Topological Concepts applied to Digital Image Processing

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ABSTRACT

This article describes an automatic method applicable to the segmentation of mediastinum Computerized Axial Tomography (CAT) images with tumors, by means of Alternating Sequential Filters (ASFs) of Mathematical Morphology, and connected components extraction based on continuous topology concepts. Digital images can be related to topological space structures, and then general topology principles can be straightforwardly implemented. This method allows not only to accurately determine the area and external boundary of the segmented structures but also to obtain their precise location.

Throughout these last years, technological development has significantly improved diagnostic imaging, enabling renal tumor and incidental hepatic tumor detection -usually small in size- in younger people and with an eventually lower malignant potential. This has led to a remarkable advance in interventionist techniques such as cryosurgery and radiofrequency ablation, preventing, in some cases, major surgeries, decreasing morbid-mortality rate, hospital stay and total treatment costs. Notwithstanding this, both cryosurgery and radiofrequency ablation, through extremely low and high temperatures, respectively, kill tumor as well as healthy cells, rendering crucial the identification of tumors with an extraordinary spatial accuracy.

Keywords: Segmentation, Topological Spaces, Connected Components, CAT.

1. INTRODUCTION

Along these last years, medical imaging has played a pivotal role in the early detection, diagnosis and treatment of cancer. Under certain circumstances, early detection through medical imaging allows tumor cure or elimination. The new interventionist techniques such as cryosurgery and radiofrequency ablation, in some cases, prevent major surgeries. However, both techniques, through extremely low and high temperatures, respectively, kill tumor as well as healthy cells, rendering crucial the identification of tumors with an extraordinary spatial accuracy.

In CAT imaging, the different tissues offer no uniformity and contain intensity variations. Conventional image segmentation techniques rely on pixels properties treated on an individual basis; and, therefore, fail to segment [1] [2] [5].

In the first stage, opening and closing Alternating Sequential Filters (ASFs) are applied based on Mathematical Morphology Reconstruction [8]. The advantage of this kind of sequential morphologic filters is that they filter the desired objects leaving the original shape unaltered. This results in less image distortion [12] [13] [15]. ASFs by Reconstruction consists in openings and closings iterations by Reconstruction with structuring elements of growing size, being necessary to use a second structuring element in the Reconstruction operation. In this way, the regions of the image describing meaningful details remain connected[9].

In the second stage, an algorithm is applied along the lines of the connected components criterion on the basis of continuous topology. Once the connected components are obtained, the boundary and interior of each of them is calculated, so yielding the desired image segmentation.

This work is organized as follows: next Section covers the theoretical principles applied; Section 3 deals with the methods suggested; in Section 4, the results yielded by the different images employed are presented; and finally Section 5 puts forward the conclusions drawn by this paper.

2. BASIC NOTIONS OF TOPOLOGICAL SPACES

This section presents some general definitions and general topological results, which will be used along the development of this article.

2.1- Topological Spaces

In a set $X \neq \varnothing$, a topology (or topological structure) consists in a collection $\Im$ of subsets of X, which meet the following conditions:

a) $\varnothing, X \in \Im$.

b) The finite intersection of $\Im$ members is a member of $\Im$.

c) The union (finite or otherwise) of $\Im$ members is a member of $\Im$.

Set X is called $\Im$ topology space; and $\Im$ is a topology for X [11]. The pair $(X, \Im)$ is a topological space. Topology members are called $\Im$-open or open with regard to topology $\Im$. A set F is $\Im$-closed or closed, if its relative complement is open, i.e., if $(X - F) \in \Im$ (when there is no room for mistake, we will refer to topology open and closed sets).

Among the topologies which can be regarded in a set X, there are two representing opposed ends. One is the discrete topology $\Im = P(X)$, in which all parts of X are regarded as open sets, and the other is the indiscrete or chaotic topology, $\Im' = \{\varnothing, X\}$, in which $\varnothing$ and X are the only open sets. Indeed, $\Im$ and $\Im'$ represent, respectively, the major and minor topologies that can be defined in a set.

A set U of a topological space $(X, \Im)$ is a neighborhood ($\Im$-neighborhood) of a point x if and only if U contains an open set to which x belongs. $U(x)$ shall denote the neighborhood system of point x.

