

# Revision of Informant Plausibility in Multi-Agent Systems

Patricio D. Simari      Marcelo A. Falappa

Department of Computer Science  
Artificial Intelligence R&D Lab (LIDIA)  
Universidad Nacional del Sur  
Avenida Alem 1253 - (8000) Bahía Blanca - Argentina  
Tel/Fax: (54)(291)4595135  
email: [pds,mfalappa]@cs.uns.edu.ar

## Abstract

Within the context of multi-agent systems, an agent may often find itself in a position where it receives information through informants. These informants are independent agents who have their own interests and, therefore, are not necessarily completely reliable. It is natural for an agent to be more inclined to believe one informant over another, especially if the informant has proven itself reliable over a period of time. This paper proposes the organization of the informants into a partial order which compares the plausibility of the relevant informants. This partial order need not remain static throughout the agent's lifespan. The agent may choose to update its partial order relation to reflect a new perception of informant plausibility. Therefore, this paper also proposes the application of change operators, not to the beliefs themselves, but to the partial order. These operators are used to alter the structure of relative credibility of the informants. These operators are characterized through postulates and a construction is provided for them.

**Keywords:** Intelligent agent, Multi-agent systems, Belief change, Contraction, Revision, Plausibility relation.

## 1 Introduction

Within the context of multi-agent systems, an agent may often find itself in a position where it receives information from other agents or, as we will call them throughout this paper, *informants*. These informants are independent agents who have their own interests and, therefore, are not necessarily completely reliable. In fact, on occasion an informant may provide information which is contradictory with that which is provided by another. It would be natural for our agent to be more inclined to believe one informant over another, especially if the informant has proven itself reliable over a period of time.

Situations like these are not infrequent in real-world scenarios. Take for instance a stock-investment agent. Such an agent may receive financial advice from several sources including, but not limited to, human investment advisors and web pages where such financial council is posted. These would fit the role of the previously mentioned informants. It is not hard to imagine the investing agent preferring one informant's advice over another's. Our agent could track the advice of the informants and later on compare it to actual stock market events. This comparison could then be used to somehow raise the relative plausibility of the informants who were correct and lower that of those who were not.

Another real-world scenario could be seen in the case of an agent in need of weather prediction information. There might be several weather-predicting informants available to the agent. As in the previous case, there are for example, a number of web pages providing such services. However, for a given period in time, different informants could predict different conditions. Once again, in such a case the agent would act based on the prediction of the most reliable informant and then use historic information to possibly update these relations.

What we propose in this paper is the organization of the informants into a partial order which compares the plausibility of the relevant informants. Upon noticing contradictory information from two informants, the agent need only check which informant supercedes the other in plausibility according to the partial order (i.e. which one is more *reliable*) and then act upon that informant's information.

This partial order need not remain static throughout the agent's life-span. Our agent may start out believing that one informant has a higher plausibility than another only to find that the first informant's information is consistently inaccurate. It follows that the agent would want to update its partial order relation to reflect this new perception.

Our proposal is the application of change operators. However, these operators are not to be applied to the beliefs themselves as provided by the informants. The change operators we define act upon the partial order. In other words they alter the structure of relative credibility of the informants that *provide* the beliefs. As we will show, these operators can be characterized through postulates, in some cases similar to those provided for belief change operators of the AGM model.

This paper is organized as follows. Section 2 provides an overview of belief change theory. It includes an introduction to, as well as motivations for, belief change. It also briefly reviews the AGM model and exposes the main concepts related to belief revision. Finally it presents the notion of plausibility and its application in belief revision systems. Section 3 presents our proposal for representation of plausibility relations for agents relying on informants. Section 4 defines revision of plausibility based relations, the operations involved, their characteristic postulates and their construction. Finally, section 5 includes this paper's conclusions as well as future work.

For a thorough presentation of the AGM model for belief revision we refer the reader to [Alchourrón *et al.* 1985]. Some interesting related work on comparing the credibility of information sources can be found in [Parra 98, Rescher 1976].

## 2 Concepts of Belief Change

### 2.1 The need for Belief Change

Current reasoning systems attempt to model an agent's knowledge and interaction with its environment in a symbolic manner. This environment, its *world*, is generally dynamic and changing due to natural evolution or the actions of other agents that are a part of it. In consequence, an agent that is a part of a reasoning system must have the following components: a *knowledge base* where its knowledge of the world is stored, a *communication mechanism* with the environment and other agents in it, and a means of modifying its knowledge of the environment.

