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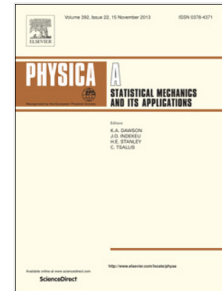
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Research highlights:

- We study statistical features and long-range dependence of Bitcoin returns.
- Hurst exponent is computed for sliding windows.
- Liquidity does not affect the level of long-range dependence
- Similar behavior of Hurst exponent at different time scales

Some stylized facts of the Bitcoin market

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Abstract

In recent years a new type of tradable assets appeared, generically known as cryptocurrencies. Among them, the most widespread is Bitcoin. Given its novelty, this paper investigates some statistical properties of the Bitcoin market. This study compares Bitcoin and standard currencies dynamics and focuses on the analysis of returns at different time scales. We test the presence of long memory in return time series from 2011 to 2017, using transaction data from one Bitcoin platform. We compute the Hurst exponent by means of the Detrended Fluctuation Analysis method, using a sliding window in order to measure long range dependence. We detect that Hurst exponents changes significantly during the first years of existence of Bitcoin, tending to stabilize in recent times. Additionally, multiscale analysis shows a similar behavior of the Hurst exponent, implying a self-similar process.

Keywords: Bitcoin, Hurst, DFA, Bitcoin, long memory

1. Introduction

2 According to the traditional definition, a currency has three main prop-
3 erties: (i) it serves as a medium of exchange, (ii) it is used as a unit of

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4 account and (iii) it allows to store value. Along economic history, monies
5 were related to political power. In the beginning, coins were minted in pre-
6 cious metals. Therefore, the value of a coin was intrinsically determined by
7 the value of the metal itself. Later, money was printed in paper bank notes,
8 but its value was linked somewhat to a quantity in gold, guarded in the
9 vault of a central bank. Nation states have been using their political power
10 to regulate the use of currencies and impose one currency (usually the one
11 issued by the same nation state) as legal tender for obligations within their
12 territory. In the twentieth century, a major change took place: abandoning
13 gold standard. The detachment of the currencies (specially the US dollar)
14 from the gold standard meant a recognition that the value of a currency
15 (specially in a world of fractional banking) was not related to its content
16 or representation in gold, but to a broader concept as the confidence in the
17 economy in which such currency is based. In this moment, the value of
18 a currency reflects the best judgment about the monetary policy and the
19 “health” of its economy.

20 In recent years, a new type of currencies, a synthetic one, emerged. We
21 name this new type as “synthetic” because it is not the decision of a na-
22 tion state, nor represents any underlying asset or tangible wealth source. It
23 appears as a new tradable asset resulting from a private agreement and facili-
24 tated by the anonymity of internet. Among this synthetic currencies, Bitcoin
25 (BTC) emerges as the most important one, with a market capitalization of
26 15 billions, as of December 2016. There are other cryptocurrencies, based
27 on blockchain technology, such as Litecoin (LTC), Ethereum (ETH), Ripple
28 (XRP). The website <https://coinmarketcap.com/currencies/> counts up
29 to 641 of such monies. However, as we can observe in Figure 1, Bitcoin rep-
30 represents 89% of the capitalization of the market of all cryptocurrencies. One
31 open question today is if Bitcoin is in fact a, or may be considered as a, cur-
32 rency. Until now, we cannot observe that Bitcoin fulfills the main properties
33 of a standard currency. It is barely accepted as a medium of exchange (e.g.
34 to buy some products online), it is not used as unit of account (there are
35 no financial statements valued in Bitcoins), and we can hardly believe that,
36 given the great swings in price, anyone can consider Bitcoin as a suitable
37 option to store value. Given these characteristics, Bitcoin could fit as an
38 ideal asset for speculative purposes. There is no underlying asset to relate
39 its value to and there is an open platform to operate round the clock.

40 The aim of this paper is to study some statistical characteristics of Bit-
41 coin *et al.* vis-à-vis some major currencies, during the period 2011-2017. We
42 will focus our attention on the evolution of the long memory of the time
43 series. This article contributes to the literature in three important aspects.

