Segmentation of Medical Images using Fuzzy Mathematical Morphology

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ABSTRACT

Currently, Mathematical Morphology (MM) has become a powerful tool in Digital Image Processing (DIP). It allows processing images to enhance fuzzy areas, segment objects, detect edges and analyze structures.

The techniques developed for binary images are a major step forward in the application of this theory to gray level images. One of these techniques is based on fuzzy logic and on the theory of fuzzy sets.

Fuzzy sets have proved to be strongly advantageous when representing inaccuracies, not only regarding the spatial localization of objects in an image but also the membership of a certain pixel to a given class. Such inaccuracies are inherent to real images either because of the presence of indefinite limits between the structures or objects to be segmented within the image due to noisy acquisitions or directly because they are inherent to the image formation methods.

Our approach is to show how the fuzzy sets specifically utilized in MM have turned into a functional tool in DIP.

Keywords: Fuzzy Logic. Mathematical morphology. Segmentation.

1. INTRODUCTION

Mathematical Morphology, created to characterize the physical and structural properties of diverse materials, relies on geometric, algebraic and topologic concepts as well as on the theory of sets. The main idea lying behind MM is assessing images geometric structures by overlapping small patterns, called structuring elements, in different parts of the same, [1]-[3].

The basic operations of MM are erosion and dilation. The remaining morphological operators are defined based on the combination of these two.

A primary difference between Traditional Mathematical Morphology (TMM) and Fuzzy Mathematical Morphology (FMM) is the structuring element. While TMM considers it as an image, FMM does so as a fuzzy set when evaluating morphological operators.

This paper introduces an automatic method for blood vessels detection in angiographic images based on the different FMM operators. The detection of such structures is crucial for the diagnosis of a vast number of illnesses in which thickenings and blockages, among other pathologies, are analyzed. However, being blurred in nature, with little contrast or immerse in noise, most standard techniques of Digital Image Processing, like Top-Hat (see seccion 2-3), do not yield optimum results in these images.

A theoretical description of the main concepts is herein presented, and the results are illustrated in low-contrast angiographic images, in which the correct segmentation obtained can be appreciated.

2. METHODS

2.1- Boolean Logic and Fuzzy Logic

As a first step to approaching this topic, the main concepts related to the logic of predicates should be introduced. According to the Boolean logic, a predicate is a function p defined in a set X that gets its values from set $\{0,1\}$ [4].

Given two predicates p and q, respectively, the basic operations used in this theory are as follows:

Conjunction: $p \wedge q$

Disjunction: $p \vee q$

Negation: $\sim p$

Their definition results from the truth tables which determine their truth values on the basis of p and q corresponding values. (See Table 1).

Table 1

Truth Tables

Conjunction		
p	***q***	$p \wedge q$
0	0	0
0	1	0
1	0	0
1	1	1

Disjunction		
p	***q***	$p \vee q$
0	0	0
0	1	1
1	0	1
1	1	1

Negation	
p	$\sim p$
0	1
1	0

Table 2

Truth Table

Implication				
p	q	$\sim p$	$p \to q$	$\sim p \vee q$
0	0	1	1	1
0	1	1	1	1
1	0	0	0	0
1	1	0	1	1

Table 3
Categories

Category	Truth Value
False	0
Almost False	0.1
Quite False	0.2
Somewhat False	0.3
More False than True	0.4
As True as False	0.5
More True than False	0.6
Somewhat True	0.7
Quite True	0.8
Almost True	0.9
True	1

The implication $p \to q$ is obtained from disjunction and negation combination, where: $p \to q = \sim p \vee q$. (See Table 2).
On given occasions, it is necessary to introduce a new scale of values in which values are assigned to the predicates depending on their truthfulness, i.e., a predicate p is a function $p:[0,1]\times[0,1]\to[0,1]$.
Therefore, the previous operations can be extended as follows:

Conjunction: $u\,(p \wedge q) = \min\,(u\,(p), u\,(q))$

Disjunction: $u\,(p \vee q) = \max\,(u\,(p), u\,(q))$

Negation: $u\,(\sim p) = 1 - u\,(p)$

where p and q are predicates and $u(\cdot)$ is their truth value.
Indeed, these operations are an extension of those above. In this case, the categories will take the values listed in Table 3.
We are dealing with FUZZY LOGIC.
In fuzzy logic, the operators Conjunction and Implication extend from the Boolean domain $\{0,1\}\times\{0,1\}$ to the new domain $[0,1]\times[0,1]$.

