Polyhedra associated with identifying codes*

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Many practical applications can be stated as set covering problems, among them newly emerging search problems for identifying codes. Consider a graph $G = (V, E)$ and denote by $N[i] = \{i\} \cup N(i)$ the closed neighborhood of i. A subset $C \subseteq V$ is *dominating* (resp. *identifying*) if $N[i] \cap C$ are non-empty (resp. distinct) sets for all $i \in V$. An *identifying code* of G is a vertex subset which is dominating and identifying, and the *identifying code number* $\gamma^{ID}(G)$ of a graph G is the minimum cardinality of and identifying code of G.

Determining a minimum identifying code in a graph $G = (V, E)$ can be formulated as set covering problem $\min \mathbf{1}^T x, M_{ID}(G) \geq \mathbf{1}, x \in \{0,1\}^{|V|}$ by

$$\begin{array}{rll} & \min \mathbf{1}^T x & \\ x(N[j]) = \sum_{i \in N[j]} x_i \geq 1 & \forall j \in V & \text{(domination)} \\ x(N[j] \triangle N[k]) = \sum_{i \in N[j] \triangle N[k]} x_i \geq 1 & \forall j,k \in V, j \neq k & \text{(identification)} \\ x \in \{0,1\}^{|V|} & & \end{array}$$

In this work we study the associated polyhedra and present some general results on their combinatorial structure. We demonstrate how the polyhedral approach can be applied to find minimum identifying codes for special bipartite graphs and cycles, and discuss further lines of research in order to obtain strong lower bounds stemming from linear relaxations of the identifying code polyhedron, enhanced by suitable cutting planes to be used in a B&C framework.

Keywords: identifying code polyhedron, identifying code clutter, odd hypercycles

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