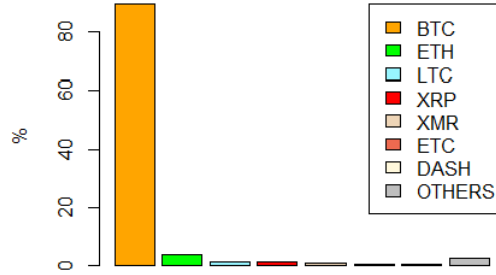


Figure 1: Cryptocurrencies. Share of market capitalization of each currency. Own elaboration based on data from [1]

44 First, we expand the empirical studies by analyzing the long memory of a
 45 new asset. Second, we compare the behavior of Bitcoin with some major
 46 currencies. Third, we highlight the evolution in the underlying dynamics of
 47 this new market. The rest of the paper is organized as follows: Section 2
 48 describes the recent emerging literature on Bitcoin, Section 3 describes the
 49 methodology used in the paper, Section 4 presents the data and results of
 50 our empirical analysis and, finally Section 5 draws the main conclusions.

51 2. Brief literature review

52 2.1. Bitcoin

53 Speculation has a long history and it seems inherent to capitalism. One
 54 common feature of speculative assets in history has been the difficulty in
 55 valuation. Tulipmania, the South Sea bubble, and more others, reflect on
 56 one side human greedy behavior, and on the other side, the difficulty to set
 57 an objective value to an asset. All speculative behaviors were reflected in a
 58 super-exponential growth of the time series [2].

59 Cryptocurrencies can be seen as the libertarian response to central bank
 60 failure to manage financial crises, as the one occurred in 2008. Also cryp-
 61 tocurrencies can bypass national restrictions to international transfers, prob-
 62 ably at a cheaper cost. Bitcoin was created by a person or group of persons
 63 under the pseudonym Satoshi Nakamoto. The description of Bitcoin Core,
 64 i.e. the open source client of the Bitcoin cryptocurrency, is described in [3]

65 The discussion of Bitcoin has several perspectives. The computer science
 66 perspective deals with the strengths and weaknesses of blockchain technol-
 67 ogy. In fact, according to [4], the introduction of a “distributed ledger” is
 68 the key innovation. Traditional means of payments (e.g. a credit card), rely
 69 on a central clearing house that validate operations, acting as “middleman”

70 between buyer and seller. On contrary, the payment validation system of
71 Bitcoin is decentralized. There is a growing army of miners, who put their
72 computer power at disposal of the network, validating transactions by gath-
73 ering together blocks, adding them to the ledger and forming a 'block chain'.
74 This work is remunerated by giving the miners Bitcoins, what makes (until
75 now) the validating costs cheaper than in a centralized system. The valida-
76 tion is made by solving some kind of algorithm. With the time solving the
77 algorithm becomes harder, since the whole ledger must be validated. Con-
78 sequently it takes more time to solve it. Contrary to traditional currencies,
79 the total number of Bitcoins to be issued is beforehand fixed: 21 million.
80 In fact, the issuance rate of Bitcoins is expected to diminish over time. Ac-
81 cording to [5], validating the public ledger was initially rewarded with 50
82 Bitcoins, but the protocol foresee halving this quantity every four years. At
83 the current pace, the maximum number of Bitcoins will be reached in 2140.
84 Taking into account the decentralized character, Bitcoin transactions seem
85 secure. All transactions are recorded in several computer servers around
86 the world. In order to commit fraud, a person should change and validate
87 (simultaneously) several ledgers, which is almost impossible. Additional,
88 ledgers are public, with encrypted identities of parties, making transactions
89 "pseudonymous, not anonymous" [6].

90 The legal perspective of Bitcoin is fuzzy. Bitcoin is not issued, nor
91 endorsed by a nation state. It is not an illegal substance. As such, its
92 transaction is not regulated.

93 The economic perspective is still under study. The use of Bitcoin in daily
94 life is marginal. At the time of writing this paper, there were only 8367
95 retailers worldwide who accepted Bitcoins as a means of payment, mostly
96 concentrated in North America, western Europe, and some major cities in
97 South America and South East Asia [7]. There is not too much information
98 regarding Bitcoin exchanges. This gray situation raises some concerns about
99 a possible Ponzi scheme. There are no savings accounts in Bitcoins and
100 consequently no interest rates. All these elements together contribute to
101 its difficulty to assess a fair value. Cheung *et al.* [8] detect several price
102 bubbles over the period 2010-2014. Three of them lasted from 66 to 106
103 days to burst. Ciaian and coworkers [9] find no macro-financial indications
104 driving Bitcoin price, and they do not discard that investor speculation
105 affects significantly the price evolution.