A function $C:[0,1]\times[0,1]\to[0,1]$ given by $C(s,t) = s \wedge t$ is called a fuzzy conjunction if it increases in both arguments, and satisfies:

$$C(0,0) = C(0,1) = C(1,0) = 0 \quad \text{y} \quad C(1,1) = 1 \qquad (1)$$

A function $I:[0,1]\times[0,1]\to[0,1]$ given by $I(s,t) = s \to t$ is called a fuzzy implication if it decreases in the first argument, increase in the second, and it satisfies the following:

$$I(0,0) = I(0,1) = I(1,1) = 1 \quad \text{y} \quad I(1,0) = 0 \qquad (2)$$

The formulae below are examples of the most used conjunctions and implications [5]:

Godel-Brouwer:

$$C(a,t) = a \wedge t$$
$$I(a,s) = \begin{cases} s, & s < a \\ 1, & s \ge a \end{cases} \qquad (3)$$

Kleene-Dienes:

$$C(a,t) = \begin{cases} 0, & t \le 1-a \\ t, & t > 1-a \end{cases} \qquad (4)$$
$$I(a,s) = (1-a) \vee s$$

By way of example, it is herein demonstrated that the Kleene-Dienes formulae meet the definition of conjunction and implication provided above. First, it is proven that Equation (1) is satisfied:

Let $a=0$ and $t=0$; therefore, $t \le 1-a$. Then: $C(a,t) = C(0,0) = 0$.
Let $a=0$ and $t=1$; therefore, $t \le 1-a$. Then: $C(a,t) = C(0,1) = 0$.
Let $a=1$ and $t=0$; therefore, $t \le 1-a$. Then: $C(a,t) = C(1,0) = 0$.
Let $a=1$ and $t=1$; therefore, $t > 1-a$. Then: $C(a,t) = t = C(1,1) = 1$.

Second, it is shown that $C(\bullet,t)$ increases; and, therefore, $a_1 < a_2$:
Case 1: If $t \le 1-a_1$ and $t \le 1-a_2$ then $C(a_1,t) = 0$ and $C(a_2,t) = 0$; i.e.,: $C(a_1,t) = C(a_2,t)$.
Case 2: If $t \le 1-a_1$ and $t > 1-a_2$ then $C(a_1,t) = 0$ and $C(a_2,t) = t > 0$; i.e.,: $C(a_1,t) < C(a_2,t)$.
Case 3: If $t > 1-a_1$ and $t > 1-a_2$ then $C(a_1,t) = t$ and $C(a_2,t) = t$; i.e.,: $C(a_1,t) = C(a_2,t)$.
Then, $C(a_1,t) \le C(a_2,t)$; and therefore function C increases in the first argument.

Lastly, it is proven that $C(a,\bullet)$ increases; and, therefore, $t_1 < t_2$:
Case 1: If $t_1 \le 1-a$ and $t_2 \le 1-a$ then $C(a,t_1) = 0$ and $C(a,t_2) = 0$; i.e.,: $C(a,t_1) = C(a,t_2)$.
Case 2: If $t_1 \le 1-a$ and $t_2 > 1-a$ then $C(a,t_1) = 0$ and $C(a,t_2) = t_2$; i.e.,: $C(a,t_1) < C(a,t_2)$.
Case 3: If $t_1 > 1-a$ and $t_2 > 1-a$ then $C(a,t_1) = t_1$ and $C(a,t_2) = t_2$; i.e.,: $C(a,t_1) < C(a,t_2)$.
Then, $C(a,t_1) \le C(a,t_2)$; and, therefore, function C increases in the second argument.
As C satisfies Equation (1) and it also increases in both arguments, then it is a conjunction.
Similarly, it is demonstrated that $I(a,s) = (1-a) \vee s$ is an implication.

2.2- Theoretical concepts of Binary Mathematical Morphology

This section defines first the main concepts regarding binary MM, which are later applied to gray level images.

To begin with, binary images should be defined. Binary images are functions f defined from a subset $G \subseteq Z^2$ in the set $\{0,1\}$ [6].

Based on this definition, binary operators are defined as follows [2][7][8]:

The morphological erosion of image f by the structuring element B is defined as:

$$f \,\Theta\, B = \{ y \in f \,/\, B_y \subseteq f \} \quad (5)$$

where $B_y = \{b + y/b \in B\}$ and $+$ represent the vector sum.

