

FERImage: An Interactive Program for Fractal Dimension, d_{per} and d_{min} Calculation

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Summary: A computer program has been written for the determination of the D fractal dimension at low scale, of the d_{per} representative parameter of the periodical region at high scale, and the d_{min} representative parameter of the minimum elemental cell which is repeated in the periodical structure from the variogram. It carries out the simultaneous obtention of the three previous parameters developed by Bonetto and Ladaga. The program also allows to obtain fractal dimension values from the Fourier power spectrum. FERImage has been developed so that the users could choose the rank where the behavior is fractal, not only in the variogram method but also in the Fourier spectrum method.

Key words: computer program, variogram, fractal dimension, periodical images, Fourier power spectrum

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Introduction

The fractal geometry (Feder 1988) supplies a description and a mathematical model for many shapes found in nature that look very complex. Shapes such as coasts, mountains, and clouds are not easily described by traditional Euclidean geometry; however, they often have a significant invariance under magnifying changes. This statistic self-similarity is an essential quality of fractals in nature. This can be quantified by a fractal dimension, a number that agrees with our intuitive notion of dimension but it does not need to be an integer.

Generally, when talking about dimension of a set, the topological dimension is referred to. From Euclidean

geometry, we know that the topological dimension equals 1 for a segment of line (with area = volume = 0 and length $M1$), 2 for a surface (with length = ∞ , area = $M2$ and volume = 0), and 3 for a volume (with length = area = ∞ and volume = $M3$). However, there are closed sets that, for example, have topological dimension equal to 1, infinite lengths, and null areas. Thus, we can speak about a new dimension type (real and noninteger) between 1 and 2 for which the measure of the set would be finite and different from 0. This is the Hausdorff-dimension basic concept (Hausdorff 1919).

Lately, a wide range of complex structures interesting to scientists and engineers has been characterized quantitatively using the fractal dimension idea: a dimension that corresponds univocally to the geometrical shape under study and that often is noninteger. The key to this progress is recognizing that many random structures respond to a symmetry defined as the ones that respond to regular structures. This scale symmetry implies that the objects look the same or similar in many different scales of observation.

When talking about fractals, an immediate question is how do they differ from usual Euclidean shapes? At least three marked differences can be indicated: first, while Euclidean shapes have only one or few specific scale sizes or lengths (the cube side, the sphere radius, etc.), the fractals do not have specific size or scale. Independent from the scale, the fractals are self-similar (the different space components are scales with the same scale factor or self-affine [at least one of the different space components is scaled with a space factor different from the others]). Second, the Euclidean geometry allows to give precise descriptions of the man-made objects, but it becomes unsuitable for representing natural scenery; in contrast, the fractals supply an excellent description of many natural shapes. Third, while the Euclidean shapes are usually described by simple algebraic formulae, the fractals generally can result from an algorithm or building procedure that is often iterative (or recursive).

The fractal dimension calculation has been added to the several methods used for studying the surface roughness for the last decades. Many of the fractal surfaces observed in nature are the result of complex physical phenomena that modify the morphology at random. The variogram method is used to calculate the fractal dimension from the V vari-

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ance of the grey level distribution of the image corresponding to the studied surface sample:

$$V = \langle \Delta z \rangle \cong s^{2H} \quad (1)$$

Here the angle brackets $\langle \rangle$ denote the expectation value, Δz_i is the gray level difference between two different positions in the digital image for a step i of s length measured in pixels or microns. The graph slope in logarithmic scale from V versus s is related to the D fractal dimensions as follows:

$$D = 3 - H \quad (2)$$

The variogram of several surfaces, either natural or manufactured, shows a fractal behavior at a certain scale and simultaneously it produces minimums at other scales. Such minimums result from the presence of a periodical structure in the gray levels.

Bonetto and Ladaga (1998) suggest that the periodic region should be described as a spectrum of space frequencies represented by “ k wave vectors”. Projecting the “ k_i wave vector” on the pair of orthogonal coordinate axes we obtain k_{xi} and k_{yi} , and therefore

$$\langle k^2 \rangle = \langle k_x^2 \rangle + \langle k_y^2 \rangle \quad (3)$$

The expectation value for k_x^2 (k_y^2) may be defined as

$$\langle k_x^2 \rangle = \sum_i \omega_{xi} k_{xi}^2$$

$$\langle k_y^2 \rangle = \sum_i \omega_{yi} k_{yi}^2$$

where

$$k_{xi} = 2\pi / s_{xi} \text{ and } k_{yi} = 2\pi / s_{yi}.$$

The s_{xi} (s_{yi}) step values correspond to the minimums of the variogram and the ω_{xi} (ω_{yi}) values denote their probable weight:

$$\omega_{xi} = (\Delta V_{xi} / V_{Mxi}) / \sum_j \Delta V_{xj} / V_{Mxj}$$

$$\omega_{yi} = (\Delta V_{yi} / V_{Myi}) / \sum_j \Delta V_{yj} / V_{Myj}$$

The V_{Mxi} and V_{Myi} values are the variance values of the maximum nearest to s_{xi} and s_{yi} , respectively, and to their left. ΔV_{xi} and ΔV_{yi} are obtained from the difference between these nearest maximum and minimum. The authors also define the parameter:

$$d_{per} = \frac{2\pi}{\sqrt{\langle k_x^2 \rangle + \langle k_y^2 \rangle}} \quad (4)$$