106 2.2. *The Efficient Market Hypothesis*

107 As recalled in the previous section, the nature of the Bitcoin is not yet
108 clear. In particular, given the nonexistence of saving accounts in Bitcoin,

109 and consequently the absence of a Bitcoin interest rate, precludes the idea
110 of studying the price behavior in relation with cash flows generated by Bit-
111 coins. As a consequence, we aim to analyze the underlying dynamics of the
112 price signal, using the Efficient Market Hypothesis as a theoretical frame-
113 work. The Efficient Market Hypothesis (EMH) is the cornerstone of financial
114 economics. One of the seminal works on the stochastic dynamics of specula-
115 tive prices is due to Bachelier [10], who in his doctoral thesis developed the
116 first mathematical model concerning the behavior of stock prices. The sys-
117 tematic study of informational efficiency begun in the 1960s, when financial
118 economics was born as a new area within economics. The classical defini-
119 tion due to Eugene Fama [11] says that a market is informationally efficient
120 if it “fully reflect all available information”. Therefore, the key element in
121 assessing efficiency is to determine the appropriate set of information that
122 impels prices. Following [12], informational efficiency can be divided into
123 three categories: (i) weak efficiency, if prices reflect the information con-
124 tained in the past series of prices, (ii) semi-strong efficiency, if prices reflect
125 all public information and (iii) strong efficiency, if prices reflect all public
126 and private information. As a corollary of the EMH, one cannot accept the
127 presence of long memory in financial time series, since its existence would
128 allow a riskless profitable trading strategy. If markets are informationally
129 efficient, arbitrage prevent the possibility of such strategies.

130 An important part of the literature focused its attention on studying the
131 long-range dependence. If we consider the financial market as a dynamical
132 structure, short term memory can exist (to some extent) without contradict-
133 ing the EMH. In fact, the presence of some mispriced assets is the necessary
134 stimulus for individuals to trade and reached an (almost) arbitrage free sit-
135 uation. However, the presence of long range memory is at odds with the
136 EMH, because it would allow a stable trading rule to beat the market.

137 Given the novelty of Bitcoin, this is one of the first papers (probably
138 with the single exception of [13]) to study the Hurst exponent of this mar-
139 ket. Previous works on long range dependence focused their attention in
140 stocks, bonds or commodities markets. In particular, [14] and [15] use the
141 Hurst exponent to detect the presence of long memory in the US and the
142 UK stock markets, respectively. In [16] positive short term autocorrelation
143 and negative long term autocorrelation is found, after examining the re-
144 turns of a diversified portfolio of the NYSE. This result reinforces the idea
145 of an underlying mean-reverting process. Long memory is also found in
146 the Spanish stock market [17] and the Turkish stock market [18]. In the
147 same line, Barkoulas *et al.* [19] finds evidence of long memory in the weekly
148 returns of the Athens Stock Exchange during the period 1981-1990, and

149 suggest that the strength of the memory could be influenced by the market
150 size. Also long memory behavior in the Greek market was found by Panas
151 [20]. Cajueiro and Tabak [21] find that developed markets are more infor-
152 mationally efficient than emerging markets and that the level of efficiency is
153 influenced by market size and trading costs. Cajueiro and Tabak [22] relate
154 long-range dependence with specific financial variables of the firms under
155 examinations. Zunino and coworkers [23] find that the long-range memory
156 in seven Latin-American markets is time varying. In this line, Bariviera
157 [24] finds evidence of a time varying long-range dependence in daily returns
158 of Thai Stock Market during the period 1975-2010 and concludes that it is
159 weakly influenced by the liquidity level and market size. Vodenska *et al.*
160 [25] show that volatility clustering in the S & P 500 index produces memory
161 in returns. [26] finds long memory in the sign of transactions but not in the
162 signs of returns. Ureche-Rangau and de Rorthays,[27] investigate the pres-
163 ence of long memory in volatility and trading volume of the Chinese stock
164 market. Cajueiro and Tabak [28] present empirical evidence of time-varying
165 long-range dependence for US interest rates. It concludes that long memory
166 has reduced over time. Moreover, Cajueiro and Tabak[29] find that this
167 long-range dependence, is affected by the monetary policy. Similarly, Ca-
168 jueiro and Tabak [30] find long range dependence in Brazilian interest rates
169 and their volatility, providing important implications for monetary stud-
170 ies. Time-varying long range dependence in Libor interest rates is found
171 in [31, 32]. The authors conclude that such behavior is consistent with the
172 Libor rate rigging scandal.

