An Adaptive Variable Structure Controller for the Trajectory Tracking of a Nonholonomic Mobile Robot with Uncertainties and Disturbances

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ABSTRACT

In this paper, a trajectory tracking control for a nonholonomic mobile robot subjected to uncertainties and disturbances in the kinematic model is proposed. An adaptive variable structure controller based on the sliding mode theory is used, and applied to compensate these uncertainties and disturbances. To minimize the problems found in practical implementation using classical variable structure controllers, and eliminate the chattering phenomenon as well as compensate disturbances a neural compensator is used, which is nonlinear and continuous, in lieu of the discontinuous portion of the control signals present in classical forms. The proposed neural compensator is designed by a modeling technique of Gaussian radial basis function neural networks and does not require the time-consuming training process. Stability analysis is guaranteed with basis on the Lyapunov method. Simulation results are provided to show the effectiveness of the proposed approach.

Keywords: nonholonomic mobile robot, trajectory tracking, kinematic model, uncertainties and disturbances, adaptive variable structure controller, neural networks, Lyapunov method.

1. INTRODUCTION

The wheeled mobile robot of the type (2,0) is usually studied as a typical example of the nonholonomic system [1]. Many approaches have been proposed to treat the motion control design of this type of mobile robot [2]. From a review of the literature, most of results on the tracking problem of this nonholonomic system are proposed based on the assumption that the parameters of the model were known exactly or by selecting a special control target, i.e. the linear and angular velocities of the mobile robot, to avoid this problem. However, considering practical applications of this nonholonomic system, the difficulty in modeling practical systems exactly, and the unavoidable disturbances in control, effective tracking control design of uncertain nonholonomic systems needs be studied. Thus, this paper describes the design of a kinematic controller for this mobile robot, which is based on the sliding mode theory, considering the presence of uncertainties and disturbances in the kinematic model.

Variable structure control design (VSC) utilizes a high speed switching control law to drive the nonlinear predefined states trajectories onto a specified surface (called the sliding or switching surface), to attain the conventional goals of control such as stabilization and tracking.

Due to robustness properties against uncertainties, modeling imprecision and disturbances, the VSC has become very popular and used in many application areas [3-5]. However, this control scheme has important drawbacks that limit its practical applicability, such as high frequency switching (chattering) and large authority control, which deteriorate the system performance [6]. The first drawback mentioned is due to control actions that are discontinuous on the sliding surfaces, which causes the high frequency switching in a boundary of the sliding surfaces. This high frequency switching might excite unmodeled dynamics and impose undue wear on the actuators, so that the control law would not be considered acceptable. The second drawback mentioned, is based on the requirement of a priori knowledge of the boundary of uncertainty in compensators. If boundary is unknown, a large value has to be applied to the gain of discontinuous part of control signal and this large control gain may intensify the high frequency switching on the sliding surfaces.

Researches have been developed using softcomputing methodologies, such as artificial neural networks, in order to improve the performance and alleviate the problem found in practical implementation of VSC's as mentioned in [7].

In this paper, the radial basis function neural networks (RBFNNs) are applied to compensate the disturbances, since the structure of an RFBNN is simpler than a multi-layer perceptron (MLP), the learning rate of a RBFNN is generally faster than a MLP, and a RBFNN is easily mathematically tractable [8].

Unlike other works that consider the kinematics of mobile robots without uncertainties and/or disturbances, and using the sliding mode theory applied to mobile robots [9]-[15], the contributions of this paper are:

- An adaptive variable structure controller (AVSC) in Cartesian coordinates to estimate the uncertainties and compensate disturbances in the kinematic model, based on the sliding mode theory;

- A neural compensator (NC) used to replace of the discontinuous portion of the classical VSC avoiding the chattering as well as suppressing the disturbances without the knowledge of its limits;
- The implementation of the NC is based on the partitioning of the RBFNNs into several smaller subnets in order to obtain a more efficient computation;
- The estimated parameters as well as the weights of the hidden layer of RBFNNs are updated in an online manner to ensure the stability of the overall system without having any prior knowledge of the uncertainties and disturbances in the kinematic model;
- The stability analysis of the mobile robot control system, the adaptation and learning algorithms are proved using the Lyapunov theory.

2. PROBLEM FORMULATION

2.1. Kinematics of a Nonholonomic Mobile Robot

A typical example of a nonholonomic mobile robot is shown in Fig. 1.

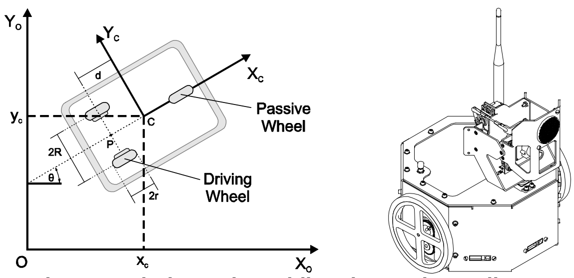

Fig. 1. Nonholonomic mobile robot and coordinate systems

The mobile robot has two driving wheels mounted on the same axis and a free front wheel. The two driving wheels are independently driven by two actuators to achieve the motion and orientation. The position of the mobile robot in the Cartesian inertial frame $\{X_o, O, Y_o\}$ can be described by a vector $\overrightarrow{OC}$, and the orientation θ between the mobile robot base frame $\{X_c, C, Y_c\}$ and the Cartesian inertial frame, where C is the center of mass coordinates (guidance point), with P, d, r, and $2R$ being the intersection of the axis of symmetry with the driven wheel axis, the distance from the point C to the point P, the radius of the wheels, and the distance between the driven wheels, respectively.