Knowledge may be represented by a logic language which is propositional, first order, modal or extensions of these. Each one of these alternatives has advantages as well as disadvantages. The higher the expressive power of a given language, the more computational problems there are regarding complexity and decidability.

Communication mechanisms can be varied, depending on the environment being modeled. They can be multimedia mechanisms such as microphones, speakers, video cameras, infrared sensors, motion detectors and even wired or wireless systems where information is transmitted without any kind of preprocessing. They are irrelevant, however, for the purpose of this work.

Mechanisms for modifying knowledge may be modeled by what is known as *belief change theory*. Belief change theory assumes that the underlying language is at least propositional. An agent's knowledge is represented as a set of sentences and new information as a single sentence. In turn, every change operator takes a set of sentences and a single sentence and produces a new set of sentences as a result.

For this section, we will adopt a propositional language  $\mathcal{L}$  with a complete set of boolean connectives: “ $\neg$ ”, “ $\wedge$ ”, “ $\vee$ ”, “ $\rightarrow$ ”, “ $\leftrightarrow$ ”. Formulae in  $\mathcal{L}$  will be denoted by lowercase Greek characters:  $\alpha, \beta, \dots, \omega$ . Sets of sentences in  $\mathcal{L}$  will be denoted by uppercase Latin characters:  $A, B, C, \dots, Z$ . The symbol “ $\top$ ” represents a tautology or *truth*. The symbol “ $\perp$ ” represents a contradiction or *falsum*. We also use a consequence operator  $Cn$ .  $Cn$  takes sets of sentences in  $\mathcal{L}$  and produces new sets of sentences. The operator  $Cn$  satisfies *inclusion* ( $A \subseteq Cn(A)$ ), *iteration* ( $Cn(A) = Cn(Cn(A))$ ), and *monotonicity* (if  $A \subseteq B$  then  $Cn(A) \subseteq Cn(B)$ ). To simplify notation, we write  $Cn(\alpha)$  instead of  $Cn(\{\alpha\})$  where  $\alpha$  is any sentence in  $\mathcal{L}$ . We also write  $\alpha \in Cn(A)$  as  $A \vdash \alpha$ . Moreover, belief bases will be finite sets of sentences.

### 2.2 The AGM approach to Belief Change

Belief revision is the process by which an agent changes its set of beliefs, making a transition from one epistemic state to another. When such an agent learns new information it may realize that this information clashes with its old beliefs. In this case the agent has to revise its belief set and decide which of the old beliefs need to be eliminated in favor of the new information.

One of the most fundamental approaches to the formalization of the dynamics of beliefs is the AGM model [Alchourrón *et al.* 1985], proposed by Carlos Alchourrón, Peter Gärdenfors and David Makinson. In the AGM approach, the epistemic states

are represented by belief sets, that is, sets of sentences closed under logical consequence.

Let  $\mathbf{K} = Cn(\mathbf{K})$  be a belief set and  $\alpha$  a sentence in a propositional language  $\mathcal{L}$ . The three main types of changes are the following [Gärdenfors, Rott 1992]:

**Expansion:** A new sentence is added to an epistemic state regardless of the consequences of the so formed larger set. If “+” is an expansion operator, then  $\mathbf{K}+\alpha$  denotes the belief set  $\mathbf{K}$  expanded by  $\alpha$ .

**Contraction:** Some sentence in the epistemic state is retracted without adding any new belief. If “-” is a contraction operator, then  $\mathbf{K}-\alpha$  denotes the belief set  $\mathbf{K}$  contracted by  $\alpha$ .

**Revision:** A new sentence is consistently added to an epistemic state. In order to make this operation possible, some sentences may be retracted from the original epistemic state. If “\*” is a revision operator, then  $\mathbf{K}*\alpha$  denotes the belief set  $\mathbf{K}$  revised by  $\alpha$ .