The morphological dilation of image f by the structuring element B is defined as:

$$f \oplus B = \{ y \in f \,/\, B_y \cap f \neq \varnothing \} \quad (6)$$

There are different types of structuring elements that can be employed. The most common are shown in Figure 1.

For instance, Figure 2 depicts, in the first place, the original image; then, the image with a cross shape eroded by a structuring element; and lastly, the dilated image using the same structuring element.

On the basis of the combination of these operations, the following operators are defined:

Morphological opening:

$$\gamma(f,B) = (f \,\Theta\, B) \oplus B \quad (7)$$

Morphological closing:

$$\Phi(f,B) = (f \oplus B) \,\Theta\, B \quad (8)$$

Morphological gradient:

$$Gradm(f,B) = (f \oplus B) - (f \,\Theta\, B) \quad (9)$$

As a consequence, binary morphological operators analyze the following propositions:

p: "the structuring element is completely contained in the image"

q: "there is intersection between the structuring element and the image"

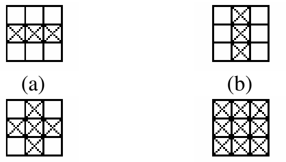
Erosion and dilation are defined in agreement with the truth value of these propositions (0 or 1); therefore, Boolean logic is present in these definitions.

2.3- Theoretical concepts of Mathematical Morphology in levels of gray

Next step is to develop the theory underlying gray level images. A gray level image I is a subset of $\Re^3$ whose graph is given by the set:

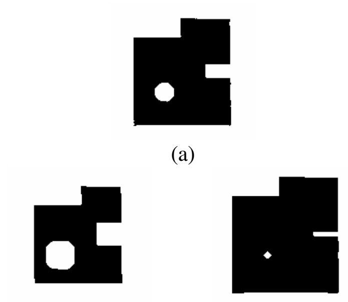
$$G(I) = \{ (x, f(x)) / x \in Z^2 \quad y \quad f(x) \in \{0,1,2,....,255\} \} \quad (10)$$

where $f : G \subset Z^2 \to [N_{mín}, N_{máx}]$ is a function denoting gray intensities in each pixel, being $[N_{mín}, N_{máx}]$ the natural interval in which the minor and major levels of gray are represented by its ends [6].



(a) (b) (c) (d)

Figure 1: (a) Horizontal Line; (b) Vertical Line; (c) Cross; (d) Square.



(a) (b) (c)

Figure 2: (a) Original Image; (b) Eroded Image; (c) Dilated Image.

As in the binary case, erosion and dilation are defined to introduce subsequently the other morphological operators that arise as a combination of these.

Then the morphological erosion of image f is defined by the structuring element B like [3][9]:

$$\varepsilon(f,B)(s,t) = \min\{ f(s+x, t+y) - B(x,y) / (s+x, t+y) \in D_f \,;\, (x,y) \in D_B \} \quad (11)$$

where D_f and D_B are the domains of f and B, respectively.

Morphological dilation is defined in a similar way for image f by the structuring element B as:

$$\delta(f,B)(s,t) = \max\{ f(s-x, t-y) + B(x,y) / (s-x, t-y) \in D_f \,;\, (x,y) \in D_B \} \quad (12)$$

Based on the combination of these operators, opening and closing are defined as:

Opening:

$$\gamma(f,B) = \delta(\varepsilon(f,B),B) \quad (13)$$

Closing:

$$\Phi(f,B) = \varepsilon(\delta(f,B),B) \quad (14)$$

Top-Hat transformation is a TMM technique used to locally extract brilliant or dark objects from a gray level image [10]. Top-Hat by opening aims at extracting brilliant objects and is given by:

$$Top - Hat_{\gamma}(f,B) = \gamma(f,B) - f \quad (15)$$

Top-Hat by closing aims at extracting dark objects, and is defined as:

$$Top - Hat_{\Phi}(f,B) = f - \Phi(f,B) \quad (16)$$

Unlike binary images, when working with gray level images, Boolean logic cannot be applied. As a consequence, it is necessary to consider fuzzy logic.

Every level of gray is associated with a value between zero and one (See Table 4). To this end, a continuous and strictly decreasing function $\vartheta:\{0,1,2,....,255\}\rightarrow[0,1]$, called fuzzy function, is defined, which, when composed with the image function, results in a fuzzification of the image, in other words: $\vartheta\circ f : Z^2 \rightarrow [0,1]$.

In this framework, the operations defined in fuzzy logic can be properly applied.

