	$G/G=-1 \times 10^{-11}$	$G/G=-1 \times 10^{-12}$
1.0	1.50 (6.0)	1.15 (2.3)	1.020 (1.12)
1.0	1.40 (4.0)	1.10 (1.6)	1.015 (1.07)
1.0	1.30 (2.6)	1.08 (1.4)	
2.0	1.50 (4.8)	1.20 (1.9)	1.10 (1.4)
2.0	1.40 (3.4)	1.10 (1.4)	1.05 (1.2)
2.0	1.30 (2.5)	1.05 (1.2)	1.02 (1.1)
2.0	1.20 (1.8)	1.03 (1.1)	

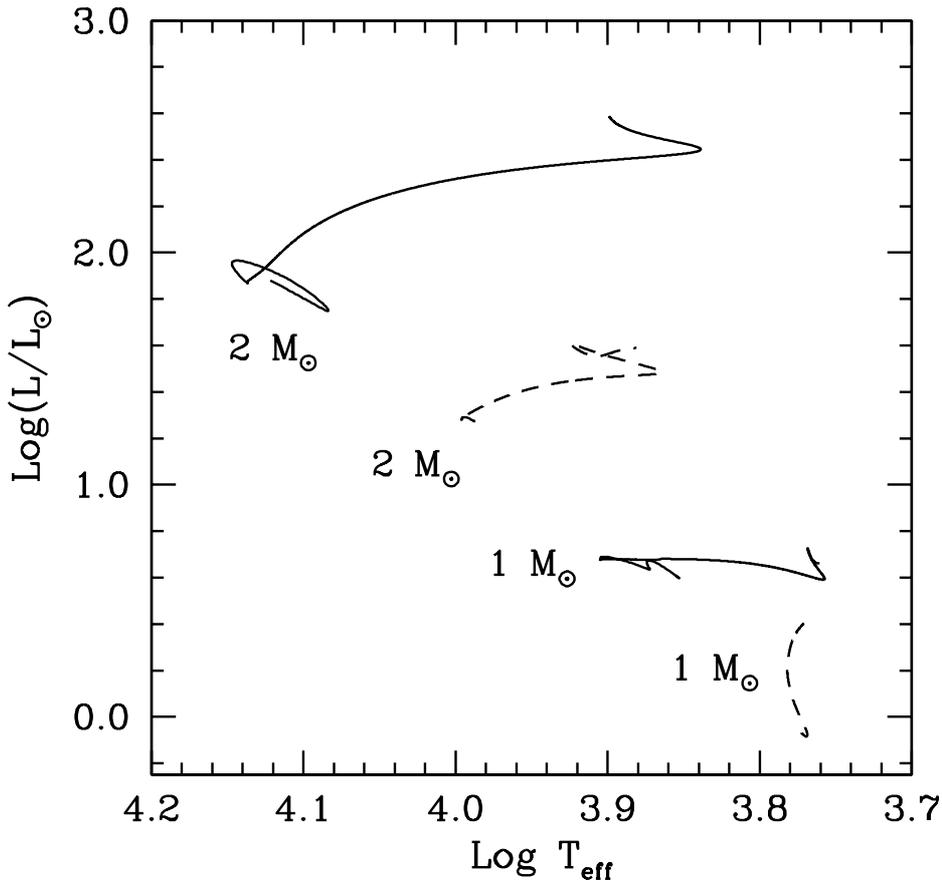
a carbon-oxygen degenerate core, but we did not follow the thermally pulsing AGB phase, except in one case — see below.

## 2.1 Overview of the evolution

Despite the small rates of change of  $G$  adopted here, the evolution of white dwarf progenitor stars is severely modified. Indeed, a varying  $G$  markedly affects the main sequence evolution. This is illustrated in Fig. 1, which shows the Hertzsprung-Russell diagram for two stars with masses 1.0 and 2.0  $M_{\odot}$  respectively, assuming a rate of change of the gravitational constant  $\dot{G}/G = -5 \times 10^{-11} \text{ yr}^{-1}$ , and an initial value of  $G$  at the ZAMS  $G_i/G_0 = 1.3$ . In this figure, for the sake of clarity, we only show the evolution from the onset of core hydrogen burning at the ZAMS to the stage at which the central hydrogen abundance has decreased down to  $\approx 10^{-9}$  by mass. Note that the evolutionary sequences are substantially different from those obtained in the standard case of constant  $G$ . In particular, the evolution in the case of a varying  $G$  occurs at much higher central temperatures, which are a consequence of the stronger gravitational pull. This results in an enhanced nuclear burning and, consequently, in larger luminosities than in the standard case. This is in agreement with earlier, pioneering studies [29, 30], since from simple theoretical grounds it can be proven that the stellar luminosity scales as a high power of  $G$ . Specifically, for a solar model  $L \propto G^7$  [30]. Finally, we note that the 1.0  $M_{\odot}$  sequence with varying  $G$  depicted in Fig. 1 is characterized by an initial convective core which encompasses  $\simeq 30\%$  of the mass of the stars. This explains the hook during the core hydrogen burning phase.