2.1.1- Topology based on neighborhood systems

Let $(X, \Im)$ be a topological space, for each $x \in X$, let $U(x)$ be the neighborhood system of point x. Then:

i) If $U \in U(x)$ then $x \in U$

ii) If U and V are members of $U(x)$, then $U \cap V \in U(x)$.

iii) If $U \in U(x)$ and $U \subset V$, then $V \in U(x)$.

iv) If $U \in U(x)$, then there is a neighborhood $V \in V(x)$, such that $V \subset U$ and $V \in U(y)$ for each y of V (i.e., V is neighborhood of each of its points).

If $\aleph$ is a function assigning a non-empty collection $U(x)$ to each $x \in X$, of parts of X satisfying i), ii), iii), then the collection $\Im$ of those sets U such that $U \in U(x)$ for each x of U is a topology of X. If iv) is also met, then $U(x)$ is precisely the neighborhood system of x with respect to topology $\Im$ [3].

2.1.2- Topology based on distance function

There are many topological spaces in which topology is deducted from a distance notion. Distance, defined in a set X, is a function $d : X \times X \to \mathbb{R}$ such that for all $x, y, z \in X$, the following is verified:

a) $d(x, y) \geq 0$

b) $d(x, y) = 0 \;\; sii \;\; x = y$ (separation)

c) $d(x, y) = d(y, x)$ (symmetry)

d) $d(x, y) \leq d(x, z) + d(z, y)$ (triangle inequality)

A metrical space is a pair (X,d), such that d is a metrics for X [7].

Let's see now how, from a given distance, topology can be generated. If a distance d is defined, it is possible to define the following: $B(x,r) = \{y{\in}X; d(x,y){<}r\}$ open ball of center x and radius r. By means of these, opens set can be generated: a set $A \subset X$ is open, if and only if, for every $x \in A$ there is an open ball of center x and radius r included in A. Then the collection made up of all open sets constitutes a topology termed *topology associated to distance d* [7]. Given the fact that the infinite collection of open sets generated is different for each distance, consideration should be given to the likelihood of associating a different distance topology to each of them.

If $(X, \Im)$ constitutes a topological space, and Y is part of X, it is possible to build a topology $\Im_Y$ of Y called relative topology or relativization of $\Im$ to Y. Relative topology $\Im_Y$ is defined as the collection of all member intersections of $\Im$ with Y. Every member U of relative topology $\Im_Y$ is regarded as open in Y. The topological space $(Y, \Im_Y)$ is called subspace of $(X, \Im)$ space.

A partition of a set X is defined as its own division in classes $X_1, X_2, X_3, \ldots, X_n$, such that if $x \in X$, x belongs to one and only one of these classes. Formally, a partition of X is defined as a function $p : X \to \Im$, such that $\forall x \in X$, $x \in p(x)$ and $\forall x, y \in X \;\; p(x) = p(y)$ o $p(x) \cap p(y) = \varnothing$ being $p(x)$ the partition of X containing x (See figure 1).

A set $S \subset X$ is said to be connected if it cannot be written as the union of two disjoint non-empty open sets. Another possible way of expressing set connection is by stating that it is possible to go from one set point to another one by means of a continuous movement, not leaving the set. This leads to the definition of a path-connected space, or simply, connected space. A path in a topological space X is a continuous function $f : [0,1] \to X$. Points $a = f(0)$ and $b = f(1)$ are the path ends f. In this case, the path is said to link point a with point b. When $a = b$, f is said to be a closed path. Given the paths $f, g : [0,1] \to X$ with $f(1) = g(0)$, path $f \vee g : [0,1] \to X$ can be defined as resulting from following path f first, and afterwards path g (See figure 2).

Formally, a $S \subset X$ set is regarded as path-connected when any two points of S can be linked by path f in S.

A topological space X is regarded locally path-connected when for every $x \in X$ and every neighborhood U of x there exists a path-connected neighborhood V such that $x \in V \subset U$.

Interior, set boundary, and isolated point are basic concepts in a topological space. A point $x \in D \subset X$ of a topological space is an interior point of D if and only if D is a neighborhood of x, and the set of all interior points of D is the interior of D, denoted by: $\text{int}(D)$. The interior of a set D can also be defined as the union of all open sets contained in D, i.e., *int(D) i*s the major open set in D.