Expansions can be defined as the logical closure of  $\mathbf{K}$  and  $\alpha$ :

$$\mathbf{K}+\alpha = Cn(\mathbf{K} \cup \{\alpha\})$$

It is not possible to give a similarly explicit definition of contractions and revisions using logical and set-theoretical notions only. These operations can be defined using logical notions and some selection mechanism. Contractions and revisions are interdefinable by the following identities:

$$\text{Levi Identity: } \mathbf{K}*\alpha = (\mathbf{K}-\neg\alpha)+\alpha.$$

$$\text{Harper Identity: } \mathbf{K}-\alpha = \mathbf{K} \cap \mathbf{K}*\neg\alpha.$$

Thus, given a definition for one of these operators we can obtain the other by using the above identities. Gärdenfors [Gärdenfors 1988] proposed the following *basic rationality postulates* for contraction operators:

$$(K^-1) \text{ Closure: } \mathbf{K}-\alpha = Cn(\mathbf{K}-\alpha).$$

$$(K^-2) \text{ Inclusion: } \mathbf{K}-\alpha \subseteq \mathbf{K}.$$

$$(K^-3) \text{ Vacuity: If } \alpha \notin \mathbf{K} \text{ then } \mathbf{K}-\alpha = \mathbf{K}.$$

$$(K^-4) \text{ Success: If } \not\vdash \alpha \text{ then } \alpha \notin \mathbf{K}-\alpha.$$

$$(K^-5) \text{ Recovery: } \mathbf{K} \subseteq (\mathbf{K}-\alpha)+\alpha.$$

$$(K^-6) \text{ Extensionality: If } \vdash \alpha \leftrightarrow \beta \text{ then } \mathbf{K}-\alpha = \mathbf{K}-\beta.$$

and the following *basic rationality postulates* for revision operators:

$$(K^*1) \text{ Closure: } \mathbf{K}*\alpha = Cn(\mathbf{K}*\alpha).$$

$$(K^*2) \text{ Success: } \alpha \in \mathbf{K}*\alpha.$$

(K\*3) **Inclusion:**  $\mathbf{K}*\alpha \subseteq \mathbf{K}+\alpha$ .

(K\*4) **Vacuity:** If  $\mathbf{K} \not\vdash \neg\alpha$  then  $\mathbf{K}*\alpha = \mathbf{K}+\alpha$ .

(K\*5) **Consistency:** If  $\not\vdash \neg\alpha$  then  $\mathbf{K}*\alpha \not\vdash \perp$ .

(K\*6) **Extensionality:** If  $\vdash \alpha \leftrightarrow \beta$  then  $\mathbf{K}*\alpha = \mathbf{K}*\beta$ .

All of these operations have some controversial postulates. A thorough presentation of the different belief change models can be found in [Falappa 1999].

## 2.3 Plausibility in the context of Belief Change

Among the operations for belief change, we have two that warrant special attention: contractions and revisions. Both of these operations require the elimination of sentences from a knowledge base. Therefore, in addition to a set of sentences which represent an agent's knowledge state, we need a *selection mechanism* to determine which beliefs are eliminated in the change process and which are not. In order to make this possible, we use some method that assigns an *informational value* to sentences. In a process of pure contraction the sentences with the least informational value will be selected among the candidates for elimination. Models that use this technique are known as *information-theoretic approaches*.

Usually, theories that assign informational value to beliefs are based on the Bayesian Model. In this type of model, an agent's epistemic state requires that each sentence be assigned a measure of probability reflecting the belief's degree of certainty. Then, if a belief must be eliminated from an epistemic state, the one with the lowest value can be selected.

However, this kind of modification cannot be modeled within classic models for belief change. This is because those beliefs which are accepted in an epistemic state (whether they are belief bases or belief sets) are completely true. This is to say, they have a maximum degree of certainty (a probability of 1). If this certainty value were changed, a new value as close to 1 as possible should be assigned. It is not possible to represent this epistemic attitude in the classic models of theory change.

The other possibility consists of assigning a different value to beliefs, one that represents their epistemic importance to the agent. This is called *epistemic entrenchment*. [Gärdenfors, Makinson 1988] [Gärdenfors, Rott 1992]. This measure is completely external to the belief, not referring to the confidence to be had in it. What it represents is the importance (or weight) that this belief can have on an agent's decision processes.

Let us contrast this to the case of probability. When we assign a probability  $x$  to a belief  $\alpha$ , within a probabilistic model it is assumed that the probability of  $\neg\alpha$  is  $1 - x$ . This does not apply to epistemic entrenchment because if an agent believes  $\alpha$  it assumes maximum certainty for this belief, even though its weight can be low for a given decision process.

### 3 Representation of informant plausibility relations

#### 3.1 The concept of generator set

Let us assume that we have a universal set of informants,  $\mathcal{J}$ , and that, of these informants, some are to be considered more reliable than others. This is to say, in any case in which two distinct informants provide an agent with contradictory information the more trustworthy one is to be believed over the other. The agent must, therefore, have a mechanism by means of which the set  $\mathcal{J}$  is ordered. To this end we present the following concept.

