On Improved Deformable Template Matching For Polygonal Objects

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ABSTRACT

In this paper, an improvement of deformable template matching algorithm for polygonal objects in grayscale images using two-dimensional deformable templates along orthogonal curves is presented. In the process of pre-computing extensions of the deformable template along orthogonal curves, the novel matching approach incorporates adapting knowledge-specific template discretization techniques appropriate for different polygonal objects and minimizing the improved internal and external energy terms containing inter-shape information of polygonal objects. In our application, this energy optimization problem of the deformable template is efficiently solved by a genetic algorithm (GA). Our algorithm has been successfully applied on synthetic images and real images. The experiment results show that the new approach provides more robust and accurate matching method.

Keywords: Deformable template, Orthogonal curve, Polygonal object, Matching, Genetic algorithm.

1. INTRODUCTION

There have been numerous attempts to build models describing shape and appearance and employ them for automated object identification in the analyzed images. Among them, the deformable template model has many favorable shape representation properties and becomes a very powerful and flexible technique.

H. D. Tagare in [1] proposed a new formulation of the 2-D deformable template model along orthogonal curves which extends the strategy of deforming the template along normal as curves rather than as straight lines [2]. It uses a lower-dimensional search space than conventional methods. The reduction in search space allows the use of dynamic programming to obtain globally optimal solutions. However, due to the complexity of the energy function in a high dimensional parameter space and the existence of many local minima, the computational complexity of the algorithm, which utilizes dynamic programming to optimize the energy function, increases exponentially with the scale of the problem. These reduce the practicability of the deformable template model in [1]. In addition, the algorithm matches polygonal objects not very well due to its smoothness internal energy.

In this paper, we present an improvement of matching algorithm using two-dimensional deformable templates along orthogonal curves. Our deformable template model is designed to match polygonal object (or multiple polygonal objects) in a grayscale image. The search procedure developed for our approach to image matching is based on a model fitting strategy that substantially differs from the deformable template model in [1]. The differences are twofold. First, more flexible knowledge-specific template discretization techniques appropriate for the different polygonal objects are adapted in our approach. Second, the improved internal and external image energy terms containing inter-shape information of polygonal objects are given and the corresponding globally optimal solution of the total energy function is efficiently obtained by using a genetic algorithm (GA). The advantage of our improved deformable templates along orthogonal curves is emphasized by provision of a knowledge-specific and compact representation of polygonal object features, particularly for simply connected polygonal target shapes, hence will be essential for their shape matching. These differences improve robustness and accuracy of the shape model fitting process. For those images where the shapes of polygonal objects do not conform to the prior information, active contour models [3] can be used.

Our method presented below focus on describing the fundamentals of 2-D deformable models along orthogonal curves and their application to 2-D polygonal objects template matching. We attempt to import and improve the deformable template model along orthogonal curves. First, an appropriate polygonal deformable template along orthogonal curves is constructed according to the polygonal object matching problem. Then, we define a new energy function to govern the polygonal template to deform and use genetic algorithm to search its global minima. The main contribution of this paper is to combine adapting flexible template discretization technique to the different polygonal objects and minimizing the improved image energy terms containing inter-shape information of polygonal objects. This approach is particularly useful if the object contour is close to a polygon and the variation in shape is small, as is often the case with deformable template model matching.

The paper is organized as follows. The next section briefly introduces a two-dimensional deformable template along orthogonal curves and defines discrete orthogonal curves appropriate for the polygonal objects template matching in grayscale images. Section 3 describes the improved internal and external energy terms containing inter-shape information of polygonal objects. In Section 4, we provide the details of the genetic algorithm used to minimize the improved total energy. Section 5 provides the experimental results and a discussion of these results.

Finally, we conclude the paper and point out future research directions.

2. DEFORMABLE TEMPLATE AND ORTHOGONAL CURVES

In this section, we define an appropriate discrete deformable template along orthogonal curves according to the 2-D polygonal objects template matching problem. In the template coordinate system, an ordinary deformable template is parameterized as a curve

$$C : \theta \mapsto (x(\theta), y(\theta))$$

with which external and internal energies are associated, and functions $x_0(\theta), y_0(\theta)$ are sought which minimize a weighted sum of the internal and external energies. By pre-computing some curves along which the template is deformed, the regions in which the template is deformed are specified while avoiding singular points. These curves, called orthogonal curves, are perpendicular to the template. After establishing the orthogonal curves, the template is deformed by restricting every point on the template to move only along its orthogonal curve. Deformable template model and orthogonal curves ([1]) are illustrated in Fig. 1.