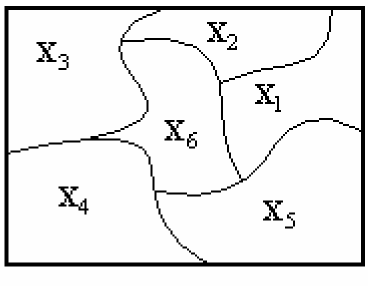

Fig. 1 Example of partition of a set *X*.

Set D closure is defined as the minor closed set $C \subset X$, such that $D \subset C$.

The boundary of set D is set:

$$\partial(D) = closure(D) - \mathrm{int}(D)$$

A point x of $D \subset X$ is isolated if there is a neighborhood $U \in U(x)$, such that $U \setminus \{x\} \cap D = \varnothing$.

Given an arbitrary point $x \in D \subset X$, there are three possibilities mutually excluding: there can either be a neighborhood $U \in U(x)$ contained in D, (i.e., $x \in \mathrm{int}(D)$), a neighborhood $U \in U(x)$ contained in $(X - D)$, (i.e., $x \in \mathrm{int}(X - D)$) or every x neighborhood contains points of D and of $(X - D)$, (i.e., $x \in \partial(D)$). Then, every D set determines space decomposition in three disjoints subsets two to two [14]:

$$X = closure(D) \cup \mathrm{int}(D) \cup \mathrm{int}(X - D)$$

2.2.2- Topology in a Discrete Space $\mathbb{Z}^2$

A digital image is a function $f : \mathbb{Z} \times \mathbb{Z} \to [0, \dots N-1]$ in which $N-1$ is a positive whole belonging to the natural interval [1,256]. Considering $\mathbb{Z} \times \mathbb{Z}$ with topologies defined by metrics d' and d'' restricted to $\mathbb{Z} \times \mathbb{Z}$ defined by:

$$d'(x, y) = \sum_{i=1}^{2} |x_i - y_i|$$

$$d''(x, y) = \max_{1 \le i \le 2} |x_i - y_i|$$

the following neighborhoods of $x \in \mathbb{Z} \times \mathbb{Z}$ can be defined, respectively (See figure 3):

$$U_4(x) = \{ y \in \mathbb{Z}^2 / |y_1 - x_1| + |y_2 - x_2| \le 1 \} = \\ = \{ y \in \mathbb{Z}^2 / d'(x, y) \le 1 \}$$

$$U_8(x) = \{ y \in \mathbb{Z}^2 / \max[|y_1 - x_1|, |y_2 - x_2|] \le 1 \} = \\ = \{ y \in \mathbb{Z}^2 / d''(x, y) \le 1 \}$$

Generally, neighborhoods characterization is geometrically understood as an "approximation".

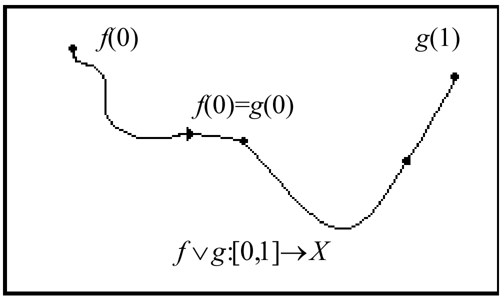

Fig. 2 Example of path-connected spaces.

However, when images in levels of gray are employed, neighborhoods depend on them. Neighborhoods can be defined by means of intensity, as follows:

$$f : G \subset \mathbb{Z}2 \to [0, \dots N-1]$$

$$U_f(v) = f^{-1}\left(f(v) - c, f(v) + c\right) \cap U(v), \quad c \in [0, \dots N-1]$$

In a discrete plane, a path is made up of a sequence of points $a = x_0, \dots, x_k = b$ in which x_i lies adjacent to x_{i+1} for $i = 1, \dots, k$ (a y point is adjacent to x, if there is a U_x neighborhood, such that $y \in U_x$).

Let $X \subset \mathbb{Z} \times \mathbb{Z}$ be an image, two points x and y are said to be connected in X if there exists a path included in X, joining x with y.