**Definition 3.1.1:** Given a set of informants  $\mathcal{J}$  we will call any binary relation  $G \subseteq \mathcal{J}^2$  a generator set over  $\mathcal{J}$ . An informant  $i$  is less trustworthy than an informant  $j$  according to  $G$  if  $(i, j) \in G^*$ .  $\square$

Graphically, we may represent a generator set as we would a directed graph. We represent the informants in  $\mathcal{J}$  as nodes. The tuples in  $G$  are then represented as directed arcs: for each tuple  $(i, j) \in G$  we add an arc from node  $i$  to node  $j$ .

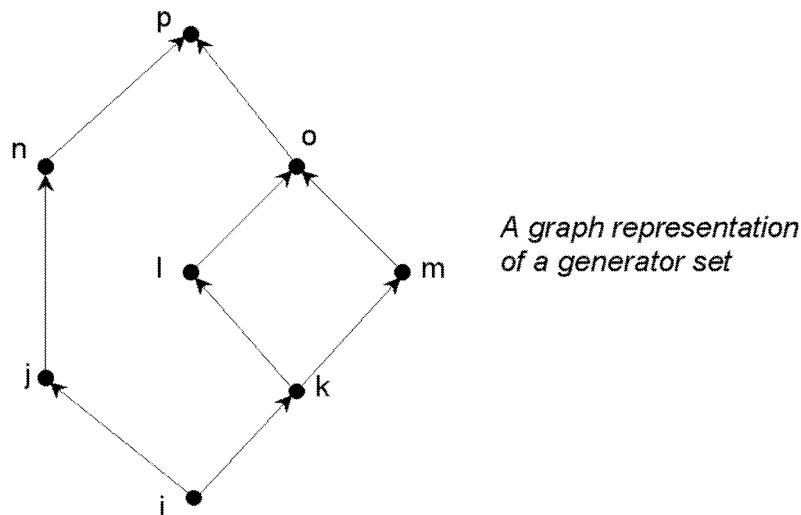


Figure 1: A graph representation of a generator set over a set of informants  $\mathcal{J} = \{i, j, k, l, m, n, o, p\}$ .

$G^*$  represents the reflexive transitive closure of  $G$ . It is desirable for  $G^*$  to be a partial order over  $\mathcal{J}$ , although according to the preceding definition this need not always be the case. We address this matter in the following definition.

**Definition 3.1.2:** A generator set  $G \subseteq \mathcal{J}^2$  is said to be sound if  $G^*$  is a partial order over  $\mathcal{J}$ .  $\square$

**Example 3.1.1:** For example the generator set  $G_1 = \{(i, j), (j, k), (i, l)\}$  is sound. However  $G_2 = G_1 \cup \{(k, i)\}$  is *not* sound because  $(i, k) \in G_2^*$  and  $(k, i) \in G_2^*$ . This violates the antisymmetry condition for partial orders.  $\square$

Why is it desirable for a generator set to be sound? For a relation to be a partial order it must obey reflexivity, antisymmetry and transitivity. Given a generator set  $G$  it is obvious that its reflexive transitive closure,  $G^*$ , will obey reflexivity and transitivity. However if antisymmetry is not respected then there is at least one pair of distinct informants,  $i$  and  $j$  such that both  $(i, j) \in G^*$  and  $(j, i) \in G^*$ . This would mean that both  $i$  is less trustworthy than  $j$  and that  $j$  is less trustworthy than  $i$ . Given that these beliefs are contradictory, believing them simultaneously would lead the believing agent to inconsistencies.

Throughout the discussions in the remainder of this paper we will sometimes speak of a tuple as being *entailed* by a generator set. This is no more than a shorthand for saying that the tuple belongs to the reflexive transitive closure of the generator set. We express this formally in the following definition.