This parameter is a measurement of the average diameter between the nearest elementary cells of enough statistic weight so as to produce periods.

The intersection between the fractal and periodic zones is the definition of d_{min} and accounts for the highest value of the smallest cell size of enough statistic weight. These two new parameters, together with the fractal dimension, provide a satisfactory characterization of surface roughness (Bonetto *et al.* 2000).

The FERImage program reported here has been designed to supply the users with an interactive program that allows to achieve the three previous parameters, varying the conditions according to the particular problem being studied. The program has been developed and it is available free of charge at the web site: <http://dalton.quimica.unlp.edu.ar/cindecaprograma.htm>.

Outline of the Program

FERImage allows work with two types of graphic files: bitmap and raw (file in which the pixel values are found without any heading). In both cases, the image width and height must coincide, which is necessary to keep the same dimensions in x and y for the treatment of periodical regions.

This program provides four options for image analysis:

1. *Variogram at fixed angle.* This option allows the user to achieve the variogram for any angle between 0° and 180° . The calculation can be done according to three process options: (a) Averaged: to obtain good statistics and to avoid the preference of a particular starting point, the variance calculus scans the images at different origins; (b) not averaged: this makes it start from a unique origin; and (c) averaged $0/90$ degrees: the variance calculus scans the images at different origins and in two orthogonal directions to decrease anisotropy and directional problems of their surfaces. In any case a square or a circle can be taken as image limits. This last option is suggested since, when the rotation takes place, the scanning is always done with the same image. In the expression for the variance of Eq. 1, it is assumed that $\langle \Delta z \rangle$ is zero. The program allows subtracting this mean value for those cases where such value is different from zero. In this option, the variance versus the s step is graphed and, by means of the linear regression option, the fractal dimension value can be obtained according to different possibilities. In the three processes, the program allows the user to choose the histogram calculus of gray levels for each step. If the fractal is Brownian, this histogram must account for a Gaussian function centered in zero.

2. *Polar variogram.* This option allows the user to have an idea of the sample anisotropy, since fractal dimension

Slope: maximum: 0.69891, minimum: 0.57162
 Intercept: maximum: 2.2389, minimum: 2.0569