## 2.2 Evolutionary timescales

Of relevance for the present work is the impact on the evolutionary times. In particular, the larger central temperatures of models with initially larger values of  $G$  directly translate in shorter main sequence lifetimes, due to the enhanced thermonuclear rates previously discussed. The reduction in the evolutionary times strongly depends on the initial value of  $G$  at the beginning of evolution, as can be seen from Table 1, which, in addition to the evolutionary sequences computed, lists the factors by which the main sequence times are reduced



**Figure 1.** Solid lines display the evolution in the Hertzsprung-Russell diagram for the main sequence evolution of stars with 1.0 and 2.0  $M_{\odot}$  assuming a rate of change of  $G$  of  $\dot{G}/G = -5 \times 10^{-11} \text{ yr}^{-1}$  and an initial value of  $G$  at the ZAMS of  $G_i/G_0 = 1.3$ . Dashed lines display the main sequence evolution for the standard case of constant  $G$ . See text for details.

from those predicted by the standard case of constant  $G$ . Note that for those sequences with initially larger values of  $G$  the evolutionary timescales are considerably reduced.

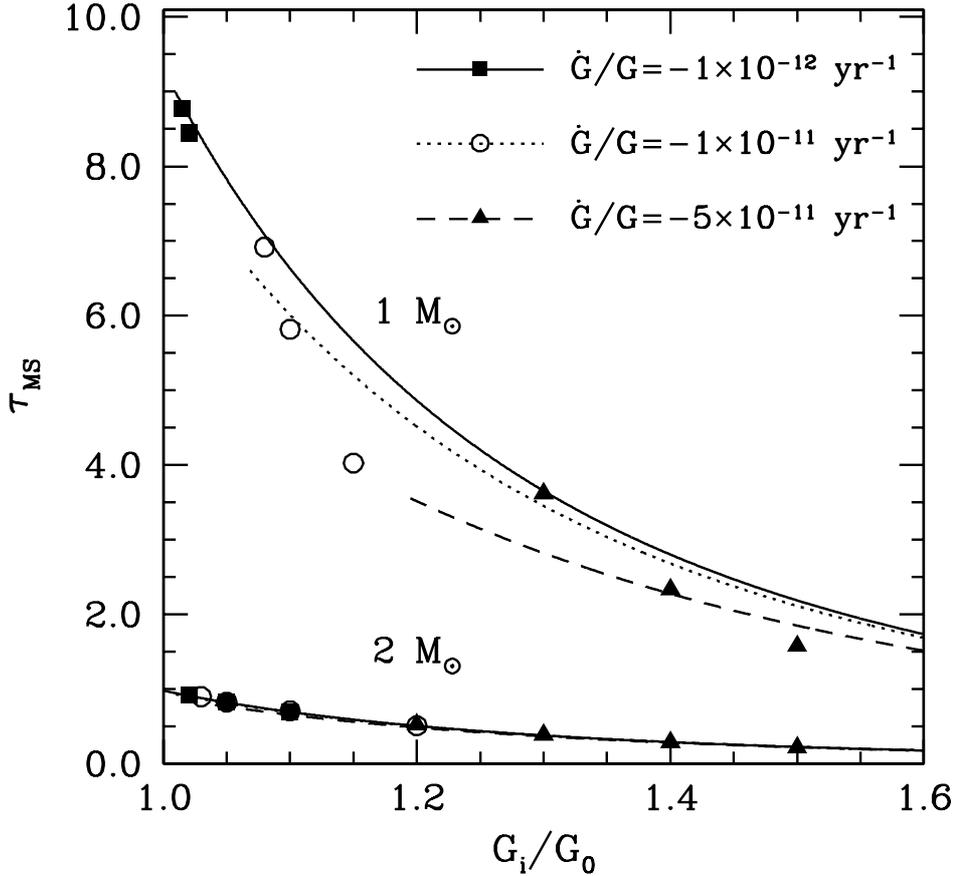
The evolutionary timescales can be modelled using rather simple arguments. We follow closely the treatment of Ref. [26]. In particular, we assume that the luminosity of a main sequence star can be factorized as  $L \propto f(Y)h(G)$ , where  $Y$  is the central helium abundance. Furthermore, we assume that  $h(G) \propto G^{\gamma}$ , being  $\gamma$  a constant. Since the luminosity is proportional to the helium production it is clear that