**Definition 3.1.3:** We will say that a tuple  $(i, j)$  is *entailed* by a generator set  $G$  if  $(i, j) \in G^*$ .  $\square$

**Example 3.1.2:** The tuple  $(i, l)$  is among those entailed by the generator set  $G = \{(i, j), (j, k), (k, l)\}$ . When we represent a generator set graphically we will show entailed tuples of interest by using a dashed line.  $\square$

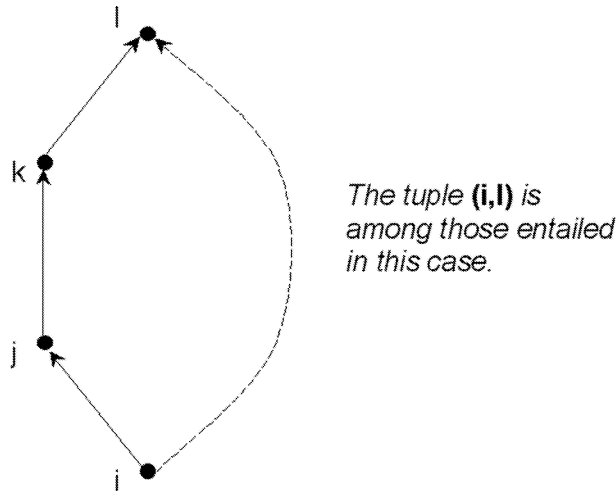


Figure 2: An entailed tuple.

Sometimes we may find tuples in a generator set which, if removed, would still be entailed by the remaining tuples. In this case we say that the tuple is *redundant* with respect to the generator set. We may also say that the generator set itself is redundant because it contains a redundant tuple. These concepts are introduced by the following definition.

**Definition 3.1.4:** Given a tuple  $(i, j)$  and a generator set  $G$  it is said that  $(i, j)$  is *redundant* in  $G$  if  $(i, j) \in (G \setminus \{(i, j)\})^*$ . A generator set is said to be *redundant* if it contains a redundant tuple. Otherwise, the generator set is said to be *non-redundant*.  $\square$

**Example 3.1.3:** The generator set  $G = \{(i, j), (j, k), (i, k)\}$  is redundant because it contains the redundant tuple  $(i, k)$ .  $\square$

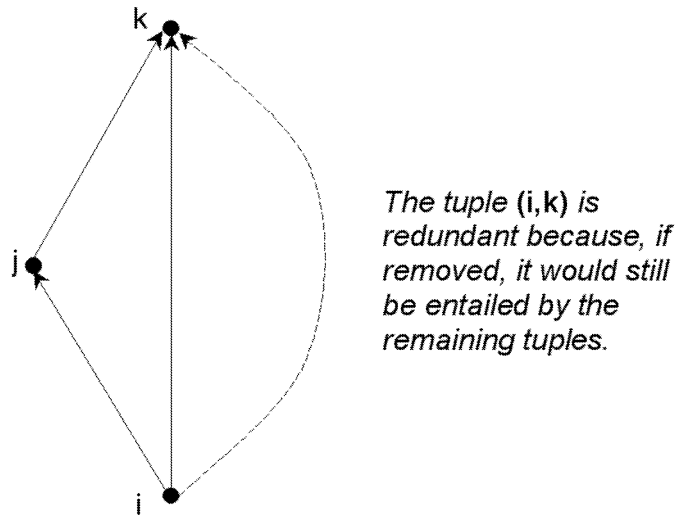


Figure 3: A redundant tuple.

## 3.2 Some interesting properties of Generator Sets

The following are interesting properties associated with generator sets.

(G1) : A generator set  $G$  is sound iff  $G$  can be represented by a directed *acyclic* graph.

The fact that  $G$  may be represented by a directed graph is trivial. The fact that it must be acyclic arises from the following argument. Let us assume that  $G$  contains a cycle of length longer than one. Now let  $i, j \in \mathcal{J}$ ,  $i \neq j$  be two vertices of said cycle. Then there exists a path from  $i$  to  $j$  and from  $j$  to  $i$ . This would imply that both  $(i, j) \in G^*$  and  $(j, i) \in G^*$ . Since this violates antisymmetry  $G^*$  cannot be a partial order and therefore  $G$  cannot be sound. By a similar argument the reverse implication may also be proven.

(G2) : If  $G$  is a sound generator set and  $(j, i) \notin G^*$  then  $G \cup \{(i, j)\}$  is a sound generator set.

Let us assume that  $G$  is a sound generator set,  $(j, i) \notin G^*$  and that  $G \cup \{(i, j)\}$  is *not* sound. If  $G$  is sound then it has no cycle. And if  $G \cup \{(i, j)\}$  is not sound then it has a cycle and it follows that  $(i, j)$  completed it. Therefore, there was a path from  $j$  to  $i$  in  $G$  and hence  $(j, i) \in G^*$ . This contradicts our initial assumption.