The posture vector $q \in \Re^3$ of the mobile robot is described by three generalized coordinates as:

$$q = [x_c \quad y_c \quad \theta]^T , \tag{1}$$

where x_c and y_c are the coordinates of C.

Under the condition of pure rolling and non-slipping and considering $d = 0$, the kinematic model of the mobile robot can be expressed as:

$$\dot{q} = S(q) v(t) , \tag{2}$$

with:

$$S(q) = \begin{bmatrix} \cos(\theta) & 0 \\ \sin(\theta) & 0 \\ 0 & 1 \end{bmatrix}, \tag{3}$$

and $v(t) = [v_l \quad \omega_a]^T$ representing the linear and angular velocities of the mobile robot, respectively. However such kinematic model, Eq. (3), does not take into account the measurement noise, modeling uncertainties and disturbances. As there are input disturbances in v_l and ω_a, a more realistic kinematic model of the mobile robot can addressed by:

$$\dot{q} = S(q)\big(v(t) + d_v(t)\big) , \tag{4}$$

where $d_v(t) = \begin{bmatrix} \delta_{v_l} & \delta_{\omega_a} \end{bmatrix}^T$ represents the disturbances in $v(t)$ only, which are assumed to be upper bounded by:

$$\left|\delta_{v_l}\right| < \varepsilon_{v_l} , \qquad \left|\delta_{\omega_a}\right| < \varepsilon_{\omega_a} , \tag{5}$$

with ε_{v_l} and ε_{ω_a} being positive bounded constants.

Another form of representing of the kinematic model, Eq. (4), in the Cartesian coordinates system is selecting the angular velocity of the wheel as the kinematic control target. So, it is possible to describe the linear and angular velocities ($v(t)$) of the mobile robot in function of the angular velocity ($\varphi(t) = [\varphi_r \quad \varphi_l]^T$) of the wheels through the following relationship:

$$\underbrace{\begin{bmatrix} v_l + \delta_{v_l} \\ \omega_a + \delta_{\omega_a} \end{bmatrix}}_{v(t)+d_v(t)} = \begin{bmatrix} \frac{r}{2} & \frac{r}{2} \\ \frac{r}{2R} & -\frac{r}{2R} \end{bmatrix} \underbrace{\begin{bmatrix} \varphi_r + \delta_{\varphi_r} \\ \varphi_l + \delta_{\varphi_l} \end{bmatrix}}_{\varphi(t)+d_\varphi(t)} , \tag{6}$$

and conversely:

$$\begin{bmatrix} \varphi_r + \delta_{\varphi_r} \\ \varphi_l + \delta_{\varphi_l} \end{bmatrix} = \begin{bmatrix} \frac{1}{r} & \frac{R}{r} \\ \frac{1}{r} & -\frac{R}{r} \end{bmatrix} \begin{bmatrix} v_l + \delta_{v_l} \\ \omega_a + \delta_{\omega_a} \end{bmatrix}, \tag{7}$$

with $d_\varphi(t) = \begin{bmatrix} \delta_{\varphi_r} & \delta_{\varphi_l} \end{bmatrix}^T$ represents the disturbances in $\varphi(t)$ only, which are assumed to be upper bounded by:

$$\left|\delta_{\varphi_r}\right| < \varepsilon_{\varphi_r} , \qquad \left|\delta_{\varphi_l}\right| < \varepsilon_{\varphi_l} , \tag{8}$$

where ε_{φ_r} and ε_{φ_l} are positive bounded constants.

Replacing Eq. (6) in Eq. (4) and multiplying by Eq. (3), results in the following kinematic model $S_d(q)$ in Cartesian coordinates system:

$$\dot{q} = S_d(q)\big(\varphi(t) + d_\varphi(t)\big) , \tag{9}$$

$$S_d(q) = \begin{bmatrix} \frac{r}{2}\cos(\theta) & \frac{r}{2}\cos(\theta) \\ \frac{r}{2}\sin(\theta) & \frac{r}{2}\sin(\theta) \\ \frac{r}{2R} & -\frac{r}{2R} \end{bmatrix}. \tag{10}$$

2.2. Error Dynamics with Disturbances

In order to formulate the trajectory tracking control problem, a reference trajectory is generated by the following reference kinematic model:

$$\dot{q}_r = S(q_r) v_r ,$$

$$\dot{x}_r = v_{l_r}\cos(\theta_r), \quad \dot{y}_r = v_{l_r}\sin(\theta_r), \quad \dot{\theta}_r = \omega_{a_r}, \quad (11)$$

where $q_r = \begin{bmatrix} x_r & y_r & \theta_r \end{bmatrix}^T \in \Re^3$ denotes the reference posture of the mobile robot, the structure of $S(q_r)$ is similarly defined as in Eq. (3), and $v_r = \begin{bmatrix} v_{l_r} & \omega_{a_r} \end{bmatrix}^T$ denotes the reference linear and angular velocities of the mobile robot, respectively. With regard to Eq. (11), it is assumed that the signal $v_r(t)$ is chosen to produce the desired motion and that $v_r(t)$, $\dot{v}_r(t)$, $q_r(t)$, and $\dot{q}_r(t)$ are bounded for all time.