173 Cheung and Lai [33] use the fractional differencing test for long memory
174 by [34] and find evidence of long memory in 5 out of the 18 markets under
175 study. Using a different methodology, [35] applies spectral regression to
176 time series of 30 firms, 7 sector indices and 2 broad stock indices at daily
177 and monthly frequency, and finds evidence of long memory only in 5 of the
178 individual firms. Wright [36] compares the memory content of the time series
179 in developed and emerging stock markets, finding that the latter exhibits
180 short term serial correlation in addition to long-range memory. Henry [37]
181 concludes that there is strong evidence of long-range memory in the Korean
182 market and some weak evidence on the German, Japanese and Taiwanese
183 markets, after analyzing monthly returns of nine stock markets. Also, Tolvi
184 [38] uses a sample of 16 stock markets of OECD countries and finds evidence
185 of long memory only in 3 of them and Kasman *et al.* [39] finds that among
186 the four main central European countries (Czech Republic, Hungary, Poland
187 and Slovak Republic), only the last one exhibits long memory. Cheong [40]
188 computes the Hurst exponent by means of three heuristic methods and find

189 evidence of long memory in the returns of five Malaysian equity market
190 indices. This study finds that the Asian economic crisis affected the extent
191 of long-range memory of the Malaysian stock market.

192 With respect to the fixed income market, Carbone [41] finds local vari-
193 ability of the correlation exponent in the German stock and sovereign bond
194 markets. Bariviera *et al.* [42] finds empirical evidence of long memory in
195 corporate and sovereign bond markets and detects that the current finan-
196 cial crisis affects more the informational efficiency of the corporate than
197 sovereign market. Zunino *et al.* [43], using the complexity-entropy causal-
198 ity plane for a sample of thirty countries, finds that informational efficiency
199 is related to the degree of economic development. Recently, Bariviera *et*
200 *al.* [44] finds that the long range memory of corporate bonds at European
201 level are affected unevenly during the financial crisis. In particular, sectors
202 closely related to financial activities were the first to exhibit a reduction in
203 the informational efficiency.

204 There are some works that find no evidence of long memory in the fi-
205 nancial time series. Among others we can cite Lo [45], in the returns of
206 US stocks, and Grau-Carles [46] in the stock indices of US, UK, Japan and
207 Spain.

208 As we can appreciate, the empirical studies on sovereign and corpo-
209 rate bond markets and stock markets are abundant. Giving the increasing
210 amounts involved in Bitcoin trading, we believe that this topic deserves a
211 detailed study.

212 3. Long range dependence

213 The presence of long range dependence in financial time series generates
214 a vivid debate. Whereas the presence of short term memory can stimu-
215 late investors to exploit small extra returns, making them disappear, long
216 range correlations poses a challenge to the established financial model. As
217 recognized by [9], Bitcoin price is not driven by macro-financial indicators.
218 Consequently a detailed analysis of the underlying dynamics becomes im-
219 portant to understand its emerging behavior.

220 There are several methods (both parametric and non parametric) to
221 calculate the Hurst exponent. For a survey on the different methods for
222 estimating long range dependences see [47] and [48]. Serinaldi [49] makes a
223 critical review on the different estimation methods of the Hurst exponent,
224 concluding that an inappropriate application of the estimation method could
225 lead to incorrect conclusions about the persistence or anti-persistence of fi-
226 nancial series. Although R/S method is probably one of the most extended

227 methods to approximate long run memory in time series, it is not robust to
 228 departures from stationarity. Consequently, if the process under scrutiny
 229 exhibits short memory, the R/S statistic could indicate erroneously the
 230 presence of long memory. In this sense, [50] develops the method called De-
 231 trended Fluctuation Analysis (DFA) that is more appropriate when dealing
 232 with nonstationary data. As recognized by [51], this method avoids spurious
 233 detection of long-range dependence due to nonstationary data. Due to this
 234 reason we select the DFA method in order to assess the existence of long
 235 memory in this paper.