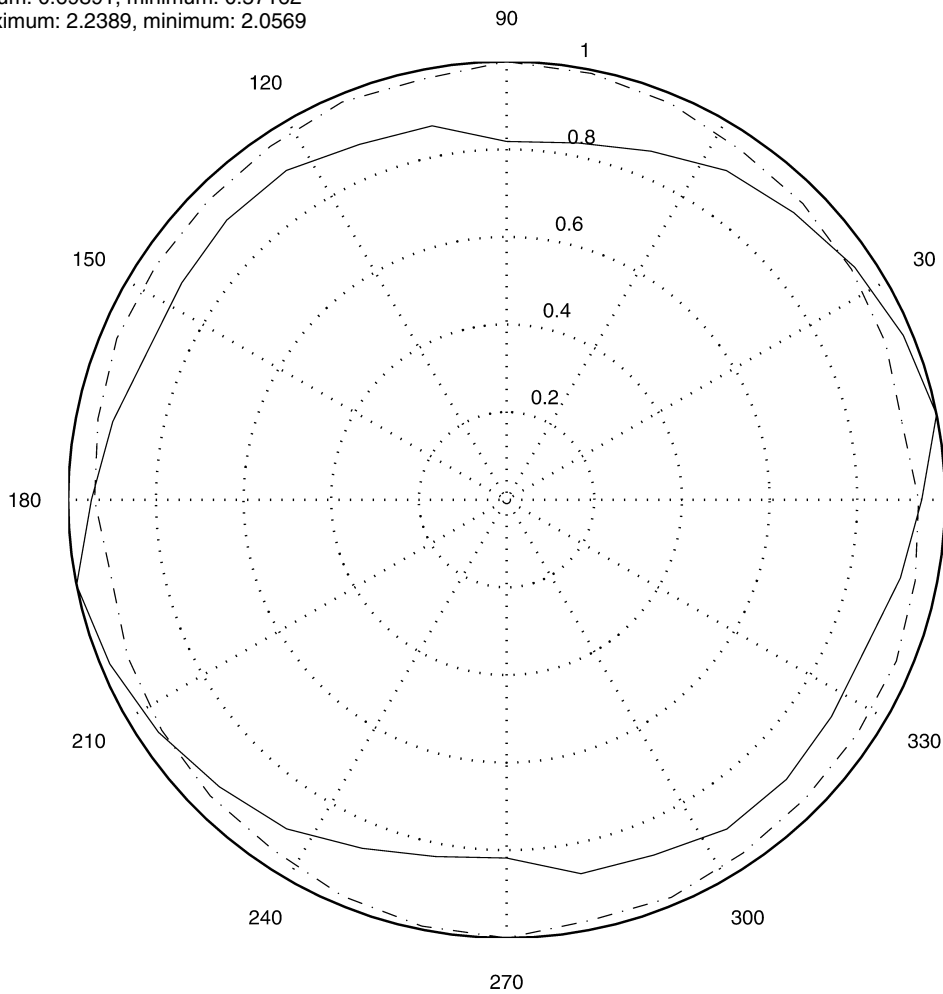


FIG. 1 Polar graph for slope values obtained from the variogram calculated at 32 different angles. The image corresponds to the theoretical Brownian image of 2.7 fractal dimension. The fractal dimension is obtained as $D=3-slope/2$. — = Slope, - - - - = intercept.

value is calculated at different angles. For each rotation angle of the image, a variogram is calculated with the 0/90 degree method. From this, the slope and the intercept of the straight line that fits the fractal region are obtained. Both slope and intercept are shown in a polar diagram, where each angle of the polar diagram corresponds to the rotation angle of the image (Fig. 1). The initial value can be changed and the calculation is made at a fixed correlation coefficient. It also provides a table with the angle values, slope, intercept, number of used points, error in the slope, error in the intercept, and correlation coefficient.

3. *Periods*. This option allows the user to calculate the d_{per} parameter (Eq. 4) for different rotation angles, thus obtaining an average value of the calculated values. In this option, a variogram plot is presented that is obtained of the average of the variance values of two orthogonal directions. The lineal regression option is used to obtain the straight line slope value, and with that, the fractal dimension at low scale (Fig. 2). The intersection of this straight line and the

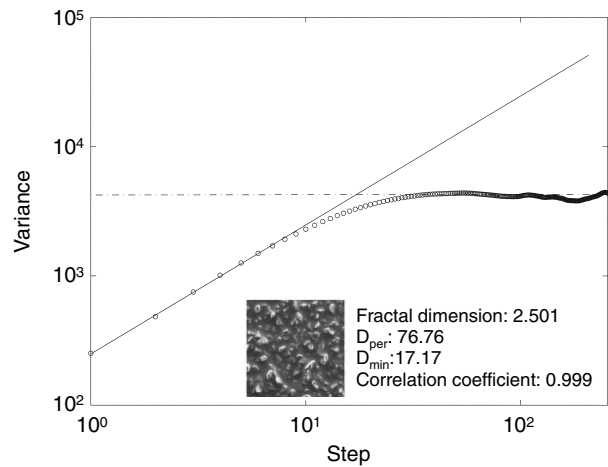


FIG. 2 Variogram corresponding to a scanning electron microscopy image of emery paper. The fractal dimension values at a low scale can be seen in the graph. The d_{per} parameter calculation is also visible. The d_{min} parameter is obtained as intersection between the straight line which defines the fractal region and a line which is tangent to the maximums in the periodical zone.

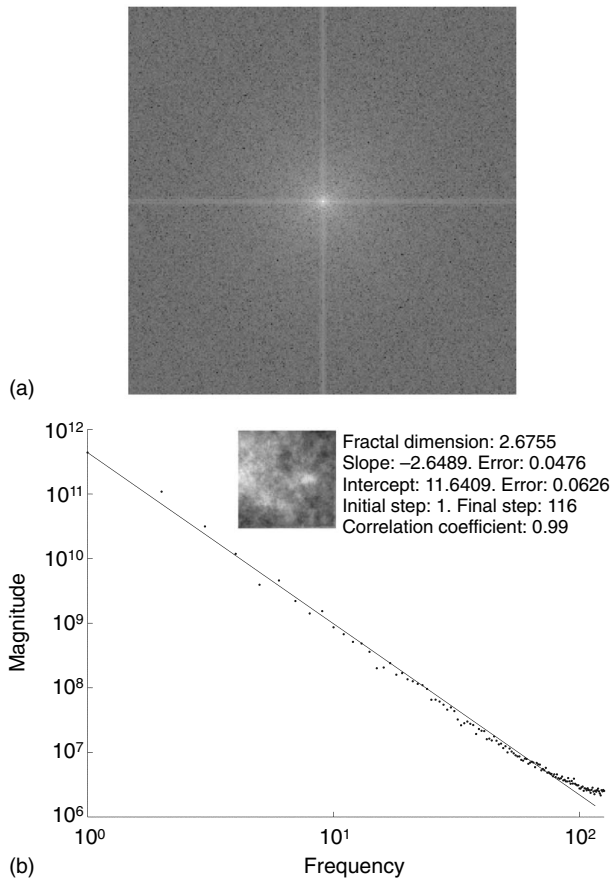
tangent at the maximums of the periodical region determines the d_{min} parameter.