$$\frac{dY}{dt} \propto f(Y)h(G), \quad (2.1)$$

and, hence, we have that the main sequence lifetime can be obtained from

$$\int_{t_0}^{t_0+\tau_{\text{MS}}} G(t)^{\gamma} dt \propto \int_{Y_0}^1 \frac{dY}{f(Y)}, \quad (2.2)$$

where  $Y_0$  is the initial helium abundance, and  $t_0$  is the time at which the star was born. This expression is valid for both the standard case and that in which  $G$  secularly varies. Thus, the left-hand side of this equation can be integrated in the case of a constant  $G$ , for which we



**Figure 2.** Main sequence lifetimes as a function of the initial value of the gravitational constant,  $G_i$ , and different rates of variation  $\dot{G}/G$ . The solid, dotted and dashed lines correspond to the predictions of Eq. (2.3) for  $\dot{G}/G = -1 \times 10^{-12} \text{ yr}^{-1}$ ,  $\dot{G}/G = -1 \times 10^{-11} \text{ yr}^{-1}$ , and  $\dot{G}/G = -5 \times 10^{-11} \text{ yr}^{-1}$ . Additionally, the squares, circles and triangles correspond to the same set of values of  $\dot{G}/G$ , and different values of  $G_i/G_0$ . See text for details.

choose the present-day value. In the following we will assume that  $\dot{G}/G$  remains constant. Consequently, if  $\tau_{\text{MS}}^0$  is the main sequence lifetime in the standard case, adopting  $t_0 = 0$ , and taking into account that in our evolutionary calculations we have adopted a negative value of  $\dot{G}/G$ , after some elementary algebra it turns out that:

$$\tau_{\text{MS}} = \frac{1}{\gamma \left| \frac{\dot{G}}{G} \right|} \ln \left[ \gamma \left| \frac{\dot{G}}{G} \right| \left( \frac{G_0}{G_i} \right)^\gamma \tau_{\text{MS}}^0 + 1 \right]. \quad (2.3)$$

The value of  $\gamma$  must be determined from fits to the detailed evolutionary sequences computed so far. Fig. 2 shows that this treatment is relatively accurate, and — as it will be shown in Sect. 5 — enough for our purposes. In this figure we show, for different values of  $G_i/G_0$ , the main sequence lifetimes of two stars of masses 1 and  $2 M_\odot$ , respectively, for different values of  $G_i/G_0$  and  $\dot{G}/G$ . The main sequence lifetimes in the standard case in which  $G$  is assumed to be constant are, respectively, 9.46 Gyr for the  $1 M_\odot$  star, and 0.98 Gyr for the  $2 M_\odot$  star. The solid, dotted and dashed lines show, respectively, the predictions of Eq. (2) for  $\dot{G}/G = -1 \times 10^{-12} \text{ yr}^{-1}$ ,  $\dot{G}/G = -1 \times 10^{-11} \text{ yr}^{-1}$ , and  $\dot{G}/G = -5 \times 10^{-11} \text{ yr}^{-1}$ . The results

of our evolutionary calculations are shown as circles, squares and triangles which correspond, respectively, to the same set of values of  $\dot{G}/G$ . The best fit corresponds to  $\gamma = 3.6$ . Clearly, the results obtained using Eq. (2.3) are in good agreement with the evolutionary results, especially for the more massive star. It should be noted that the value of  $\gamma$  derived here is somewhat smaller than that obtained in previous evolutionary calculations [26]. This, in part, is due to the different microphysics used in the stellar evolutionary codes. However, the largest difference comes from the fact that in our calculations we start from a different value of  $G$ , to account for its present-day value.