236 The algorithm, described in detail in [52], begins by computing the mean
 237 of the stochastic time series $y(t)$, for $t = 1, \dots, M$. Then, an integrated time
 238 series $x(i)$, $i = 1, \dots, M$ is obtained by subtracting mean and adding up to
 239 the i -th element, $x(i) = \sum_{t=1}^i [y(t) - \bar{y}]$. Then $x(i)$ is divided into M/m
 240 non overlapping subsamples and a polynomial fit $x_{pol}(i, m)$ is computed in
 241 order to determine the local trend of each subsample. Next the fluctuation
 242 function

$$F(m) = \sqrt{\frac{1}{M} \sum_{i=1}^M [x(i) - x_{pol}(i, m)]^2} \quad (1)$$

243 is computed. This procedure is repeated for several values of m . The fluctu-
 244 ation function $F(m)$ behaves as a power-law of m , $F(m) \propto m^H$, where H is
 245 the Hurst exponent. Consequently, the exponent is computed by regressing
 246 $\ln(F(m))$ onto $\ln(m)$. According to the literature the maximum block size
 247 to use in partitioning the data is $(length(window)/2)$, where $window$ is the
 248 time series window vector. Consequently, in this paper we use six points
 249 to estimate the Hurst exponent. The points for regression estimation are:
 250 $m = \{4, 8, 16, 32, 64, 128\}$.

251 There are other methodologies to verify the presence of long-range mem-
 252 ory. Rosso *et al.* [53] introduces the complexity-causality plane in order to
 253 discriminate between Gaussian from non-Gaussian processes. Zunino *et al.*
 254 [54] shows that this innovative approach could be used to rank stock markets
 255 according to their stage of development. In Zunino *et al.* [55], the appli-
 256 cation of the complexity-entropy causality plane was extended to the study
 257 of the efficiency of commodity prices. This method reveals that it is not
 258 only useful to produce a ranking of efficiency of different commodities, but
 259 it also allows to identify periods of increasing and decreasing randomness
 260 in the price dynamics. Zunino *et al.* [43] uses this representation space to
 261 establish an efficiency ranking of different markets and distinguish different
 262 bond market dynamics and concludes that the classification derived from
 263 the complexity-entropy causality plane is consistent with the qualifications

264 assigned to sovereign instruments by major rating companies.

265 4. Data and results

266 The period under study goes from 2011 until 2017 for daily data and
 267 from 2013 until 2016 for intraday data. We downloaded the daily prices
 268 of Bitcoin and exchange rates of Euro and Sterling Pound, in US dollars.
 269 These daily data were downloaded from Datastream. Additionally, we down-
 270 loaded Bitcoin intraday transaction data from Bitcoin charts website [56].
 271 The original dataset comprises a total of 9540332 transactions. Given that
 272 transactions take place irregularly in time, we sampled data each $\{5, 6, \dots,$
 273 $12\}$ hours. The minimum sample space corresponds to the maximum time
 274 without transactions in our dataset.

275 We compute the instantaneous return, measured a $r_t = \log(P_t) - \log(P_{t-1})$.
 276 With this values we calculate the Hurst exponent using DFA method. In or-
 277 der to assess the change in time in long range memory, following [57, 58], we
 278 use sliding windows. We estimate the Hurst exponent using two year slid-
 279 ing windows (500 datapoints). In particular, we use overlapping windows,
 280 moving forward by 1 datapoint, in order to allow for smooth transitions.

281 4.1. Daily returns

282 Our first analysis focuses on the descriptive statistics of daily returns
 283 of Bitcoin (BTC) *vis-à-vis* two major currencies such as Euro (EUR) and
 284 the British Pound (GBP). Results are presented in Table 1. Whereas EUR
 285 and GBP exhibit similar mean, median and standard deviation values, BTC
 286 presents a significant positive mean and median. Moreover, BTC standard
 287 deviation is 10 times greater than of the other currencies. All three curren-
 288 cies are clearly non-normal according to the Jarque-Bera test [59].

289 We continue our analysis computing the long-range memory of all three
 290 assets using the DFA method. Figure 2, shows important difference with
 291 respect to the stochastic behavior of all three assess. On the one hand,
 292 EUR and GBP wanders roughly within the interval $H = (0.45, 0.55)$, which
 293 reflects an approximate random walk behavior. Except for the last period
 294 in GBP, we can say that both currencies behaves accordingly the Efficient
 295 Market Hypothesis. Taking into account that both are very liquid markets,
 296 we can expect such behavior. On the other hand, BTC returns exhibits long
 297 range correlations for most of the period under study. The convergence in
 298 memory behavior begins in 2014, where all three currencies meets around
 299 $H = 0.5$.