The trajectory tracking control problem of a mobile robot is solved designing a control input $v(t) = \begin{bmatrix} v_l & \omega_a \end{bmatrix}^T$ such that the system, Eq. (4), follows the reference, Eq. (11), despite of disturbances. In fact, the aim is to converge the tracking errors ($e_x = x_r - x_c$, $e_y = y_r - y_c$, $e_\theta = \theta_r - \theta$) to zero, respecting the following constraints:

$$|v_l| \le v_{l_{\max}}, \qquad |\omega_a| \le \omega_{a_{\max}}, \quad (12)$$

where $v_{l_{\max}}$ and $\omega_{a_{\max}}$ are positive bounded constants.

Converting the tracking errors in the inertial frame to the mobile robot frame, the posture error equation of the mobile robot can be denoted as:

$$\tilde{z} = \begin{bmatrix} \tilde{x} \\ \tilde{y} \\ \tilde{\theta} \end{bmatrix} = \begin{bmatrix} \cos(\theta) & \sin(\theta) & 0 \\ -\sin(\theta) & \cos(\theta) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} e_x \\ e_y \\ e_\theta \end{bmatrix}. \quad (13)$$

The error dynamics can be obtained from the time derivative of Eq. (13) as:

$$\dot{\tilde{z}} = \begin{bmatrix} \dot{\tilde{x}} \\ \dot{\tilde{y}} \\ \dot{\tilde{\theta}} \end{bmatrix} = \begin{bmatrix} v_{l_r}\cos(\tilde{\theta}) \\ v_{l_r}\sin(\tilde{\theta}) \\ \omega_{a_r} \end{bmatrix} + \begin{bmatrix} -1 & \tilde{y} \\ 0 & -\tilde{x} \\ 0 & -1 \end{bmatrix} \begin{bmatrix} v_l + \delta_{v_l} \\ \omega_a + \delta_{\omega_a} \end{bmatrix}. \quad (14)$$

Now, considering the wheel angular velocity ($\varphi(t) = \begin{bmatrix} \varphi_r & \varphi_l \end{bmatrix}^T$) as the control input, and using Eq. (6), the error dynamics, Eq. (14), can also be expressed by:

$$\dot{\tilde{z}} = \begin{bmatrix} v_{l_r}\cos(\tilde{\theta}) \\ v_{l_r}\sin(\tilde{\theta}) \\ \omega_{a_r} \end{bmatrix} + \begin{bmatrix} -\frac{r}{2} + \frac{r}{2R}\tilde{y} & -\frac{r}{2} - \frac{r}{2R}\tilde{y} \\ -\frac{r}{2R}\tilde{x} & \frac{r}{2R}\tilde{x} \\ -\frac{r}{2R} & \frac{r}{2R} \end{bmatrix} \begin{bmatrix} \varphi_r + \delta_{\varphi_r} \\ \varphi_l + \delta_{\varphi_l} \end{bmatrix}, \quad (15)$$

respecting the following constraints:

$$|\varphi_r| \le \varphi_{r_{\max}}, \qquad |\varphi_l| \le \varphi_{l_{\max}}, \quad (16)$$

where $\varphi_{r_{\max}}$ and $\varphi_{l_{\max}}$ are positive bounded constants.

2.3. Error Dynamics with Parametric Uncertainties and Disturbances

From kinematics, Eq. (9) and Eq. (10), if the parameters, r and R, are unknown, the kinematic control targets, angular velocity of the wheels, can not be obtained from the selected velocity input because of the relationship, Eq. (7), between φ_r, φ_l and v_l, ω_a. But it is possible to use the estimates of these parameters in Eq. (7) and design adaptation laws for an adaptive controller to estimate these parameters. Assume:

$$\alpha_1 = \frac{1}{r}, \qquad \alpha_2 = \frac{R}{r}. \quad (17)$$

Then, Eq. (7) can be rewritten as the following:

$$\begin{bmatrix} \varphi_r + \delta_{\varphi_r} \\ \varphi_l + \delta_{\varphi_l} \end{bmatrix} = \begin{bmatrix} \alpha_1 - \tilde{\alpha}_1 & \alpha_2 - \tilde{\alpha}_2 \\ \alpha_1 - \tilde{\alpha}_1 & -(\alpha_2 - \tilde{\alpha}_2) \end{bmatrix} \begin{bmatrix} v_l + \delta_{v_l} \\ \omega_a + \delta_{\omega_a} \end{bmatrix}, \quad (18)$$

where $\hat{\alpha}_1$ and $\hat{\alpha}_2$ are the estimates of α_1 and α_2; $\tilde{\alpha}_1 = \alpha_1 - \hat{\alpha}_1$ and $\tilde{\alpha}_2 = \alpha_2 - \hat{\alpha}_2$ are the parameter errors.

Replacing Eq. (17), and Eq. (18) in Eq. (15), the error dynamics becomes:

$$\dot{\tilde{z}} = \begin{bmatrix} v_{l_r}\cos(\tilde{\theta}) \\ v_{l_r}\sin(\tilde{\theta}) \\ \omega_{a_r} \end{bmatrix} + \begin{bmatrix} -\left(1 - \frac{\tilde{\alpha}_1}{\alpha_1}\right) & \left(1 - \frac{\tilde{\alpha}_2}{\alpha_2}\right)\tilde{y} \\ 0 & -\left(1 - \frac{\tilde{\alpha}_2}{\alpha_2}\right)\tilde{x} \\ 0 & -\left(1 - \frac{\tilde{\alpha}_2}{\alpha_2}\right) \end{bmatrix} \begin{bmatrix} v_l + \delta_{v_l} \\ \omega_a + \delta_{\omega_a} \end{bmatrix}. \quad (19)$$

3. CONTROL DESIGN-TRAJECTORY TRACKING

3.1. Choice of Sliding Surfaces

The VSC is a feedback control with high-speed switching, whose action takes place in two phases: the reaching phase and the sliding phase. In the reaching phase, the states trajectories of the system (linear or nonlinear) are lead to a place in the states space chosen by the designer. In general, this place is defined by linear surfaces of the control errors ($\tilde{z} = \begin{bmatrix} \tilde{x} & \tilde{y} & \tilde{\theta} \end{bmatrix}^T$), known as switching or sliding surfaces (σ), which are described by:

$$\sigma(\tilde{z}, t) = c^T \tilde{z} = 0. \quad (20)$$

In the sliding phase, the states trajectories are forced to remain on the sliding surfaces. Therefore, during this phase, the errors tend exponentially to zero according to a standard determined by matrix of positive constants c^T of Eq. (20), which is chosen by the designer.