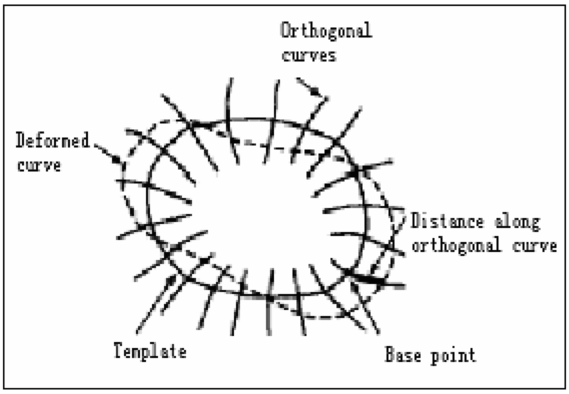


Fig.1. Deformable template and orthogonal curves

Suppose that the template is a closed curve and always non-self intersecting. Each orthogonal curve intersects the template at a single point and the single point is called the base point of the orthogonal curve (Fig. 1). Further suppose that the region of deformation is small and can be bounded by two pre-defined curves C_{in} and C_{out}, where C_{in} is located inside C and C_{out} is located outside C.

In order to realize the template matching algorithm, it is necessary to use discrete versions of orthogonal and deformed curves. The general discrete orthogonal and deformed curves in [1] are sampled everywhere uniformly and do not take the characteristic shapes of polygonal objects into account.

Here, let us put a vehicle as one of typical examples of polygonal objects. According to the generic shape of the vehicles and the discretization method of the prototype templates in [4], we adapt an especial discretization way which considers carefully the peculiar polygonal shape character of a vehicle (Fig. 2).

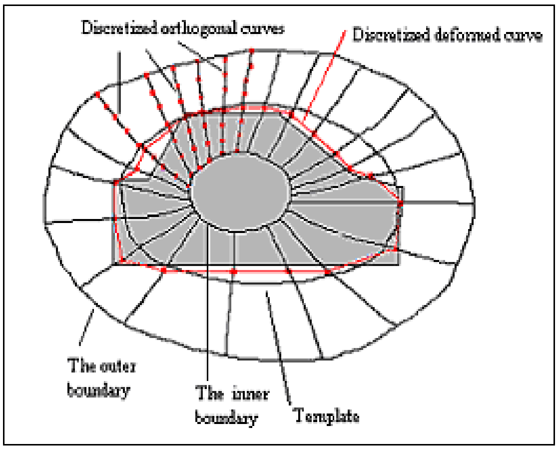


Fig.2. Discrete version of deformable template

By studying a database of images containing typical vehicles which can be considered as our matching targets, we find that those vertices on orthogonal curves whose base points are close to the bottom edge of the vehicle usually distribute on a straight line and thus can be fixed by few vertices such as start point and end point of the straight line. On the contrary, those vertices on orthogonal curves whose base points are close to the else edges of the vehicle are correlative with the different shapes of different vehicles.

In view of these reasons and the flexibility of template discretization, we will adopt the following discretization strategy:

(1) First, the corners of a polygonal object are detected by Kitchen-Rosenfeld gray level corner detection measurement in [5]

$$m(x, y) = \left| \frac{I''_{yy} I'^{2}_{x} - 2I''_{xy} I'_x I'_y + I''_{xx} I'^{2}_{y}}{(I'^{2}_{x} + I'^{2}_{y})^{3/2}} \right|$$

(2) Second, two kinds of discrete sampling rates are exerted: one is high and to the top half of the template, the other is low and to the bottom half of the template (Fig. 2). In the concrete, we sample the top half and the bottom half of the template uniformly at N_1 and N_2 $(N_2 < N_1)$ base points respectively.

If we regard C, C_{in}, C_{out} as three contours of a 2-D continuous function such as three different gray-scale curves, the gradient trajectories of the function are orthogonal to these contours everywhere and thus can be admissible as orthogonal curves of the template C.

Suppose we have found a 2-D continuous function $\psi(x, y)$ which is defined in the above region G and subject to

$$\begin{cases} \psi(x, y) = 100, & (x, y) \in C_{out} \\ \psi(x, y) = 255, & (x, y) \in C \end{cases} \quad (1)$$

Where $\psi(x, y)$ denotes a user-defined image gray-level function and so has well-defined gradients.