Table 1: Descriptive statistics of daily returns of BTC, EUR and GBP, from 2011 until 2017

	GBP	EUR	BTC
Observations	1404	1404	1404
Mean	0.0205	0.0219	0.3172
Median	0.0000	0.0033	0.2151
Std Deviation	0.5701	0.5731	6.2416
Skewness	2.2166	-0.0418	-1.1775
Kurtosis	36.1865	4.8014	25.5677
Jarque Bera	65578.4593	190.2491	30118.6642

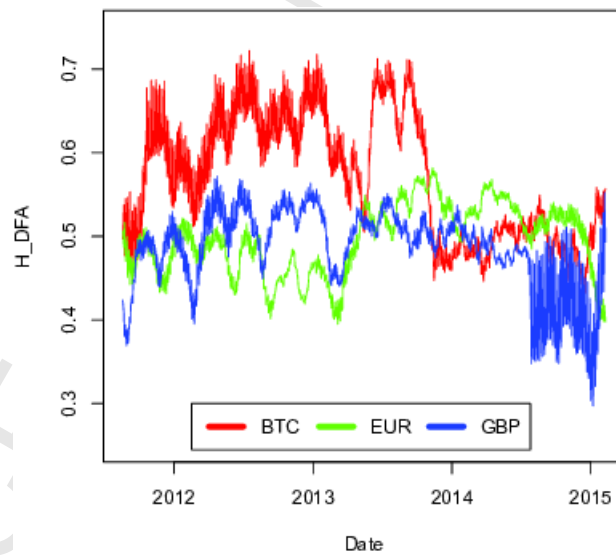


Figure 2: Hurst exponent of BTC, EUR and GBP daily values, using a sliding window of 500 datapoints and stepping forward 1 datapoint.

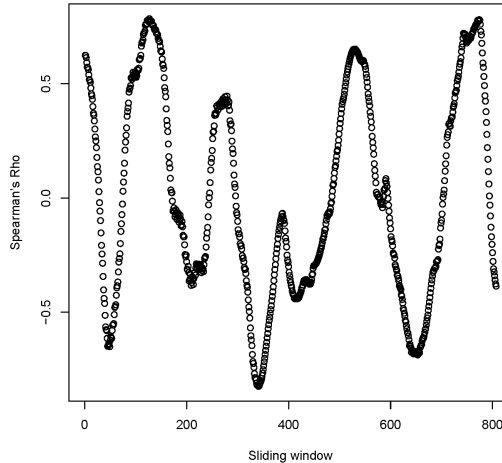


Figure 3: Spearman's Rho between Hurst exponent and turnover by volume of BTC.

300 We test if the Hurst exponent is, specially in recent times, related to
 301 the liquidity level of the market. In order to do so, we run the Spearman's
 302 non parametric test, to assess the association between the Hurst exponent
 303 and BTC turnover by volume. If we consider the whole period, there is
 304 no significant association between both variables. However, if we study this
 305 association over time, we observe a time-varying relationship. This situation
 306 (see Figure 3) could reflect a detachment of the underlying dynamics from
 307 one important market liquidity indicator.

308 4.2. Intraday returns

309 Taking into account that one of the advantages of Bitcoin is its open
 310 source philosophy, there is much available data, in order to analyze. Con-
 311 sequently we obtained transaction data from the 31th March 2013 to 2nd
 312 August 2016, and we sampled it in order to generate returns by hours, with
 313 the aim of dissecting the behavior at different time scales.

314 In Figure 4, we appreciate the sometimes meteoric runs-up and down of
 315 price. In less than a year, between 2013 and 2014, the price rocketed from
 316 less than 100 USD to more than 1000 USD, followed by a several falls and
 317 rebounds, without reaching an stability zone.

318 Another aspect we detect is that price volatility (sample variance) shows
 319 a diminishing trend. This situation is reflected in Figure 5.

320 Table 2 shows the descriptive statistics of Bitcoin returns, for each of
 321 the sampling intervals. We observe that, whereas the mean return increases
 322 *pari passu* with the interval length, the median return remains around 0.03.