Thus, from the error dynamics, Eq. (19), are selected the following sliding surfaces:

$$\sigma(\tilde{z}, t) = \begin{bmatrix} \sigma_1 \\ \sigma_2 \end{bmatrix} = \begin{bmatrix} k_1 & 0 & 0 \\ 0 & k_2 & k_3 \end{bmatrix} \begin{bmatrix} \tilde{x} \\ \tilde{y} \\ \tilde{\theta} \end{bmatrix} = \begin{bmatrix} k_1\tilde{x} \\ k_2\tilde{y} + k_3\tilde{\theta} \end{bmatrix}, \quad (21)$$

where k_1, k_2, k_3 are positive constants.

3.2. Generic Model for Nonlinear Systems

The derivation of the VSC and their properties are made directly for an important class of nonlinear systems, whose model, in the form of state equations, is given by:

$$\dot{\tilde{z}}(t) = A(\tilde{z},\rho,t) + B(\tilde{z},\rho,t)v(\tilde{z},t) + d_b(t)\,, \qquad (22)$$

with $A(\tilde{z},\rho,t) = A_0(\tilde{z},t) + \Delta A(\tilde{z},\rho,t)$ and $B(\tilde{z},\rho,t) = B_0(\tilde{z},t) + \Delta B(\tilde{z},\rho,t)$, where $\tilde{z}(t)$ is the vector of states; $A(\tilde{z},\rho,t)$ is the vector of nonlinear functions; $v(\tilde{z},t)$ is the vector of control inputs; $\rho(\tilde{z},t)$ is the vector of parametric uncertainties; $B(\tilde{z},\rho,t)$ is the matrix of nonlinear functions; $\Delta A(\tilde{z},\rho,t)$ and $\Delta B(\tilde{z},\rho,t)$ are the vector and the matrix representing the disturbances in the system arising from the parametric uncertainties, respectively; $d_b(t)$ is the vector of external disturbances; and $A_0(\tilde{z},t)$, $B_0(\tilde{z},t)$ refers to the vector and the matrix of nominal parameters, respectively.

The aim of this study is the derivation of a VSC robust to the present disturbances in the kinematic model, Eq. (4). To ensure the robustness of the controller, the disturbances should be bounded, the matrix $B(\tilde{z},\rho,t)$ should be nonsingular and the following conditions must be satisfied:

$$\Delta A(\tilde{z},\rho,t) = B_0(\tilde{z},t)\tilde{a}\,, \qquad \Delta B(\tilde{z},\rho,t) = B_0(\tilde{z},t)\tilde{b}\,,$$
$$d_b(t) = B_0(\tilde{z},t)\tilde{d}_0\,, \qquad (23)$$

which means that $\Delta A(\tilde{z},\rho,t)$, $\Delta B(\tilde{z},\rho,t)$, and $d_b(t)$ must belong to the image of $B_0(\tilde{z},t)$; $\tilde{a}$ and $\tilde{b}$ are the vector and the matrix that incorporate the parametric uncertainties, respectively; $\tilde{d}_0$ represents the external disturbances.

So, the error dynamics, Eq. (19), can be rewritten based in Eq. (22) and Eq. (23) as:

$$\dot{\tilde{z}} = A_0(\tilde{z},t) + B_0(\tilde{z},t)\left(I_n + \tilde{b}\right)\left(v(t) + d_v(t)\right), \qquad (24)$$

since $\Delta A = 0$, $\tilde{d}_0 = \left(I_n + \tilde{b}\right)d_v$, and I_n is identity matrix. Compared with Eq. (22), it is important to emphasize that appears an additional term $B_0(\tilde{z},t)\tilde{b}d_v(t)$ in Eq. (24), because the dynamic behavior of the disturbances also suffers the influence of parametric uncertainties.

3.3. Adaptive Variable Structure Controller (AVSC)

In order to have influences also on the process of reaching of the sliding surfaces, the control $v(\tilde{z},t)$ will be chosen in such a way to impose $\sigma(\tilde{z},t)$ to have the dynamics given by the following first order differential equation:

$$\dot{\sigma}(\tilde{z},t) = -Gsign(\sigma) - K_p h(\sigma)\,, \qquad (25)$$

where G and K_p are positive definite diagonal matrices, $h(\sigma) = \sigma$ (could be another function, since that $\sigma^T h(\sigma) > 0$), and $sign(\sigma) \geq \dfrac{\sigma}{|\sigma|}$ is a discontinuous function.