If a topological space X is not connected, we can wonder how many "parts" X does have. Space parts are its connected components.

Let X be a topological space and x a point of X, a connected component of $x \in X$ is the union C_x of all connected subsets of X containing an x.

There is at least a connected subset of X with an x: $\{x\}$. Then C_x is not empty. C_x is the major connected subset of X containing an x.

2.2.3. Grouping Criterion

Let X be an image in levels of gray and $\Im$ a topology associated to X; and let $S \subset \Im$ be defined as $\varphi : S \times X \to \mathbb{R}$, such that:

$$\varphi(A, x) = d(\gamma(A), x) \qquad (1)$$

where $\gamma(A)$ is a characterization of A, and d a metrics.

Then φ is said to be a function yielding the set similarity relationship $A \in S$ with $x \in X$, after considering $\gamma(A)$. This function depends on the application under consideration.

Given a fixed ε and δ and $S \subset \Im$. Let X be an image in levels of gray and $A \in S$, an element $x \in X$ is said to belong to A if the following is met:

$$B(x, \varepsilon) \cap A \setminus \{x\} \ne \varnothing \quad y \quad \varphi(A \setminus \{x\}, x) < \delta$$

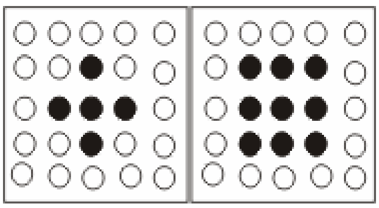

Fig. 3 Different neighborhoods of a point x depending on the metrics employed.

i.e., S is a covering of X. To put it in other words, for each $x \in X, \exists A \in S$ such that $x \in A$.

Topologically, this definition determines that an element in the image will belong to an element $A \in S$ if there is a ball with a center in x and a given radius, such that interception occurs; and that said element is related to it.

3. MATERIALS AND METHODS

In the first stage, reconstruction opening and closing Alternating Sequential Filters (ASFs) were applied by reconstruction. AFSs consist in the iteration of morphologic closing and opening operations by reconstruction, with structuring elements of growing size, being necessary to use a second structuring element in the Reconstruction operation. The advantage of this new type of filters is that they filter the desired objects leaving the original shape unaltered, thus resulting in less image distortion. In this way, the regions of the image describing meaningful details remain connected.

In the second stage, the proposed algorithm employs the grouping criterion defined in the prior section to obtain the desired image segmentation. The algorithm starts by generating a set with an image point (the first pixel found at the beginning of the image, or that pixel verifying a certain condition) assuming certain order or direction. Based on this, it creates the image connected components according to the grouping criterion above defined in equation 1 for an specific function φ defined in this article.

Algorithm to extract the image connected components:

To begin with, function φ should be defined as it determines the grouping criterion, i.e., distance d and $\gamma(A)$. Let d_G be the geodesic distance [4] [6] [12] and $\gamma(A) = \{a_C \in A: a_C \text{ is center of } A\}$. Given a fixed ε, a first set $A_n = \{x\}$ is created, such that $x \in X$, with $n=1$.

Step 1: If $\exists y \in X$ such that $y \notin A_i \; \forall i = 1 \ldots n$ g, go to steps 2 and 3, if there is no $y \in X$ go to step 4.

Step 2: For $i = 1 \ldots n$ if $B(y, \varepsilon) \cap A_i \neq \varnothing$ and $\varphi(A_i, y) < \infty$, in $I = I \cup \{i\}$ (being I a set of indexes).

Step 3: If $I \neq \varnothing$ then $A_i = A_i \cup \{y\} \forall i \in I$; otherwise $n = n + 1$; $A_n = A_n \cup \{y\}$

Step 4: Once $S = \bigcup_{i \in I} A_i$ covering is obtained, the connected components C_j are generated.

Step 5: Once the resulting connected components C_j are obtained, the boundary and interior of each of them is obtained.

Step 6: Calculation of the mass center of the connected components j-esima. (whose localization information is of interest when it comes to practical ends).

Step 7: Visualization.

The algorithms were programmed according to Matlab® 5.3. Standard functions of this language and a specific library called SDC Morphology Toolbox (SDC, 2001) were employed.