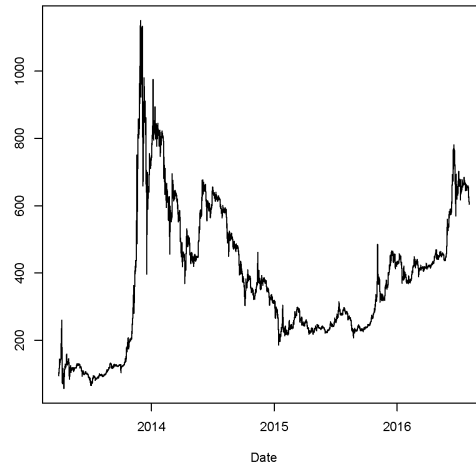


Figure 4: Bitcoin price in USD, sampled every 5 hours.

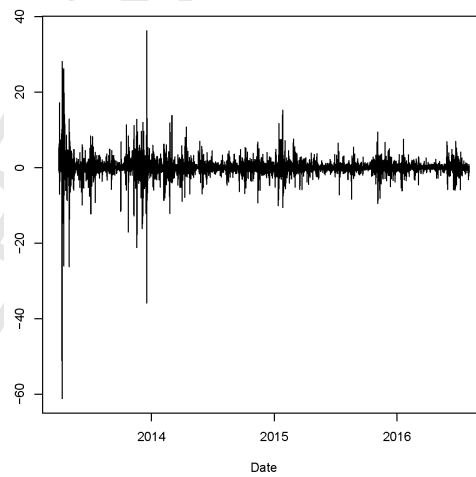


Figure 5: Bitcoin returns, sampled every 5 hours.

323 Another feature about returns is that they exhibit huge volatility, either
 324 measured by the standard deviation or the return range (max-min). In
 325 particular, large range values are reflected in the presence of great swings
 326 in returns, which can be observed in Figure 5. Finally, we detect that data
 327 is negatively skewed and present an acute excess of kurtosis, which lead to
 328 a rejection of the null hypothesis of normality according to the Jarque-Bera
 329 statistic. Skewness and kurtosis seem to reduce with greater time spans,
 330 which could reflect a slow trend toward a more Gaussian behavior.

Table 2: Descriptive statistics of returns, sampled at different time spans

	5h	6h	7h	8h	9h	10h	11h	12h
Length	5746	4879	4182	3659	3252	2927	2661	2439
Mean	0.0325	0.0382	0.0445	0.0508	0.0572	0.0632	0.0695	0.0751
Median	0.0359	0.0252	0.0323	0.0246	0.0395	0.0302	0.0235	0.0630
Min	-61.1397	-46.4425	-61.1258	-40.1405	-50.4934	-63.3724	-40.5581	-53.6354
Max	36.2219	40.3414	46.7465	48.5574	47.7417	47.5930	29.8259	51.3806
Std. Dev.	2.5994	2.6907	3.0265	3.2340	3.1859	3.6885	3.4752	3.9545
Skewness	-3.6037	-2.0001	-2.9456	-1.1589	-1.3924	-1.8665	-1.2430	-2.1920
Kurtosis	107.5232	70.1941	85.9471	45.6609	53.0422	61.2545	27.2200	52.9933
Jarque-Bera	2775514	1003188	1292625	320676	384041	460864	83211	287323

331 The analysis of the long range dependence is rather similar for the dif-
 332 ferent time scales. The Hurst exponent profiles for the different subsamples
 333 are close regarding temporal behavior and range. In all cases, we notice a
 334 marked persistent (procyclical) behavior until 2014. After such year, the
 335 time series of Hurst exponents seem to stabilize around a value of 0.5 ± 0.05 ,
 336 inducing to think in a more informational efficient market. However we
 337 cannot find the reason for such change in the dynamics, giving the uncon-
 338 nectedness of price behavior with market fundamentals.

339 5. Conclusions

340 In this paper we study the long range memory and other statistical
 341 properties of Bitcoin daily and intraday prices. The period under study
 342 goes from 2011 until 2017. We compute the Hurst exponent by means of
 343 the Detrended Fluctuation Analysis method, using a sliding window in order
 344 to assess the time-varying long range dependence. We detect that:

- 345 1. In spite of the fact that Bitcoin presents large volatility, it is reducing
 346 over time.
- 347 2. We find that the long range memory is not related to market liquidity.