Using the Eq. (20), Eq. (21), and Eq. (25), and taking into account the Eq. (24), results in:

$$\begin{aligned}\dot{\sigma}(\tilde{z},t) &= \frac{\partial\sigma(\tilde{z},t)}{\partial\tilde{z}}\dot{\tilde{z}} + \frac{\partial\sigma(\tilde{z},t)}{\partial t}\\ &= \frac{\partial\sigma}{\partial\tilde{z}}\left(A_0(\tilde{z},t) + B_0(\tilde{z},t)\left(v(t) + d_v(t)\right)\right)\\ &\quad + \frac{\partial\sigma}{\partial\tilde{z}}B_0(\tilde{z},t)\tilde{b}\left(v(t) + d_v(t)\right)\end{aligned}$$

$$\begin{aligned}\dot{\sigma}(\tilde{z},t) &= \frac{\partial\sigma}{\partial\tilde{z}}\left(A_0(\tilde{z},t) + B_0(\tilde{z},t)v(t)\right)\\ &\quad + \frac{\partial\sigma}{\partial\tilde{z}}B_0(\tilde{z},t)\tilde{b}v(t) + \frac{\partial\sigma}{\partial\tilde{z}}B_0(\tilde{z},t)\tilde{d}_0\end{aligned}\,, \qquad (26)$$

with

$$\frac{\partial\sigma(\tilde{z},t)}{\partial t} = 0\,, \quad \frac{\partial\sigma(\tilde{z},t)}{\partial\tilde{z}} = c^T = \begin{bmatrix} k_1 & 0 & 0 \\ 0 & k_2 & k_3 \end{bmatrix}, \qquad (27)$$

whence is derived the following control law:

$$v = -B_{0_\sigma}^{-1}\left(A_{0_\sigma} + Gsign(\sigma) + K_p\sigma\right), \qquad (28)$$

in which

$$A_{0_\sigma} = \frac{\partial\sigma}{\partial\tilde{z}}A_0 = \begin{bmatrix} k_1 v_{l_r}\cos(\tilde{\theta}) \\ k_2 v_{l_r}\sin(\tilde{\theta}) + k_3\omega_{a_r} \end{bmatrix}, \qquad (29)$$

$$B_{0_\sigma} = \frac{\partial\sigma}{\partial\tilde{z}}B_0 = \begin{bmatrix} -k_1 & k_1\tilde{y} \\ 0 & -k_2\tilde{x} - k_3 \end{bmatrix}, \qquad (30)$$

$$B_{0_\sigma}^{-1} = \begin{bmatrix} -\dfrac{1}{k_1} & -\dfrac{\tilde{y}}{k_2\tilde{x} + k_3} \\ 0 & -\dfrac{1}{k_2\tilde{x} + k_3} \end{bmatrix}, \qquad (31)$$

and $k_2 = k_3\gamma$, $0 \leq \gamma \leq \dfrac{1}{\|\tilde{x}\| + 1}$ similar to [16].

Defining

$$v^* = -\left(Gsign(\sigma) + K_p\sigma\right), \qquad (32)$$

and replacing Eq. (28) in Eq. (26), results in:

$$\begin{aligned}\dot{\sigma} &= A_{0_\sigma} - B_{0_\sigma}B_{0_\sigma}^{-1}\left(A_{0_\sigma} - v^*\right) + B_{0_\sigma}\tilde{b}v(t) + B_{0_\sigma}\tilde{d}_0(t)\\ &= -Gsign(\sigma) - K_p\sigma + B_{0_\sigma}\tilde{b}v(t) + \psi\end{aligned}\,, \qquad (33)$$

where

$$\tilde{b} = \begin{bmatrix} -\dfrac{\tilde{\alpha}_1}{\alpha_1} & 0 \\ 0 & -\dfrac{\tilde{\alpha}_2}{\alpha_2} \end{bmatrix}, \qquad (34)$$

$B_{0_\sigma}B_{0_\sigma}^{-1} = I_n$, and $\psi = B_{0_\sigma}\tilde{d}_0(t)$ are the disturbances in the system.

3.4. Stability Analysis

Choosing the Lyapunov function candidate in the form:

$$V = \frac{1}{2}\left(\sigma^T\sigma + \frac{\tilde{\alpha}_1^{\,2}}{\beta_1\alpha_1} + \frac{\tilde{\alpha}_2^{\,2}}{\beta_2\alpha_2}\right), \quad \beta_1 > 0\,, \ \beta_2 > 0\,, \qquad (35)$$

which is positive definite, the sliding surface will be attractive since that the control law, Eq. (28), ensures that $\dot{V}$ is negative definite. Using the result described by Eq. (33), an expression for $\dot{V}$ is immediately obtained, that is,

$$\dot{V} = \dot{V}_1 + \dot{V}_2 = \sigma^T\dot{\sigma} - \frac{\tilde{\alpha}_1}{\beta_1\alpha_1}\dot{\hat{\alpha}}_1 - \frac{\tilde{\alpha}_2}{\beta_2\alpha_2}\dot{\hat{\alpha}}_2\,, \qquad (36)$$

$$\dot{V}_1 = \sigma^T B_{0_\sigma} \tilde{b} v(t) - \frac{\tilde{\alpha}_1}{\beta_1 \alpha_1} \dot{\hat{\alpha}}_1 - \frac{\tilde{\alpha}_2}{\beta_2 \alpha_2} \dot{\hat{\alpha}}_2 , \quad (37)$$

$$\dot{V}_2 = -\sigma^T G sign(\sigma) - \sigma^T K_p \sigma + \sigma^T \psi . \quad (38)$$