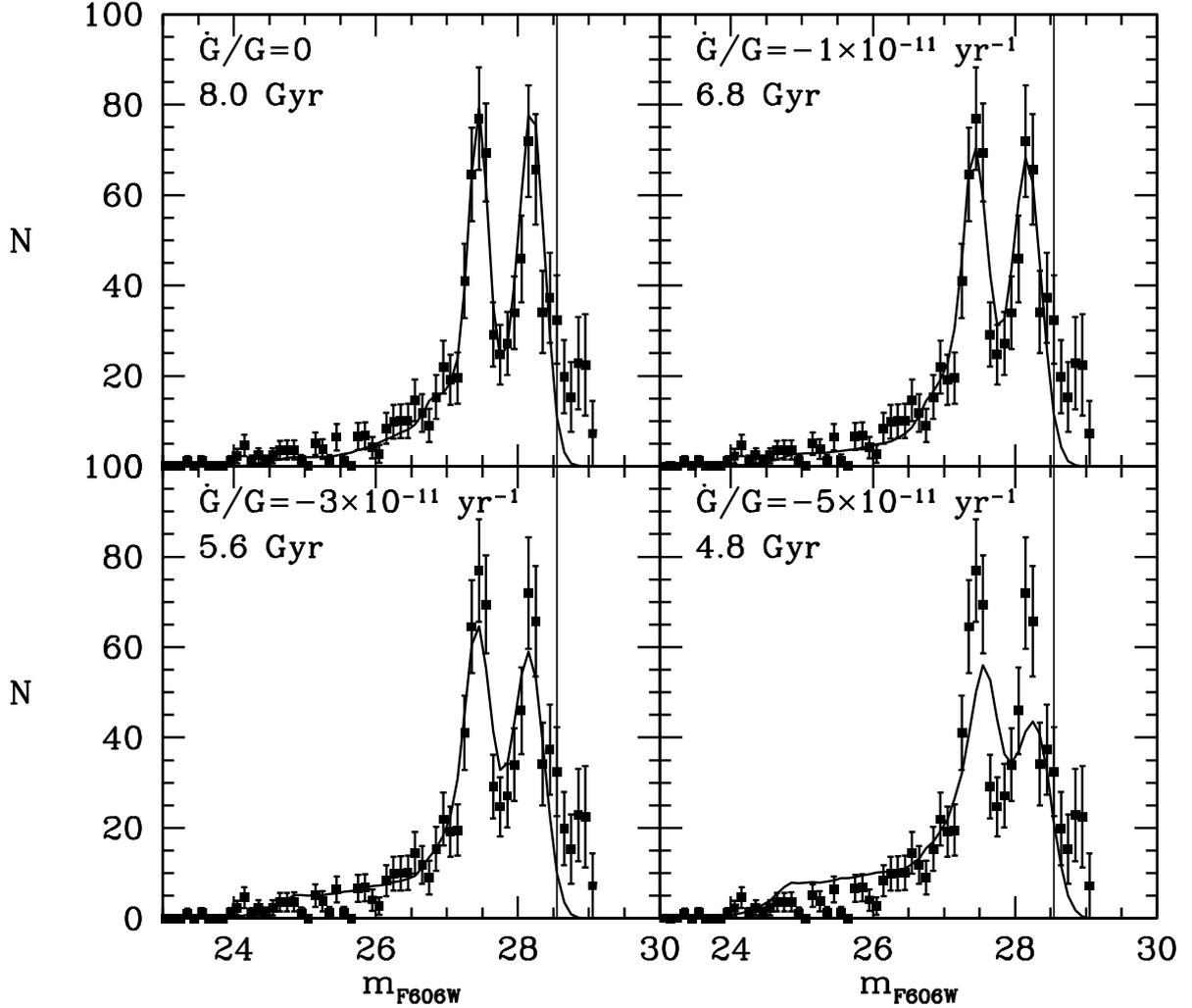
### 2.3 Other effects

Finally, we have also explored the possibility that a varying  $G$  could change the initial-to-final-mass relationship. To this end, we have computed the full evolution of an initially  $2.0 M_{\odot}$  sequence adopting  $\dot{G}/G = -5 \times 10^{-11} \text{ yr}^{-1}$  and  $G_i/G_0 = 1.3$ . The evolution was followed all the way from the ZAMS, through the hydrogen and helium core burning phases to the thermally pulsing AGB phase, where mass loss was modeled following the standard treatment. We find that the mass of the hydrogen-free core at the first thermal pulse,  $0.624 M_{\odot}$ , turns out to be substantially larger than that obtained in the standard case,  $0.52 M_{\odot}$ . However, the resulting white dwarf masses are quite similar in both cases. This is because in the case of the sequence with varying  $G$ , the mass of the hydrogen-free core grows only slightly with further evolution as a result of the larger mass losses that occur in this sequence during the thermally-pulsing AGB phase. With regard to the core chemical composition of this sequence, a larger carbon abundance is expected in the case of a varying  $G$ , resembling the composition that results from a  $\approx 4.0 M_{\odot}$  sequence computed assuming constant  $G$ , but this effect is expected to be of minor importance in the white dwarf cooling phase.