In view of the fact that traditional morphology relies on the theory of sets, fuzzy morphological operators can be defined by means of fuzzy logic [5].

Let A and B belong to the set of images parts, then:

$$\begin{aligned} A\subseteq B &\Leftrightarrow \forall y\in f,\quad y\in A\Rightarrow y\in B \\ &\Leftrightarrow \forall y\in f,\quad A(y)\Rightarrow B(y) \\ &\Leftrightarrow \forall y\in f,\quad I(A(y),B(y))=1 \end{aligned} \tag{17}$$

where I denotes the binary implication.

The fuzzy erosion of an image f can be defined by a structuring element B, in a point x like:

$$\varepsilon^F(f,B)(x) := \inf_{y\in f}\{I(B(y)),f(y)\} \tag{18}$$

In a similar way, being A and B part of f set of parts, it is known that:

$$\begin{aligned} A\cap B\neq\varnothing &\Leftrightarrow \exists\, y\in f,\quad y\in A \quad \text{y} \quad y\in B \\ &\Leftrightarrow \exists\, y\in f,\quad C(A(y),B(y))=1 \end{aligned} \tag{19}$$

where C is the binary conjunction.

Then, the fuzzy dilation of an image f can be defined by a structuring element B, in a point x like:

$$\delta^F(f,B)(x) := \sup_{y\in f}\{C(B(y)),f(y)\} \tag{20}$$

Following the steps of the morphological theory, fuzzy opening is described as:

$$\gamma^F(f,B)=\delta^F\left(\varepsilon^F(f,B),B\right) \tag{21}$$

and fuzzy closing as:

$$\Phi^F(f,B)=\varepsilon^F\left(\delta^F(f,B),B\right) \tag{22}$$

On the basis of these equations, the fuzzy inner edge is defined as:

$$\partial^F_{Int}f = f-\varepsilon^F(f,B) \tag{23}$$

and the fuzzy outer edge as:

$$\partial^F_{Ext}f = \delta^F(f,B)-f \tag{24}$$

Table 4
Categories

Category	Truth value
Absolutely Black	0
Almost Black	0.1
Quite Black	0.2
Somewhat Black	0.3
More Black than White	0.4
As White as Black	0.5
More White than Black	0.6
Somewhat White	0.7
Quite White	0.8
Almost White	0.9
Absolutely White	1

Fuzzy gradient is described as:

$$Gradm^F(f)=\delta^F(f,B)-\varepsilon^F(f,B) \tag{25}$$

and fuzzy Top-Hat by opening as:

$$Top-Hat^F_\gamma(f)=\gamma^F(f,B)-f \tag{26}$$

Lastly, fuzzy Top-Hat by closing is defined as:

$$Top-Hat^F_\Phi(f)=f-\Phi^F(f,B) \tag{27}$$

2.4- Structuring element

As mentioned in the introduction, the main difference between TMM and FMM is the way in which the structuring element is regarded. TMM considers it with an image; i.e., the structuring element is a $B: Dom\, B\subseteq Z^2\rightarrow\{0,1,2,....,255\}$ function. Conversely, FMM considers it as a fuzzy set in which the rules mentioned in Table 1 are met.

A quick way to obtain fuzzy structuring elements is to compose function B with the fuzzy function ϑ. By so doing, building fuzzy structuring elements becomes an easy task. In this work, such elements are cone shaped and shown in Figure 3 together with other shapes.

2.5- Proposed method

The proposed method follows these steps:
Step 1: Image Fuzzification.
Step 2: Image fuzzy dilation.
Step 3: Calculation of fuzzy Top-Hat transform.
Step 4: Image Defuzzification.
Step 5: Visualization.

Step 1: To be able to operate with fuzzy logic in gray level images, the first step is to "fuzzify" the image. This consists in taking the values of each pixel from the original image to values between 0 and 1. There are several techniques that allow so. The one employed in this work was the sigmoid function:

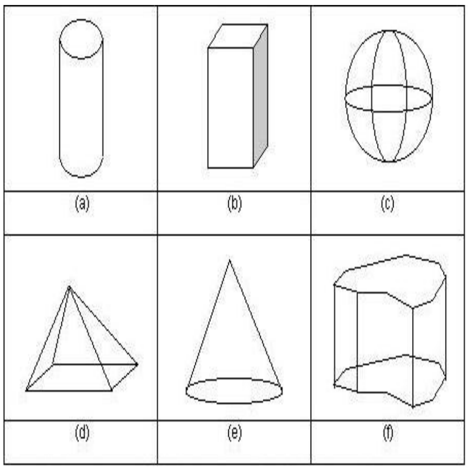


Figure 3: 3-D structuring elements: (a) Cylinder; (b) Parallelepiped; (c) Sphere; (d) Pyramid; (e) Cone; (f) Arbitrary shape.