4. *Fourier*. This option allows the user to determine the fractal dimensions in 20 different directions from the Fourier power spectrum. For each direction, the slope and the intercept of the straight line that better fits in the spectrum are determined, and they are graphed in a polar diagram. The spectrum of average power of all directions is also graphed in a log-log graph in function of the f frequency ($P^2 \approx f^{slope}$), and by means of the linear regression option, the average fractal dimension value can be obtained which accounts for $D = 4 + slope/2$ (Fig. 3).

Even though the maximum step available in the variance calculation equals half the image size, the program allows the user to take the amount of points necessary for each case. Either the value of the initial or the final step can be varied to exclude regions that do not account for fractal behavior.

In cases in which the fractal region extends to almost the whole scale under study, the largest steps are statistically heavier than the rest because of a larger population density. To avoid this, the program takes only those steps that are nearly equi-spaced in a logarithmic scale.

The program has the possibility of exporting the results in a file in ASCII format. This file can be read by other calculus programs.



Slope: maximum: -2.2999, minimum: -2.7079
 Intercept: maximum: 11.9903, minimum: 10.961

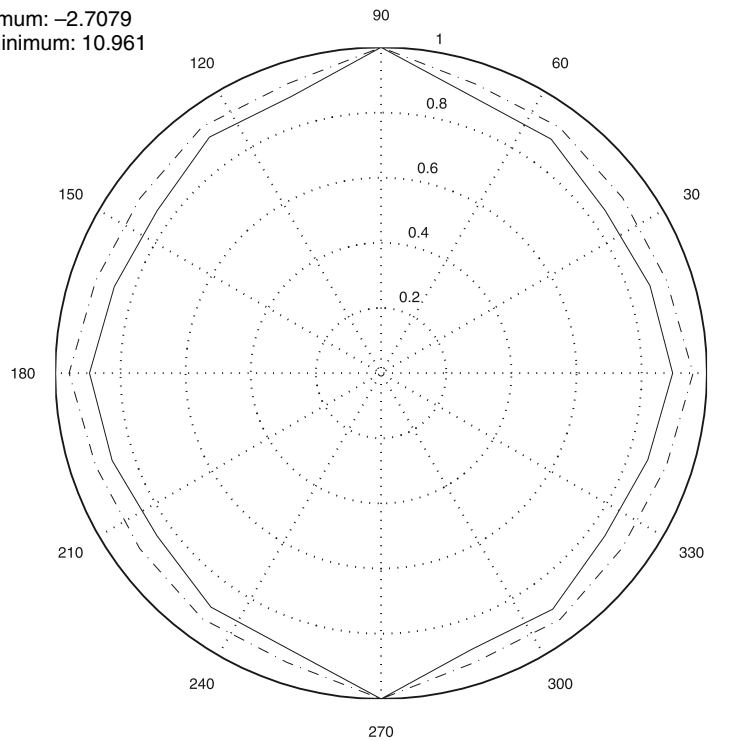


FIG. 3 (a) Fourier transformed image of the theoretical Brownian image of 2.7 fractal dimension; (b) average power spectrum from which the average fractal dimension value is obtained with a correlation index $r = 0.99$; fractal dimension is obtained as $D=4+slope/2$; (c) polar graph for the fractal dimension values obtained for different angles. — = Slope, - - - = intercept.

Conclusions

A program has been developed to calculate fractal dimension by means of a variogram and by means of a Fourier power spectrum. The program also calculates the d_{per} parameter, representative of periodical region in an image, and the d_{min} parameter representative of the minimum elemental cell which is repeated in the periodical structure from the variogram. These two parameters and the fractal dimension allow the user to characterize the roughness of the sample under study by means of the obtained digitalized image, either by means of a scanning electron microscope or by means of an optical microscope according to the type of information being sought.

An important characteristic of the program is that it is interactive, that is, the user can modify the limits for the fractal dimension calculation. This makes it very valuable since there are several samples which do not present a unique fractal dimension but different fractal dimensions at different scales.

Another important characteristic of the program is that it allows the user to obtain fractal dimensions at different angles with the variogram method as well as with the spectrum of Fourier powers. This enables the user to study

anisotropies in the roughness of samples. In this case, the possibility of using a circle as image limit assures that the work is always being done with the same image at different rotations.

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