## 3 White dwarf cooling tracks

In this study we adopt the most recent and reliable white dwarf cooling tracks available so far [21]. These cooling sequences incorporate the most up-to-date description of the stellar plasma and incorporate self-consistently the effects of a secularly varying  $G$ . In particular these cooling sequences consider  $^{22}\text{Ne}$  diffusion and its associated energy release [20, 27, 31]. The energy sources arising from crystallization of the white dwarf core — namely, the release of latent heat and of gravitational energy associated with carbon-oxygen phase separation [32–34] — are fully taken into account. Non-gray model atmospheres were used to provide accurate outer boundary conditions for our models. Our atmospheres include non-ideal effects in the gas equation of state and chemical equilibrium based on the occupation probability formalism. They also consider collision-induced absorption caused by  $\text{H}_2\text{-H}_2$ ,  $\text{H}_2\text{-He}$ , and  $\text{H-He}$  pairs, and the  $\text{Ly}\alpha$  quasi-molecular opacity that results from perturbations of hydrogen atoms by interactions with other particles, mainly  $\text{H}$  and  $\text{H}_2$ . Our white dwarf cooling tracks also incorporate element diffusion in the outer layers, as well as many other important physical inputs, which we do not detail here for the sake of conciseness. Instead, we refer the interested reader to Ref. [21], where a detailed description of all them can be found.

The set of white dwarf cooling tracks employed here display a marked dependence of the white dwarf ages on the assumed rate of change of  $G$ . As a matter of fact, it turns out that a secularly decreasing  $G$  accelerates the cooling of the white dwarf. This fact reflects the energetic demand required for the star to expand against gravity in response to a smaller value



**Figure 3.** The white dwarf luminosity function of NGC 6791 for different values of  $\dot{G}/G$ . The observational data of Ref. [23] are fitted using the procedures outlined in Sect. 4. The thin vertical solid line corresponds to the magnitude limit beyond which observations are severely incomplete. Note that the shape of the luminosity function strongly depends on the specific value of  $\dot{G}/G$ .

of  $G$ . This dependence is more pronounced for massive white dwarfs, owing to their larger gravitational forces. However, it is worth mentioning that for  $|\dot{G}/G| \lesssim 1 \times 10^{-12} \text{ yr}^{-1}$  the evolution is almost indistinguishable from that of the standard case of constant  $G$ , particularly for low-mass white dwarfs. Consequently, this value of  $\dot{G}/G$  constitutes a lower limit for the rate of change of  $G$  above which we can expect that the evolution of white dwarfs will be influenced by a varying  $G$ .

#### 4 Building the white dwarf luminosity function

The white dwarf luminosity function of NGC 6791 has been simulated using a Monte Carlo technique [35–37]. In our Monte Carlo simulator synthetic main sequence stars are randomly drawn according to a Salpeter-like initial mass function that in the mass range relevant to