$$\theta(t) = \frac{1}{2} + \frac{1}{\pi}\arctan(t) \tag{28}$$

Step 2: The image was dilated by means of Kleene-Dienes conjunction. The structuring element used is 5×5 in size and cone shaped. Its values range from 0 to 1, and it is obtained from the following formula:

$$\frac{1}{f(x,y)} \cdot \begin{bmatrix} f(x,y)-2j & f(x,y)-k & f(x,y)-2i & f(x,y)-k & f(x,y)-2j \\ f(x,y)-k & f(x,y)-j & f(x,y)-i & f(x,y)-j & f(x,y)-k \\ f(x,y)-2i & f(x,y)-i & f(x,y) & f(x,y)-i & f(x,y)-2i \\ f(x,y)-k & f(x,y)-j & f(x,y)-i & f(x,y)-j & f(x,y)-k \\ f(x,y)-2j & f(x,y)-k & f(x,y)-2i & f(x,y)-k & f(x,y)-2j \end{bmatrix} \tag{29}$$

If any value is below zero, it is assigned a zero value.

Step 3: The Fuzzy Top-Hat transform was calculated. The objective is to extract the locally brilliant elements from the image. To do so, fuzzy Top-Hat by opening was used.

The purpose is to obtain the graph peak representing the object to be segmented (brilliant object). The strategy is to create a new image with the irrelevant information, that is to say, to eliminate the peak applying an opening to the original image. Then, by subtracting it from the original image, a new one is obtained built from the information of interest. This is graphically shown in Figure 4.

Step 4: To achieve "defuzzification", the inverse function of the sigmoid function was used.

Step 5: Visualization of the segmented structures.

The algorithm was developed in Matlab® 6.5. Standard functions of this language, and a specific library called SDC Morphology Toolbox with functions of Mathematical Morphology [11] were employed.

3. RESULTS AND DISCUSSION

Medical imaging constitutes an ideal field to apply fuzzy morphology techniques. The segmentation of tree-structure images such as retinal angiographic images is not simple if TMM traditional methods are employed (See Figure 5), the reason being that lines are fuzzy and their contours not sharp. The application of the above described algorithm to this type of images enables lines enhancement and optimum results.

As an example of its potential applications, this paper proposes the use of fuzzy morphological operators to segment branching images.

The algorithm was applied to 50 retinal angiographic images. This type of images accounts for a great deal of noise which leads to notorious difficulty when it comes to applying traditional segmentation techniques. When processing these images with the proposed algorithm, the lines of interest were well segmented.

Below, several images to which fuzzy operators were applied using Kleene-Dienes conjunction are shown. The efficiency of the proposed method can be clearly appreciated (See Figures 6-9).

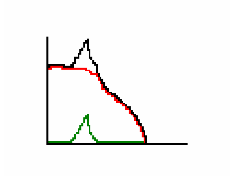
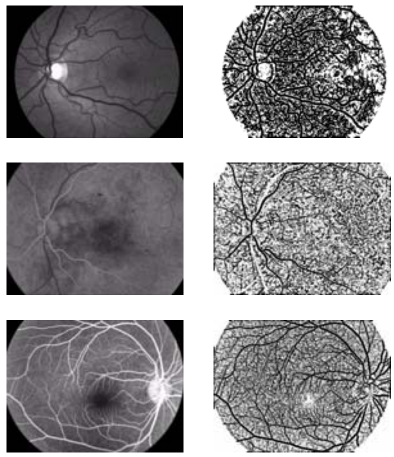
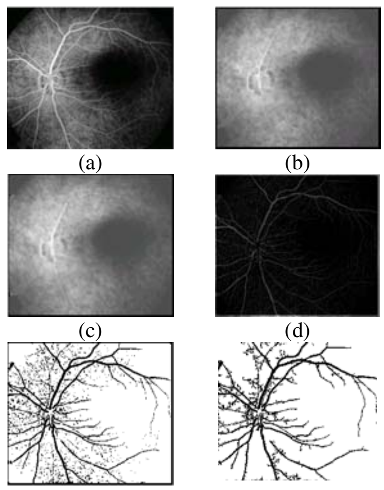


Figure 4: The original image appears in black. The image in red shows the irrelevant information; and the one in green, the image Top-Hat.