After the necessary mathematical manipulations in $\dot{V}_1$ of Eq. (37), the parameter adaptation laws are defined as:

$$\dot{\hat{\alpha}}_1 = \beta_1 k_1{}^2 \tilde{x} v_l ,$$
$$\dot{\hat{\alpha}}_2 = -\beta_2 \left(k_1{}^2 \tilde{x}\tilde{y} - \left(k_2 \tilde{y} + k_3 \tilde{\theta} \right)\left(k_2 \tilde{x} + k_3 \right) \right) \omega_a . \quad (39)$$

In $\dot{V}_2$ of Eq. (38), as $\sigma^T K_p \sigma \ge 0$, the condition $\dot{V}_2 \le 0$ can be expressed by:

$$\sigma^T G sign(\sigma) \ge \sigma^T \psi , \quad (40)$$

which is satisfied if the diagonal elements of G meet the following constraint:

$$g > |\bar{\psi}| , \quad (41)$$

with g being the minimum singular value of G and $\bar{\psi}$ representing the maximum effect of the uncertainties and/or disturbances. Thus, $\dot{V}_2$ is negative definite. According to a standard Lyapunov theory, the signals $\sigma(\tilde{z},t)$, $\tilde{\alpha}_1$, and $\tilde{\alpha}_2$ are bounded.

3.5. Neural Compensator (NC)

Due to delays, physical limitations of actuators and imperfections of switching, it is not possible to switch the control from a value to another instantaneously. Because of this, the states trajectory varies in a vicinity around the sliding surface, instead of sliding over it. This phenomenon, known as chattering [3-5], as well as disturbances, can be avoided or at least reduced using RBFNNs, which are nonlinear and continuous functions, to approximate $G \operatorname{sgn}(\sigma)$ in Eq. (28) [17]. Then v stays,

$$v = -B_{0_\sigma}^{-1} \left(A_{0_\sigma} + \hat{Q}(\sigma) + K_p \sigma \right) ,$$
$$v = -B_{0_\sigma}^{-1} \left(A_{0_\sigma} + \left[\left\{ \hat{W}_\sigma \right\}^T \bullet \left\{ \xi_\sigma(\sigma) \right\} \right] + K_p \sigma \right) , \quad (42)$$

where $\{\hat{W}_\sigma\}$, $\{\xi_\sigma(\sigma)\}$ are GL vectors of weights and Gaussian radial basis functions [18], and their respective elements are $\hat{W}_{\sigma_k}$, and $\xi_{\sigma_k}(\sigma)$; with $\hat{Q}(\sigma)$ being an $n \times 1$ output vector of the RBFNNs. The stability of the RBFNNs can be analyzed, using Ge-Lee (GL) matrix and vector [18], which are defined by {.}, and by its product operator '•'. The ordinary matrix and vector are denoted by [.].

Substituting Eq. (42) in Eq. (26), results in:

$$\dot{\sigma} = -\left[\left\{ \hat{W}_\sigma \right\}^T \bullet \left\{ \xi_\sigma(\sigma) \right\} \right] - K_p \sigma + \psi + B_{0_\sigma} \tilde{b} v(t) . \quad (43)$$

To analyze the stability, the Lyapunov function candidate, Eq. (35), is modified, that is,

$$V = \frac{1}{2} \left(\sigma^T \sigma + \frac{\tilde{\alpha}_1{}^2}{\beta_1 \alpha_1} + \frac{\tilde{\alpha}_2{}^2}{\beta_2 \alpha_2} + \sum_{k=1}^{n} \tilde{W}_{\sigma_k}{}^T \Gamma_{\sigma_k}{}^{-1} \tilde{W}_{\sigma_k} \right) , \quad (44)$$

where Γ_{σ_k} is dimensional compatible symmetric positive definite matrix, and $\{\tilde{W}_{\sigma_k}\} = \{W_{\sigma_k}\} - \{\hat{W}_{\sigma_k}\}$ is vector of weight estimation errors.

Differentiating Eq. (44), and replacing (43), $\dot{V}$ is obtained as:

$$\dot{V} = -\sigma^T \left[\left\{ \hat{W}_\sigma \right\}^T \bullet \left\{ \xi_\sigma(\sigma) \right\} \right] - \sigma^T K_p \sigma + \sigma^T \psi - \sum_{k=1}^{n} \tilde{W}_{\sigma_k}{}^T \Gamma_{\sigma_k}{}^{-1} \dot{\hat{W}}_{\sigma_k} + \dot{V}_1 , \quad (45)$$

with $\dot{V}_1$ defined as in Eq. (37).

Recall that:

$$\sigma^T \left[\left\{ \tilde{W}_\sigma \right\}^T \bullet \left\{ \xi_\sigma(\sigma) \right\} \right] = \sum_{k=1}^{n} \tilde{W}_{\sigma_k}{}^T \xi_{\sigma_k}(\sigma) \sigma_k , \quad (46)$$

and choosing the learning law of RBFNNs as:

$$\dot{\hat{W}}_{\sigma_k} = \Gamma_{\sigma_k} \xi_{\sigma_k}(\sigma) \sigma_k , \quad (47)$$

and substituting Eq. (46), and Eq.(47) into Eq. (45), $\dot{V}$ stays:

$$\dot{V} \le -K_{p\min} |\sigma|^2 + \sigma^T \psi - \sigma^T \left[\left\{ W_\sigma \right\}^T \bullet \left\{ \xi_\sigma(\sigma) \right\} \right] + \dot{V}_1 , \quad (48)$$

where $K_{p\min}$ is the minimum singular value of K_p, and the solution for $\dot{V}_1$ is given by Eq. (39).