## 4 Revision of informant plausibility relations

### 4.1 The expansion operator

Let us assume that an agent learns that, of a pair of informants, one is more reliable than the other. This would warrant the modification of its knowledge accordingly. For this purpose, we define the operator  $\oplus : \mathcal{P}(\mathcal{J}^2) \times \mathcal{J}^2 \longrightarrow \mathcal{P}(\mathcal{J}^2)$ . This operator adds new tuples to a generator set in order to establish relations between informants. Given a pair of informants and a generator set, this function returns a new generator set in which said agents are now related. According to this new generator set we may say that the first informant is “less reliable” than the second.

#### Postulates for the expansion operator

(E1) **Success:**  $(i, j) \in (G \oplus (i, j))^*$ .

Determining new relations among informants is most likely a costly process for the agent. Consequently a desirable property of expansions is that the new relation given will indeed be added to the agents beliefs, and not somehow lost.

(E2) **Inclusion:**  $G^* \subseteq (G \oplus (i, j))^*$ .

Here the case of equality between the previous set and the new one occurs in the event of an expansion by a relation which was already entailed by the generator set. This leads us to the following postulate for expansions.

(E3) **Vacuity:** if  $(i, j) \in G^*$  then  $(G \oplus (i, j))^* = G^*$ .

What this postulate states is that there is no information to be lost or gained by the addition of redundant data to the generator set.

(E4) **Commutativity:**  $((G \oplus (k, l)) \oplus (i, j))^* = ((G \oplus (i, j)) \oplus (k, l))^*$ .

The order in which tuples are added to the generator set does not affect the final, closed relation. This is important because sometimes we will use  $G \oplus A$  as a shorthand for the expansion of  $G$  by every tuple in  $A$ . Such is the case of the following postulate.

(E5) **Extensionality:** if  $A^* = B^*$  then  $(G \oplus A)^* = (G \oplus B)^*$ .

The expansion of a generator set by two sets whose reflexive transitive closure is equal yields generator sets whose closure is also equal.

(E6) **Conditional Soundness Preservation:** if  $G$  is a sound generator set and  $(j, i) \notin G^*$  then  $G \oplus (i, j)$  is a sound generator set.

## Construction

In this subsection, we will introduce a construction of expansions on plausibility relations.

**Definition 4.1.1:** Given a pair of informants  $i, j \in \mathcal{J}$  and generator set  $G \subseteq \mathcal{J}^2$ , we define the expansion of  $G$  by  $(i, j)$  as  $G \oplus (i, j) = G \cup \{(i, j)\}$ .  $\square$

The following lemma summarizes some interesting properties of the expansion operator.

**Lemma 4.1.1:** Let  $\oplus$  be an expansion operator as defined in Definition 4.1.1. Then  $\oplus$  satisfies *success*, *inclusion*, *vacuity*, *commutation*, and *extensionality*.  $\square$

Expansion does not preserve soundness *per se*, but is conditioned as stated in the postulate. This property is a consequence of the properties of sound generator sets and the definition of expansion that we have provided.

## 4.2 The contraction operator

At the beginning of the previous subsection, we said that an agent may need to assert the fact that one informant is less reliable than another. In a similar fashion the opposite may also become true. This is to say, we may wish to reflect the fact that an informant is no longer more reliable than another. For this purpose we define a contraction operator  $\ominus : \mathcal{P}(\mathcal{J}^2) \times \mathcal{J}^2 \longrightarrow \mathcal{P}(\mathcal{J}^2)$ .

Assume we have a pair of informants  $i$  and  $j$  and a generator set  $G$  such that  $(i, j) \in G^*$ . The basic task of the  $\ominus$  function is to construct a new generator set in which this is no longer the case while losing as little information as possible. However we cannot simply remove the pair  $(i, j)$  from  $G$ . In fact,  $(i, j)$  may not even be in  $G$ . Care must be taken to also remove pairs that, through transitivity, would imply the pair  $(i, j)$  in  $G^*$ . As long as there is a path in the generator set from  $i$  to  $j$ ,  $(i, j)$  will be found in its transitive closure. It is therefore necessary to eliminate a set of pairs so that no path is left from  $i$  to  $j$  in  $G$ . This set is desirably minimal.

### Postulates for the contraction operator

(C1) **Inclusion:**  $(G \ominus (i, j))^* \subseteq G^*$ .