At the same time, the smoothness of the function $\psi(x, y)$ can not be neglected. The function $\psi(x, y)$ must minimize $\iint_G \|\nabla \psi(x, y)\|^2 dxdy$ which reflects the summation of gradient magnitude of $\psi(x, y)$ in the region G. That is to say, $\psi(x, y)$ is the solution to the corresponding second elliptic partial differential equation (PDE) ----Euler-Lagrange Equation

$$\nabla^2 \psi(x, y) = \frac{\partial^2 \psi(x, y)}{\partial x^2} + \frac{\partial^2 \psi(x, y)}{\partial y^2} = 0 \quad (2)$$

with the boundary condition of Eq. (1).

First, in a $H_1 \times H_2$ image, we draw the initial template C on which the gray level of the points is 255 and an outer curve C_{out} on which the gray level of the points is 100. The region G is limited and specified by the two curves. Then, the region G is divided and plotted by planar rectangular grids. Finally, the numerical solution $\psi_{i,j}$ of the function $\psi(x, y)$ is obtained by a standard finite-difference successive over-relaxation iterative method (FD-SOR) ([7]):

$$\psi_{i,j}^{(m+1)} = \psi_{i,j}^{(m)} + \frac{\omega}{4}(\psi_{i-1,j}^{(m+1)} + \psi_{i+1,j}^{(m)} + \psi_{i,j-1}^{(m+1)} + \psi_{i,j+1}^{(m)} - 4\psi_{i,j}^{(m)})$$

$$1 \le i \le H_1 - 1, 1 \le j \le H_2 - 1, 1 < \omega < 2 \quad (3)$$

Where ω is a relaxation factor and $\psi_{i,j}^{(m)}$ will converge at $\psi_{i,j}$ $(1 \le i \le H_1 - 1, 1 \le j \le H_2 - 1)$ when $m \to +\infty$. Here, the iteration can be executed by row-column order or odd-even order. In addition, if the region boundary curve C or C_{out} is irregular, the region G can be divided by planar triangular grids and we can adapt more current variational calculus method----Finite Element Method ([6]).

Apparently, the tangent line of gradient trajectory $t \to (u(t), v(t))$ at arbitrary point $(\bar{x}, \bar{y})$ in G is parallel to the gradient direction at this point. Furthermore, the gradient direction at any point $(\bar{x}, \bar{y})$ in G can be given by

$$\mathbf{n}(\bar{x}, \bar{y}) = \left. \frac{\nabla \psi(x, y)}{\|\nabla \psi(x, y)\|} \right|_{\substack{x=\bar{x} \\ y=\bar{y}}} = (p, q) \quad (4)$$

Where p and q are constants only dependent on $\bar{x}, \bar{y}$.

Therefore, after sampling the top half and the bottom half of the template uniformly at N_1 and N_2 $(N_2 < N_1)$ base points respectively, we will calculate $N_1 + N_2$ gradient trajectories by numerical method. Starting from a certain base point $(x(\theta), y(\theta))$ on the template, the gradient trajectory $t \to (u(t), v(t))$ is given by the solution of the coupled differential equations

$$\begin{cases} du(t)/dt = p \\ dv(t)/dt = q \end{cases} \quad (5)$$

with the boundary conditions

$$\left. \begin{pmatrix} u(t) \\ v(t) \end{pmatrix} \right|_{t=0} = \begin{pmatrix} x(\theta) \\ y(\theta) \end{pmatrix} \quad (6)$$

Then, using finite difference method ([7]), the numerical form of gradient trajectory between C and C_{out} can be obtained.

The above argument and method can be used in the region between C and C_{in}. Furthermore, the orthogonal curves in the two regions can be joined at C to obtain curves which have a continuous tangent vectors and extend from the inner curve C_{in} to the outer curve C_{out}.

Then, when $N_1 + N_2$ orthogonal curves have been established at $N_1 + N_2$ discrete base points, each orthogonal curve can be traced inwards and outwards from its base point and sampled uniformly at $2M + 1$ points. The deformed curve is discretized as an $(N_1 + N_2)$ -sided polygon

$$P(V_1, V_2, \cdots, V_{N_1+N_2})$$

and the *k*-th vertex V_k of the polygon is constrained to lie on the *k*-th orthogonal curve. Here $N_1 + N_2$ is denoted by N.

3. INTERNAL AND EXTERNAL ENERGY FUNCTION

The energy function $E(P(V_1, \cdots, V_N))$ which is associated with the deformed curve $P(V_1, \cdots, V_N)$ is composed of the internal energy $I(P(V_1, \cdots, V_N))$ and the external energy $X(P(V_1, \cdots, V_N))$.