(a) (b)

Figure 5: (a) Original Images; (b) Morphological Top-Hat.



(a) (b) (c) (d) (e) (f)

Figure 6: (a) Original Image; (b) Fuzzy Erosion; (c) Fuzzy Dilation; (d) Fuzzy Top-Hat; (e) Binarization; (f) Final Image.

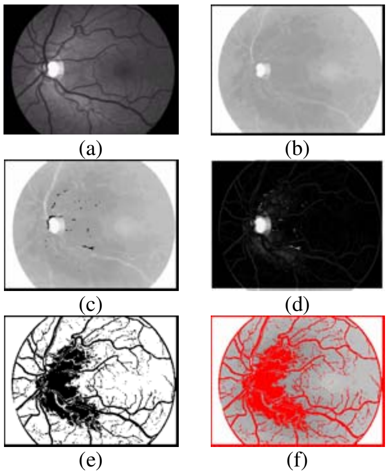


Figure 7: (a) Original Image; (b) Fuzzy Erosion; (c) Fuzzy Dilation; (d) Fuzzy Top-Hat; (e) Final Image; (f) Visualization.

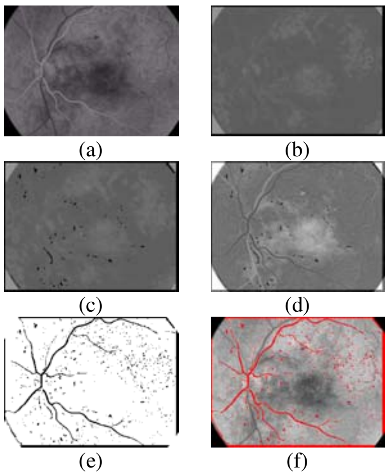


Figure 8: Original Image; (b) Fuzzy Erosion; (c) Fuzzy Dilation; (d) Fuzzy Top-Hat; (e) Final Image; (f) Visualization.

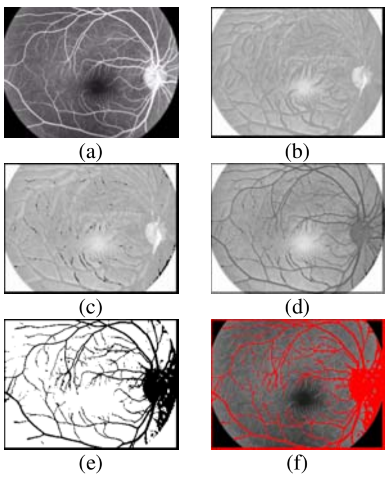


Figure 9: (a) Original Image; (b) Fuzzy Erosion; (c) Fuzzy Dilation; (d) Fuzzy Top-Hat; (e) Final Image; (f) Visualization.

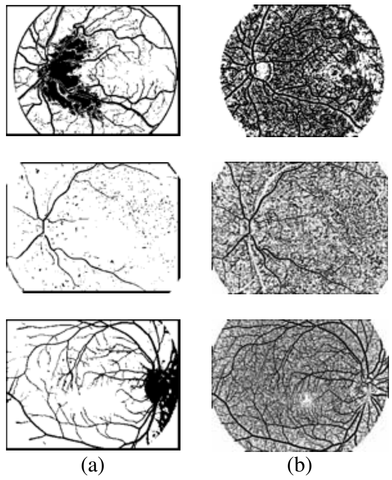


Figure 10: Comparison: (a) Fuzzy Top-Hat; (b) Traditional Top-Hat.

4. CONCLUSIONS

Fuzzy Mathematical Morphology constitutes a valid tool to segment tree-structure images such as angiographic images, in which the edges of the blood vessels are not clearly cut. These operators constitute a considerable contribution to Digital Image Processing.

Even though the mathematical bases for these techniques are complex, their implementation is simple, quick and easier on the user.

This paper presents an algorithm providing a far more efficient way of recognizing ramifications in angiographic retinal images as compared with the segmentation achieved by traditional DIP techniques (See Figure 10).

Lines segmentation in this kind of images is complex due to the fuzzy areas which hinder object extraction. The method herein proposed offers an efficient segmentation applicable to similar images, thereby facilitating professionals´ work by reducing analysis subjectivity.

This method appears to be a robust approach and a novel contribution to medical images quantification.

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