If a tuple is entailed by a generator set, then its contraction by said tuple removes at least one element from the set: the tuple itself. The sets are equal in the case in which  $(i, j) \notin G^*$ . This is expressed in the following postulate.

(C2) **Vacuity:** if  $(i, j) \notin G^*$  then  $G \ominus (i, j) = G$ .

That is, if a tuple is not a consequence of a given generator set then its contraction by said tuple produces no change.

(C3) **Success:** if  $i \neq j$  then  $(i, j) \notin (G \ominus (i, j))^*$ .

A tuple cannot be entailed by the generator set resulting from its contraction. In the case of  $i = j$ , the tuple will trivially be in the reflexive transitive closure of any generator set due to reflexivity.

(C4) **Path Disruption:** for all  $k \in \mathcal{J}$ , if  $(i, k) \in G^*$  and  $(k, j) \in G^*$  then either  $(i, k) \notin (G \ominus (i, j))^*$  or  $(k, j) \notin (G \ominus (i, j))^*$ .

Let  $k \in \mathcal{J}$  be such that both  $(i, k)$  and  $(k, j)$  are in  $G^*$ , and assume that we contract  $G$  by  $(i, j)$ . If both  $(i, k)$  and  $(k, j)$  are still entailed then by transitivity  $(i, j)$  would also be entailed. This would go against the success postulate.

(C5) **Recovery:** if  $(i, j) \notin G^*$  then  $G^* \subseteq ((G \ominus (i, j)) \oplus (i, j))^*$ .

This postulate is a direct consequence of the vacuity postulate for contraction and the inclusion postulate for expansion.

(C6) **Reverse Recovery:** if  $(i, j) \in G^*$  then  $((G \ominus (i, j)) \oplus (i, j))^* \subseteq G^*$ .

Here the case of equality between the sets arises when the contraction of  $G$  by  $(i, j)$  only causes the deletion of this tuple and no other implying pairs must also be removed. If other pairs were removed to avoid the appearance of  $(i, j)$  in the closure then they would not reappear with the expansion of  $G \ominus (i, j)$  by  $(i, j)$ .

(C7) **Soundness Preservation:** if  $G$  is a sound generator set then  $G \ominus (i, j)$  is a sound generator set.

## Construction

In this subsection we will introduce a construction for contractions on plausibility relations. However, before we do so, we will need to present a few concepts.

First let us briefly review the concept of path. We say that a set of tuples  $P$  is a *path* from  $i$  to  $j$  if  $(i, j) \in P$ , or  $(i, k) \in P$  and there is a path from  $k$  to  $j$  in  $P$ . We say that  $P$  is a *nonredundant path* from  $i$  to  $j$  if it is a path from  $i$  to  $j$  and there is no path from  $i$  to  $j$  in any  $P' \subset P$ .

**Definition 4.2.1:** Given a pair of informants  $i, j \in \mathcal{J}$  and generator set  $G \subseteq \mathcal{J}^2$ , we define the *path set* from  $i$  to  $j$  in  $G$ , and we will note it  $G_{ij}$ , as

$$G_{ij} = \{C \subseteq G : C \text{ is a nonredundant path from } i \text{ to } j \text{ in } G\}$$

□

Notice that according to this definition the path set from  $i$  to  $j$  in a generator set  $G$  is a set of sets. Each set represents a path from  $i$  to  $j$ . In the contraction of  $G$  by

$(i, j)$ , in order to avoid the appearance of this tuple, none of these paths may remain complete. Therefore, we need a selection mechanism to decide which tuples will be erased from each path in  $G_{ij}$ .

**Definition 4.2.2:** Given a path set  $G_{ij}$ , we say that  $\gamma$  is a *cut function* for  $G_{ij}$  if and only if:

1.  $\gamma(G_{ij}) \subseteq G$ .
2. For each  $C \in G_{ij}$ ,  $\gamma(G_{ij}) \cap C \neq \emptyset$ .

□

Now we may present our definition of contraction.

**Definition 4.2.3:** Given a pair of informants  $i, j \in \mathcal{J}$  and generator set  $G \subseteq \mathcal{J}^2$ , we define the contraction of  $G$  by  $(i, j)$  as  $G \ominus (i, j) = G \setminus \gamma(G_{ij})$  □

The following result gives a summary of the properties of the contraction operator.