The internal energy measures the proximity between the deformed curve $P(V_1, \cdots, V_N)$ and the initial template C, which is the overall effect of the displacements of the vertices from their base points weighted with γ_k:

$$I(P(V_1, \cdots, V_N)) = \sum_{k=1}^{N} \gamma_k H(V_k - \tilde{X}_{k,0}) \quad (7)$$

Where the weights are correlative with the dissimilarity of distances of three consecutive vertices from their base points:

$$\gamma_k = \exp\{-\left|2H(V_k - \tilde{X}_{k,0}) - H(V_{k-1} - \tilde{X}_{k-1,0}) - H(V_{k+1} - \tilde{X}_{k+1,0})\right|\} \quad (8)$$

for $k = 1, \cdots, N$. Here $H()$ denotes the Hamming distance. Apparently, the greater the weight coefficient γ_k is, the stronger the proximity restriction on the vertex V_k is. The kind of constraint incorporation using prior knowledge can limit the deformable template from deforming to some irrelevant shapes and so reduce the matching accuracy.

Since the shape of a vehicle is always a polygon and the deformed curve is discretized as an *N*-sided polygon $P(V_1, V_2, \cdots, V_N)$, some polygon vertices on the final deformed curve will overlap to a certainty some vertices on a vehicle contour curve and keep away from their base points. By studying and comparing carefully the relationship between polygon vertices and vehicle contour, we find that:

(1) If $H(V_{k-1} - \tilde{X}_{k-1,0}), H(V_k - \tilde{X}_{k,0})$ and $H(V_{k+1} - \tilde{X}_{k+1,0})$, the Hamming distances of three consecutive vertices V_{k-1}, V_k, V_{k+1} from their base points $\tilde{X}_{k-1,0}, \tilde{X}_{k,0}, \tilde{X}_{k+1,0}$, increase or decrease gradually, the middle vertex V_k usually will not correspond with a vehicle vertex and thus be endowed with a weight coefficient γ_k which is relatively great in the internal energy expression;

(2) If $H(V_k - \tilde{X}_{k,0})$ is the largest or smallest among $H(V_{k-1} - \tilde{X}_{k-1,0}),\ H(V_k - \tilde{X}_{k,0})$ and $H(V_{k+1} - \tilde{X}_{k+1,0})$, the middle vertex V_k usually will correspond with a vehicle vertex or an edge point on the vehicle contour and thus be endowed with a weight coefficient γ_k which is relatively little in the internal energy expression.

The external energy $X(P(V_1, \cdots, V_N))$ measures the matching degree between the vehicle object in the image and the deformed curve $P(V_1, \cdots, V_N)$. In the paper, only the image edge information is considered. First, the edge image is obtained by using the Roberts Edge Operator. Then, by defining the edge energy function based on the pixel displacements, we can establish the image edge potential energy field. The value of the edge energy at an arbitrary pixel (x, y) in the image is given by

$$\delta(x, y) = 1 - \exp\{-\rho(\delta_x^2 + \delta_y^2)^{1/2}\} \quad (9)$$

Here δ_x, δ_y are the displacements between the pixel (x, y) and the closest boundary point in the image and ρ is a smoothness factor.

According to Eq. (9), in the image potential energy field, each point in the edge image can be convolved with a two-dimensional Gaussian mask to obtain potential energy field. The external energy of the deformed curve is the sum of edge energies of the vertices on the deformed curve $P(V_1, \cdots, V_N)$:

$$X(P(V_1, \cdots, V_N)) = \sum_{k=1}^{N} \delta(V_k) \quad (10)$$

So, the total energy $E(P(V_1, \cdots, V_N))$ which is associated with the deformed curve $P(V_1, \cdots, V_N)$ takes the form

$$E(P(V_1, \cdots, V_N)) = \lambda_1 I(P(V_1, \cdots, V_N)) + \lambda_2 X(P(V_1, \cdots, V_N)) \quad (11)$$

Where λ_1 *and* λ_2 are nonnegative constants.

4. GENETIC ALGORITHM AND OPTIMIZATION

The objective function to be minimized in Eq. (11) is a complex function with some local extrema over the deformation parameter space. The computational complexity of the algorithm in [1] increases exponentially with the scale of the problem, which utilizes dynamic programming to search the really global minimum of the energy function. A global search is usually impossible due to the size of the configuration space. Instead, in this paper, we implement a genetic-algorithm-based optimization technique to obtain the globally optimal solution of the objective function.