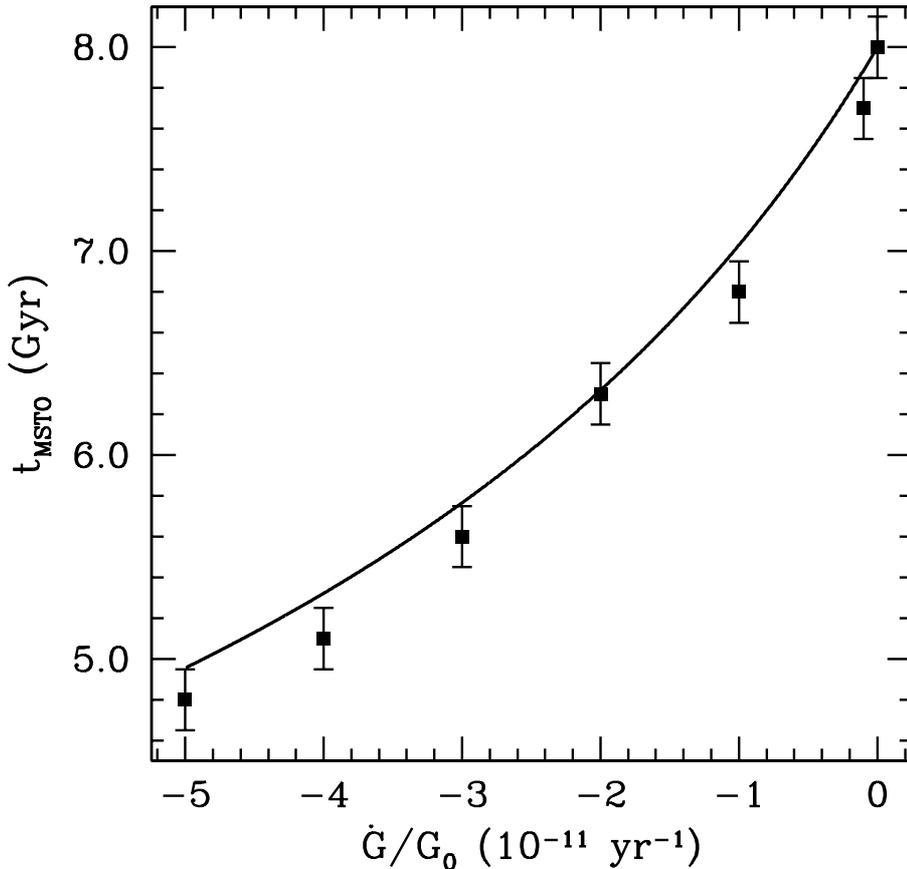
NGC 6791 white dwarf progenitors ( $M > 1.0 M_{\odot}$ ) is essentially identical to the “universal” initial mass function [38], and a burst of star formation with a small age dispersion of 0.1 Gyr. At this point of our discussion it is important to mention that as it will be shown in detail in Sect. 5 the age of the cluster depends on the adopted value of  $\dot{G}/G$ , and thus the time at which the burst of star formation occurred also does. In accordance with previous studies of this cluster [24, 25], we account for a population of unresolved detached binary white dwarfs and we adopt a fraction of binary systems in the main sequence of 54%. The distribution of secondary masses is the same adopted in Ref. [20]. We also used an up-to-date white dwarf initial-final mass relationship [39].

Given the age of the cluster, and the value of  $\dot{G}/G$  we know which was the initial value of the gravitational constant,  $G_i/G_0$ , when the cluster was formed. Moreover, once the time at which each synthetic star is randomly assigned within the burst of star formation, we can compute its associated main sequence lifetime when a varying  $G$  is adopted using Eq. (2.3) and up-to-date main sequence evolutionary times [40]. Thus, we know which synthetic stars were able to evolve to the white dwarf stage and we know which was the value of  $G$  when the corresponding white dwarfs were formed. Consequently, we can interpolate their colors and luminosities using the theoretical cooling sequences described in the previous section. For unresolved binary systems we performed the same calculation for the secondary and we added the fluxes and computed the corresponding colors. The standard photometric errors in magnitude and color were assumed to increase linearly with magnitude [23–25]. Finally, we added the distance modulus of NGC 6791,  $(m - M)_{\text{F606W}} = 13.44$ , and its color excess  $E(\text{F606W-F814W}) = 0.14$  [23] to obtain a synthetic white dwarf color-magnitude diagram, and from it the corresponding white dwarf luminosity function was computed.

## 5 Results

Using the procedure outlined in the previous sections the white dwarf luminosity function for an arbitrary value of  $\dot{G}/G$  can be computed. In Fig. 3 we show several luminosity functions corresponding to increasing absolute values of the secular rate of change of the gravitational constant. Each of these panels is clearly labeled with the specific value of  $\dot{G}/G$  employed in the calculation. As can be seen, the white dwarf luminosity function of NGC 6791 presents two peaks. The bright peak corresponds to the population of unresolved binary white dwarfs, whilst the faint peak is due to finite age of the cluster — see Ref. [20] for a detailed explanation of these features. For the purpose of this work it is important to realize that white dwarfs with magnitudes larger than  $\sim 28.7^{\text{mag}}$  have not had time to cool enough. In fact, the position of the cut-off — or, alternatively, of the faint peak — of the white dwarf luminosity function can be used to estimate the age of the cluster. Moreover, this age can be compared with the age obtained from the position of the main sequence turn-off. It has been recently shown [20] that both ages agree to a few percent when an appropriate description of the degenerate plasma of cooling white dwarfs is considered. Accordingly, we have fitted the age of the cluster looking for the age that best fits the position of the faint peak of the white dwarf luminosity function. These ages are also displayed in each of the panels. Note that as the absolute value of  $\dot{G}/G$  increases, the age of the cluster decreases. This is a consequence of the accelerated cooling of typical white dwarfs — see Sect. 3.