4. RESULTS AND DISCUSSION

Figure 3 lists the results obtained with the proposed method in a CAT image of the mediastinum, in which a liver tumor can be observed. First the original image is presented, then image b) after applying the Alternating Sequential Filters with structuring elements of growing size. This allows to homogenize the areas to be detected without subjecting them to deformations. Image c) depicts different connected components marked in different colors. Finally, Figure 3 d) shows the correct tumor segmentation. The exact boundaries of the segmented tumor as well as its localization can be calculated in relation to it.

Figure 4 depicts several CAT mediastinum images and their corresponding segmented images with the proposed method. Original images can be observed in the first column; while the second column shows the images resulting from the segmentation of the different tumors (lung, kidney and liver, respectively)

The third column depicts the contour extraction of said tumors. As seen in the figure, the proposed segmentation is genuinely satisfactory in all cases, having been tested in a group of more than 50 testing images.

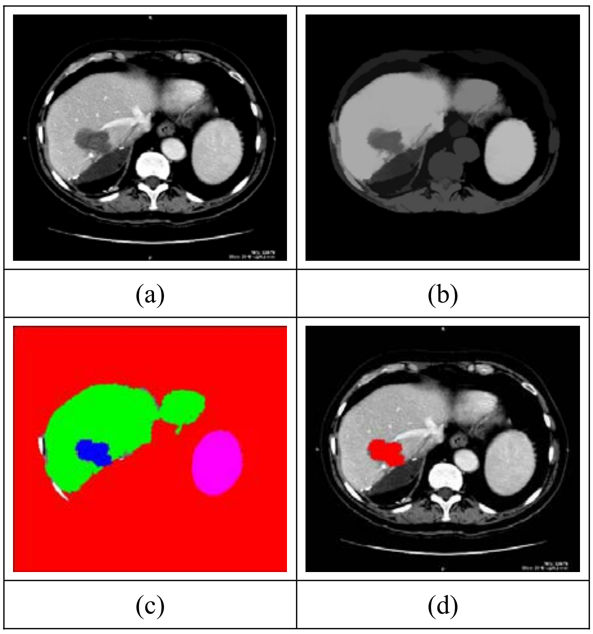

Fig. 3. CAT mediastinum image. a) Original image b) Image obtained after applying sequential filters, c) Image with different connected components; and d) Image resulting from liver tumor segmentation.

5. CONCLUSIONS

This article describes an automatic method applicable to the segmentation of mediastinum Computerized Axial Tomography (CAT) images with tumors. It has proved to be accurate and efficient, thus enabling final users to know the tumor area and localization with remarkable spatial accuracy. This segmentation method turns out to be optimum in tests made prior to cryosurgery or radio frequency ablation.

The potential of this method lies in the possibility of employing different distances and characterizations of mounting interest in the segmentation of other types of images. For instance when φ is defined by the Euclidean distance and $\gamma(A)$ by a level of gray, the grouping algorithm is that known as image *labeling*.

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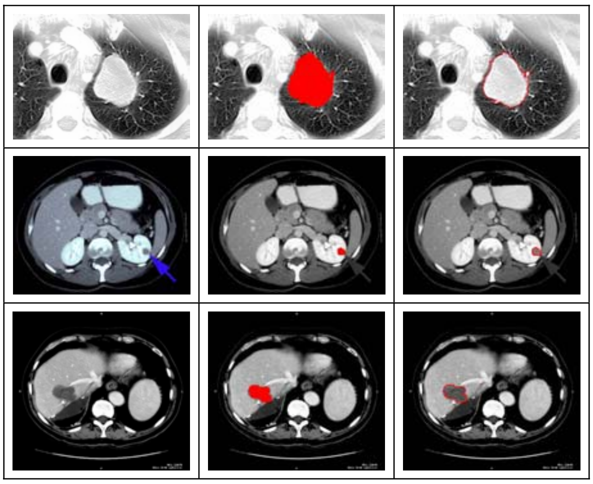

Fig. 4 Different CAT images. The first column shows original images. The second introduces the images resulting from the segmentation of different tumors (lung, kidney and liver, respectively). Lastly, in the third column, the perimeter extraction of the tumors previously segmented can be observed.

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