Genetic Algorithm (GA) ([8], [9]) is a recent optimization technique originally introduced by John Holland in 1975 and developed by his students and colleges. The principle of this technique tries to adapt the natural process of evolution and selection for optimization. As the natural populations evolve according to the Darwin's principles of natural selection and "survival of the fittest", so by simulating this evolution process, a genetic algorithm is able to evolve solutions to real-world problems, if they have been suitably encoded. GA doesn't search from one single point, but from a population of points. Using an iterative procedure that consists of a constant-size population of individuals, each one represented by a finite string of symbols, known as the *genome*, The genetic algorithm encodes a possible solution in a given problem space. This space, referred to as the search space, comprises all possible solutions to the problem at hand.

GA is inherently a parallel optimization procedure. A basic GA uses stochastic reproduction instead of deterministic rules and employs selection, crossover and mutation operators. The whole process can be referred to as "reproduction". Algorithm is started with a set of random solutions (represented by chromosomes) called subpopulation. Solutions from one subpopulation are taken and used to form a new subpopulation by a hope that the new population will be better, at least not worse than the old one. Solution is selected according to its fitness score which gives an evaluation for each chromosome as to how suitable a solution it is. The more suitable it is, the more chance it has to reproduce. Therefore, the most important part of a GA is the fitness function.

The standard genetic algorithm proceeds as follows:

Step 1 Generate random initial population P_0 containing t chromosomes (suitable solutions for the problem) and $i \leftarrow 0$.

Step 2 Evaluate the fitness $f(x)$ of each chromosome *x* in the population P_i .

Step 3 Create a new population P_{i+1} by repeating the following steps until the new population P_{i+1} is complete: (1) Select two parent chromosomes from population P_i according to their fitness score (the bigger fitness score, the more chance to be selected); (2) With a crossover probability p_c , the parents is crossover to form a new offspring. If no crossover was performed, offspring is an exact copy of parents; (3) With a mutation probability p_m , new offspring is mutated at each locus (position in chromosome); (4) Place new offspring in a new population P_{i+1} .

Step 4 Use a new generated population P_{i+1} for a further run of algorithm: $i \leftarrow i+1$.

Step 5 If the end condition is unsatisfied (Either a tolerable error margin and/or the maximum of iterations to run), Go to step 2; if the end condition is satisfied, stop and return the best solution in current population P_i .

In order to use Genetic Algorithm to obtain the globally optimal solution of the total energy function $E(P(V_1,\cdots,V_N))$ of the deformed curve $P(V_1,\cdots,V_N)$, the encoding representation and the fitness evaluation method of the chromosome should be ascertained:

(1) Encoding representation

The first thing we have to do is encode the independent parameters into a chromosome in an appropriate manner. There are different ways of encoding the parameters used in the genetic algorithm: binary encoding, floating-point encoding, integer encoding, character encoding, real-valued encoding, and tree representation ([8], [9]). In this paper, since the deformed curve is denoted by a polygon $P(V_1,\cdots,V_N)$ with N edges, integer-based encoding representation is adapted. The encoding representation of a chromosome x is expressed by $a_1a_2\cdots a_N$, where N stands for the length of the string and a_i is the sequence number of the *i*-th vertex V_i on the *i*-th discretized orthogonal curve. The polygon $P(V_1,\cdots,V_N)$ corresponding to the chromosome x is denoted by $P_x(V_1,\cdots,V_N)$.

(2) Fitness measure

Since the fitness scores in GA fall into R^+ , the set of positive natural numbers, and the optimal solution is chosen according to the magnitude of the fitness score, the problem of minimizing the energy function $E(P(V_1,\cdots,V_N))$ must be transformed into a problem of maximization. The fitness evaluation function is given by

$$F: x \mapsto E_{\max} - E(P_x(V_1,\cdots V_N)) \qquad (12)$$

Where $E_{\max}$ denotes the maximum of all the energy functions of the chromosomes in the same generation.

Genetic algorithm based on integer encoding is used in this work and the best suited size of population in each generation $(t=20)$, crossover rate $(r_c=0.8)$ and mutation rate $(r_m=0.02)$ are determined after a few tries, respectively. In this paper, the maximal generation is set to 200 and the EPSILON-DELTA termination criteria are adapted. That is to say, the program is terminated if the fitness changes less than EPSILON over DELTA generations or the maximum number of iterations is up to 200.