Given that both the main sequence turn-off ages and the white dwarf cooling ages depend on the assumed value of  $\dot{G}/G$  it is interesting to compare how the white dwarf age of NGC 6791 compares with the main sequence turn-off age, for different values of  $\dot{G}/G$ .



**Figure 4.** Comparison of the white dwarf cooling age and the main sequence turn-off age of NGC 6791 for several values of  $\dot{G}/G$ .

This can be done inspecting Fig. 4, where we display a comparison of the age of the main-sequence turn-off, which corresponds to a mass  $M_{\text{MSTO}} \simeq 1,137 M_{\odot}$ . The main sequence lifetime of this star can be scaled using Eq. (2.3) and compared to the ages derived from a fit to the observational white dwarf luminosity function of NGC 6791, which in this figure are represented using solid squares. Note that the error bars are very small, a consequence of the narrowness of the faint peak of the white dwarf luminosity function. As can be seen, the overall agreement is excellent, independently of the adopted value of  $\dot{G}/G$ . Thus, we are confident that the assumptions used to build white dwarf luminosity functions with a varying  $G$  are valid.

Now we turn our attention again to Fig. 3. Clearly, the theoretical white dwarf luminosity functions strongly depend on the adopted value of  $\dot{G}/G$ . Actually, the agreement between the theoretical luminosity functions and the observational data for NGC 6791 steadily degrades as the absolute value of  $\dot{G}/G$  is increased. This can be used to place an upper limit to the rate of variation of the gravitational constant. To quantify the maximum value of  $\dot{G}/G$  allowed by the observational white dwarf luminosity function of NGC 6791 we have conducted a  $\chi^2$  test, and we have found that for  $\dot{G}/G = 0$  a probability of  $P \simeq 0.90$  is obtained, while for  $\dot{G}/G \simeq -2.5 \times 10^{-11} \text{ yr}^{-1}$  this value drops to  $P \simeq 0.66$  — see table 2, where we show in addition to the age of the cluster obtained from the position of the faint

**Table 2.** Age of the cluster for different values of the rate of change of  $G$ ,  $\dot{G}/G$  (in units of  $\text{yr}^{-1}$ ) and the corresponding probability using the  $\chi^2$  test.

$t$ (Gyr)	$\dot{G}/G$ ( $\text{yr}^{-1}$ )	$P$
8.0	0	0.901
7.7	$-1 \times 10^{-12}$	0.809
6.8	$-1 \times 10^{-11}$	0.795
6.3	$-2 \times 10^{-11}$	0.717
5.6	$-3 \times 10^{-11}$	0.588
5.1	$-4 \times 10^{-11}$	0.264
4.8	$-5 \times 10^{-11}$	0.002

peak of the white dwarf luminosity function, the probability given by the  $\chi^2$  test for several values of  $\dot{G}/G$ . Hence, the white dwarf luminosity function of this open cluster can be used to constrain the secular rate of variation of the gravitational constant, although the upper bound obtained with this method is not very tight. However, this upper limit compares well with other stellar constraints. For instance, the upper bound derived from the luminosity function of disk white dwarfs is  $\dot{G}/G \sim -3.0 \times 10^{-11} \text{ yr}^{-1}$  [22], that derived from globular clusters is  $\dot{G}/G \sim -3.2 \times 10^{-11} \text{ yr}^{-1}$  [26], the pulsating white dwarf G117-B15A provides an upper limit of  $\dot{G}/G \sim -2.5 \times 10^{-11} \text{ yr}^{-1}$  [41], while helioseismology provides a much better (but purely local) upper bound,  $\dot{G}/G \sim -1.6 \times 10^{-12} \text{ yr}^{-1}$  [42].