The $\dot{V}$ can be rewritten as:

$$\dot{V} \le -K_{p\min} |\sigma|^2 + |\Delta f - Q| |\sigma| , \quad (49)$$

with $Q = \left[\left\{ W_\sigma \right\}^T \bullet \left\{ \xi_\sigma(\sigma) \right\} \right]$ being the optimal compensation for $\Delta f = \psi$. According to the property of universal approximation of RBFNNs [19], there exists $\mu > 0$ satisfying $|\Delta f - Q| \le \mu$, where μ is arbitrary and can be chosen as small as possible. Assuming that $\mu \le \eta |\sigma|$ with $0 < \eta < 1$, one obtains that $|\Delta f - Q| |\sigma| \le \eta |\sigma|^2 = \eta \sigma^2$, therefore, $\dot{V}$ results in:

$$\dot{V} \le -\left(K_{p\min} - \eta \right) \sigma^2 . \quad (50)$$

Because of $K_{p\min} > \eta$, $\dot{V}$ is negative definite. According to a standard Lyapunov theory, the signals $\sigma(\tilde{z},t)$, $\tilde{\alpha}_1$, $\tilde{\alpha}_2$, and $\{\tilde{W}_{\sigma_k}\}$ are bounded.

4. SIMULATION RESULTS

In the simulation, the same kinematic model of the mobile robot described in [20] is used, where geometric parameters are $r = 0.057$ m and $R = 0.18$ m. A circular trajectory was used as reference trajectory and the results were obtained using MATLAB/Simulink. The parameters of the circular trajectory are: $v_{l_r} = 0.5$ m/s, $\omega_{a_r} = 0.5$ rad/s, and the initial coordinates are given by $\left[x_r, \, y_r, \, \theta_r \right]^T = \left[1, \, 2, \, 26.56^\circ \right]^T$. The initial position of the mobile robot is $\left[x_c, \, y_c, \, \theta \right]^T = \left[1, \, 1, \, 10^\circ \right]^T$.

For this case, the AVSC with NC, Eq. (18), Eq. (39), Eq. (42), and Eq. (47), is considered.

The control gains of simulation are: $k_1 = 1.0$, $k_2 = 1.0$, $k_3 = 1.0$, $K_{p11} = 1.5$, $K_{p22} = 3.0$, $\Gamma_{\sigma_k} = 0.2$, $\beta_1 = 19.0$ and $\beta_2 = 2.9$.

Moreover, the number of hidden neurons used are 25. For simplicity, the centres of the localized Gaussian radial basis functions [18] are evenly distributed in order to span the input space of the neural network, and the variance value of the Gaussian radial basis functions is fixed at $\sqrt{1.5}$. The weights of the RBFNNs as well as the estimated parameters ($\hat{\alpha}_1(0)$ and $\hat{\alpha}_2(0)$) were initialized to zero, without to have any prior knowledge of the system uncertainties and disturbances.

A bounded periodic disturbance term for all time (Fig. 2) is added to the velocity vector v of the mobile robot, Eq. 4, (similar to [21]), which is given by:

$$d_v = \begin{bmatrix} \delta_{vl} \\ \delta_{\omega_a} \end{bmatrix} = \begin{bmatrix} 0.5 + 0.1\sin(0.01t) \\ 0.2 + 0.1\cos(0.01t) \end{bmatrix}.$$

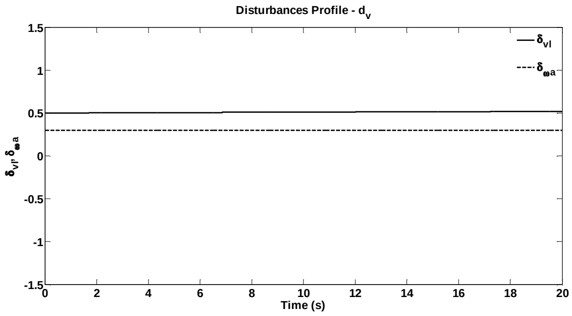

Fig. 2. Disturbances applied in the mobile robot

The Figs. 3-10 illustrate the simulation results. It is seem in Fig. 3 that the mobile robot naturally describes a smooth path tracking over the reference trajectory. The tracking errors tends to zero as shown in Fig. 4.

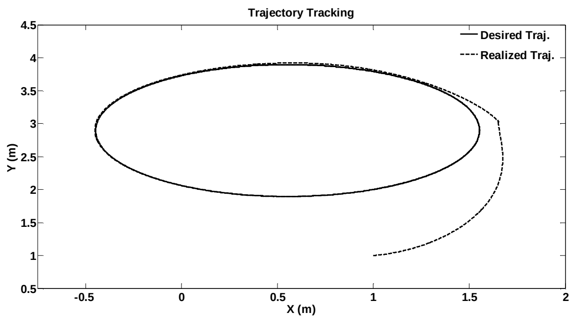

Fig. 3. Trajectory tracking

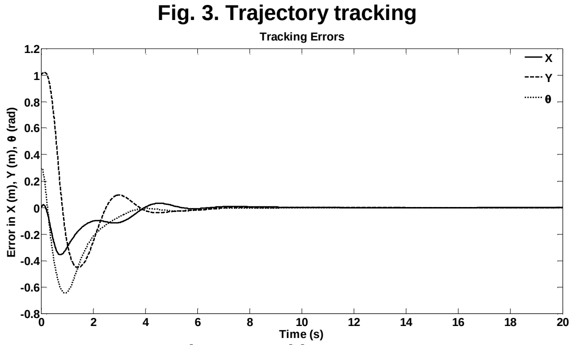

Fig. 4. Tracking errors

Observe that in Figs. 5-6 are demonstrated that there are not chattering in the linear and angular velocities as well as angular velocities of the wheels, which represents the control signals. Both sliding surfaces σ (Fig. 7) and their derivatives $\dot{\sigma}$ (Fig. 8) converge to zero as well as the chattering is eliminated. In the Fig. 9 is shown that the responses of $\hat{\alpha}_1$ and $\hat{\alpha}_2$ are bounded and tends to true parameters. The values of RBFNNs outputs $\hat{Q}$ presents behaviors similar to the disturbances (magnitudes in absolute values, see Fig. 2) in the steady-state as shown in Fig. 10, thus demonstrating the efficiency of the NC, Eq. (42) and Eq. (47).