5. EXPERIMENTAL RESULTS

Our method presented above has been tested on a variety of grayscale images, some of which are shown below. In all cases, the method was independently tested in separate images. First, the approach was employed in detecting cars which usually in the form of polygonal shapes in images containing cars and the scene. As shown in Fig. 3, our approach correctly matched and detected the polygonal shapes of cars ($N_1=30,\ N_2=15,\ M=20$).

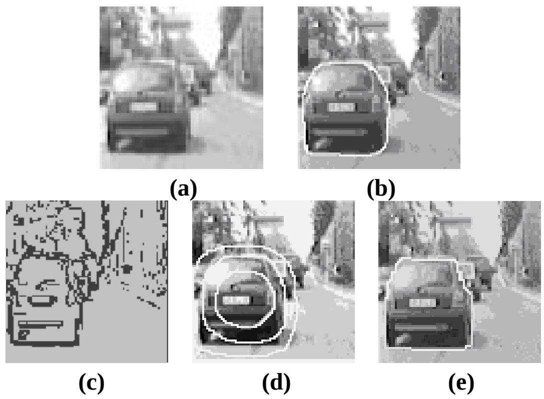
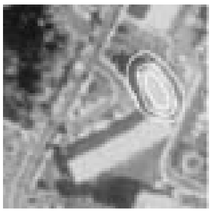


Fig. 3: (a) Original image (512*512). (b) The final deformed contour using the deformable template in [1]. (c) Edge detected image. (d) The initial deformable contour and its deformable region boundaries. (e) The final converged deformed contour result.

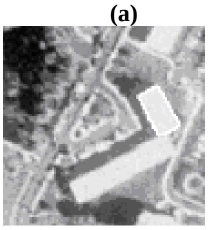
Fig.3 shows an example of the computer-detected contours for the vehicle models. This image was taken using a KODAK DC40 digital camera which provides 512*512 pixel resolutions. It demonstrates the ability of our algorithm to cope with polygonal object template matching. The optimization based on genetic algorithm runs about 10-20 seconds for a single image on a standard PIII 550 MHz PC.

Note that, although the model was detected on the single object template matching, it was also able to accurately detect multiple polygonal objects in a single image, as shown in Fig.4 ($N_1 = N_2 = 15, M = 20$), where a digital orthophoto aerial photograph from our lab database of images is employed. It can be observed that two rectangular buildings are successfully detected by our technique.

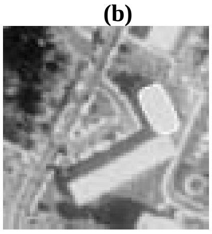
(a)



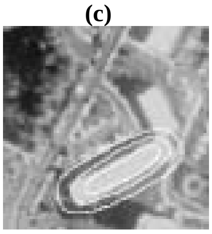
(b)



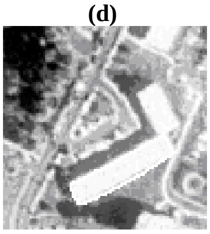
(c)



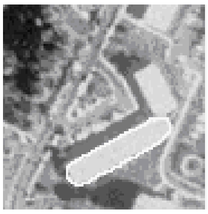
(d)



(e)



(f)



(g)

Fig. 4: (a) Original image (8-bit gray-level orthophoto). (b) The initial deformable contour of one building and its deformable region boundaries. (c) One detected building with the rectangular contour. (d) The compared matching result of (c). (e) The initial deformable contour of another building and its deformable region boundaries. (f) Another detected building with the rectangular contour. (g) The compared matching result of (f).

6. CONCLUSIONS AND FUTURE WORK

In this paper, we have described a promising matching method for real grayscale images, which use application-specific prior knowledge and/or global shape properties to obtain more robust and accurate results. Our experimental results have shown that the use of the improved deformable template along orthogonal curves increases the robustness and accuracy of the algorithm in situations where the boundaries of the matched objects are approximately polygons [4]. These template matching results demonstrate the ability of our algorithm to cope with polygonal object template matching and represent a substantial improvement over the study [1].

The above algorithm can handle the template matching of polygonal objects well. However, it will be more desirable to speed up the algorithm further and to deal with certain limitations of the model. Also, the current approach is only able to deal with limited scale variations. We anticipate that more sophisticated combination of high-level priori knowledge and key low-level image knowledge will help solve the problems. In addition, there are many other optimal techniques which we have not yet explored which offer more principled ways to speed up and improve the matching algorithm.

7. ACKNOWLEDGEMENTS

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