However, this is not the best constraint that can be obtained using the properties of NGC 6791. As previously mentioned, there is a degeneracy between the age of the cluster derived from the termination of the cooling sequence (or, equivalently, from the main sequence turn-off) and the upper bound to  $\dot{G}/G$  obtained from the white dwarf luminosity function. This degeneracy can be broken if the true distance of the cluster is known by other means. Indeed, if  $\dot{G}/G \sim -2.5 \times 10^{-11} \text{ yr}^{-1}$  is adopted, the resulting age of the cluster would be  $\sim 6$  Gyr — see Fig. 4 and Table 2 — but then the position of the main sequence turn-off in the color-magnitude diagram would be significantly different if the same distance modulus is adopted. Accordingly, the distance modulus necessary to fit the position in the color-magnitude diagram of the main sequence turn-off of the cluster needs to be changed as well. In particular, if an age of  $\sim 6$  Gyr is adopted for NGC 6791, its distance modulus needs to be decreased by about  $0.5^{\text{mag}}$  when  $\dot{G}/G = 0$  is adopted [20]. However, the distance modulus derived using a totally independent and reliable method (eclipsing binaries) which does not make use of theoretical models turns out to be  $13.46 \pm 0.1$  [43]. Thus, large errors in the distance modulus seem to be quite implausible. Accordingly, such a small age for NGC 6791 (6 Gyr) can be safely discarded. This argument can be put all the way around, and given the uncertainty in the distance modulus ( $\simeq 0.1^{\text{mag}}$ ), and the measured value of the distance modulus an upper limit to  $\dot{G}/G$  can be placed. Specifically, since  $\Delta t_{\text{MSTO}}/\Delta(m-M)_{\text{F606W}} \approx 4 \text{ Gyr/mag}$ , the maximum age difference with respect to the case in which a constant  $G$  is adopted should be  $\sim 0.4$  Gyr, which translates into an upper bound  $\dot{G}/G \sim -1.8 \times 10^{-12} \text{ yr}^{-1}$ , see again Fig. 4 and Table 2. This upper limit considerably improves the other existing upper bounds to the rate of variation of  $G$  and is equivalent to the upper limit set by helioseismology.

## 6 Conclusions

In this paper we have analyzed the effect of a varying gravitational constant on the evolutionary properties of white dwarf progenitors. We have found that the main sequence lifetimes and other properties of these stars are strongly dependent not only on the rate of variation of the gravitational constant but also on the adopted initial value of  $G$ . Specifically, we have found that a negative value of  $\dot{G}/G$  implies larger temperatures in the central regions of these stars. This, in turn, translates in enhanced thermonuclear rates, larger luminosities, and significantly shorter main sequence lifetimes, in agreement with previous studies [26, 29]. However, these pioneering studies only took into account the effects of a varying gravitational constant and overlooked the fact that if  $G$  decreases with time its value should have been larger in the past. Our study has been the first one to address and quantify this issue. Furthermore, we have derived a simple analytical expression that reproduces with reasonable accuracy our numerical results.

With these tools, and with appropriate white dwarf cooling sequences that also take into account the effects of a varying  $G$ , we have built up-to-date white dwarf luminosity functions for the old, metal-rich, well-populated, nearby, Galactic open cluster NGC 6791. For this cluster we have a reliable observational color-magnitude diagram and a well determined white dwarf luminosity function. Comparing our results with the extant observational data we have been able to derive an upper bound to the secular rate of variation of  $G$ . This upper bound,  $\dot{G}/G \sim -2.5 \times 10^{-11} \text{ yr}^{-1}$ , is on the order of other constraints obtained previously using the evolutionary properties of either main sequence or white dwarf stars, and can be easily improved if a reanalysis of the white dwarf luminosity function of NGC 6791 is undertaken, and smaller observational uncertainties are achieved at magnitudes within  $26.5^{\text{mag}}$  and  $28.5^{\text{mag}}$  — something perfectly feasible using the Hubble Space Telescope. Moreover, we also have found that combined observations of the white dwarf cooling sequence and main sequence stars provide an improved upper bound if the true distance to the cluster is independently determined. In particular, using the the distance modulus measured using eclipsing binaries — a totally independent and reliable method that does not use theoretical models — we find that the upper bound to the rate of variation of the gravitational constant can be improved to  $\dot{G}/G \sim -1.8 \times 10^{-12} \text{ yr}^{-1}$ , which compares favorably with the rest of upper bounds obtained using stellar evolutionary arguments.

Finally, we would like to mention that NGC 6791 is not the only cluster for which we have white dwarf luminosity functions. There are other Galactic clusters, either open or globular, for which the Hubble Space Telescope has imaged the white dwarf cooling sequence, and for which we expect to have in the near future very reliable white dwarf luminosity functions. These clusters include M67, NGC 2099, NGC 188, M4, and NGC 6397. Their metallicities are very different of that of NGC 6791, and consequently, appropriate white dwarf cooling sequences and progenitor lifetimes should be computed to obtain sound results and accurate upper limits to the rate of variation of  $G$ . Additionally, if reliable upper bounds are to be obtained independent distance determinations are needed as well. Nevertheless, our study of NGC 6791 already paved the way for improved determinations of an upper limit to the rate of variation of Newton’s constant.

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