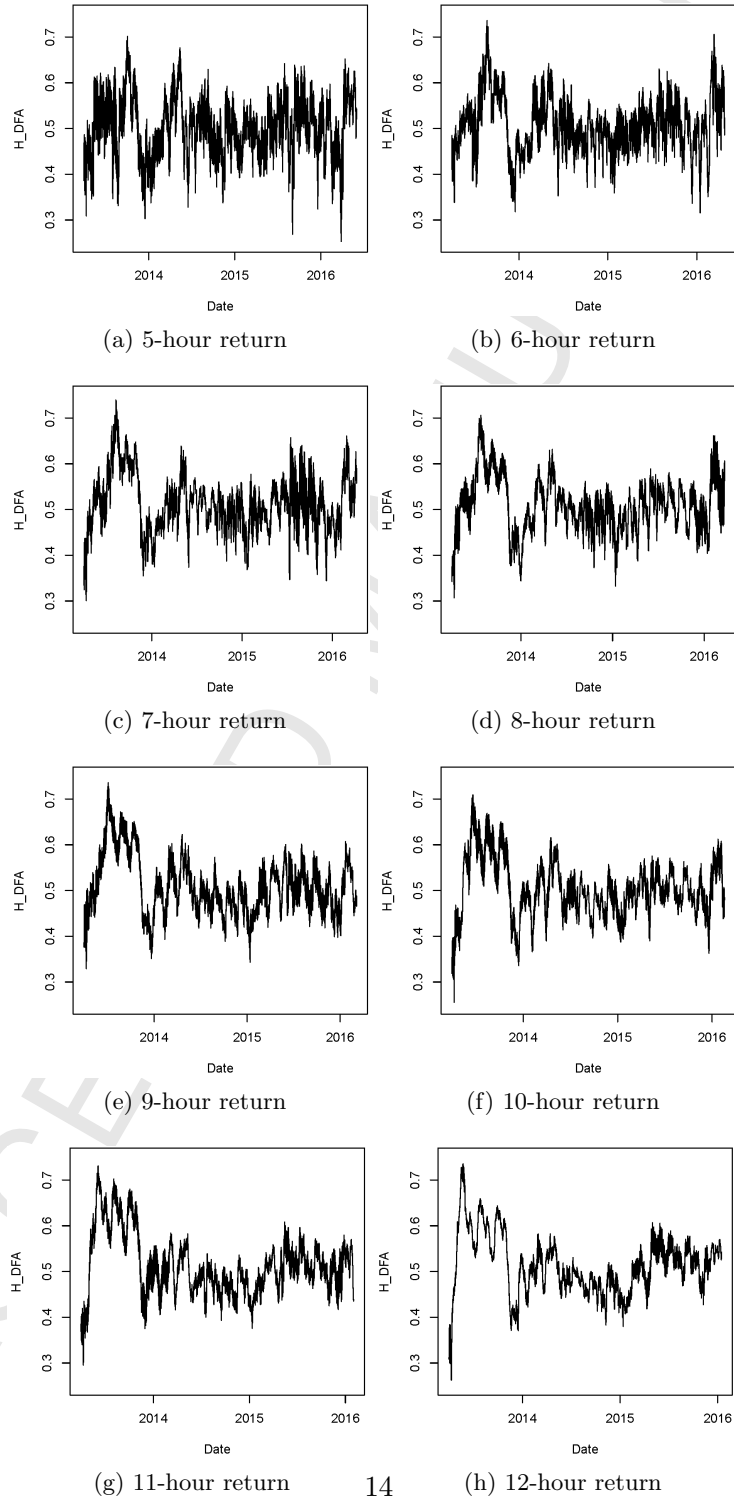


Figure 6: Hurst exponent using DFA method, for 5 to 12 hour BTC returns, using a sliding window of 500 datapoints and one datapoint step forward. Period: 2013-2016

- 348 3. The behavior across different time scales (5 to 12 hours) is essentially
 349 similar, in terms of long range memory.
 350 4. Until 2014 the time series had a persistent behavior ($H > 0.5$), whereas
 351 after such date, the Hurst exponent tended to move around 0.5.

352 In light of our results, more research should be done in order to uncover the
 353 reason for the change in Bitcoin dynamics across time.

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