**Lemma 4.2.1:** Let  $\ominus$  be a contraction operator as defined as in Definition 4.2.3. Then  $\ominus$  satisfies *inclusion, vacuity, success, path disruption, recovery, reverse recovery* and *soundness preservation*. □

Notice that here, in contrast to the case of expansion, the soundness preservation property of contraction is not conditioned. This is due to the way we define contraction. Since contraction is basically a process of elimination, it is impossible for this operation to introduce cycles if there were none with which to begin.

### 4.3 Revision operator

Suppose that an agent learns that an informant is less reliable than another. The agent's current generator set should be modified to reflect this new information. However, it would be convenient if the generator set were also modified, when necessary, so that the opposite can no longer hold. That is to say, if up to now the agent believed that the second informant was less reliable then this should be retracted.

For this purpose we define the revision operator  $\otimes : \mathcal{P}(\mathcal{J}^2) \times \mathcal{J}^2 \rightarrow \mathcal{P}(\mathcal{J}^2)$ . Assume we have a pair of informants  $i$  and  $j$  and a generator set  $G$ , and the agent now has reason to believe that  $i$  is less reliable than  $j$ . The basic task of the  $\otimes$  operator is to construct a new generator set in which  $(i, j)$  is entailed but  $(j, i)$  is not.

#### Postulates for the revision operator

(R1) **Success:**  $(i, j) \in (G \otimes (i, j))^*$ .

This is basically a consequence of the definition given for revision and the success postulate for expansion.

**(R2) Inclusion:**  $(G \otimes (i, j))^* \subseteq (G \oplus (i, j))^*$ .

This is due to the fact that expansion simply inserts the new tuple into the generator set while revision may need to remove tuples before adding the new one. The border case of equality presents itself when  $(j, i) \notin G^*$ , which leads us to our next postulate.

**(R3) Vacuity:** if  $(j, i) \notin G^*$  then  $G \otimes (i, j) = G \oplus (i, j)$ .

When there is no path from  $j$  to  $i$  then revising  $G$  by  $(i, j)$  is equal to expanding  $G$  by this same tuple.

**(R4) Path Disruption:** for all  $k \in \mathcal{J}$ , if  $(j, k) \in G^*$  and  $(k, i) \in G^*$  then either  $(j, k) \notin (G \otimes (i, j))^*$  or  $(k, i) \notin (G \otimes (i, j))^*$ .

This postulate is analogous to the one presented for contraction. After revising  $G$  by  $(i, j)$  there can be no path from  $j$  to  $i$  remaining in  $G$ .

**(R5) Soundness Preservation:** if  $G$  is a sound generator set then  $G \otimes (i, j)$  is a sound generator set.

## Construction

In this subsection, we will introduce a construction of revisions on plausibility relations.

**Definition 4.3.1:** Given a pair of informants  $i, j \in \mathcal{J}$  and generator set  $G \subseteq \mathcal{J}^2$ , we define the revision of  $G$  by  $(i, j)$  as  $G \otimes (i, j) = (G \ominus (j, i)) \oplus (i, j)$ .  $\square$

The following lemma enunciates some interesting properties of the operator.

**Lemma 4.3.1:** Let  $\otimes$  be a revision operator as defined in Definition 4.3.1. Then  $\otimes$  satisfies *success*, *inclusion*, *vacuity*, *path disruption*, and *soundness preservation*.  $\square$

Again here, as in the case of contraction, soundness preservation is not conditioned. In the case that the new tuple to be inserted,  $(i, j)$  were to complete a cycle, the previous contraction of  $(j, i)$  would insure that there is no link between  $j$  and  $i$ . Hence, it is impossible for revision to introduce cycles.

## 5 Conclusions and Future Work

We have introduced a model for representing plausibility relations among informants in the context of a multi-agent system. This was done through the use of *generator sets* which, when sound, establish a partial order over the set of an agent's informants. Thus, when faced with contradicting information, the agent can solve the inconsistency by believing the more reliable informant as determined by the generator set.

Given the dynamic nature of multi-agent systems, we also allowed for the modification of the plausibility relation. This was done with the introduction of change operators defined on the generator sets. These change operators were formally characterized by means of postulates, analogous in some cases to those of the AGM model, and unique in others. Finally, a construction was provided for each of these change operators, thus completing their definition.

Clearly, what follows is to devise ways of handling the perception of changing plausibilities in real sources of information. Such is the case of the weather forecasting systems and predictors of stock market behavior mentioned in this paper's introduction. From these examples, and others, we will seek to understand the complexities of dynamic updating in decision making and advising systems.

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