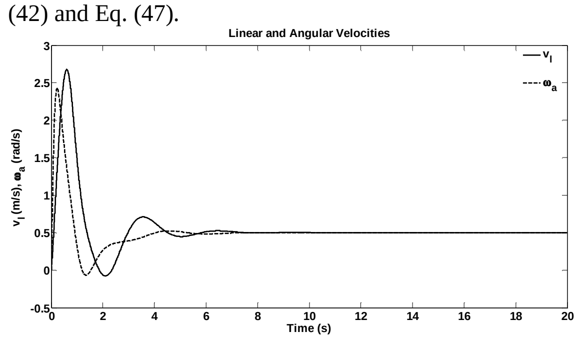

Fig. 5. Linear and angular velocities

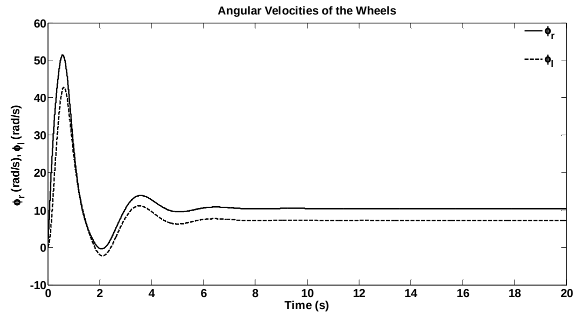

Fig. 6. Angular velocities of the wheels

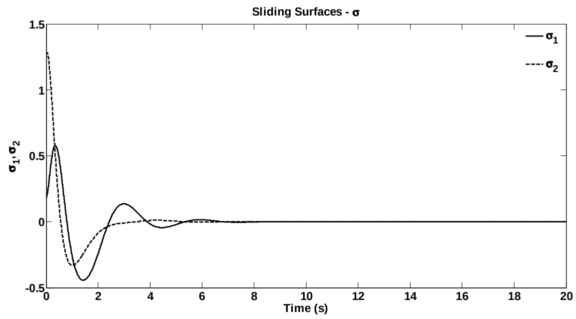

Fig. 7. Sliding surfaces

5. CONCLUSIONS

An AVSC with NC considering uncertainties and disturbances in the kinematic model were proposed in this work, and used as an alternative solution to the trajectory tracking control problem applied to nonholonomic mobile robot. The VSC was considered because the invariance principle is applicable to it, but this technique exhibits the chattering phenomenon, that is highly undesirable. To avoid such a phenomenon, RBFNNs were used in the replacement of the discontinuous portion of the classical VSC, without compromising robustness. Due to this replacement, the invariance principle was not more verified, however the smooth control signal is achieved. The simulation results of the proposed approach were satisfactory.

As future works, it is validation of the AVSC with NC in real-time applications of a nonholonomic mobile robot, as well as to realize the integration of torque controllers of the literature with these kinematic controllers proposed here.

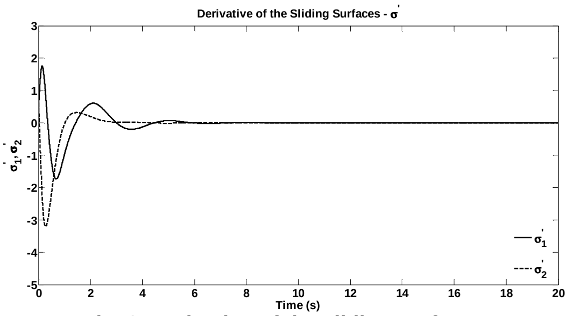

Fig. 8. Derivative of the sliding surfaces

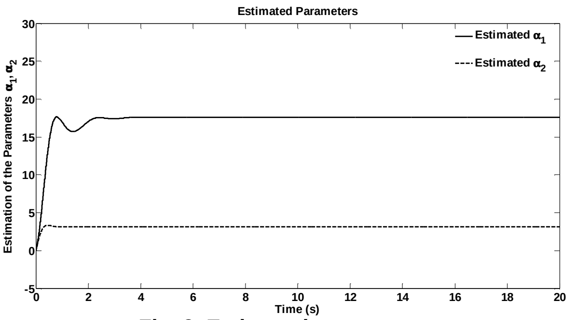

Fig. 9. Estimated parameters

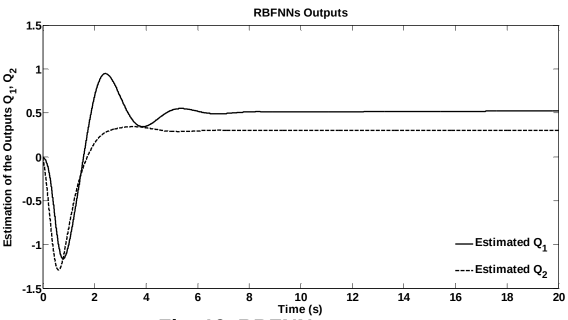

Fig. 10. RBFNNs outputs

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